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29 S.Ct. 192
Supreme Court of the United States

WILLIAM R. WILLCOX et al.,
Constituting the Public Service
Commission, etc., of New York, Appts.,
v.
CONSOLIDATED GAS COMPANY OF NEW YORK.

No. 396. No. 397. No. 398. | Nos. 396, 397, 398. | Argued November 4, 5, 6, 1908. | Decided January 4, 1909.

APPEALS from the Circuit Court of the United States for the Southern District of New York to review a decree enjoining the enforcement of legislative regulation of gas rates. Reversed with directions to dismiss the bill without prejudice.

See same case below 157 Fed. 849.

West Headnotes (15)

[1] **Federal Civil Procedure**
Effect

The dismissal of a bill seeking to enjoin enforcement of legislative regulation of gas rates as confiscatory in advance of any actual experience of the practical result of such rates should be without prejudice.

9 Cases that cite this headnote

[2] **Eminent Domain**
Oil and gas

Legislative regulation of gas rates is invalid, where such rates are plainly unreasonable to the extent that their enforcement will be equivalent to the taking of property for public use without such compensation as, under the circumstances, is just, both to the owner and the public.

12 Cases that cite this headnote

[3] **Gas**
Injunction

The case must be a clear one before the courts should be asked to interfere by injunction in advance of any actual experience of the practical result of such rates.

35 Cases that cite this headnote

[4] **Gas**
Injunction

A court of equity ought not interfere by injunction before a fair trial has been made of continuing the business under such rates, where the rates complained of show a very narrow line of division between possible confiscation and proper regulation, as based upon the findings as to the value of the property and the division depends upon variant opinions as to value and upon the results in the future of operating under such rates.

32 Cases that cite this headnote

[5] **Gas**
Statutory and municipal regulation in general

In determining the reasonableness of Laws N.Y.1905, p. 2091, c. 736, and Laws 1906, p. 235, c. 125, fixing gas rates in New York City, a discrimination between individual consumers and the city is not material to the inquiry, if the total profits from the gas supplied to all consumers is sufficient to insure the requisite return upon the property used by the gas company in its business.

6 Cases that cite this headnote

[6] **Gas**
Mains, pipes, and appliances

The requirements as to gas pressure made by Laws N.Y.1905, p. 2091, c. 736, and Laws 1906, p. 235, c. 125, are confiscatory, where to put this pressure upon the mains and other service pipes, in their present condition, is to run a great risk of explosion and consequent disaster, and to eliminate such danger requires an expenditure of many millions of dollars from which no return can be had at the rates established by those acts.

4 Cases that cite this headnote

[7] Gas

Determination of rate base in general

The valuation of the property of the company, upon which it is entitled to a fair return must, as a general rule, be determined as of the time when the inquiry is made regarding the reasonableness of rates fixed by statute, giving the company the benefit of any increase in the value of the property since it was acquired.

132 Cases that cite this headnote

[8] Gas

Determination of rate base in general

Increase since consolidation of the tangible assets and in the amount of gas supplied by it does not justify the court, in attributing a proportional increase to the value of the franchises as fixed by the constituent companies at the time of consolidation.

10 Cases that cite this headnote

[9] Gas

Determination of rate base in general

The valuation of the franchises of the constituent gas companies as fixed by them when consolidating pursuant to Laws N.Y. 1884, p. 448, c. 367, which valuation was included in the total sum for which the consolidated corporation issued its stock, must be accepted by the courts as conclusive of such value at the time of consolidation, where the validity of the agreement, fixing the valuation has always been recognized, and the stock has earned large dividends and has been largely dealt in for many years on the basis of the validity of the valuation and of the stock.

16 Cases that cite this headnote

[10] Gas

Method of valuation

The assessed value for taxation of the franchises furnishes no criterion by which to ascertain their value, where the taxes are treated by the company as part of its operating expenses, to be paid out of its earnings before the net amount applicable to dividends can be ascertained.

10 Cases that cite this headnote

[11] Gas

Good will and going value

No allowance for the value of the good will should be made in estimating the value of the property of the company upon which it is entitled to earn a fair return, where such company is secure from possible competition.

10 Cases that cite this headnote

[12] Gas

Earnings and reasonableness of return

Gas rates which will yield a return of 6 per cent. upon the fair value of the property actually used by such company in its business are not confiscatory.

45 Cases that cite this headnote

[13] Gas

Earnings and reasonableness of return

There is no particular rate of compensation which any corporation subject to legislative control respecting rates has the right to obtain without legislative interference.

4 Cases that cite this headnote

[14] Courts

Comity in general

A federal Circuit Court, if properly appealed to, cannot decline, on the ground of discretion or comity, to take jurisdiction of a suit to enjoin the enforcement of state statutes fixing gas rates which are asserted to violate the federal Constitution.

93 Cases that cite this headnote

[15] **Statutes**

Carriers and public utilities

The invalidity of provisions as to gas pressure and penalties in Laws N.Y.1905, p. 2091, c. 736, and Laws 1906, p. 235, c. 125, regulating rates in New York City, does not avoid provisions as to rates, from which the invalid provisions are clearly separable.

24 Cases that cite this headnote

Attorneys and Law Firms

****194** ***25** Messrs. **Edward B. Whitney** and George S. Coleman for the Public Service Commission.

***29** Messrs. **Alton B. Parker,** William P. Burr, and Francis K. Pendleton for the city of New York.

31** Mr. **William S. Jacksonin propria persona* for the Attorney General.

***33** Messrs. **James M. Beck, John A. Garver, Charles F. Mathewson,** and Shearman & Sterling for the Consolidated Gas Company.

***39** Messrs. W. Bourke Cochran and Nathan Mattews as *amici curice*.

Opinion

Statement by Mr. Justice **Peckham:**

***23** The appellee, complainant below, filed its bill May 1, 1906, in the United States circuit Court for the southern district of New York, against the city of New York, the attorney general of the state, the district attorney of New York county, and the gas commission of the state, to enjoin the enforcement of certain acts of the legislature of the state, as well as of an order made by the gas commission, February 23, 1906, to take effect May 1, 1906, relative to rates for gas in New York city.

Since the commencement of the suit, the gas commission has been abolished and the public service commission has been created by the legislature in its stead. The official term of Attorney General Meyer has also expired, and Attorney General Jackson, his successor, has been substituted in his place.

***24** The ground for the relief asked for in the bill was the alleged unconstitutionality of the acts and the order, because the rates fixed were so low as to be confiscatory. Upon filing the bill a preliminary injunction was granted (146 Fed. 150), and, after issue was joined, the case was referred to one of the standing masters of the court to take testimony, in conformity to the practice indicated in Chicago, M. & St. P. R. Co. v. Tompkins, 176 U. S. 167, 179, 44 L. ed. 417, 422, 20 Sup. Ct. Rep. 336.

A hearing was had before the master, who reported in favor of the complainant. The case then came before the circuit court, and, after argument, a final decree was entered, restraining defendants from enforcing the provisions of the acts and the order relating to rates or penalties. 157 Fed. 849. These various defendants, except the district attorney, have taken separate appeals directly to this court from the decree so entered. The acts which are declared void as unconstitutional are chapter 736 of the Laws of 1905, which limits the price of gas sold to the city of New York to a sum not to exceed 75 cents per thousand cubic feet. The act also requires that the gas sold shall have a specified illuminating power, and a certain pressure at all distances from the place of manufacture. Penalties are attached to a violation of the act. The other act is chapter 125 of the Laws of 1906, limiting the prices of gas in the boroughs of Manhattan and the Bronx, to other consumers than the city of New York, to 80 cents per thousand cubic feet, with like penalties as in the act of 1905, and with the same provisions as to illuminating power and the pressure in the service mains. The order which was declared invalid was one made by the gas commission created under and by virtue of chapter 737 of the Laws of 1905, the order providing that the price of gas in the city should be not more than 80 cents to consumers other than the city of New York. The order had the same provisions as to illuminating power and pressure as the acts above mentioned. The master and the court below found that the 80-cent rate was so low as to amount to confiscation, and hence the acts and the order were invalid as in violation of the Federal Constitution.

****195** Mr. Justice **Peckham,** after making the foregoing statement, delivered the opinion of the court:

At the outset it seems to us proper to notice the views regarding the action of the court below, which have been stated ***40** by counsel for the appellants, the public service commission, in their brief in this court. They assume to criticize that court for taking jurisdiction of this case, as precipitate, as if it were a question of discretion or comity, whether or not that court should have heard the case. On the contrary, there was no discretion or comity about it. When a Federal court is properly appealed to in a case over which it has by law jurisdiction, it is its duty to take such jurisdiction (Cohens v. Virginia, 6 Wheat. 264, 404, 5 L. ed. 257, 291), and, in taking it, that court cannot be truthfully spoken of as precipitate in its conduct. That the case may be one of local interest only is entirely immaterial, so long as the parties are citizens of different states or a question is involved which, by law, brings the case within the jurisdiction of a Federal court. The right of a party plaintiff to choose a Federal court where there is a choice cannot be properly denied. Re Metropolitan R. Receivership, 208 U. S. 90–110, 52 L. ed. 403–412, 28 Sup. Ct. Rep. 219; Prentis v. Atlantic Coast Line R. Co. 211 U. S. 210, ante, 67, 29 Sup. Ct. Rep. 67. In the latter case it was said that a plaintiff could not be forbidden to try the facts upon which his right to relief is based before a court of his own choice, if otherwise competent. It is true an application for an injunction was denied in that case because the plaintiff should, in our opinion, have taken the appeal allowed him by the law of Virginia while the rate of fare in litigation was still at the legislative stage, so as to make it absolutely certain that the officials of the state would try to establish and enforce an unconstitutional rule.

The case before us is not like that. It involves the constitutionality, with reference to the Federal Constitution, of two acts of the legislature of New York, and it is one over which the circuit court undoubtedly had jurisdiction under the act of Congress, and its action in taking and hearing the case cannot be the subject of proper criticism.

An examination of the record herein, with reference to the questions involved in the merits, shows that the act under which the gas commission was appointed was, subsequently to the commencement and trial of this suit, declared, on grounds ***41** not here material, to be unconstitutional by the court of appeals of New York. Saratoga Springs v. Saratoga Gas, Electric Light, & P. Co. 191 N. Y. 123, 83 N. E. 693, February 18, 1908. The order made by the commission must therefore be regarded as invalid. It is not important in this case, because the act of the legislature of 1906 makes the same provision as to the price of gas to consumers other than the city that the order does. We have, as remaining to be considered, the above mentioned two acts of the legislature.

The question arising is as to the validity of the acts limiting the rates for gas to the prices therein stated. The rule by which to determine the question is pretty well established in this court. The rates must be plainly unreasonable to the extent that their enforcement would be equivalent to the taking of property for public use without such compensation as, under the circumstances, it just both to the owner and the public. There must be a fair return upon the reasonable value of the property at the time it is being used for the public. San Diego Land & Town Co. v. National City, 174 U. S. 739, 757, 43 L. ed. 1154, 1161, 19 Sup. Ct. Rep. 804; San Diego Land & Town Co. v. Jasper, 189 U. S. 439, 442, 47 L. ed. 892, 894, 23 Sup. Ct. Rep. 571.

Many of the cases are cited in Knoxville v. Knoxville Water Co. just decided. [212 U. S. 1, 53 L. ed. 371, 29 Sup. Ct. Rep. 148.] The case must be a clear one before the courts ought to be asked to interfere with state legislation upon the subject of rates, especially before there has been any actual experience of the practical result of such rates. In this case the rates have not been enforced as yet, because the bill herein was filed, and an injunction obtained restraining their enforcement, before they came into actual operation.

In order to determine the rate of return upon the reasonable value of the property at the time it is being used for the public, it, of course, becomes necessary to ascertain what that value is. A very great amount of evidence was taken before the master upon that subject, which is included in five large volumes of the record. Valuations by expert witnesses were given as to the value of the real estate owned by the complainant, and as to the value of the mains, service pipes, plants, meters, and miscellaneous personal property.

***42** The value of real estate and plant is, to a considerable extent, matter of opinion; and the same may be said of personal estate when not based upon the actual cost of material and construction. Deterioration of the value of the plant, mains, and pipes is also, to some extent, based upon opinion. All these matters make questions of value somewhat uncertain; while added to this is an alleged prospective loss of income from a reduced rate,—a matter also of much uncertainty, ****196** depending upon the extent of the reduction and the probable increased consumption,—and we have a problem as to the character of a rate which is difficult to answer without a practical test from actual operation of the rate. Of course, there may be cases where the rate is so low, upon any reasonable basis of valuation, that there can be no just doubt as to its confiscatory nature; and, in that event,

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there should be no hesitation in so deciding and in enjoining its enforcement without waiting for the damage which must inevitably accompany the operation of the business under the objectionable rate. But, where the rate complained of shows, in any event, a very narrow line of division between possible confiscation and proper regulation, as based upon the value of the property found by the court below, and the division depends upon opinions as to value, which differ considerably among the witnesses, and also upon the results in the future of operating under the rate objected to, so that the material fact of value is left in much doubt, a court of equity ought not to interfere by injunction before a fair trial has been made of continuing the business under that rate, and thus eliminating, as far as is possible, the doubt arising from opinions as opposed to facts.

A short history of the complainant, as to its incorporation and its capital, and the method by which the value of its franchises was arrived at, will render the further examination of the case more intelligible.

Prior to 1884 there were seven gaslight companies in New York city, each operated under separate charters, granted at different times between the years 1823 and 1865 or 1871. They ***43** each had the right to use the streets of certain portions of the city for the purpose of laying their mains and service pipes in order to furnish gas to the city and the citizens. Not one of the companies had ever been called upon to pay a penny for such right, but the grant to each was, in that aspect, a gratuity. It was not, at the time of granting franchises such as these, the custom to pay for them.

In 1884, by chapter 367 of the laws of that year, authority to consolidate manufacturing corporations was granted upon conditions mentioned in the act. The directors of the corporations proposing to consolidate were to make an agreement for consolidation, embracing, among other things, the amount of capital and the number of shares of stock into which it should be divided, the capital not to be in amount more 'than the fair aggregate value of the property, franchises, and rights of the several companies to be consolidated.' The agreement was not to be valid until submitted to the stockholders of each of the companies and approved by two thirds of each. The constituent companies, which were afterwards consolidated under their agreement, and pursuant to the act mentioned, were six in number, the seventh, the Mutual Company, withdrawing. The companies agreed upon the valuation of their property, which was to be paid for in the stock of the consolidated company, and the original stock held by the stockholders of each company was surrendered to the consolidated company. The value of the franchise of all the companies was set at the figure of $7,781,000. The court below said that the master reported there was little direct evidence before him as to the value of the franchises, to which the court added that if the master, by direct evidence, meant testimony of the same kind regarding their value as had been offered regarding every item of tangible property, there was none at all.

The court further stated that it does not appear in the evidence how the valuation of the franchises was measured, or why the figures selected were chosen, but that it was true that, when complainant was organized, in 1884, under the consolidation ***44** statute, which, in terms, permitted it to acquire the property and franchises of the other companies, it issued stock of the par value of $7,781,000, representing the franchises it then acquired and nothing else, and that the stock was held by purchasers who, I am compelled to think, had a right to rely upon legal protection for legally issued stock. It is not, of course, contended there was special stock issued for this particular item, but it was included in the total sum for which the consolidated company issued its stock, and, upon its receipt, the stockholders in the various companies surrendered their stock in those companies. The result was that the amount of the stock issued by the consolidated company was increased by $7,781,000, representing a value of franchises which was agreed upon by the stockholders in the companies, and which had never cost any of them a single penny.

It cannot be disputed that franchises of this nature are property and cannot be taken or used by others without compensation. Monogahela Nav. Co. v. United States, 148 U. S. 312, 37 L. ed. 463, 13 Sup. Ct. Rep. 622; People v. O'Brien, 111 N. Y. 1, 2 L.R.A. 255, 7 Am. St. Rep. 684, 18 N. E. 692, and cases cited. The important question is always one of value. Taking their value in this case as arrived at by agreement of their owners, at the time of the consolidation, that value has been increased by the finding of the court below to the sum of $12,000,000 at the time of the commencement of this suit. The trial court said: 'If, however, complainant's ****197** franchises were worth $7,781,000 in 1884, and its tangible property, at the same time, was appraised (as appears in evidence) at $30,000,000 (in round figures), then since complainant's business (in sales volume) has, in twenty-three years, almost quadrupled, and its tangible assets grown to $47,000,000, it appears to me that a fair method of fixing value of the franchises in 1905 is to assume the same growth in value for the franchise as is demonstrated by the evidence in the case of tangible property. If, therefore, the franchise valuation of 1884 was proportioned to personalty and realty of $30,000,000, a franchise valuation proportioned to $47,000,000 in 1905 would be over $12,000,000. This, I think, a logical result ***45** from the assumption I am compelled to start with, *i. e.*, that franchises have a separable and independent value. But there is, however, no method of valuing franchises, except by a consideration of earnings. Earnings must be proportioned to assets; and both kinds of assets, tangible and intangible, must stand upon the same plane of valuation. Having, therefore, a measure of growth of tangible assets from 1884 to 1905, the franchise assets must be assumed to have grown in the same proportion. I find that the value of complainant's franchises at the date of inquiry was not less than $12,000,000, making a total valuation of $59,000,000, upon which the probable return is $3,030,000, or very considerably less than 6 per cent.' The judge stated his own views as opposed to including these franchises in the property upon the value of which a return is to be calculated in fixing the amount of rates, but held that he was bound by decided cases to hold against his personal views.

