Ibbotson® SBBI®
2013 Valuation Yearbook

Market Results for
Stocks, Bonds, Bills, and Inflation
1926–2012

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Nai-fu Chen, with Roll and Ross, conducted an empirical investigation of APT relating stock returns to macroeconomic factors.[6] They found five factors to be important: 1) changes in industrial production, 2) changes in anticipated inflation, 3) unanticipated inflation, 4) the return differential between low-grade corporate bonds and government bonds (both with long maturities), and 5) the return differential between long-term government bonds and short-term Treasury bills. APT risk premia are additive, as in the CAPM; therefore, differences of arithmetic means should be used as estimates of future risk premia.

The cost of capital for a stock, bond, or company can be estimated using APT. This is generally accomplished by estimating the size of the payoffs for each risk factor and the amount of each risk factor inherent in the given security.

A standard APT formulation is:

$$k_s = r_f + \beta_{s1} RP_1 + \beta_{s2} RP_2 + \ldots + \beta_{sn} RP_n$$

where:

k_s = the cost of equity for company **s**;
r_f = the riskless rate;
$RP_1, RP_2, \ldots RP_n$ = the various risk premia; and
$\beta_{s1}, \beta_{s2}, \ldots \beta_{sn}$ = the factor loadings (or exposure of the security to each of the risks).

Fama-French Three Factor Model

Other models for computing the cost of equity capital rely on "anomalies"—or apparent violations of the CAPM or other equilibrium models—such as the size effect, described above. Professors Eugene Fama and Kenneth French developed one such model. They found that returns on stocks are better explained as a function of a company's size (capturing the size effect) and its book-to-market ratio (capturing the financial distress of a firm) in addition to the single market factor of the CAPM.[7]

Specifically, they found that the return on a firm's cost of equity is negatively related to its size and positively related to its book-to-market ratio. In other words, firms with smaller equity capitalization have higher expected cost of equity, and firms with higher book value relative to market value also have higher expected cost of equity. This finding suggests a predictive model in which these variables—size and book-to-market ratio—are used (in conjunction with beta) to estimate the expected return or cost of equity capital. Chapter 8 covers this model in detail.

The Discounted Cash Flow Model

The discounted cash flow model, or income method, was developed by John Burr Williams and elaborated by Myron J. Gordon and Eli Shapiro.[8] The model uses the cost of capital to discount the expected cash flows to the present value. There are several different forms of the discounted cash flow model. The most general form of the model can be written as follows:

$$PV_s = \frac{CF_1}{(1+k_s)^1} + \frac{CF_2}{(1+k_s)^2} + \ldots + \frac{CF_i}{(1+k_s)^i}$$

where:

PV_s = the present value of the expected cash flows for company **s**;
CF_i = the dividend or cash flow expected to be received at the end of period **i**; and
k_s = the cost of capital for company **s**.

In order to solve for the cost of capital, one must forecast each of the future cash flows, and the present value of the company must be known. Solving for the cost of capital, **k**, is an iterative process that generally requires the use of a computer program. The model in the long form is difficult to work with since each and every future cash flow must be forecasted. A simplification of the long form of the model is required to make it more useful. This can be accomplished by assuming that the cash flows grow at a constant rate.

The Single-Stage Growth Model

In its simplest form, this model describes the cost of equity capital for a dividend-paying stock that has a constant expected dividend growth rate in perpetuity. This form of the discounted cash flow approach is known as the single-stage growth model or the Gordon Growth Model. We can simplify the equation by replacing the cash flows of the long-form discounted cash flow model with the following:

$$CF_i = CF_1(1+g_s)^{(i-1)}$$

where:

CF_i = the dividend or cash flow expected to be received at the end of period **i**; and
g_s = the expected dividend or cash flow growth rate into perpetuity.

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That is, each cash flow is now assumed to grow at a constant rate, g_s. The discounted cash flow equation simplifies to the following:

$$PV_s = \frac{CF_1}{(k_s - g_s)}$$

Rearranging the terms to solve for the equity cost of capital results in:

$$k_s = \frac{CF_1}{PV_s} + g_s$$

where:

$$CF_1 = CF_0(1+g_s)$$

k_s = the cost of equity for company **s**;
CF_0 = the current period dividend or cash flow earned by shareholders in company **s**;
CF_1 = the expected dividend or cash flow to be earned in the next period by shareholders in company **s**;
PV_s = the current market value of company **s**; and
g_s = the expected dividend or cash flow growth rate into perpetuity.