We are not prepared to hold with the court below as to the increased value which it attributes to the franchises. It is not only too much a matter of pure speculation, but we think it is also opposed to the principle upon which such valuation should be made. This corporation is one of that class which is subject to regulation by the legislature in the matter of rates, provided they are not made so low as to be confiscatory. The franchises granted the various companies and held by complainant consisted in the right to open the streets of the city and lay down mains and use them to supply gas, subject to the legislative right to so regulate the price for the gas as to permit not more than a fair return (regard being had to the risk of the business) upon the reasonable value of the property at the time it is being used for the public.

The evidence shows that from their creation, down to the consolidation in 1884, these companies had been free from legislative regulation upon the amount of the rates to be charged for gas. The had been most prosperous and had divided very large earnings in the shape of dividends to their stockholders,—dividends which are characterized by the Senate committee, ***46** appointed in 1885 to investigate the fact surrounding the consolidation, as enormous. The report of that committee shows that several of the companies had averaged, from their creation, dividends over 16 per cent, and the six companies in the year 1884 paid a dividend upon capital which had been increased by earnings, as in the case of the Manhattan and the New York, of 18 per cent; and, had it been upon the money actually paid in, it would have been nearly 25 per cent.

The committee also said in the same report that these 'franchises were in force November 10, 1884, the time of the consolidation, and the money invested in them was earning the same enormous dividends. So far as the evidence shows, there was nothing in the condition of affairs on the 10th of November to indicate that these franchises would not be as valuable for the next twenty years as they had been in the past. There were gas companies enough in the city with a capacity capable of supplying the demands for the next twenty years. A law was on our statute books that virtually prohibited the laying of any more gas pipes in the streets. The gas companies had an agreement among themselves, fixing the price of gas at a figure that paid these dividends. The people were paying this price, as they had in the past, without objection or protest. This price may have been too high, and the dividends were excessive, but they were not illegal, and the valuation of the franchises computed upon these dividends and that state of facts cannot be called a violation of a law that expressly authorized it to be done, unless such valuation was too high.'

The committee, upon these facts, were of opinion that the valuation of $7,781,000 for the franchises was not more than their fair aggregate value.

Assuming, as the committee did, that the company would be permitted to charge the same prices in the future which in the past had resulted in these 'enormous' or 'excessive' dividends, it need not be matter of surprise that a franchise by ***47** means of which such dividends had been possible was not regarded as overvalued at the sum stated in 1884.

We think that, under the above facts, the courts ought to accept the valuation of the franchises fixed and agreed upon under the act of 1884 as conclusive at that time. The valuation was provided for in the act, which was followed by the companies, and the ****198** agreement regarding it has been always recognized as valid, and the stock has been largely dealt in for more than twenty years past on the basis of the validity of the valuation and of the stock issued by the company.

But, although the state ought, for these reasons, to be bound to recognize the value agreed upon in 1884 as part of the property upon which a reasonable return can be demanded, we do not think an increase in that valuation ought to be allowed upon the theory suggested by the court below. Because the amount of gas supplied has increased to the extent stated, and the other and tangible property of the corporations has increased so largely in value, is not, as it seems to us, any reason for attributing a like proportional increase in the value of the franchise. Real estate may have increased in value very largely, as also the personal property, without any necessary increase in the value of the franchise. Its past value was founded upon the opportunity of obtaining these enormous and excessive returns upon the property of the company, without legislative interference with the price for the supply of gas; but that immunity for the future was, of course, uncertain; and the moment it ceased, and the legislature reduced the earnings to a reasonable sum, the great value of the franchise would be at once and unfavorably affected, but how much so it is not possible for us now to see. The value would most certainly not increase. The question of the regulation of rates did, from time to time thereafter, arise in the legislature, and finally culminated in these acts which were in existence when the court below found this increased value of the franchises. We cannot, in any view of the case, concur in that finding.

***48** This increase in value did, however, form part of the sum upon which the court below held the complainant was entitled to a return. That court found the value of the tangible assets actually employed at the time of the commencement of this suit in the business of supplying gas by the complainant to be $47,831,435, to which it added the $12,000,000 as the value of the franchises as found by it, making the total of $59,831,435, upon which it held that the company was entitled to a return of 6 per cent, being $3,589,886.10. It also found its total net income for the year 1905 amounted to $5,881,192.45, almost 10 per cent upon the sum above named. Altering the finding of the court so far only as to place the value of the franchises at the time agreed upon in 1884, $7,781,000, the total value upon that basis of the property employed by the company would be $55,612,435, upon which 6 per cent would be $3,336,746.10, while the sum estimated as the return on 80-cent gas would have been $3,024,592.14, which is nearly 5 ½ per cent on the above total of $55,612,435.

What has been said herein regarding the value of the franchises in this case has been necessarily founded upon its own peculiar facts, and the decision thereon can form no precedent in regard to the valuation of franchises generally, where the facts are not similar to those in the case before us. We simply accept the sum named as the value under the circumstances stated.

There is no particular rate of compensation which must, in all cases and in all parts of the country, be regarded as sufficient for capital invested in business enterprises. Such compensation must depend greatly upon circumstances and locality; among other things, the amount of risk in the business is a most important factor, as well as the locality where the business is conducted, and the rate expected and usually realized there upon investments of a somewhat similar nature with regard to the risk attending them. There may be other matters which, in some cases, might also be properly taken into account in determining the rate which an investor might properly expect ***49** or hope to receive and which he would be entitled to without legislative interference. The less risk, the less right to any unusual returns upon the investments. One who invests his money in a business of a somewhat hazardous character is very properly held to have the right to a larger return, without legislative interference, than can be obtained from an investment in government bonds or other perfectly safe security. The man that invested in gas stock in 1823 had a right to look for and obtain, if possible, a much greater rate upon his investment than he who invested in such property in the city of New York years after the risk and danger involved had been almost entirely eliminated.

In an investment in a gas company, such as complainant's, the risk is reduced almost to a minimum. It is a corporation which, in fact, as the court below remarks, monopolizes the gas service of the largest city in America, and is secure against competition under the circumstances in which it is placed, because it is a proposition almost unthinkable that the city of New York would, for purposes of making competition, permit the streets of the city to be again torn up in order to allow the mains of another company to be laid all through them to supply gas which the present company can adequately supply. And, so far as it is given us to look into the future, it seems as certain as anything of such a nature can be, that the demand for gas will increase, ****199** and, at the reduced price, increase to a considerable extent. An interest in such a business is as near a safe and secure investment as can be imagined with regard to any private manufacturing business, although it is recognized at the same time that there is a possible element of risk, even in such a business. The court below regarded it as the most favorably situated gas business in America, and added that all gas business is inherently subject to many of the vicissitudes of manufacturing Under the circumstances, the court held that a rate which would permit a return of 6 per cent would be enough to avoid the charge of confiscation, and for the reason that a return of such an amount was the return ordinarily ***50** sought and obtained on investments of that degree of safety in the city of New York.

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Taking all facts into consideration, we concur with the court below on this question, and think complainant is entitled to 6 per cent on the fair value of its property devoted to the public use. But, assuming that the company is entitled to 6 per cent upon the value of its property actually used for the public, the total fixed by the court below is, as we have seen, much too large. We must first strike out the increased value of the franchises asserted by the court over the amount agree upon in 1884, when the company was consolidated. We also find that the total value of the tangible property is made up of several items, two of which are——

Real Estate..	$11,985,435
Plants..	15,000,000

Both depend largely upon the opinions of expert witnesses as to the value of that kind of property. Where a large amount of the total value of a mass of different properties consists in the value of real estate, which is only ascertained by the varying opinions of expert witnesses, and where the opinions of the plaintiffs' witnesses differ quite radically from those of the defendants, it is apparent that the total value must necessarily be more or less in doubt. It, in other words, becomes matter of speculation or conjecture to a great extent. It may be, as already suggested, that in many cases, the rates objected to might be so low that there could be no reasonable doubt of their inadequacy upon any fair estimate of the value of the property. In such event the enforcement of the rates should be enjoined even in a case where the value of the property depends upon the value to be assigned to real estate by the evidence of experts. But there may be other cases where the evidence as to the probable result of the rates in controversy would show they were so nearly adequate that nothing but a practical test could satisfy the doubt as to their sufficiency.

In this case a slight reduction in the estimated value of the real estate, plants, and mains, as given by the witnesses for ***51** complainant, would give a 6 per cent return upon the total value of the property, as above stated. And again, increased consumption at the lower rate might result in increased earnings, as the cost of furnishing the gas would not increase in proportion to the increased amount of gas furnished.

The elevated railroads in New York, when first built, charged 10 cents for each passenger; but, when the rate was reduced to 5 cents, it is common knowledge that their receipts were not cut in two, but that, from increased patronage, the earnings increased from year to year, and soon surpassed the highest sum ever received upon the 10-cent rate.

Of course, there is always a point below which a rate could not be reduced, and, at the same time, permit the proper return on the value of the property; but it is equally true that a reduction in rates will not always reduce the net earnings, but, on the contrary, may increase them. The question of how much an increased consumption under a less rate will increase the earnings of complainant, if at all, at a cost not proportioned to the former cost, can be answered only by a practical test. In such a case as this, where the other *data* upon which the computation of the rate of return must be based, are from the evidence, so uncertain, and where the margin between possible confiscation and valid regulation is so narrow, we cannot say there is no fair or just doubt about the truth of the allegation that the rates are insufficient.

The complainant also contends that the state, having taxed it upon its franchises, cannot be heard to deny their existence or their value as taxed.

The fact that the state has taxed the company upon its franchises at a greater ualue than is awarded them here is not material. Those taxes, even if founded upon an erroneous valuation, were properly treated by the company as part of its operating expenses, to be paid out of its earnings be fore the net amount could be arrived at applicable to dividends, and, if such latter sums were not sufficient to permit the proper return on the property used by the company for the public, ***52** then the rate would be inadequate. The future assessment of the value of the franchises, it is presumed, will be much lessened if it is seen that the great profits upon which that value was ****200** based are largely reduced by legislative action. In that way the consumer will be benefited by paying a reduced sum (although indirectly) for taxes.

We are also of opinion that it is not a case for a valuation of 'good will.' The master combined the franchise value with that of good will, and estimated the total value at $20,000,000.

The complainant has a monopoly in fact, and a consumer must take gas from it or go without. He will resort to the 'old stand,' because he cannot get gas anywhere else. The court below excluded that item, and we concur in that action.

And we concur with the court below in holding that the value of the property is to be determined as of the time when the inquiry is made regarding the rates. If the property which legally enters into the consideration of the question of rates has increased in value since it was acquired, the company is entitled to the benefit of such increase. This is, at any rate, the general rule. We do not say there may not possibly be an exception to it where the property may have increased so enormously in value as to render a rate permitting a reasonable return upon such increased value unjust to the public. How such facts should be treated is not a question now before us, as this case does not present it. We refer to the matter only for the purpose of stating that the decision herein does not prevent an inquiry into the question when, if ever, it should be necessarily presented.

The matter of the increased cost of the gas, resulting from the provisions of the acts as to making the gas equal to 22 candle power, is also alleged as a reason for in adequacy of rate.

It appears that the everage candle power actually produced in the first six months of the year 1905 was 22, while but 20 candle power was exacted by law, and, for the last six months of that year, while 22 candle power was exacted, the everage ***53** amount was 24.19. This expense was included in the operating expense of that year, which resulted in the net earnings above mentioned, while the company was complying with the requirements of the act in this particular.

It is unnecessary, therefore, to further inquire as to the additional expense caused by this requirement.

Again, it has been asserted that the laws are unconstitutional because of the provision as to pressure, and also by reason of the penalties which a violation of the acts may render a corporation liable to.

The acts provide that the pressure of the gas in the service mains at any distance from the place of manufacture shall not be less than 1 inch nor more than 2 ½ inches.

The evidence shows that, to put a pressure such as is demanded by the acts upon the mains and other service pipes in their present condition would be to run a great risk of explosion, and consequent disaster. Before compliance with this provision would be safe, the mains and other pipes would have to be strengthened throughout their whole extent, and at an expenditure of many millions of dollars, from which no return could be obtained at the rates provided in the acts. This would take from the complainant the ability to secure the return to which it is entitled upon its property, used for supplying gas, and the provision as to the amount of pressure is therefore void. This particular duty imposed by the acts is, however, clearly separable from the enactments as to rates, and we have no doubt that the remainder of the statute would have been enacted, even with that provision omitted.

The obligation would remain upon the company to have a pressure sufficient to insure a light of 22 candle power, as provided in the acts.

We are of the same opinion as to the penalties provided for a violation of the acts. They are not a necessary or inseparable part of the acts, without which they would not have been passed. If these provisions as to penalties have been properly construed by the court below, they are undoubtedly void, ***54** within the principle decided in Ex parte Young, 209 U. S. 123, 52 L. ed. 714, 13 L.R.A.(N.S.) 932, 28 Sup. Ct. Rep. 441, and cases there cited, because so enormous and overwhelming in their amount.

When the objectionable part of a statute is eliminated, if the balance is valid and capable of being carried our, and if the court can conclude it would have been enacted if that portion which is illegal had been omitted, the remainder of the statute thus treated is good. Reagan v. Farmers' Loan & T. Co. 154 U. S. 362, 395, 38 L. ed. 1014, 1022, 4 Inters. Com. Rep. 560, 14 Sup. Ct. Rep. 1047; Berea College v. Kentucky, 211 U. S. 45–54, 53 L. ed. ——, 29 Sup. Ct. Rep. 33. This is a familiar principle.

Lastly, it is objected that there is an illegal discrimination as between the city and the consumers individually. We see no discrimination which is illegal or for which good reasons could not be given. But neither the city nor the consumers are finding any fault with it, and the only interest of the complainant in the question is to find out whether, by the reduced price to the city, the complainant is, upon the whole, unable to realize a return sufficient to comply with what it has the right to demand. What we have already said applies to the facts now in question.

****201** We cannot see, from the whole evidence, that the price fixed for gas supplied to the city by wholesale, so to speak, would so reduce the profits from the total of the gas supplied as to thereby render such total profits insufficient as a return upon the property used by the complainant. So long as the total is enough to furnish such return, it is not important that, with relation to some customers, the price is not enough. Minneapolis & St. L. R. Co. v. Minnesota, 186 U. S. 257, 46 L. ed. 1151, 22 Sup. Ct. Rep. 900; Atlantic Coast line R. Co. v. North Carolina Corp. Commission, 206 U. S. 1, 51 L. ed. 933, 27 Sup. Ct. Rep. 585.

Upon a careful consideration of the case before us we are of opinion that the complainant has failed to sustain the burden cast upon it of showing beyond any just or fair doubt that the acts of the legislature of the state of New York are in fact confiscatory.

It may possibly be, however, that a practical experience of the effect of the acts by actual operation under them might ***55** prevent the complainant from obtaining a fair return, as already described, and, in that event, complainant ought to have the opportunity of again presenting its case to the court. To that end we reverse the decree with directions to dismiss the bill without prejudice; and it is so ordered.

All Citations

212 U.S. 19, 29 S.Ct. 192, 53 L.Ed. 382, 48 L.R.A.N.S. 1134, 15 Am.Ann.Cas. 1034

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43 S.Ct. 675
Supreme Court of the United States

BLUEFIELD WATERWORKS
& IMPROVEMENT CO.
v.
PUBLIC SERVICE COMMISSION
OF WEST VIRGINIA et al.

No. 256. | Argued January 22, 1923. | Decided June 11, 1923.

In Error to the Supreme Court of Appeals of West Virginia.

Proceedings by the Bluefield Waterworks & Improvement Company against the Public Service Commission of the State of West Virginia and others to suspend and set aside an order of the Commission fixing rates. From a judgment of the Supreme Court of West Virginia, dismissing the petition, and denying the relief (89 W. Va. 736, 110 S. E. 205), the Waterworks Company bring error. Reversed.

West Headnotes (6)

[1] **Constitutional Law**
Charges and prices in general

Rates which are not sufficient to yield a reasonable return on the value of the property used in public service at the time it is being so used to render the service are unjust, unreasonable, and confiscatory, and their enforcement deprives the public utility company of its property, in violation of the Fourteenth Amendment of the Constitution.

109 Cases that cite this headnote

[2] **Constitutional Law**
Water, sewer, and irrigation

Under the due process clause of the Fourteenth Amendment of the Constitution, U.S.C.A., a waterworks company is entitled to the independent judgment of the court as to both law and facts, where the question is whether the rates fixed by a public service commission are confiscatory.

33 Cases that cite this headnote

[3] **Water Law**
Methodologies; establishment of rate base

It was error for a state public service commission, in arriving at the value of the property used in public service, for the purpose of fixing the rates, to fail to give proper weight to the greatly increased cost of construction since the war.

25 Cases that cite this headnote

[4] **Water Law**
Rate of return

A public utility is entitled to such rates as will permit it to earn a return on the value of the property which it employs for the convenience of the public equal to that generally being made at the same time and in the same general part of the country on investments in other business undertakings which are attended by corresponding risks and uncertainties, but it has no constitutional right to such profits as are realized or anticipated in highly profitable enterprises or speculative ventures.

150 Cases that cite this headnote

[5] **Water Law**
Rate of return

Since the investors take into account the result of past operations as well as present rates in determining whether they will invest, a waterworks company which had been earning a low rate of returns through a long period up to the time of the inquiry is entitled to return of more than 6 per cent. on the value of its property used in the public service, in order to justly compensate it for the use of its property.

154 Cases that cite this headnote

[6] **Federal Courts**
Particular Cases, Contexts, and Questions

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A proceeding in a state court attacking an order of a public service commission fixing rates, on the ground that the rates were confiscatory and the order void under the federal Constitution, is one where there is drawn in question the validity of authority exercised under the state, on the ground of repugnancy to the federal Constitution, and therefore is reviewable by writ of error.

21 Cases that cite this headnote

Attorneys and Law Firms

****675** ***680** Messrs. Alfred G. Fox and Jos. M. Sanders, both of Bluefield, W. Va., for plaintiff in error.

Mr. Russell S. Ritz, of Bluefield, W. Va., for defendants in error.

Opinion

***683** Mr. Justice BUTLER delivered the opinion of the Court.

Plaintiff in error is a corporation furnishing water to the city of Bluefield, W. Va., ****676** and its inhabitants. September 27, 1920, the Public Service Commission of the state, being authorized by statute to fix just and reasonable rates, made its order prescribing rates. In accordance with the laws of the state (section 16, c. 15–O, Code of West Virginia [sec. 651]), the company instituted proceedings in the Supreme Court of Appeals to suspend and set aside the order. The petition alleges that the order is repugnant to the Fourteenth Amendment, and deprives the company of its property without just compensation and without due process of law, and denies it equal protection of the laws. A final judgment was entered, denying the company relief and dismissing its petition. The case is here on writ of error.

[1] 1. The city moves to dismiss the writ of error for the reason, as it asserts, that there was not drawn in question the validity of a statute or an authority exercised under the state, on the ground of repugnancy to the federal Constitution.

The validity of the order prescribing the rates was directly challenged on constitutional grounds, and it was held valid by the highest court of the state. The prescribing of rates is a legislative act. The commission is an instrumentality of the state, exercising delegated powers. Its order is of the same force as would be a like enactment by the Legislature. If, as alleged, the prescribed rates are confiscatory, the order is void. Plaintiff in error is entitled to bring the case here on writ of error and to have that question decided by this court. The motion to dismiss will be denied. See ***684** Oklahoma Natural Gas Co. v. Russell, 261 U. S. 290, 43 Sup. Ct. 353, 67 L. Ed. 659, decided March 5, 1923, and cases cited; also Ohio Valley Co. v. Ben Avon Borough, 253 U. S. 287, 40 Sup. Ct. 527, 64 L. Ed. 908.

2. The commission fixed $460,000 as the amount on which the company is entitled to a return. It found that under existing rates, assuming some increase of business, gross earnings for 1921 would be $80,000 and operating expenses $53,000 leaving $27,000, the equivalent of 5.87 per cent., or 3.87 per cent. after deducting 2 per cent. allowed for depreciation. It held existing rates insufficient to the extent of 10,000. Its order allowed the company to add 16 per cent. to all bills, excepting those for public and private fire protection. The total of the bills so to be increased amounted to $64,000; that is, 80 per cent. of the revenue was authorized to be increased 16 per cent., equal to an increase of 12.8 per cent. on the total, amounting to $10,240.

As to value: The company claims that the value of the property is greatly in excess of $460,000. Reference to the evidence is necessary. There was submitted to the commission evidence of value which it summarized substantially as follows:

a.	Estimate by company's engineer on basis of reproduction new, less depreciation, at prewar prices......................................	$ 624,548 00
b.	Estimate by company's engineer on basis of reproduction new, less depreciation, at 1920 prices..	1,194,663 00

c.	Testimony of company's engineer fixing present fair value for rate making purposes...	900,000 00
d.	Estimate by commissioner's engineer on basis of reproduction new, less depreciation at 1915 prices, plus additions since December 31, 1915, at actual cost, excluding Bluefield Valley waterworks, water rights, and going value..	397,964 38
e.	Report of commission's statistician showing investment cost less depreciation..	365,445 13
f.	Commission's valuation, as fixed in case No. 368 ($360,000), plus gross additions to capital since made ($92,520.53)...	452,520 53

***685** It was shown that the prices prevailing in 1920 were nearly double those in 1915 and pre-war time. The company did not claim value as high as its estimate of cost of construction in 1920. Its valuation engineer testified that in his opinion the value of the property was $900,000—a figure between the cost of construction in 1920, less depreciation, and the cost of construction in 1915 and before the war, less depreciation.

The commission's application of the evidence may be stated briefly as follows:

As to 'a,' supra: The commission deducted $204,000 from the estimate (details printed in the margin), [1] leaving approximately $421,000, which it contrasted with the estimate of its own engineer, $397,964.38 (see 'd,' supra). It found that there should be included $25,000 for the Bluefield Valley waterworks plant in Virginia, 10 per cent. for going value, and $10,000 for working capital. If these be added to $421,000, there results $500,600. This may be compared with the commission's final figure, $460,000.

***686** As to 'b' and 'c,' supra: These were given no weight by the commission in arriving at its final figure, $460,000. It said:

> 'Applicant's plant was originally constructed more than twenty years ago, and has been added to from time to time as the progress and development of the community required. For this reason, it would be unfair to its consumers to use as a basis for present fair value the abnormal prices prevailing during the recent war period; but, when, as in this case, a part of the plant has been constructed or added to during that period, in fairness to the applicant, consideration must be given to

the cost of such expenditures made to meet the demands of the public.'

****677** As to 'd,' supra: The commission, taking $400,000 (round figures), added $25,000 for Bluefield Valley waterworks plant in Virginia, 10 per cent. for going value, and $10,000 for working capital, making $477,500. This may be compared with its final figure, $460,000.

As to 'e,' supra: The commission, on the report of its statistician, found gross investment to be $500,402.53. Its engineer, applying the straight line method, found 19 per cent. depreciation. It applied 81 per cent. to gross investment and added 10 per cent. for going value and $10,000 for working capital, producing $455,500. [2] This may be compared with its final figure, $460,000.

As to 'f,' supra: It is necessary briefly to explain how this figure, $452,520.53, was arrived at. Case No. 368 was a proceeding initiated by the application of the company for higher rates, April 24, 1915. The commission made a valuation as of January 1, 1915. There were presented two estimates of reproduction cost less depreciation, one by a valuation engineer engaged by the company, ***687** and the other by a valuation engineer engaged by the city, both 'using the same method.' An inventory made by the company's engineer was accepted as correct by the city and by the commission. The method 'was that generally employed by courts and commissions in arriving at the value of public utility properties under this method.' and in both estimates 'five year average unit prices' were applied. The estimate of the company's engineer was $540,000 and of the city's engineer, $392,000. The principal differences as given by the commission are shown in the margin. [3] The commission disregarded both estimates and arrived at $360,000. It held that the best basis of valuation was the net investment, i. e., the total cost of the property less depreciation. It said:
'The books of the company show a total gross investment, since its organization, of $407,882, and that there has been charged off for depreciation from year to year the total sum of $83,445, leaving a net investment of $324,427. * * * From an examination of the books * * * it appears that the records of the company have been remarkably well kept and preserved. It therefore seems that, when a plant is developed under these conditions, the net investment, which, of course, means the total gross investment less depreciation, is the very best basis of valuation for rate making purposes and that the other methods above referred to should ***688** be used only when it is impossible to arrive at the true investment. Therefore, after making due allowance for capital necessary for the conduct of the business and considering the plant as a going concern, it is the opinion of the commission that the fair value for the purpose of determining reasonable and just rates in this case of the property of the applicant company, used by it in the public service of supplying water to the city of Bluefield and its citizens, is the sum of $360,000, which sum is hereby fixed and determined by the commission to be the fair present value for the said purpose of determining the reasonable and just rates in this case.'

In its report in No. 368, the commission did not indicate the amounts respectively allowed for going value or working capital. If 10 per cent. be added for the former, and $10,000 for the latter (as fixed by the commission in the present case), there is produced $366,870, to be compared with $360,000, found by the commission in its valuation as of January 1, 1915. To this it added $92,520.53, expended since, producing $452,520.53. This may be compared with its final figure, $460,000.

The state Supreme Court of Appeals holds that the valuing of the property of a public utility corporation and prescribing rates are purely legislative acts, not subject to judicial review, except in so far as may be necessary to determine whether such rates are void on constitutional or other grounds, and that findings of fact by the commission based on evidence to support them will not be reviewed by the court. City of Bluefield v. Waterworks, 81 W. Va. 201, 204, 94 S. E. 121; Coal & Coke Co. v. Public Service Commission, 84 W. Va. 662, 678, 100 S. E. 557, 7 A. L. R. 108; Charleston v. Public Service Commission, 86 W. Va. 536, 103 S. E. 673.

In this case (89 W. Va. 736, 738, 110 S. E. 205, 206) it said: 'From the written opinion of the commission we find that it ascertained the value of the petitioner's property for rate making [then quoting the commission] 'after ***689** maturely and carefully considering the various methods presented for the ascertainment of fair value and giving such weight as seems proper to every element involved and all the facts and circumstances disclosed by the record.''

[2] [3] The record clearly shows that the commission, in arriving at its final figure, did not accord proper, if any, weight to the greatly enhanced costs of construction in 1920 over those prevailing about 1915 and before the war, as established by uncontradicted ****678** evidence; and the company's detailed estimated cost of reproduction

new, less depreciation, at 1920 prices, appears to have been wholly disregarded. This was erroneous. Missouri ex rel. Southwestern Bell Telephone Co. v. Public Service Commission of Missouri, 262 U. S. 276, 43 Sup. Ct. 544, 67 L. Ed. 981, decided May 21, 1923. Plaintiff in error is entitled under the due process clause of the Fourteenth Amendment to the independent judgment of the court as to both law and facts. Ohio Valley Co. v. Ben Avon Borough, 253 U. S. 287, 289, 40 Sup. Ct. 527, 64 L. Ed. 908, and cases cited.

We quote further from the court's opinion (89 W. Va. 739, 740, 110 S. E. 206):

'In our opinion the commission was justified by the law and by the facts in finding as a basis for rate making the sum of $460,000.00. * * * In our case of Coal & Coke Ry. Co. v. Conley, 67 W. Va. 129, it is said: 'It seems to be generally held that, in the absence of peculiar and extraordinary conditions, such as a more costly plant than the public service of the community requires, or the erection of a plant at an actual, though extravagant, cost, or the purchase of one at an exorbitant or inflated price, the actual amount of money invested is to be taken as the basis, and upon this a return must be allowed equivalent to that which is ordinarily received in the locality in which the business is done, upon capital invested in similar enterprises. In addition to this, consideration must be given to the nature of the investment, a higher rate ***690** being regarded as justified by the risk incident to a hazardous investment.'

'That the original cost considered in connection with the history and growth of the utility and the value of the services rendered constitute the principal elements to be considered in connection with rate making, seems to be supported by nearly all the authorities.'

[4] The question in the case is whether the rates prescribed in the commission's order are confiscatory and therefore beyond legislative power. Rates which are not sufficient to yield a reasonable return on the value of the property used at the time it is being used to render the service are unjust, unreasonable and confiscatory, and their enforcement deprives the public utility company of its property in violation of the Fourteenth Amendment. This is so well settled by numerous decisions of this court that citation of the cases is scarcely necessary:

'What the company is entitled to ask is a fair return upon the value of that which it employs for the public convenience.' Smyth v. Ames (1898) 169 U. S. 467, 547, 18 Sup. Ct. 418, 434 (42 L. Ed. 819).

'There must be a fair return upon the reasonable value of the property at the time it is being used for the public. * * * And we concur with the court below in holding that the value of the property is to be determined as of the time when the inquiry is made regarding the rates. If the property, which legally enters into the consideration of the question of rates, has increased in value since it was acquired, the company is entitled to the benefit of such increase.' Willcox v. Consolidated Gas Co. (1909) 212 U. S. 19, 41, 52, 29 Sup. Ct. 192, 200 (53 L. Ed. 382, 15 Ann. Cas. 1034, 48 L. R. A. [N. S.] 1134).

'The ascertainment of that value is not controlled by artificial rules. It is not a matter of formulas, but there must be a reasonable judgment having its basis in a proper consideration of all relevant facts.' Minnesota Rate Cases (1913) 230 U. S. 352, 434, 33 Sup. Ct. 729, 754 (57 L. Ed. 1511, 48 L. R. A. [N. S.] 1151, Ann. Cas. 1916A, 18).

***691** 'And in order to ascertain that value, the original cost of construction, the amount expended in permanent improvements, the amount and market value of its bonds and stock, the present as compared with the original cost of construction, the probable earning capacity of the property under particular rates prescribed by statute, and the sum required to meet operating expenses, are all matters for consideration, and are to be given such weight as may be just and right in each case. We do not say that there may not be other matters to be regarded in estimating the value of the property.' Smyth v. Ames, 169 U. S., 546, 547, 18 Sup. Ct. 434, 42 L. Ed. 819.

'* * * The making of a just return for the use of the property involves the recognition of its fair value if it be more than its cost. The property is held in private ownership and it is that property, and not the original cost of it, of which the owner may not be deprived without due process of law.'

Minnesota Rate Cases, 230 U. S. 454, 33 Sup. Ct. 762, 57 L. Ed. 1511, 48 L. R. A. (N. S.) 1151, Ann. Cas. 1916A, 18.

In Missouri ex rel. Southwestern Bell Telephone Co., v. Public Service Commission of Missouri, supra, applying the principles of the cases above cited and others, this court said:

> 'Obviously, the commission undertook to value the property without according any weight to the greatly enhanced costs of material, labor, supplies, etc., over those prevailing in 1913, 1914, and

> 1916. As matter of common knowledge, these increases were large. Competent witnesses estimated them as 45 to 50 per centum. * * * It is impossible to ascertain what will amount to a fair return upon properties devoted to public service, without giving consideration to the cost of labor, supplies, etc., at the time the investigation is made. An honest and intelligent forecast of probable future values, made upon a view of all the relevant circumstances, is essential. If the highly important element of present costs is wholly disregarded, such a forecast becomes impossible. Estimates for to-morrow cannot ignore prices of to-day.'

[5] ***692** It is clear that the court also failed to give proper consideration to the higher cost of construction in 1920 over that in 1915 and before the war, and failed to give weight to cost of reproduction less depreciation on the basis of 1920 prices, or to the testimony of the company's valuation engineer, based on present and past costs of construction, that the property in his opinion, was worth $900,000. The final figure, $460,000, was arrived ****679** at substantially on the basis of actual cost, less depreciation, plus 10 per cent. for going value and $10,000 for working capital. This resulted in a valuation considerably and materially less than would have been reached by a fair and just consideration of all the facts. The valuation cannot be sustained. Other objections to the valuation need not be considered.

3. Rate of return: The state commission found that the company's net annual income should be approximately $37,000, in order to enable it to earn 8 per cent. for return and depreciation upon the value of its property as fixed by it. Deducting 2 per cent. for depreciation, there remains 6 per cent. on $460,000, amounting to $27,600 for return. This was approved by the state court.

[6] The company contends that the rate of return is too low and confiscatory. What annual rate will constitute just compensation depeds upon many circumstances, and must be determined by the exercise of a fair and enlightened judgment, having regard to all relevant facts. A public utility is entitled to such rates as will permit it to earn a return on the value of the property which it employs for the convenience of the public equal to that generally being made at the same time and in the same general part of the country on investments in other business undertakings which are attended by corresponding, risks and uncertainties; but it has no constitutional right to profits such as are realized or anticipated in ***693** highly profitable enterprises or speculative ventures. The return should be reasonably sufficient to assure confidence in the financial soundness of the utility and should be adequate, under efficient and economical management, to maintain and support its credit and enable it to raise the money necessary for the proper discharge of its public duties. A rate of return may be reasonable at one time and become too high or too low by changes affecting opportunities for investment, the money market and business conditions generally.

In 1909, this court, in Willcox v. Consolidated Gas Co., 212 U. S. 19, 48–50, 29 Sup. Ct. 192, 53 L. Ed. 382, 15 Ann. Cas. 1034, 48 L. R. A. (N. S.) 1134, held that the question whether a rate yields such a return as not to be confiscatory depends upon circumstances, locality and risk, and that no proper rate can be established for all cases; and that, under the circumstances of that case, 6 per cent. was a fair return on the value of the property employed in supplying gas to the city of New York, and that a rate yielding that return was not confiscatory. In that case the investment was held to be safe, returns certain and risk reduced almost to a minimum—as nearly a safe and secure investment as could be imagined in regard to any private manufacturing enterprise.

In 1912, in Cedar Rapids Gas Co. v. Cedar Rapids, 223 U. S. 655, 670, 32 Sup. Ct. 389, 56 L. Ed. 594, this court declined to reverse the state court where the value of the plant considerably exceeded its cost, and the estimated return was over 6 per cent.