The discounted cash flow model in this form is simple to use. The value of a stock is directly observable as its price in the market. One difficulty with this model, however, is obtaining an accurate perpetual dividend or cash flow growth forecast because dividends and cash flows do not in fact grow at stable rates forever. It is typically easier to forecast a company-specific or project-specific growth rate over the short run than over the long run. One way of obtaining such a forecast is to use a consensus of security analysts' estimates, which generally cover a short period of time.

For example, assume that a company has a current market price of \$50 and a recent annual dividend of \$2, and that the consensus of the security analysts' growth estimates is 8 percent. The estimated cost of capital would be:

$$CF_1 = CF_0(1+g_s) = \$2(1+0.08) = \$2.16$$

$$k_s = \frac{CF_1}{PV_s} + g_s = \frac{\$2.16}{\$50} + 0.08 = 0.0432 + 0.08 = 12.32 \text{ percent}$$

In this example, we made the assumption that the analysts' growth rate is constant.

Another difficulty with implementing the single-stage growth model is that it does not allow the growth rate to exceed the cost of equity. Recall that in the original equation, the term $(k_s - g_s)$ was in the denominator. If g_s exceeds k_s, the result is a negative present value. Growth can exceed the cost of equity for some rapidly growing firms. A model that allows the growth rate to change over time and to exceed the cost of equity can produce a better estimate of the equity cost of capital.

The Two-Stage Growth Model

To produce a better estimate of the equity cost of capital, one can use a multi-stage discounted cash flow model. All multi-stage discounted cash flow models allow for the growth rate to exceed the cost of equity in all but the last stage. The two-stage growth model can be expressed as follows:

$$PV_s = \sum_{i=1}^{n} \frac{CF_0(1+g_1)^i}{(1+k_s)^i} + \frac{\frac{CF_n(1+g_2)}{(k_s - g_2)}}{(1+k_s)^n}$$

where:

k_s = the cost of equity for company **s**;
PV_s = the current market value of company **s**;
i = a measure of time (in this example the unit of measure is a year);
n = the number of years in the first stage of growth;
CF_0 = the dividend or cash flow amount (in \$) in year 0;
CF_n = the expected dividend or cash flow amount (in \$) in year **n**;
g_1 = the expected dividend or cash flow growth rate from year 1 to year **n**; and
g_2 = the expected perpetual dividend or cash flow growth rate starting in year (n + 1).

The equity cost of capital is given by the value of $\mathbf{k_s}$, which makes the right-hand side of the above equation equal to the current stock price ($\mathbf{PV_s}$). The first summation term denotes the present value of dividends expected over the first **n** years, and the second term denotes the present value of dividends expected over all the years thereafter. For the resulting cost of capital estimate to be useful, the growth rate over the latter period should be sustainable indefinitely. An example of an indefinitely sustainable growth rate is the expected long-run growth rate of the economy.

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To illustrate the two-stage growth model, we can alter the growth assumptions of the example found under the single-stage model. Assume that the analysts' growth rate of 8 percent applies only to years one through five. For years six and onwards, assume a growth rate of 5 percent.

Year	Growth Rate (%)	Annual Dividend ($)	Present Value Factor @ 9.78 %	Present Value of Dividend ($)
0		2.00	1.00	
1	8.0	2.16	0.91	1.97
2	8.0	2.33	0.83	1.94
3	8.0	2.52	0.76	1.90
4	8.0	2.72	0.69	1.87
5	8.0	2.94	0.63	1.84
6–forever	5.0	3.09	13.12	40.48
			Total	$50.00

We arrive at the current stock price of $50 by discounting this stream of cash flows at an estimated rate of 9.78 percent. This is a considerably different estimate compared to the 12.32 percent we arrive at using a constant growth rate of 8 percent. Therefore, the growth rate assumptions can have a significant impact on the cost of equity estimate.