In 1915, in Des Moines Gas Co. v. Des Moines, 238 U. S. 153, 172, 35 Sup. Ct. 811, 59 L. Ed. 1244, this court declined to reverse the United States District Court in refusing an injunction upon the conclusion reached that a return of 6 per cent. per annum upon the value would not be confiscatory.

In 1919, this court in Lincoln Gas Co. v. Lincoln, 250 U. S. 256, 268, 39 Sup. Ct. 454, 458 (63 L. Ed. 968), declined on the facts of that case to approve a finding that no rate yielding as much as 6 per cent. ***694** on the invested capital could be regarded as confiscatory. Speaking for the court, Mr. Justice Pitney said:

> 'It is a matter of common knowledge that, owing principally to the World War, the costs of labor and supplies of every

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> kind have greatly advanced since the ordinance was adopted, and largely since this cause was last heard in the court below. And it is equally well known that annual returns upon capital and enterprise the world over have materially increased, so that what would have been a proper rate of return for capital invested in gas plants and similar public utilities a few years ago furnishes no safe criterion for the present or for the future.'

In 1921, in Brush Electric Co. v. Galveston, the United States District Court held 8 per cent. a fair rate of return.[4]

In January, 1923, in City of Minneapolis v. Rand, the Circuit Court of Appeals of the Eighth Circuit (285 Fed. 818, 830) sustained, as against the attack of the city on the ground that it was excessive, 7 ½ per cent., found by a special master and approved by the District Court as a fair and reasonable return on the capital investment—the value of the property.

[7] Investors take into account the result of past operations, especially in recent years, when determining the terms upon which they will invest in such an undertaking. Low, uncertain, or irregular income makes for low prices for the securities of the utility and higher rates of interest to be demanded by investors. The fact that the company may not insist as a matter of constitutional right that past losses be made up by rates to be applied in the present and future tends to weaken credit, and the fact that the utility is protected against being compelled to serve for confiscatory rates tends to support it. In ***695** this case the record shows that the rate of return has been low through a long period up to the time of the inquiry by the commission here involved. For example, the average rate of return on the total cost of the property from 1895 to 1915, inclusive, was less than 5 per cent.; from 1911 to 1915, inclusive, ****680** about 4.4 per cent., without allowance for depreciation. In 1919 the net operating income was approximately $24,700, leaving $15,500, approximately, or 3.4 per cent. on $460,000 fixed by the commission, after deducting 2 per cent. for depreciation. In 1920, the net operating income was approximately $25,465, leaving $16,265 for return, after allowing for depreciation. Under the facts and circumstances indicated by the record, we think that a rate of return of 6 per cent. upon the value of the property is substantially too low to constitute just compensation for the use of the property employed to render the service.

The judgment of the Supreme Court of Appeals of West Virginia is reversed.

Mr. Justice BRANDEIS concurs in the judgment of reversal, for the reasons stated by him in Missouri ex rel. Southwestern Bell Telephone Co. v. Public Service Commission of Missouri, supra.

Parallel Citations

P.U.R. 1923D 11, 43 S.Ct. 675, 67 L.Ed. 1176

Footnotes

1

Difference in depreciation allowed	$ 49,000
Preliminary organization and development cost	14,500
Bluefield Valley waterworks plant	25,000
Water rights	50,000
Excess overhead costs	39,000
Paving over mains	28,500
	$204,000

2 As to 'e': $365,445.13 represents investment cost less depreciation. The gross investment was found to be $500,402.53, indicating a deduction on account of depreciation of $134,957.40, about 27 per cent., as against 19 per cent. found by the commission's engineer.

3

		Company Engineer.	City Engineer.
1.	Preliminary costs	$14,455	$1,000
2.	Water rights	50,000	Nothing
3.	Cutting pavements over		

	mains..	27,744	233
4.	Pipe lines from gravity springs..	22,072	15,442
5.	Laying cast iron street mains..	19,252	15,212
6.	Reproducing Ada springs..	18,558	13,027
7.	Superintendence and engineering..	20,515	13,621
8.	General contingent cost..	16,415	5,448
		$189,011	$63,983

4 This case was affirmed by this court June 4, 1923, 262 U. S. 443, 43 Sup. Ct. 606, 67 L. Ed. 1076.

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64 S.Ct. 281
Supreme Court of the United States

FEDERAL POWER COMMISSION et al.
v.
HOPE NATURAL GAS CO.
CITY OF CLEVELAND
v.
SAME.

Nos. 34 and 35. | Argued Oct. 20, 21, 1943. | Decided Jan. 3, 1944.

Separate proceedings before the Federal Power Commission by such Commission, by the City of Cleveland and the City of Akron, and by Pennsylvania Public Utility Commission wherein the State of West Virginia and its Public Service Commission were permitted to intervene concerning rates charged by Hope Natural Gas Company which were consolidated for hearing. An order fixing rates was reversed and remanded with directions by the Circuit Court of Appeals, 134 F.2d 287, and Federal Power Commission, City of Akron and Pennsylvania Public Utility Commission in one case and the City of Cleveland in another bring certiorari.

Reversed.

Mr. Justice REED, Mr. Justice FRANKFURTER and Mr. Justice JACKSON, dissenting.

On Writs of Certiorari to the United States Circuit Court of Appeals for the Fourth Circuit.

West Headnotes (26)

[1] **Public Utilities**
Nature and extent in general

Rate-making is only one species of price-fixing which, like other applications of the police power, may reduce the value of the property regulated, but that does not render the regulation invalid.

25 Cases that cite this headnote

[2] **Public Utilities**
Reasonableness of charges in general

Rates cannot be made to depend upon fair value, which is the end product of the process of rate-making and not the starting point, when the value of the going enterprise depends on earnings under whatever rates may be anticipated.

101 Cases that cite this headnote

[3] **Gas**
Federal Power Commission

The rate-making function of the Federal Power Commission under the Natural Gas Act involves the making of pragmatic adjustments, and the Commission is not bound to the use of any single formula or combination of formulae in determining rates. Natural Gas Act, §§ 4(a), 5(a), 6, 15 U.S.C.A. §§ 717c(a), 717d(a), 717e.

46 Cases that cite this headnote

[4] **Gas**
Scope of review and trial de novo

When order of Federal Power Commission fixing natural gas rates is challenged in the courts, the question is whether order viewed in its entirety meets the requirements of the Natural Gas Act. Natural Gas Act, §§ 4(a), 5(a), 6, 19(b), 15 U.S.C.A. §§ 717c(a), 717d(a), 717e, 717r(b).

8 Cases that cite this headnote

[5] **Gas**
Reasonableness of Charges

Under the statutory standard that natural gas rates shall be "just and reasonable" it is the result reached and not the method employed that is controlling. Natural Gas Act §§ 4(a), 5(a), 15 U.S.C.A. §§ 717c(a), 717d(a).

69 Cases that cite this headnote

[6] **Gas**
Scope of review and trial de novo

If the total effect of natural gas rates fixed by Federal Power Commission cannot be said to be unjust and unreasonable, judicial inquiry under the Natural Gas Act is at an end. Natural Gas Act, §§ 4(a), 5(a), 6, 19(b), 15 U.S.C.A. §§ 717c(a), 717d(a), 717e, 717r(b).

74 Cases that cite this headnote

[7] Gas

Presumptions

An order of the Federal Power Commission fixing rates for natural gas is the product of expert judgment, which carries a presumption of validity, and one who would upset the rate must make a convincing showing that it is invalid because it is unjust and unreasonable in its consequences. Natural Gas Act, §§ 4(a), 5(a), 6, 19(b), 15 U.S.C.A. §§ 717c(a), 717d(a), 717e, 717r(b).

118 Cases that cite this headnote

[8] Gas

Reasonableness of Charges

The fixing of just and reasonable rates for natural gas by the Federal Power Commission involves a balancing of the investor and the consumer interests. Natural Gas Act, §§ 4(a), 5(a), 15 U.S.C.A. §§ 717c(a), 717d(a).

52 Cases that cite this headnote

[9] Gas

Depreciation and depletion

As respects rates for natural gas, from the investor or company point of view it is important that there be enough revenue not only for operating expenses but also for the capital costs of the business, which includes service on the debt and dividends on stock, and by such standard the return to the equity owner should be commensurate with the terms on investments in other enterprises having corresponding risks, and such returns should be sufficient to assure confidence in the financial integrity of the enterprise so as to maintain its credit and to attract capital. Natural Gas Act, §§ 4(a), 5(a), 15 U.S.C.A. §§ 717c(a), 717d(a).

265 Cases that cite this headnote

[10] Gas

Depreciation and depletion

The fixing by the Federal Power Commission of a rate of return that permitted a natural gas company to earn $2,191,314 annually was supported by substantial evidence. Natural Gas Act, §§ 4(a), 5(a), 6, 19(b), 15 U.S.C.A. §§ 717c(a), 717d(a), 717e, 717r(b).

3 Cases that cite this headnote

[11] Gas

Depreciation and depletion

Rates which enable a natural gas company to operate successfully, to maintain its financial integrity, to attract capital and to compensate its investors for the risks assumed cannot be condemned as invalid, even though they might produce only a meager return on the so-called "fair value" rate base. Natural Gas Act, §§ 4(a), 5(a), 6, 19(b), 15 U.S.C.A. §§ 717c(a), 717d(a), 717e, 717r(b).

155 Cases that cite this headnote

[12] Gas

Method of valuation

A return of only 3 27/100 per cent. on alleged rate base computed on reproduction cost new to natural gas company earning an annual average return of about 9 per cent. on average investment and satisfied with existing gas rates suggests an inflation of the base on which the rate had been computed, and justified Federal Power Commission in rejecting reproduction cost as the measure of the rate base. Natural Gas Act, §§ 4(a), 5(a), 15 U.S.C.A. §§ 717c(a), 717d(a).

64 Cases that cite this headnote

[13] Gas

Depreciation and depletion

There is no constitutional requirement that owner who engages in a wasting-asset business of limited life shall receive at the end more than he has put into it, and such rule is applicable to a natural gas company since the ultimate

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exhaustion of its supply of gas is inevitable. Natural Gas Act, §§ 4(a), 5(a), 6, 19(b), 15 U.S.C.A. §§ 717c(a), 717d(a), 717e, 717r(b).

1 Cases that cite this headnote

[14] **Gas**

Depreciation and depletion

In fixing natural gas rate the basing of annual depreciation on cost is proper since by such procedure the utility is made whole and the integrity of its investment is maintained, and no more is required. Natural Gas Act, §§ 4(a), 5(a), 6, 19(b), 15 U.S.C.A. §§ 717c(a), 717d(a), 717e, 717r(b).

13 Cases that cite this headnote

[15] **Gas**

Findings and orders

There are no constitutional requirements more exacting than the standards of the Natural Gas Act which are that gas rates shall be just and reasonable, and a rate order which conforms with the act is valid. Natural Gas Act, §§ 4(a), 5(a), 6, 19(b), 15 U.S.C.A. §§ 717c(a), 717d(a), 717e, 717r(b).

13 Cases that cite this headnote

[16] **Commerce**

Gas

The purpose of the Natural Gas Act was to provide through the exercise of the national power over interstate commerce an agency for regulating the wholesale distribution to public service companies of natural gas moving in interstate commerce not subject to certain types of state regulation, and the act was not intended to take any authority from state commissions or to usurp state regulatory authority. Natural Gas Act, § 1 et seq., 15 U.S.C.A. § 717 et seq.

25 Cases that cite this headnote

[17] **Mines and Minerals**

Oil and gas

Under the Natural Gas Act, the Federal Power Commission has no authority over the production or gathering of natural gas. Natural Gas Act, § 1(b), 15 U.S.C.A. § 717(b).

8 Cases that cite this headnote

[18] **Gas**

In general; amount and regulation

The primary aim of the Natural Gas Act was to protect consumers against exploitation at the hands of natural gas companies and holding companies owning a majority of the pipeline mileage which moved gas in interstate commerce and against which state commissions, independent producers and communities were growing quite helpless. Natural Gas Act, §§ 4, 6–10, 14, 15 U.S.C.A. §§ 717c, 717e–717i, 717m.

59 Cases that cite this headnote

[19] **Gas**

In general; amount and regulation

Apart from the express exemptions contained in § 7 of the Natural Gas Act considerations of conservation are material where abandonment or extensions of facilities or service by natural gas companies are involved, but exploitation of consumers by private operators through maintenance of high rates cannot be continued because of the indirect benefits derived therefrom by a state containing natural gas deposits. Natural Gas Act, §§ 4, 5, and § 7 as amended 15 U.S.C.A. §§ 717c, 717d, 717f.

19 Cases that cite this headnote

[20] **Commerce**

Gas

A limitation on the net earnings of a natural gas company from its interstate business is not a limitation on the power of the producing state, either to safeguard its tax revenues from such industry, or to protect the interests of those who sell their gas to the interstate operator, particularly where the return allowed the company by the Federal Power Commission was a net return after all such charges. Natural

Gas Act, §§ 4, 5, and § 7, as amended, 15 U.S.C.A. §§ 717c, 717d, 717f.

10 Cases that cite this headnote

[21] **Gas**

Reasonableness of Charges

The Natural Gas Act granting Federal Power Commission power to fix "just and reasonable rates" does not include the power to fix rates which will disallow or discourage resales for industrial use. Natural Gas Act, §§ 4(a), 5(a), 15 U.S.C.A. §§ 717c(a), 717d(a).

73 Cases that cite this headnote

[22] **Gas**

Reasonableness of Charges

The wasting-asset nature of the natural gas industry does not require the maintenance of the level of rates so that natural gas companies can make a greater profit on each unit of gas sold. Natural Gas Act, §§ 4(a), 5(a), 15 U.S.C.A. §§ 717c(a), 717d(a).

1 Cases that cite this headnote

[23] **Federal Courts**

Presentation of Questions Below or on Review; Record; Waiver

Federal Courts

Scope and Extent of Review

Where the Federal Power Commission made no findings as to any discrimination or unreasonable differences in rates, and its failure was not challenged in the petition to review, and had not been raised or argued by any party, the problem of discrimination was not open to review by the Supreme Court on certiorari. Natural Gas Act, § 4(b), 15 U.S.C.A. § 717c(b).

18 Cases that cite this headnote

[24] **Constitutional Law**

Judicial encroachment on executive acts taken under statutory authority

Gas

Power to control and regulate

Congress has entrusted the administration of the Natural Gas Act to the Federal Power Commission and not to the courts, and apart from the requirements of judicial review, it is not for the Supreme Court to advise the Commission how to discharge its functions. Natural Gas Act, §§ 1 et seq., 19(b), 15 U.S.C.A. §§ 717 et seq., 717r(b).

13 Cases that cite this headnote

[25] **Gas**

Decisions reviewable

Under the Natural Gas Act, where order sought to be reviewed does not of itself adversely affect complainant but only affects his rights adversely on the contingency of future administrative action, the order is not reviewable, and resort to the courts in such situation is either premature or wholly beyond the province of such courts. Natural Gas Act, § 19(b), 15 U.S.C.A. § 717r(b).

8 Cases that cite this headnote

[26] **Gas**

Persons entitled to relief; parties

Findings of the Federal Power Commission on lawfulness of past natural gas rates, which the Commission was without power to enforce, were not reviewable under the Natural Gas Act giving any "party aggrieved" by an order of the Commission the right of review. Natural Gas Act, § 19(b), 15 U.S.C.A. § 717r(b).

27 Cases that cite this headnote

Attorneys and Law Firms

****283 *592** Mr. Francis M. Shea, Asst. Atty. Gen., for petitioners Federal Power Com'n and others.

***593** Mr. Spencer W. Reeder, of Cleveland, Ohio, for petitioner City of cleveland.

Mr. William B. Cockley, of Cleveland, Ohio, for respondent.

Mr. M. M. Neeley, of Charleston, W. Va., for State of West Virginia, as amicus curiae by special leave of Court.

Opinion

Mr. Justice DOUGLAS delivered the opinion of the Court.

The primary issue in these cases concerns the validity under the Natural Gas Act of 1938, 52 Stat. 821, 15 U.S.C. s 717 et seq., 15 U.S.C.A. s 717 et seq., of a rate order issued by the Federal Power Commission reducing the rates chargeable by Hope Natural Gas Co., 44 P.U.R.,N.S., 1. On a petition for review of the order made pursuant to s 19(b) of the Act, the ***594** Circuit Court of Appeals set it aside, one judge dissenting. 4 Cir., 134 F.2d 287. The cases ****284** are here on petitions for writs of certiorari which we granted because of the public importance of the questions presented. City of Cleveland v. Hope Natural Gas Co., 319 U.S. 735, 63 S.Ct. 1165.

Hope is a West Virginia corporation organized in 1898. It is a wholly owned subsidiary of Standard Oil Co. (N.J.). Since the date of its organization, it has been in the business of producing, purchasing and marketing natural gas in that state.[1] It sells some of that gas to local consumers in West Virginia. But the great bulk of it goes to five customer companies which receive it at the West Virginia line and distribute it in Ohio and in Pennsylvania.[2] In July, 1938, the cities of Cleveland and Akron filed complaints with the Commission charging that the rates collected by Hope from East Ohio Gas Co. (an affiliate of Hope which distributes gas in Ohio) were excessive and unreasonable. Later in 1938 the Commission on its own motion instituted an investigation to determine the reasonableness of all of Hope's interstate rates. In March ***595** 1939 the Public Utility Commission of Pennsylvania filed a complaint with the Commission charging that the rates collected by Hope from Peoples Natural Gas Co. (an affiliate of Hope distributing gas in Pennsylvania) and two non-affiliated companies were unreasonable. The City of Cleveland asked that the challenged rates be declared unlawful and that just and reasonable rates be determined from June 30, 1939 to the date of the Commission's order. The latter finding was requested in aid of state regulation and to afford the Public Utilities Commission of Ohio a proper basic for disposition of a fund collected by East Ohio under bond from Ohio consumers since June 30, 1939. The cases were consolidated and hearings were held.

On May 26, 1942, the Commission entered its order and made its findings. Its order required Hope to decrease its future interstate rates so as to reflect a reduction, on an annual basis of not less than $3,609,857 in operating revenues. And it established 'just and reasonable' average rates per m.c.f. for each of the five customer companies.[3] In response to the prayer of the City of Cleveland the Commission also made findings as to the lawfulness of past rates, although concededly it had no authority under the Act to fix past rates or to award reparations. 44 P.U.R.,U.S., at page 34. It found that the rates collected by Hope from East Ohio were unjust, unreasonable, excessive and therefore unlawful, by $830,892 during 1939, $3,219,551 during 1940, and $2,815,789 on an annual basis since 1940. It further found that just, reasonable, and lawful rates for gas sold by Hope to East Ohio for resale for ultimate public consumption were those required ***596** to produce $11,528,608 for 1939, $11,507,185 for 1940 and $11.910,947 annually since 1940.

The Commission established an interstate rate base of $33,712,526 which, it found, represented the 'actual legitimate cost' of the company's interstate property less depletion and depreciation and plus unoperated acreage, working capital and future net capital additions. The Commission, beginning with book cost, made ****285** certain adjustments not necessary to relate here and found the 'actual legitimate cost' of the plant in interstate service to be $51,957,416, as of December 31, 1940. It deducted accrued depletion and depreciation, which it found to be $22,328,016 on an 'economic-service-life' basis. And it added $1,392,021 for future net capital additions, $566,105 for useful unoperated acreage, and $2,125,000 for working capital. It used 1940 as a test year to estimate future revenues and expenses. It allowed over $16,000,000 as annual operating expenses—about $1,300,000 for taxes, $1,460,000 for depletion and depreciation, $600,000 for exploration and development costs, $8,500,000 for gas purchased. The Commission allowed a net increase of $421,160 over 1940 operating expenses, which amount was to take care of future increase in wages, in West Virginia property taxes, and in exploration and development costs. The total amount of deductions allowed from interstate revenues was $13,495,584.

Hope introduced evidence from which it estimated reproduction cost of the property at $97,000,000. It also presented a so-called trended 'original cost' estimate which exceeded $105,000,000. The latter was designed 'to indicate what the original cost of the property would have been if

1938 material and labor prices had prevailed throughout the whole period of the piece-meal construction of the company's property since 1898.' 44 P.U.R.,N.S., at pages 8, 9. Hope estimated by the 'percent condition' method accrued depreciation at about 35% of ***597** reproduction cost new. On that basis Hope contended for a rate base of $66,000,000. The Commission refused to place any reliance on reproduction cost new, saying that it was 'not predicated upon facts' and was 'too conjectural and illusory to be given any weight in these proceedings.' Id., 44 P.U.R.,U.S., at page 8. It likewise refused to give any 'probative value' to trended 'original cost' since it was 'not founded in fact' but was 'basically erroneous' and produced 'irrational results.' Id., 44 P.U.R., N.S., at page 9. In determining the amount of accrued depletion and depreciation the Commission, following Lindheimer v. Illinois Bell Telephone Co., 292 U.S. 151, 167-169, 54 S.Ct. 658, 664—666, 78 L.Ed. 1182; Federal Power Commission v. Natural Gas Pipeline Co., 315 U.S. 575, 592, 593, 62 S.Ct. 736, 745, 746, 86 L.Ed. 1037, based its computation on 'actual legitimate cost'. It found that Hope during the years when its business was not under regulation did not observe 'sound depreciation and depletion practices' but 'actually accumulated an excessive reserve' [4] of about $46,000,000. Id., 44 P.U.R.,N.S., at page 18. One member of the Commission thought that the entire amount of the reserve should be deducted from 'actual legitimate cost' in determining the rate base. [5] The majority of the ***598** Commission concluded, however, that where, as here, a business is brought under regulation for the first time and where incorrect depreciation and depletion practices have prevailed, the deduction of the reserve requirement (actual existing depreciation and depletion) rather than the excessive reserve should be made so as to ****286** lay 'a sound basis for future regulation and control of rates.' Id., 44 P.U.R.,N.S., at page 18. As we have pointed out, it determined accrued depletion and depreciation to be $22,328,016; and it allowed approximately $1,460,000 as the annual operating expense for depletion and depreciation. [6]

Hope's estimate of original cost was about $69,735,000—approximately $17,000,000 more than the amount found by the Commission. The item of $17,000,000 was made up largely of expenditures which prior to December 31, 1938, were charged to operating expenses. Chief among those expenditures was some $12,600,000 expended ***599** in well-drilling prior to 1923. Most of that sum was expended by Hope for labor, use of drilling-rigs, hauling, and similar costs of well-drilling. Prior to 1923 Hope followed the general practice of the natural gas industry and charged the cost of drilling wells to operating expenses. Hope continued that practice until the Public Service Commission of West Virginia in 1923 required it to capitalize such expenditures, as does the Commission under its present Uniform System of Accounts. [7] The Commission refused to add such items to the rate base stating that 'No greater injustice to consumers could be done than to allow items as operating expenses and at a later date include them in the rate base, thereby placing multiple charges upon the consumers.' Id., 44 P.U.R.,N.S., at page 12. For the same reason the Commission excluded from the rate base about $1,600,000 of expenditures on properties which Hope acquired from other utilities, the latter having charged those payments to operating expenses. The Commission disallowed certain other overhead items amounting to over $3,000,000 which also had been previously charged to operating expenses. And it refused to add some $632,000 as interest during construction since no interest was in fact paid.

Hope contended that it should be allowed a return of not less than 8%. The Commission found that an 8% return would be unreasonable but that 6 1/2% was a fair rate of return. That rate of return, applied to the rate base of $33,712,526, would produce $2,191,314 annually, as compared with the present income of not less than $5,801,171.

The Circuit Court of Appeals set aside the order of the Commission for the following reasons. (1) It held that the rate base should reflect the 'present fair value' of the ***600** property, that the Commission in determining the 'value' should have considered reproduction cost and trended original cost, and that 'actual legitimate cost' (prudent investment) was not the proper measure of 'fair value' where price levels had changed since the investment. (2) It concluded that the well-drilling costs and overhead items in the amount of some $17,000,000 should have been included in the rate base. (3) It held that accrued depletion and depreciation and the annual allowance for that expense should be computed on the basis of 'present fair value' of the property not on the basis of 'actual legitimate cost'.

****287** The Circuit Court of Appeals also held that the Commission had no power to make findings as to past rates in aid of state regulation. But it concluded that those findings were proper as a step in the process of fixing future rates. Viewed in that light, however, the findings were deemed to be invalidated by the same errors which vitiated the findings on which the rate order was based.

Order Reducing Rates. Congress has provided in s 4(a) of the Natural Gas Act that all natural gas rates subject to the jurisdiction of the Commission 'shall be just and reasonable, and any such rate or charge that is not just and reasonable is hereby declared to be unlawful.' Sec. 5(a) gives the Commission the power, after hearing, to determine the 'just and reasonable rate' to be thereafter observed and to fix the rate by order. Sec. 5(a) also empowers the Commission to order a 'decrease where existing rates are unjust * * * unlawful, or are not the lowest reasonable rates.' And Congress has provided in s 19(b) that on review of these rate orders the 'finding of the Commission as to the facts, if supported by substantial evidence, shall be conclusive.' Congress, however, has provided no formula by which the 'just and reasonable' rate is to be determined. It has not filled in the ***601** details of the general prescription [8] of s 4(a) and s 5(a). It has not expressed in a specific rule the fixed principle of 'just and reasonable'.

[1] **[2]** When we sustained the constitutionality of the Natural Gas Act in the Natural Gas Pipeline Co. case, we stated that the 'authority of Congress to regulate the prices of commodities in interstate commerce is at least as great under the Fifth Amendment as is that of the states under the Fourteenth to regulate the prices of commodities in intrastate commerce.' 315 U.S. at page 582, 62 S.Ct. at page 741, 86 L.Ed. 1037. Rate-making is indeed but one species of price-fixing. Munn v. Illinois, 94 U.S. 113, 134, 24 L.Ed. 77. The fixing of prices, like other applications of the police power, may reduce the value of the property which is being regulated. But the fact that the value is reduced does not mean that the regulation is invalid. Block v. Hirsh, 256 U.S. 135, 155—157, 41 S.Ct. 458, 459, 460, 65 L.Ed. 865, 16 A.L.R. 165; Nebbia v. New York, 291 U.S. 502, 523—539, 54 S.Ct. 505, 509—517, 78 L.Ed. 940, 89 A.L.R. 1469, and cases cited. It does, however, indicate that 'fair value' is the end product of the process of rate-making not the starting point as the Circuit Court of Appeals held. The heart of the matter is that rates cannot be made to depend upon 'fair value' when the value of the going enterprise depends on earnings under whatever rates may be anticipated. [9]

***602** **[3]** **[4]** **[5]** **[6]** **[7]** We held in Federal Power Commission v. Natural Gas Pipeline Co., supra, that the Commission was not bound to the use of any single formula or combination of formulae in determining rates. Its rate-making function, moreover, involves the making of 'pragmatic adjustments.' Id., 315 U.S. at page 586, 62 S.Ct. at page 743, 86 L.Ed. 1037. And when the Commission's order is challenged in the courts, the question is whether that order 'viewed in its entirety' meets the requirements of the Act. Id., 315 U.S. at page 586, 62 S.Ct. at page 743, 86 L.Ed. 1037. Under the statutory standard of 'just and reasonable' it is the result reached not the method employed which is controlling. Cf. ****288** Los Angeles Gas & Electric Corp. v. Railroad Commission, 289 U.S. 287, 304, 305, 314, 53 S.Ct. 637, 643, 644, 647, 77 L.Ed. 1180; West Ohio Gas Co. v. Public Utilities Commission (No. 1), 294 U.S. 63, 70, 55 S.Ct. 316, 320, 79 L.Ed. 761; West v. Chesapeake & Potomac Tel. Co., 295 U.S. 662, 692, 693, 55 S.Ct. 894, 906, 907, 79 L.Ed. 1640 (dissenting opinion). It is not theory but the impact of the rate order which counts. If the total effect of the rate order cannot be said to be unjust and unreasonable, judicial inquiry under the Act is at an end. The fact that the method employed to reach that result may contain infirmities is not then important. Moreover, the Commission's order does not become suspect by reason of the fact that it is challenged. It is the product of expert judgment which carries a presumption of validity. And he who would upset the rate order under the Act carries the heavy burden of making a convincing showing that it is invalid because it is unjust and unreasonable in its consequences. Cf. Railroad Commission v. Cumberland Tel. & T. Co., 212 U.S. 414, 29 S.Ct. 357, 53 L.Ed. 577; Lindheimer v. Illinois Bell Tel. Co., supra, 292 U.S. at pages 164, 169, 54 S.Ct. at pages 663, 665, 78 L.Ed. 1182; Railroad Commission v. Pacific Gas & E. Co., 302 U.S. 388, 401, 58 S.Ct. 334, 341, 82 L.Ed. 319.

***603** **[8]** **[9]** The rate-making process under the Act, i.e., the fixing of 'just and reasonable' rates, involves a balancing of the investor and the consumer interests. Thus we stated in the Natural Gas Pipeline Co. case that 'regulation does not insure that the business shall produce net revenues.' 315 U.S. at page 590, 62 S.Ct. at page 745, 86 L.Ed. 1037. But such considerations aside, the investor interest has a legitimate concern with the financial integrity of the company whose rates are being regulated. From the investor or company point of view it is important that there be enough revenue not only for operating expenses but also for the capital costs of the business. These include service on the debt and dividends on the stock. Cf. Chicago & Grand Trunk R. Co. v. Wellman, 143 U.S. 339, 345, 346, 12 S.Ct. 400, 402, 36 L.Ed. 176. By that standard the return to the equity owner should be commensurate with returns on investments in other enterprises having corresponding risks. That return, moreover, should be sufficient to assure confidence in the financial integrity of the enterprise, so as to maintain its credit and to attract capital. See State of Missouri ex rel. Southwestern Bell Tel. Co. v. Public Service Commission, 262 U.S. 276, 291, 43 S.Ct. 544, 547, 67 L.Ed. 981, 31 A.L.R. 807 (Mr. Justice Brandeis concurring). The conditions under which more or less might be allowed are not important here. Nor is it important to this case to determine the various permissible ways in which any rate base on which the return is computed might be arrived at. For we are of the view that the end result in this case cannot be condemned under the Act as unjust and unreasonable from the investor or company viewpoint.

We have already noted that Hope is a wholly owned subsidiary of the Standard Oil Co. (N.J.). It has no securities outstanding except stock. All of that stock has been owned by Standard since 1908. The par amount presently outstanding is approximately $28,000,000 as compared with the rate base of $33,712,526 established by ***604** the Commission. Of the total outstanding stock $11,000,000 was issued in stock dividends. The balance, or about $17,000,000, was issued for cash or other assets. During the four decades of its operations Hope has paid over $97,000,000 in cash dividends. It had, moreover, accumulated by 1940 an earned surplus of about $8,000,000. It had thus earned the total investment in the company nearly seven times. Down to 1940 it earned over 20% per year on the average annual amount of its capital stock issued for cash or other assets. On an average invested capital of some $23,000,000 Hope's average earnings have been about 12% a year. And during this period it had accumulated in addition reserves for depletion and depreciation of about $46,000,000. Furthermore, during 1939, 1940 and 1941, Hope paid dividends of 10% on its stock. And in the year 1942, during about half of which the lower rates were in effect, it paid dividends of 7 1/2%. From 1939-1942 its earned surplus increased from $5,250,000 to about $13,700,000, i.e., to almost half the par value of its outstanding stock.

As we have noted, the Commission fixed a rate of return which permits Hope to earn $2,191,314 annually. In determining that amount it stressed the importance of maintaining the financial integrity of the ****289** company. It considered the financial history of Hope and a vast array of data bearing on the natural gas industry, related businesses, and general economic conditions. It noted that the yields on better issues of bonds of natural gas companies sold in the last few years were ‘close to 3 per cent’, 44 P.U.R.,N.S., at page 33. It stated that the company was a ‘seasoned enterprise whose risks have been minimized’ by adequate provisions for depletion and depreciation (past and present) with ‘concurrent high profits', by ‘protected established markets, through affiliated distribution companies, in populous and industralized areas', and by a supply of gas locally to meet all requirements, ***605** ‘except on certain peak days in the winter, which it is feasible to supplement in the future with gas from other sources.’ Id., 44 P.U.R.,N.S., at page 33. The Commission concluded, ‘The company's efficient management, established markets, financial record, affiliations, and its prospective business place it in a strong position to attract capital upon favorable terms when it is required.’ Id., 44 P.U.R.,N.S., at page 33.

[10] **[11]** **[12]** In view of these various considerations we cannot say that an annual return of $2,191,314 is not ‘just and reasonable’ within the meaning of the Act. Rates which enable the company to operate successfully, to maintain its financial integrity, to attract capital, and to compensate its investors for the risks assumed certainly cannot be condemned as invalid, even though they might produce only a meager return on the so-called ‘fair value’ rate base. In that connection it will be recalled that Hope contended for a rate base of $66,000,000 computed on reproduction cost new. The Commission points out that if that rate base were accepted, Hope's average rate of return for the four-year period from 1937-1940 would amount to 3.27%. During that period Hope earned an annual average return of about 9% on the average investment. It asked for no rate increases. Its properties were well maintained and operated. As the Commission says such a modest rate of 3.27% suggests an ‘inflation of the base on which the rate has been computed.’ Dayton Power & Light Co. v. Public Utilities Commission, 292 U.S. 290, 312, 54 S.Ct. 647, 657, 78 L.Ed. 1267. Cf. Lindheimer v. Illinois Bell Tel. Co., supra, 292 U.S. at page 164, 54 S.Ct. at page 663, 78 L.Ed. 1182. The incongruity between the actual operations and the return computed on the basis of reproduction cost suggests that the Commission was wholly justified in rejecting the latter as the measure of the rate base.

In view of this disposition of the controversy we need not stop to inquire whether the failure of the Commission to add the $17,000,000 of well-drilling and other costs to ***606** the rate base was consistent with the prudent investment theory as developed and applied in particular cases.