Year	Growth Rate (%)	Annual Dividend ($)	Present Value Factor @ 10.03%	Present Value of Dividend ($)
0		2.00	1.00	
1	8.0	2.16	0.91	1.96
2	8.0	2.33	0.83	1.93
3	8.0	2.52	0.75	1.89
4	8.0	2.72	0.68	1.86
5	8.0	2.94	0.62	1.82
6	6.5	3.13	0.56	1.76
7	6.5	3.33	0.51	1.71
8	6.5	3.55	0.47	1.65
9	6.5	3.78	0.42	1.60
10	6.5	4.03	0.38	1.55
11–forever	5.0	4.23	7.63	32.27
			Total	$50.00

The Three-Stage Growth Model

Additional growth stages can be used but, in practice, only one-, two-, or three-stage discounted cash flow models are usually employed. The three-stage model is denoted as follows:

$$PV_s = \sum_{i=1}^{n1} \frac{CF_0(1+g_1)^i}{(1+k_s)^i} + \sum_{i=n1+1}^{n2} \frac{CF_{n1}(1+g_2)^{i-n1}}{(1+k_s)^i} + \frac{\frac{CF_{n2}(1+g_3)}{(k_s-g_3)}}{(1+k_s)^{n2}}$$

where:

k_s = the cost of equity for company **s**;
PV_s = the current market value of company **s**,
i = a measure of time (in this example the unit of measure is a year),
n_1 = the number of years in the first stage of growth,
n_2 = the last year in the second stage of growth;
CF_0 = the dividend or cash flow amount (in $) in year 0,
CF_{n1} = the expected dividend or cash flow amount (in $) in year $\mathbf{n}_1$;
CF_{n2} = the expected dividend or cash flow amount (in $) in year $\mathbf{n}_2$;
g_1 = the expected dividend or cash flow growth rate from year 1 to year $\mathbf{n}_1$;
g_2 = the expected dividend or cash flow growth rate from year (n_1+1) to year $\mathbf{n}_2$; and
g_3 = the expected perpetual dividend or cash flow growth rate starting in year (n_2+1).

To illustrate the three-stage growth model, we alter the growth assumptions of the two-stage model example (see table on left). Again we assume that the analysts' growth rate of eight percent applies only to years one through five. For years 6 through 10, we assume a growth rate of 6.5 percent. In the last stage, from year 11 and beyond, we assume a perpetual growth rate of 5 percent.

By discounting this stream of cash flows at a rate of 10.03 percent, we arrive at the current stock price of $50.

Timing Differences and Discount Rates

Year	Growth Rate (%)	Annual Dividend ($)	Periodic Dividend ($)	Reinvestment ($)	Total Dividend ($)	Present Value Factor @ 9.96%	Present Value of Dividend ($)
0		2.00				1.00	
1	8.0	2.16	0.54	0.08	2.24	0.91	2.04
2	8.0	2.33	0.58	0.09	2.42	0.83	2.00
3	8.0	2.52	0.63	0.10	2.62	0.75	1.97
4	8.0	2.72	0.68	0.10	2.82	0.68	1.93
5	8.0	2.94	0.73	0.11	3.05	0.62	1.90
6–forever	5.0	3.09	0.77	0.12	3.20	12.54	40.16
						Total	$50.00

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Quarterly Dividend Adjustment

When valuing a stock, one should remember that even though dividends grow and are declared annually, they are usually paid in equal quarterly installments. In order to account for this in the discounted cash flow model, each cash flow can be replaced by the following term:

$$CF_i \times \frac{1+(1+k)^{1/4}+(1+k)^{1/2}+(1+k)^{3/4}}{4}$$

If we look at the same example that was used for the two-stage discounted cash flow model but use the quarterly dividend adjustment, the cost of equity estimate becomes 9.96 percent instead of 9.78 percent. The higher discount rate reflects the difference in timing of the cash flows, as shown below.

Estimating Growth Rates

One of the advantages of a three-stage discounted cash flow model is that it fits with life cycle theories in regards to company growth. In these theories, companies are assumed to have a life cycle with varying growth characteristics. Typically, the potential for extraordinary growth in the near term eases over time and eventually growth slows to a more stable level.

In the *Ibbotson Cost of Capital Yearbook* the three-stage growth model is used. In the first stage (the first five years), analysts' consensus estimates of earnings growth are used. These should reflect any extraordinary near-term growth potential. Over years 6 through 10, an average of the analysts' consensus estimates of growth for the entire industry is used (we assume that over a middle horizon, growth of any particular company will lie more in line with the industry as a whole). Finally, in years 11 and beyond, a growth rate estimate for the entire economy is used, reflecting the belief that even in a rapidly growing industry there will come a time when growth slows to be more in line with the overall economy.

Short-term growth rates are generally available from security analysts who follow a particular company or industry. Long-term growth rates can be estimated in a number of ways. One rudimentary estimate of long-term growth is the sustainable-growth model. This model relies on two accounting concepts: return on equity and the plow-back ratio.