[13] **[14]** **[15]** Only a word need be added respecting depletion and depreciation. We held in the Natural Gas Pipeline Co. case that there was no constitutional requirement ‘that the owner who embarks in a wasting-asset business of limited life shall receive at the end more than he has put into it.’ 315 U.S. at page 593, 62 S.C. at page 746, 86 L.Ed. 1037. The Circuit Court of Appeals did not think that that rule was applicable here because Hope was a utility required to continue its service to the public and not scheduled to end its business on a day certain as was stipulated to be true of the Natural Gas Pipeline Co. But that distinction is quite immaterial. The ultimate exhaustion of the supply is inevitable in the case of all natural gas companies. Moreover, this Court recognized in Lindheimer v. Illinois Bell Tel. Co., supra, the propriety of basing annual depreciation on cost. [10] By such a procedure the ****290** utility is made whole and the integrity of its investment maintained. [11] No more is required. [12] We cannot approve the contrary holding ***607** of United Railways & Electric Co. v. West, 280 U.S. 234, 253, 254, 50 S.Ct. 123, 126, 127, 74 L.Ed. 390. Since there are no constitutional requirements more exacting than the standards of the Act, a rate order which conforms to the latter does not run afoul of the former.

The Position of West Virginia. The State of West Virginia, as well as its Public Service Commission, intervened in the proceedings before the Commission and participated in the hearings before it. They have also filed a brief amicus curiae here and have participated in the argument at the bar. Their contention is that the result achieved by the rate order 'brings consequences which are unjust to West Virginia and its citizens' and which 'unfairly depress the value of gas, gas lands and gas leaseholds, unduly restrict development of their natural resources, and arbitrarily transfer their properties to the residents of other states without just compensation therefor.'

West Virginia points out that the Hope Natural Gas Co. holds a large number of leases on both producing and unoperated properties. The owner or grantor receives from the operator or grantee delay rentals as compensation for postponed drilling. When a producing well is successfully brought in, the gas lease customarily continues indefinitely for the life of the field. In that case the operator pays a stipulated gas-well rental or in some cases a gas royalty equivalent to one-eighth of the gas marketed. [13] Both the owner and operator have valuable property interests in the gas which are separately taxable under West Virginia law. The contention is that the reversionary interests in the leaseholds should be represented in the rate proceedings since it is their gas which is being sold in interstate ***608** commerce. It is argued, moreover, that the owners of the reversionary interests should have the benefit of the 'discovery value' of the gas leaseholds, not the interstate consumers. Furthermore, West Virginia contends that the Commission in fixing a rate for natural gas produced in that State should consider the effect of the rate order on the economy of West Virginia. It is pointed out that gas is a wasting asset with a rapidly diminishing supply. As a result West Virginia's gas deposits are becoming increasingly valuable. Nevertheless the rate fixed by the Commission reduces that value. And that reduction, it is said, has severe repercussions on the economy of the State. It is argued in the first place that as a result of this rate reduction Hope's West Virginia property taxes may be decreased in view of the relevance which earnings have under West Virginia law in the assessment of property for tax purposes. [14] Secondly, it is pointed out that West Virginia has a production tax [15] on the 'value' of the gas exported from the State. And we are told that for purposes of that tax 'value' becomes under West Virginia law 'practically the substantial equivalent of market value.' Thus West Virginia argues that undervaluation of Hope's gas leaseholds will cost the State many thousands of dollars in taxes. The effect, it is urged, is to impair West Virginia's tax structure for the benefit of Ohio and Pennsylvania consumers. West Virginia emphasizes, moreover, its deep interest in the conservation of its natural resources including its natural gas. It says that a reduction of the value of these leasehold values will jeopardize these conservation policies in three respects: (1) ****291** exploratory development of new fields will be discouraged; (2) abandonment of lowyield high-cost marginal wells will be hastened; and (3) secondary recovery of oil will be hampered. ***609** Furthermore, West Virginia contends that the reduced valuation will harm one of the great industries of the State and that harm to that industry must inevitably affect the welfare of the citizens of the State. It is also pointed out that West Virginia has a large interest in coal and oil as well as in gas and that these forms of fuel are competitive. When the price of gas is materially cheapened, consumers turn to that fuel in preference to the others. As a result this lowering of the price of natural gas will have the effect of depreciating the price of West Virginia coal and oil.

West Virginia insists that in neglecting this aspect of the problem the Commission failed to perform the function which Congress entrusted to it and that the case should be remanded to the Commission for a modification of its order. [16]

We have considered these contentions at length in view of the earnestness with which they have been urged upon us. We have searched the legislative history of the Natural Gas Act for any indication that Congress entrusted to the Commission the various considerations which West Virginia has advanced here. And our conclusion is that Congress did not.

[16] **[17]** We pointed out in Illinois Natural Gas Co. v. Central Illinois Public Service Co., 314 U.S. 498, 506, 62

S.Ct. 384, 387, 86 L.Ed. 371, that the purpose of the Natural Gas Act was to provide, 'through the exercise of the national power over interstate commerce, an agency for regulating the wholesale distribution to public service companies of natural gas moving interstate, which this Court had declared to be interstate commerce not subject to certain types of state regulation.' As stated in the House Report the 'basic purpose' of this legislation was 'to occupy' the field in which such cases as ***610** State of Missouri v. Kansas Natural Gas Co., 265 U.S. 298, 44 S.Ct. 544, 68 L.Ed. 1027, and Public Utilities Commission v. Attleboro Steam & Electric Co., 273 U.S. 83, 47 S.Ct. 294, 71 L.Ed. 549, had held the States might not act. H.Rep. No. 709, 75th Cong., 1st Sess., p. 2. In accomplishing that purpose the bill was designed to take 'no authority from State commissions' and was 'so drawn as to complement and in no manner usurp State regulatory authority.' Id., p. 2. And the Federal Power Commission was given no authority over the 'production or gathering of natural gas.' s 1(b).

[18] The primary aim of this legislation was to protect consumers against exploitation at the lands of natural gas companies. Due to the hiatus in regulation which resulted from the Kansas Natural Gas Co. case and related decisions state commissions found it difficult or impossible to discover what it cost interstate pipe-line companies to deliver gas within the consuming states; and thus they were thwarted in local regulation. H.Rep., No. 709, supra, p. 3. Moreover, the investigations of the Federal Trade Commission had disclosed that the majority of the pipe-line mileage in the country used to transport natural gas, together with an increasing percentage of the natural gas supply for pipe-line transportation, had been acquired by a handful of holding companies.[17] State commissions, independent producers, and communities having or seeking the service were growing quite helpless against these combinations.[18] These were the types of problems with which those participating in the hearings were pre-occupied.[19] Congress addressed itself to those specific evils.

***611** The Federal Power Commission was given ****292** broad powers of regulation. The fixing of 'just and reasonable' rates (s 4) with the powers attendant thereto[20] was the heart of the new regulatory system. Moreover, the Commission was given certain authority by s 7(a), on a finding that the action was necessary or desirable 'in the public interest,' to require natural gas companies to extend or improve their transportation facilities and to sell gas to any authorized local distributor. By s 7(b) it was given control over the abandonment of facilities or of service. And by s 7(c), as originally enacted, no natural gas company could undertake the construction or extension of any facilities for the transportation of natural gas to a market in which natural gas was already being served by another company, or sell any natural gas in such a market, without obtaining a certificate of public convenience and necessity from the Commission. In passing on such applications for certificates of convenience and necessity the Commission was told by s 7(c), as originally enacted, that it was 'the intention of Congress that natural gas shall be sold in interstate commerce for resale for ultimate public consumption for domestic, commercial, industrial, or any other use at the lowest possible reasonable rate consistent with the maintenance of adequate service in the public interest.' The latter provision was deleted from s 7(c) when that subsection was amended by the Act of February 7, 1942, 56 Stat. 83. By that amendment limited grandfather rights were granted companies desiring to extend their facilities and services over the routes or within the area which they were already serving. Moreover, s 7(c) was broadened so as to require certificates ***612** of public convenience and necessity not only where the extensions were being made to markets in which natural gas was already being sold by another company but in other situations as well.

[19] These provisions were plainly designed to protect the consumer interests against exploitation at the hands of private natural gas companies. When it comes to cases of abandonment or of extensions of facilities or service, we may assume that, apart from the express exemptions[21] contained in s 7, considerations of conservation are material to the issuance of certificates of public convenience and necessity. But the Commission was not asked here for a certificate of public convenience and necessity under s 7 for any proposed construction or extension. It was faced with a determination of the amount which a private operator should be allowed to earn from the sale of natural gas across state lines through an established distribution system. Secs. 4 and 5, not s 7, provide the standards for that determination. We cannot find in the words of the Act or in its history the slightest intimation or suggestion that the exploitation of consumers by private operators through the maintenance of high rates should be allowed to continue provided the producing states obtain indirect benefits from it. That apparently was the Commission's view of the matter, for the same arguments advanced here were presented to the Commission and not adopted by it.

We do not mean to suggest that Congress was unmindful of the interests of the producing states in their natural gas supplies when it drafted the Natural Gas Act. As we have said, the Act does not intrude on the domain traditionally reserved for control by state commissions; and the Federal Power Commission was given no authority over ***613** 'the production or gathering of natural gas.' s 1(b). In addition, Congress recognized the legitimate interests of the States in the conservation of natural gas. By s 11 Congress instructed the Commission to make reports on compacts between two or more States dealing with the conservation, production and transportation of natural gas.[22] The Commission was also ****293** directed to recommend further legislation appropriate or necessary to carry out any proposed compact and 'to aid in the conservation of natural-gas resources within the United States and in the orderly, equitable, and economic production, transportation, and distribution of natural gas.' s 11(a). Thus Congress was quite aware of the interests of the producing states in their natural gas supplies.[23] But it left the protection of ***614** those interests to measures other than the maintenance of high rates to private companies. If the Commission is to be compelled to let the stockholders of natural gas companies have a feast so that the producing states may receive crumbs from that table, the present Act must be redesigned. Such a project raises questions of policy which go beyond our province.

[20] It is hardly necessary to add that a limitation on the net earnings of a natural gas company from its interstate business is not a limitation on the power of the producing state either to safeguard its tax revenues from that industry[24] or to protect the interests of those who sell their gas to the interstate operator.[25] The return which ****294** the Commission ***615** allowed was the net return after all such charges.

It is suggested that the Commission has failed to perform its duty under the Act in that it has not allowed a return for gas production that will be enough to induce private enterprise to perform completely and efficiently its functions for the public. The Commission, however, was not oblivious of those matters. It considered them. It allowed, for example, delay rentals and exploration and development costs in operating expenses.[26] No serious attempt has been made here to show that they are inadequate. We certainly cannot say that they are, unless we are to substitute our opinions for the expert judgment of the administrators to whom Congress entrusted the decision. Moreover, if in light of experience they turn out to be inadequate for development of new sources of supply, the doors of the Commission are open for increased allowances. This is not an order for all time. The Act contains machinery for obtaining rate adjustments. s 4.

[21] **[22]** But it is said that the Commission placed too low a rate on gas for industrial purposes as compared with gas for domestic purposes and that industrial uses should be discouraged. It should be noted in the first place that the rates which the Commission has fixed are Hope's interstate wholesale rates to distributors not interstate rates to industrial users[27] and domestic consumers. We hardly ***616** can assume, in view of the history of the Act and its provisions, that the resales intrastate by the customer companies which distribute the gas to ultimate consumers in Ohio and Pennsylvania are subject to the rate-making powers of the Commission.[28] But in any event those rates are not in issue here. Moreover, we fail to find in the power to fix 'just and reasonable' rates the power to fix rates which will disallow or discourage resales for industrial use. The Committee Report stated that the Act provided 'for regulation along recognized and more or less standardized lines' and that there was 'nothing novel in its provisions'. H.Rep.No.709, supra, p. 3. Yet if we are now to tell the Commission to fix the rates so as to discourage particular uses, we would indeed be injecting into a rate case a 'novel' doctrine which has no express statutory sanction. The same would be true if we were to hold that the wasting-asset nature of the industry required the maintenance of the level of rates so that natural gas companies could make a greater profit on each unit of gas sold. Such theories of rate-making for this industry may or may not be desirable. The difficulty is that s 4(a) and s 5(a) contain only the conventional standards of rate-making for natural gas companies.[29] The ***617** Act of February 7, 1942, by broadening s 7 gave the Commission some additional authority to deal with the conservation aspects of the problem.[30] But s 4(a) and s 5(a) were not changed. If the standard ****295** of 'just and reasonable' is to sanction the maintenance of high rates by a natural gas company because they restrict the use of natural gas for certain purposes, the Act must be further amended.

[23] **[24]** It is finally suggested that the rates charged by Hope are discriminatory as against domestic users and in favor of industrial users. That charge is apparently based on s 4(b) of the Act which forbids natural gas companies from maintaining 'any unreasonable difference in rates, charges, service, facilities, or in any other respect, either as between localities or as between classes of service.' The power of the Commission to eliminate any such unreasonable differences or discriminations is plain. s 5(a). The Commission, however, made no findings under s 4(b). Its failure in that regard was not challenged in the petition to review. And it has not been raised or argued here by any party. Hence the problem of discrimination has no proper place in the present decision. It will be time enough to pass on that issue when it is presented to us. Congress has entrusted the administration of the Act to the Commission not to the courts. Apart from the requirements of judicial review it is not ***618** for us to advise the Commission how to discharge its functions.

Findings as to the Lawfulness of Past Rates. As we have noted, the Commission made certain findings as to the lawfulness of past rates which Hope had charged its interstate customers. Those findings were made on the complaint of the City of Cleveland and in aid of state regulation. It is conceded that under the Act the Commission has no power to make reparation orders. And its power to fix rates admittedly is limited to those 'to be thereafter observed and in force.' s 5(a). But the Commission maintains that it has the power to make findings as to the lawfulness of past rates even though it has no power to fix those rates. [31] However that may be, we do not think that these findings were reviewable under s 19(b) of the Act. That section gives any party 'aggrieved by an order' of the Commission a review 'of such order' in the circuit court of appeals for the circuit where the natural gas company is located or has its principal place of business or in the United States Court of Appeals for the District of Columbia. We do not think that the findings in question fall within that category. **[25]** **[26]** The Court recently summarized the various types of administrative action or determination reviewable as orders under the Urgent Deficiencies Act of October 22, ***619** 1913, 28 U.S.C. ss 45, 47a, 28 U.S.C.A. ss 45, 47a, and kindred statutory provisions. Rochester Tel. Corp. v. United States, 307 U.S. 125, 59 S.Ct. 754, 83 L.Ed. 1147. It was there pointed out that where 'the order sought to be reviewed does not of itself adversely affect complainant but only affects his rights adversely on the contingency of future administrative action', it is not reviewable. Id., 307 U.S. at page 130, 59 S.Ct. at page 757, 83 L.Ed. 1147. The Court said, 'In view of traditional conceptions of federal judicial power, resort to the courts in these situations is either premature or wholly beyond their province.' ****296** Id., 307 U.S. at page 130, 59 S.Ct. at page 757, 83 L.Ed. 1147. And see United States v. Los Angeles S.L.R. C/O., 273 U.S. 299, 309, 310, 47 S.CT. 413, 414, 415, 71 L.ED. 651; SHANNAHAN V. UNITED STATES, 303 U.S. 596, 58 S.CT. 732, 82 L.ED. 1039. THESE CONSIDERATIONS ARE APPOSITE HERE. THE COMMISSION HAS NO AUTHORITY TO ENFORCE THESE FINDINGS. THEY ARE 'THE EXERCISE SOLELY OF THE FUNCTION OF INVESTIGATION.' UNITED STATES V. LOS ANGELES & S.L.R. CO., SUPRA, 273 U.S. AT PAGE 310, 47 S.CT. AT PAGE 414, 71 L.ED. 651. THEY ARE ONLY A PRELIMINARY, INTERIM STEP TOWARDS POSSIBLE FUTURE ACTION—ACTION NOT BY THE COMMISSION BUT BY WHOLLY INDEPENDENT AGENCIES. THE OUTCOME OF THOSE PROCEEDINGS MAY TURN ON FACTORS OTHER THAN THESE FINDINGS. THESE FINDINGS MAY NEVER RESULT IN THE RESPONDENT FEELING THE PINCH OF ADMINISTRATIVE ACTION.

Reversed.

Mr. Justice ROBERTS took no part in the consideration or decision of this case.

Opinion of Mr. Justice BLACK and Mr. Justice MURPHY.

We agree with the Court's opinion and would add nothing to what has been said but for what is patently a wholly gratuitous assertion as to Constitutional law in the dissent of Mr. Justice FRANKFURTER. We refer to the statement that 'Congressional acquiescence to date in the doctrine of Chicago, etc., R. Co. v. Minnesota, supra (134 U.S. 418, 10 S.Ct. 462, 702, 33 L.Ed. 970), may fairly be claimed.' That was the case in which a majority of this Court was finally induced to expand the meaning ***620** of 'due process' so as to give courts power to block efforts of the state and national governments to regulate economic affairs. The present case does not afford a proper occasion to discuss the soundness of that doctrine because, as stated in Mr. Justice FRANKFURTER'S dissent, 'That issue is not here in controversy.' The salutary practice whereby courts do not discuss issues in the abstract applies with peculiar force to Constitutional questions. Since, however, the dissent adverts to a highly controversial due process doctrine and implies its acceptance by Congress, we feel compelled to say that we do not understand that Congress voluntarily has acquiesced in a Constitutional principle of government that courts, rather than legislative bodies, possess final authority over regulation of economic affairs. Even this Court has not always fully embraced that principle, and we wish to repeat that we have never acquiesced in it, and do not now. See Federal Power Commission v. Natural Gas Pipeline Co., 315 U.S. 575, 599-601, 62 S.Ct. 736, 749, 750, 86 L.Ed. 1037.

Mr. Justice REED, dissenting.

This case involves the problem of rate making under the Natural Gas Act. Added importance arises from the obvious fact that the principles stated are generally applicable to all federal agencies which are entrusted with the determination of rates for utilities. Because my views differ somewhat from those of my brethren, it may be of some value to set them out in a summary form.

The Congress may fix utility rates in situations subject to federal control without regard to any standard except the constitutional standards of due process and for taking private property for public use without just compensation. Wilson v. New, 243 U.S. 332, 350, 37 S.Ct. 298, 302, 61 L.Ed. 755, L.R.A.1917E, 938, Ann.Cas.1918A, 1024. A Commission, however, does not have this freedom of action. Its powers are limited not only by the constitutional standards but also by the standards of the delegation. Here the standard added by the Natural Gas Act is that the rate be 'just ***621** and reasonable.'[1] Section 6[2] ****297** throws additional light on the meaning of these words.

When the phrase was used by Congress to describe allowable rates, it had relation to something ascertainable. The rates were not left to the whim of the Commission. The rates fixed would produce an annual return and that annual return was to be compared with a theoretical just and reasonable return, all risks considered, on the fair value of the property used and useful in the public service at the time of the determination.

Such an abstract test is not precise. The agency charged with its determination has a wide range before it could properly be said by a court that the agency had disregarded statutory standards or had confiscated the property of the utility for public use. Cf. Chicago, M. & St. P.R. Co. v. Minnesota, 134 U.S. 418, 461—466, 10 S.Ct. 462, 702, 703—705, 33 L.Ed. 970, dissent. This is as Congress intends. Rates are left to an experienced agency particularly competent by training to appraise the amount required.

The decision as to a reasonable return had not been a source of great difficulty, for borrowers and lenders reached such agreements daily in a multitude of situations; and although the determination of fair value had been troublesome, its essentials had been worked out in fairness to investor and consumer by the time of the enactment ***622** of this Act. Cf. Los Angeles G. & E. Corp. v. Railroad Comm., 289 U.S. 287, 304 et seq., 53 S.Ct. 637, 643 et seq., 77 L.Ed. 1180. The results were well known to Congress and had that body desired to depart from the traditional concepts of fair value and earnings, it would have stated its intention plainly. Helvering v. Griffiths, 318 U.S. 371, 63 S.Ct. 636.

It was already clear that when rates are in dispute, 'earnings produced by rates do not afford a standard for decision.' 289 U.S. at page 305, 53 S.Ct. at page 644, 77 L.Ed. 1180. Historical cost, prudent investment and reproduction cost[3] were all relevant factors in determining fair value. Indeed, disregarding the pioneer investor's risk, if prudent investment and reproduction cost were not distorted by changes in price levels or technology, each of them would produce the same result. The realization from the risk of an investment in a speculative field, such as natural gas utilities, should be reflected in the present fair value.[4] The amount of evidence to be admitted on any point was of course in the agency's reasonable discretion, and it was free to give its own weight to these or other factors and to determine from all the evidence its own judgment as to the necessary rates.

***623** I agree with the Court in not imposing a rule of prudent investment alone in determining the rate base. This leaves the Commission free, as I understand it, to use any available evidence for its finding of fair value, including both prudent investment and the cost of installing at the present time an efficient system for furnishing the needed utility service.

My disagreement with the Court arises primarily from its view that it makes no ****298** difference how the Commission reached the rate fixed so long as the result is fair and reasonable. For me the statutory command to the Commission is more explicit. Entirely aside from the constitutional problem of whether the Congress could validly delegate its rate making power to the Commission, in toto and without standards, it did legislate in the light of the relation of fair and reasonable to fair value and reasonable return. The Commission must therefore make its findings in observance of that relationship.

The Federal Power Commission did not, as I construe their action, disregard its statutory duty. They heard the evidence relating to historical and reproduction cost and to the reasonable rate of return and they appraised its weight. The evidence of reproduction cost was rejected as unpersuasive, but from the other evidence they found a rate base, which is to me a determination of fair value. On that base the earnings allowed seem fair and reasonable. So far as the Commission went in appraising the property employed in the service, I find nothing in the result which indicates confiscation, unfairness or unreasonableness. Good administration of rate making agencies under this method would avoid undue delay and render revaluations unnecessary except after violent fluctuations of price levels. Rate making under this method has been subjected to criticism. But until Congress changes the standards for the agencies, these rate making bodies should continue the conventional theory of rate ***624** making. It will probably be simpler to improve present methods than to devise new ones.

But a major error, I think was committed in the disregard by the Commission of the investment in exploratory operations and other recognized capital costs. These were not considered by the Commission because they were charged to operating expenses by the company at a time when it was unregulated. Congress did not direct the Commission in rate making to deduct from the rate base capital investment which had been recovered during the unregulated period through excess earnings. In my view this part of the investment should no more have been disregarded in the rate base than any other capital investment which previously had been recovered and paid out in dividends or placed to surplus. Even if prudent investment throughout the life of the property is accepted as the formula for figuring the rate base, it seems to me illogical to throw out the admittedly prudent cost of part of the property because the earnings in the unregulated period had been sufficient to return the prudent cost to the investors over and above a reasonable return. What would the answer be under the theory of the Commission and the Court, if the only prudent investment in this utility had been the seventeen million capital charges which are now disallowed?

For the reasons heretofore stated, I should affirm the action of the Circuit Court of Appeals in returning the proceeding to the Commission for further consideration and should direct the Commission to accept the disallowed capital investment in determining the fair value for rate making purposes.

Mr. Justice FRANKFURTER, dissenting.

My brother JACKSON has analyzed with particularity the economic and social aspects of natural gas as well as ***625** the difficulties which led to the enactment of the Natural Gas Act, especially those arising out of the abortive attempts of States to regulate natural gas utilities. The Natural Gas Act of 1938 should receive application in the light of this analysis, and Mr. Justice JACKSON has, I believe, drawn relevant inferences regarding the duty of the Federal Power Commission in fixing natural gas rates. His exposition seems to me unanswered, and I shall say only a few words to emphasize my basic agreement with him.

For our society the needs that are met by public utilities are as truly public services as the traditional governmental functions of police and justice. They are not less so when these services are rendered by private enterprise under governmental regulation. Who ultimately determines the ways of regulation, is the decisive aspect in the public supervision of privately-owned utilities. Foreshadowed nearly sixty years ago, Railroad Commission Cases (Stone v. Farmers' Loan & Trust Co.), 116 U.S. 307, 331, 6 S.Ct. 334, 344, 388, 1191, 29 L.Ed. 636, it was decided more than fifty ****299** years ago that the final say under the Constitution lies with the judiciary and not the legislature. Chicago, etc., R. Co. v. Minnesota , 134 U.S. 418, 10 S.Ct. 462, 702, 33 L.Ed. 970.

While legal issues touching the proper distribution of governmental powers under the Constitution may always be raised, Congressional acquiescence to date in the doctrine of Chicago, etc., R. Co. v. Minnesota, supra, may fairly be claimed. But in any event that issue is not here in controversy. As pointed out in the opinions of my brethren, Congress has given only limited authority to the Federal Power Commission and made the exercise of that authority subject to judicial review. The Commission is authorized to fix rates chargeable for natural gas. But the rates that it can fix must be 'just and reasonable'. s 5 of the Natural Gas Act, 15 U.S.C. s 717d, 15 U.S.C.A. s 717d. Instead of making the Commission's rate determinations final, Congress ***626** specifically provided for court review of such orders. To be sure, 'the finding of the Commission as to the facts, if supported by substantial evidence' was made 'conclusive', s 19 of the Act, 15 U.S.C. s 717r; 15 U.S.C.A. s 717r. But obedience of the requirement of Congress that rates be 'just and reasonable' is not an issue of fact of which the Commission's own determination is conclusive. Otherwise, there would be nothing for a court to review except questions of compliance with the procedural provisions of the Natural Gas Act. Congress might have seen fit so to cast its legislation. But it has not done so. It has committed to the administration of the Federal Power Commission the duty of applying standards of fair dealing and of reasonableness relevant to the purposes expressed by the Natural Gas Act. The requirement that rates must be 'just and reasonable' means just and reasonable in relation to appropriate standards. Otherwise Congress would have directed the Commission to fix such rates as in the judgment of the Commission are just and reasonable; it would not have also provided that such determinations by the Commission are subject to court review.

To what sources then are the Commission and the courts to go for ascertaining the standards relevant to the regulation of natural gas rates? It is at this point that Mr. Justice JACKSON'S analysis seems to me pertinent. There appear to be two alternatives. Either the fixing of natural gas rates must be left to the unguided discretion of the Commission so long as the rates it fixes do not reveal a glaringly had prophecy of the ability of a regulated utility to continue its service in the future. Or the Commission's rate orders must be founded on due consideration of all the elements of the public interest which the production and distribution of natural gas involve just because it is natural gas. These elements are reflected in the Natural Gas Act, if that Act be applied as an entirety. See, for ***627** instance, ss 4(a)(b)(c)(d), 6, and 11, 15 U.S.C. ss 717c(a)(b)(c)(d), 717e, and 717j, 15 U.S.C.A. ss 717c(a—d), 717e, 717j. Of course the statute is not concerned with abstract theories of ratemaking. But its very foundation is the ‘public interest’, and the public interest is a texture of multiple strands. It includes more than contemporary investors and contemporary consumers. The needs to be served are not restricted to immediacy, and social as well as economic costs must be counted.

It will not do to say that it must all be left to the skill of experts. Expertise is a rational process and a rational process implies expressed reasons for judgment. It will little advance the public interest to substitute for the hodge-podge of the rule in Smyth v. Ames, 169 U.S. 466, 18 S.Ct. 418, 42 L.Ed. 819, an encouragement of conscious obscurity or confusion in reaching a result, on the assumption that so long as the result appears harmless its basis is irrelevant. That may be an appropriate attitude when state action is challenged as unconstitutional. Cf. Driscoll v. Edison Light & Power Co., 307 U.S. 104, 59 S.Ct. 715, 83 L.Ed. 1134. But it is not to be assumed that it was the design of Congress to make the accommodation of the conflicting interests exposed in Mr. Justice JACKSON'S opinion the occasion for a blind clash of forces or a partial assessment of relevant factors, either before the Commission or here.

The objection to the Commission's action is not that the rates it granted were too low but that the range of its vision was too narrow. And since the issues before the Commission involved no less than the ****300** total public interest, the proceedings before it should not be judged by narrow conceptions of common law pleading. And so I conclude that the case should be returned to the Commission. In order to enable this Court to discharge its duty of reviewing the Commission's order, the Commission should set forth with explicitness the criteria by which it is guided ***628** in determining that rates are ‘just and reasonable’, and it should determine the public interest that is in its keeping in the perspective of the considerations set forth by Mr. Justice JACKSON.

By Mr. Justice JACKSON.

Certainly the theory of the court below that ties rate-making to the fair-value-reproduction-cost formula should be overruled as in conflict with Federal Power Commission v. Natural Gas Pipeline Co.[1] But the case should, I think, be the occasion for reconsideration of our rate-making doctrine as applied to natural gas and should be returned to the Commission for further consideration in the light thereof.

The Commission appears to have understood the effect of the two opinions in the Pipeline case to be at least authority and perhaps direction to fix natural gas rates by exclusive application of the ‘prudent investment’ rate base theory. This has no warrant in the opinion of the Chief Justice for the Court, however, which released the Commission from subservience to ‘any single formula or combination of formulas' provided its order, ‘viewed in its entirety, produces no arbitrary result.’ 315 U.S. at page 586, 62 S.Ct. at page 743, 86 L.Ed. 1037. The minority opinion I understood to advocate the ‘prudent investment’ theory as a sufficient guide in a natural gas case. The view was expressed in the court below that since this opinion was not expressly controverted it must have been approved.[2] I disclaim this imputed ***629** approval with some particularity, because I attach importance at the very beginning of federal regulation of the natural gas industry to approaching it as the performance of economic functions, not as the performance of legalistic rituals.

I.

Solutions of these cases must consider eccentricities of the industry which gives rise to them and also to the Act of Congress by which they are governed.

The heart of this problem is the elusive, exhaustible, and irreplaceable nature of natural gas itself. Given sufficient money, we can produce any desired amount of railroad, bus, or steamship transportation, or communications facilities, or capacity for generation of electric energy, or for the manufacture of gas of a kind. In the service of such utilities one customer has little concern with the amount taken by another, one's waste will not deprive another, a volume of service and be created equal to demand, and today's demands will not exhaust or lessen capacity to serve tomorrow. But the wealth of Midas and the wit of man cannot produce or reproduce a natural gas field. We cannot even reproduce the gas, for our manufactured product has only about half the heating value per unit of nature's own. [3]

****301** Natural gas in some quantity is produced in twenty-four states. It is consumed in only thirty-five states, and is ***630** available only to about 7,600,000 consumers. [4] Its availability has been more localized than that of any other utility service because it has depended more on the caprice of nature.

The supply of the Hope Company is drawn from that old and rich and vanishing field that flanks the Appalachian mountains. Its center of production is Pennsylvania and West Virginia, with a fringe of lesser production in New York, Ohio, Kentucky, Tennessee, and the north end of Alabama. Oil was discovered in commercial quantities at a depth of only 69 1/2 feet near Titusville, Pennsylvania, in 1859. Its value then was about $16 per barrel. [5] The oil branch of the petroleum industry went forward at once, and with unprecedented speed. The area productive of oil and gas was roughed out by the drilling of over 19,000 'wildcat' wells, estimated to have cost over $222,000,000. Of these, over 18,000 or 94.9 per cent, were 'dry holes.' About five per cent, or 990 wells, made discoveries of commercial importance, 767 of them resulting chiefly in oil and 223 in gas only. [6] Prospecting for many years was a search for oil, and to strike gas was a misfortune. Waste during this period and even later is appalling. Gas was regarded as having no commercial value until about 1882, in which year the total yield was valued only at about $75,000. [7] Since then, contrary to oil, which has become cheaper gas in this field has pretty steadily advanced in price.

While for many years natural gas had been distributed on a small scale for lighting, [8] its acceptance was slow, ***631** facilities for its utilization were primitive, and not until 1885 did it take on the appearance of a substantial industry. [9] Soon monopoly of production or markets developed. [10] To get gas from the mountain country, where it was largely found, to centers of population, where it was in demand, required very large investment. By ownership of such facilities a few corporate systems, each including several companies, controlled access to markets. Their purchases became the dominating factor in giving a market value to gas produced by many small operators. Hope is the market for over 300 such operators. By 1928 natural gas in the Appalachian field commanded an average price of 21.1 cents per m.c.f. at points of production and was bringing 45.7 cents at points of consumption. [11] The companies which controlled markets, however, did not rely on gas purchases alone. They acquired and held in fee or leasehold great acreage in territory proved by 'wildcat' drilling. These large marketing system companies as well as many small independent owners and operators have carried on the commercial development of proved territory. The development risks appear from the estimate that up to 1928, 312,318 proved area wells had been sunk in the Appalachian field of which 48,962, or 15.7 per cent, failed to produce oil or gas in commercial quantity. [12]

***632** With the source of supply thus tapped to serve centers of large demand, like Pittsburgh, Buffalo, Cleveland, Youngstown, Akron, and other industrial communities, the distribution of natural gas fast became big business. Its advantages as a ****302** fuel and its price commended it, and the business yielded a handsome return. All was merry and the goose hung high for consumers and gas companies alike until about the time of the first. World War. Almost unnoticed by the consuming public, the whole Appalachian field passed its peak of production and started to decline. Pennsylvania, which to 1928 had given off about 38 per cent of the natural gas from this field, had its peak in 1905; Ohio, which had produced 14 per cent, had its peak in 1915; and West Virginia, greatest producer of all, with 45 per cent to its credit, reached its peak in 1917. [13]

Western New York and Eastern Ohio, on the fringe of the field, had some production but relied heavily on imports from Pennsylvania and West Virginia. Pennsylvania, a producing and exporting state, was a heavy consumer and supplemented her production with imports from West Virginia. West Virginia was a consuming state, but the lion's share of her production was exported. Thus the interest of the states in the North Appalachian supply was in conflict.