Sustainable growth is then given by:

$$g_s = b_s \times ROE_s$$

where:

g_s = the sustainable growth rate for company **s**;

b_s = the plow-back ratio of company **s** calculated as follows:

$$\frac{\text{Annual Earnings} - \text{Annual Dividends}}{\text{Annual Earnings}}\text{; and}$$

ROE_s = the return on book equity of company **s** calculated as follows:

$$\frac{\text{Annual Earnings}}{\text{Book Value of Equity}}$$

This model relies on a number of assumptions that may or may not hold. The first of these assumptions is that ROE and the plow-back of earnings are constant over time. That is, there exists a forecast of these two accounting ratios that is sustainable in the long term. Though the model appears simple to implement at first glance, finding a forecast of the ratios that is sustainable indefinitely is extremely difficult. Dividend policy and potential investment opportunities change over time and have a direct impact on these ratios.

The model assumes that the only possible source of corporate earnings growth is the reinvestment of earnings into the existing business and that any investment of funds in the firm will earn the same rate of return as existing projects. However, firms generally seek projects that have a higher return than existing projects. The sustainable growth model may therefore underestimate a firm's future growth. Other problems may arise because the model relies on accounting practices that can distort earnings.

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In addition, other sources of growth may exist that do not require the plow-back of earnings. Changes in technology can advance growth with little capital expenditure by a firm. For instance, efficiency in the transfer of information has improved tremendously over the years as a result of internet technology. Many companies benefit from this increased efficiency with little direct investment in the internet. A company may also grow at the rate of inflation without retaining any earnings. The growth rate that the model estimates is a nominal growth rate, not a real growth rate. If retained earnings are zero, the model predicts zero growth; however, a firm could still grow at the general rate of inflation.

Another approach to estimating long-term growth rates is to focus on estimating the overall economic growth rate. Again, this is the approach used in the *Ibbotson Cost of Capital Yearbook*. To obtain the economic growth rate, a forecast is made of the growth rate's component parts. Expected growth can be broken into two main parts: expected inflation and expected real growth. By analyzing these components separately, it is easier to see the factors that drive growth.

Treasury Inflation-Protected Securities (TIPS), a relatively new investment vehicle in the U.S., can be used in conjunction with traditional long-term government bonds to estimate the market expectation for inflation. Theoretically, the yield on inflation-indexed bonds is equal to the real default-free rate of return.

To estimate long-term inflation, we can start with the current yield on a government bond with approximately 20 years to maturity of 2.41 percent and subtract the current yield on an inflation-indexed bond with approximately 20 years to maturity of 0.15 percent, for an inflation estimate of 2.26 percent.

Once the long-term expected inflation rate is estimated, the real growth rate must be determined. The growth rate in real Gross Domestic Product (GDP) for the period 1929 to 2012 was approximately 3.22 percent. Growth in real GDP (with only a few exceptions) has been reasonably stable over time; therefore, its historical performance is a good estimate of expected long-term (future) performance.

By combining the inflation estimate with the real growth rate estimate, a long-term estimate of nominal growth is formed:

2.26 percent + 3.22 percent = 5.48 percent.

Endnotes

[1] This relationship does not seem to hold empirically with small company stocks. This size effect is discussed in Chapter 7.

[2] In general, small company betas are expected to be higher than large company betas. This, however, does not hold for all time periods. Chapter 6 discusses in more detail the measurement of beta for small stocks.

[3] The beta-adjusted size premia are different from the small stock premia (or non-beta-adjusted size premia) shown in previous editions of the *Ibbotson Stocks, Bonds, Bills, and Inflation Yearbook* (prior to the 1995 Yearbook). The small stock premium reported in older editions of Stocks, Bonds, Bills, and Inflation is the difference in long-term average returns between the large company stock total return series (currently represented by the S&P 500) and the small company stock total return series (currently represented by the Dimensional Fund Advisors U.S. Micro Cap Portfolio). The size premia given here are based on slightly different baskets of stocks from the CRSP (Center for Research in Security Prices) data set and, more importantly, they are adjusted for beta. That is, small stocks do have higher betas than large stocks; the return, above what might be expected because of the higher betas, is the size premium. These size premia increase as the capitalization of the company decreases. Chapter 7 describes the development of these premia in more detail.

[4] Beta estimate is based on the full information beta for SIC code 36 from the *Ibbotson Industry Cost of Capital Reports* as of December 31, 2012 and December 31, 1996. This beta estimation methodology is described in detail in Chapter 6. For more information, visit http://global.morningstar.com/IndReportsStats

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