Competition among localities to share in the failing supply and the helplessness of state and local authorities in the presence of state lines and corporate complexities is a part of the background of federal intervention in the industry. [14] West Virginia took the boldest measure. It legislated a priority in its entire production in favor of its own inhabitants. That was frustrated by an injunction ***633** from this Court. [15] Throughout the region clashes in the courts and conflicting decisions evidenced public anxiety and confusion. It was held that the New York Public Service Commission did not have power to classify consumers and restrict their use of gas.[16] That Commission held that a company could not abandon a part of its territory and still serve the rest.[17] Some courts admonished the companies to take action to protect consumers.[18] Several courts held that companies, regardless of failing supply, must continue to take on customers, but such compulsory additions were finally held to be within the Public Service Commission's discretion.[19] There were attempts to throw up franchises and quit the service, and municipalities resorted to the courts with conflicting results.[20] Public service commissions of consuming states were handicapped, for they had no control of the supply.[21]

****303** ***634** Shortages during World War I occasioned the first intervention in the natural gas industry by the Federal Government. Under Proclamation of President Wilson the United States Fuel Administrator took control, stopped extensions, classified consumers and established a priority for domestic over industrial use.[22] After the war federal control was abandoned. Some cities once served with natural gas became dependent upon mixed gas of reduced heating value and relatively higher price.[23]

Utilization of natural gas of highest social as well as economic return is domestic use for cooking and water ***635** heating, followed closely by use for space heating in homes. This is the true public utility aspect of the enterprise, and its preservation should be the first concern of regulation. Gas does the family cooking cheaper than any other fuel.[24] But its advantages do not end with dollars and cents cost. It is delivered without interruption at the meter as needed and is paid for after it is used. No money is tied up in a supply, and no space is used for storage. It requires no handling, creates no dust, and leaves no ash. It responds to thermostatic control. It ignites easily and immediately develops its maximum heating capacity. These incidental advantages make domestic life more liveable.

Industrial use is induced less by these qualities than by low cost in competition with other fuels. Of the gas exported from West Virginia by the Hope Company a very substantial part is used by industries. This wholesale use speeds exhaustion of supply and displaces other fuels. Coal miners and the coal industry, a large part of whose costs are wages, have complained of unfair competition from low-priced industrial gas produced with relatively little labor cost.[25]

Gas rate structures generally have favored industrial users. In 1932, in Ohio, the average yield on gas for domestic consumption was 62.1 cents per m.c.f. and on industrial, ***636** 38.7. In Pennsylvania, the figures were 62.9 against 31.7. West Virginia showed the least spread, domestic consumers paying 36.6 cents; and industrial, 27.7.[26] Although this spread is less than ****304** in other parts of the United States,[27] it can hardly be said to be self-justifying. It certainly is a very great factor in hastening decline of the natural gas supply.

About the time of World War I there were occasional and short-lived efforts by some hard-pressed companies to reverse this discrimination and adopt graduated rates, giving a low rate to quantities adequate for domestic use and graduating it upward to discourage industrial use.[28] ***637** These rates met opposition from industrial sources, of course, and since diminished revenues from industrial sources tended to increase the domestic price, they met little popular or commission favor. The fact is that neither the gas companies nor the consumers nor local regulatory bodies can be depended upon to conserve gas. Unless federal regulation will take account of conservation, its efforts seem, as in this case, actually to constitute a new threat to the life of the Appalachian supply.

II.

Congress in 1938 decided upon federal regulation of the industry. It did so after an exhaustive investigation of all aspects including failing supply and competition for the use of natural gas intensified by growing scarcity.[29] Pipelines from the Appalachian area to markets were in the control of a handful of holding company systems.[30] This created a highly concentrated control of the producers' market and of the consumers' supplies. While holding companies dominated both production and distribution they segregated those activities in separate ***638** subsidiaries,[31] the effect of which, if not the purpose, was to isolate ****305** some end of the business from the reach of any one state commission. The cost of natural gas to consumers moved steadily upwards over the years, out of proportion to prices of oil, which, except for the element of competition, is produced under somewhat comparable conditions. The public came to feel that the companies were exploiting the growing scarcity of local gas. The problems of this region had much to do with creating the demand for federal regulation.

The Natural Gas Act declared the natural gas business to be 'affected with a public interest,' and its regulation 'necessary in the public interest.'[32] Originally, and at the time this proceeding was commenced and tried, it also declared 'the intention of Congress that natural gas shall be sold in interstate commerce for resale for ultimate public consumption for domestic, commercial, industrial, or any other use at the lowest possible reasonable rate consistent with the maintenance of adequate service in the public interest.' [33] While this was later dropped, there is nothing to indicate that it was not and is not still an accurate statement of purpose of the Act. Extension or improvement of facilities may be ordered when 'necessary or desirable in the public interest,' abandonment of facilities may be ordered when the supply is 'depleted to the extent that the continuance of service is unwarranted, or that the present or future public convenience or necessity ***639** permit' abandonment and certain extensions can only be made on finding of 'the present or future public convenience and necessity.' [34] The Commission is required to take account of the ultimate use of the gas. Thus it is given power to suspend new schedules as to rates, charges, and classification of services except where the schedules are for the sale of gas 'for resale for industrial use only,' [35] which gives the companies greater freedom to increase rates on industrial gas than on domestic gas. More particularly, the Act expressly forbids any undue preference or advantage to any person or 'any unreasonable difference in rates * * * either as between localities or as between classes of service.' [36] And the power of the Commission expressly includes that to determine the 'just and reasonable rate, charge, classification, rule, regulation, practice, or contract to be thereafter observed and in force.' [37]

In view of the Court's opinion that the Commission in administering the Act may ignore discrimination, it is interesting that in reporting this Bill both the Senate and the House Committees on Interstate Commerce pointed out that in 1934, on a nationwide average the price of natural gas per m.c.f. was 74.6 cents for domestic use, 49.6 cents for commercial use, and 16.9 for industrial use. [38] I am not ready to think that supporters of a bill called attention to the striking fact that householders were being charged five times as much for their gas as industrial users only as a situation which the Bill would do nothing to remedy. On the other hand the Act gave to the Commission what the Court aptly describes as 'broad powers of regulation.'

***640 III.**

This proceeding was initiated by the Cities of Cleveland and Akron. They alleged that the price charged by Hope for natural gas 'for resale to domestic, commercial and small industrial consumers in Cleveland and elsewhere is excessive, unjust, unreasonable, greatly in excess of the price charged by Hope to nonaffiliated companies at wholesale for resale to domestic, commercial and small industrial consumers, and greatly in excess of the price charged by Hope to East Ohio for resale to certain favored industrial consumers in Ohio, and therefore is further unduly discriminatory between consumers and between classes of service' (italics supplied). The company answered admitting differences in prices to affiliated and nonaffiliated companies and justifying them by differences in conditions of delivery. ****306** As to the allegation that the contract price is 'greatly in excess of the price charged by Hope to East Ohio for resale to certain favored industrial consumers in Ohio,' Hope did not deny a price differential, but alleged that industrial gas was not sold to 'favored consumers' but was sold under contract and schedules filed with and approved by the Public Utilities Commission of Ohio, and that certain conditions of delivery made it not 'unduly discriminatory.'

The record shows that in 1940 Hope delivered for industrial consumption 36,523,792 m.c.f. and for domestic and commercial consumption, 50,343,652 m.c.f. I find no separate figure for domestic consumption. It served 43,767 domestic consumers directly, 511,521 through the East Ohio Gas Company, and 154,043 through the Peoples Natural Gas Company, both affiliates owned by the same parent. Its special contracts for industrial consumption, so far as appear, are confined to about a dozen big industries.

***641** Hope is responsible for discrimination as exists in favor of these few industrial consumers. It controls both the resale price and use of industrial gas by virtue of the very interstate sales contracts over which the Commission is exercising its jurisdiction.

Hope's contract with East Ohio Company is an example. Hope agrees to deliver, and the Ohio Company to take, '(a) all natural gas requisite for the supply of the domestic consumers of the Ohio Company; (b) such amounts of natural gas as may be requisite to fulfill contracts made with the consent and approval of the Hope Company by the Ohio Company, or companies which it supplies with natural gas, for the sale of gas upon special terms and conditions for manufacturing purposes.' The Ohio company is required to read domestic customers' meters once a month and meters of industrial customers daily and to furnish all meter readings to Hope. The Hope Company is to have access to meters of all consumers and to all of the Ohio Company's accounts. The domestic consumers of the Ohio Company are to be fully supplied in preference to consumers purchasing for manufacturing purposes and 'Hope Company can be required to supply gas to be used for manufacturing purposes only where the same is sold under special contracts which have first been submitted to and approved in writing by the Hope Company and which expressly provide that natural gas will be supplied thereunder only in so far as the same is not necessary to meet the requirements of domestic consumers supplied through pipe lines of the Ohio Company.' This basic contract was supplemented from time to time, chiefly as to price. The last amendment was in a letter from Hope to East Ohio in 1937. It contained a special discount on industrial gas and a schedule of special industrial contracts, Hope reserving the right to make eliminations therefrom and agreeing that others might be added from time to ***642** time with its approval in writing. It said, 'It is believed that the price concessions contained in this letter, while not based on our costs, are under certain conditions, to our mutual advantage in maintaining and building up the volumes of gas sold by us (italics supplied).'[39]

****307** The Commission took no note of the charges of discrimination and made no disposition of the issue tendered on this point. It ordered a flat reduction in the price per m.c.f. of all gas delivered by Hope in interstate commerce. It made no limitation, condition, or provision as to what classes of consumers should get the benefit of the reduction. While the cities have accepted and are defending the reduction, it is my view that the discrimination of which they have complained is perpetuated and increased by the order of the Commission and that it violates the Act in so doing.

The Commission's opinion aptly characterizes its entire objective by saying that 'bona fide investment figures now become all-important in the regulation of rates.' It should be noted that the all-importance of this theory is not the result of any instruction from Congress. When the Bill to regulate gas was first before Congress it contained ***643** the following: 'In determining just and reasonable rates the Commission shall fix such rate as will allow a fair return upon the actual legitimate prudent cost of the property used and useful for the service in question.' H.R. 5423, 74th Cong., 1st Sess. Title III, s 312(c). Congress rejected this language. See H.R. 5423, s 213 (211(c)), and H.R. Rep. No. 1318, 74th Cong., 1st Sess. 30.

The Commission contends nevertheless that the 'all important' formula for finding a rate base is that of prudent investment. But it excluded from the investment base an amount actually and admittedly invested of some $17,000,000. It did so because it says that the Company recouped these expenditures from customers before the days of regulation from earnings above a fair return. But it would not apply all of such 'excess earnings' to reduce the rate base as one of the Commissioners suggested. The reason for applying excess earnings to reduce the investment base roughly from $69,000,000 to $52,000,000 but refusing to apply them to reduce it from that to some $18,000,000 is not found in a difference in the character of the earnings or in their reinvestment. The reason assigned is a difference in bookkeeping treatment many years before the Company was subject to regulation. The $17,000,000, reinvested chiefly in well drilling, was treated on the books as expense. (The Commission now requires that drilling costs be carried to capital account.) The allowed rate base thus actually was determined by the Company's bookkeeping, not its investment. This attributes a significance to formal classification in account keeping that seems inconsistent with rational rate regulation.[40] Of ***644** course, the ****308** Commission would not and should not allow a rate base to be inflated by bookkeeping which had improperly capitalized expenses. I have doubts about resting public regulation upon any rule that is to be used or not depending on which side it favors.

***645** The Company on the other hand, has not put its gas fields into its calculations on the present-value basis, although that, it contends, is the only lawful rule for finding a rate base. To do so would result in a rate higher than it has charged or proposes as a matter of good business to charge.

The case before us demonstrates the lack of rational relationship between conventional rate-base formulas and natural gas production and the extremities to which regulating bodies are brought by the effort to rationalize them. The Commission and the Company each stands on a different theory, and neither ventures to carry its theory to logical conclusion as applied to gas fields.

IV.

This order is under judicial review not because we interpose constitutional theories between a State and the business it seeks to regulate, but because Congress put upon the federal courts a duty toward administration of a new federal regulatory Act. If we are to hold that a given rate is reasonable just because the Commission has said it was reasonable, review becomes a costly, time-consuming pageant of no practical value to anyone. If on the other hand we are to bring judgment of our own to the task, we should for the guidance of the regulators and the regulated reveal something of the philosophy, be it legal or economic or social, which guides us. We need not be slaves to a formula but unless we can point out a rational way of reaching our conclusions they can only be accepted as resting on intuition or predilection. I must admit that I possess no instinct jby which to know the 'reasonable' from the 'unreasonable' in prices and must seek some conscious design for decision.

The Court sustains this order as reasonable, but what makes it so or what could possibly make it otherwise, ***646** I cannot learn. It holds that: 'it is the result reached not the method employed which is controlling'; 'the fact that the method employed to reach that result may contain infirmities is not then important' and it is not 'important to this case to determine the various permissible ways in which any rate base on which the return is computed might be arrived at.' The Court does lean somewhat on considerations of capitalization and dividend history and requirements for dividends on outstanding stock. But I can give no real weight to that for it is generally and I think deservedly in discredit as any guide in rate cases. [41]

Our books already contain so much talk of methods of rationalizing rates that we must appear ambiguous if we announce results without our working methods. We are confronted with regulation of a unique type of enterprise which I think requires considered rejection of much conventional utility doctrine and adoption of concepts of 'just and reasonable' rates and practices and of the 'public interest' that will take account of the peculiarities of the business.

The Court rejects the suggestions of this opinion. It says that the Committees in reporting the bill which became the Act said it provided 'for regulation along recognized and more or less standardized lines' and that there was 'nothing novel in its provisions.' So saying it sustains a rate calculated on a novel variation of a rate base theory which itself had at the time of enactment of the legislation been recognized only in dissenting opinions. Our difference seems to be between unconscious innovation, [42] and the purposeful ****309** and deliberate innovation I ***647** would make to meet the necessities of regulating the industry before us.

Hope's business has two components of quite divergent character. One, while not a conventional common-carrier undertaking, is essentially a transportation enterprise consisting of conveying gas from where it is produced to point of delivery to the buyer. This is a relatively routine operation not differing substantially from many other utility operations. The service is produced by an investment in compression and transmission facilities. Its risks are those of investing in a tested means of conveying a discovered supply of gas to a known market. A rate base calculated on the prudent investment formula would seem a reasonably satisfactory measure for fixing a return from that branch of the business whose service is roughly proportionate to the capital invested. But it has other consequences which must not be overlooked. It gives marketability and hence 'value' to gas owned by the company and gives the pipeline company a large power over the marketability and hence 'value' of the production of others.

The other part of the business—to reduce to possession an adequate supply of natural gas—is of opposite character, being more erratic and irregular and unpredictable in relation to investment than any phase of any other utility business. A thousand feet of gas captured and severed from real estate for delivery to consumers is recognized under our law as property of much the same nature as a ton of coal, a barrel of oil, or a yard of sand. The value to be allowed for it is the real battleground between the investor and consumer. It is from this part of the business that the chief difference between the parties as to a proper rate base arises.

It is necessary to a 'reasonable' price for gas that it be anchored to a rate base of any kind? Why did courts in the first place begin valuing 'rate bases' in order to 'value' something else? The method came into vogue ***648** in fixing rates for transportation service which the public obtained from common carriers. The public received none of the carriers' physical property but did make some use of it. The carriage was often a monopoly so there were no open market criteria as to reasonableness. The 'value' or 'cost' of what was put to use in the service by the carrier was not a remote or irrelevant consideration in making such rates. Moreover the difficulty of appraising an intangible service was thought to be simplified if it could be related to physical property which was visible and measurable and the items of which might have market value. The court hoped to reason from the known to the unknown. But gas fields turn this method topsy turvy. Gas itself is tangible, possessible, and does have a market and a price in the field. The value of the rate base is more elusive than that of gas. It consists of intangibles—leaseholds and freeholds—operated and unoperated—of little use in themselves except as rights to reach and capture gas. Their value lies almost wholly in predictions of discovery, and of price of gas when captured, and bears little relation to cost of tools and supplies and labor to develop it. Gas is what Hope sells and it can be directly priced more reasonably and easily and accurately than the components of a rate base can be valued. Hence the reason for resort to a roundabout way of rate base price fixing does not exist in the case of gas in the field.

But if found, and by whatever method found, a rate base is little help in determining reasonableness of the price of gas. Appraisal of present value of these intangible rights to pursue fugitive gas depends on the value assigned to the gas when captured. The 'present fair value' rate base, generally in ill repute, [43] is not even ****310** urged by the gas company for valuing its fields.

***649** The prudent investment theory has relative merits in fixing rates for a utility which creates its service merely by its investment. The amount and quality of service rendered by the usual utility will, at least roughly, be measured by the amount of capital it puts into the enterprise. But it has no rational application where there is no such relationship between investment and capacity to serve. There is no such relationship between investment and amount of gas produced. Let us assume that Doe and Roe each produces in West Virginia for delivery to Cleveland the same quantity of natural gas per day. Doe, however, through luck or foresight or whatever it takes, gets his gas from investing $50,000 in leases and drilling. Roe drilled poorer territory, got smaller wells, and has invested $250,000. Does anybody imagine that Roe can get or ought to get for his gas five times as much as Doe because he has spent five times as much? The service one renders to society in the gas business is measured by what he gets out of the ground, not by what he puts into it, and there is little more relation between the investment and the results than in a game of poker.

Two-thirds of the gas Hope handles it buys from about 340 independent producers. It is obvious that the principle of rate-making applied to Hope's own gas cannot be applied, and has not been applied, to the bulk of the gas Hope delivers. It is not probable that the investment of any two of these producers will bear the same ratio to their investments. The gas, however, all goes to the same use, has the same utilization value and the same ultimate price.

To regulate such an enterprise by undiscriminatingly transplanting any body of rate doctrine conceived and ***650** adapted to the ordinary utility business can serve the 'public interest' as the Natural Gas Act requires, if at all, only by accident. Mr. Justice Brandeis, the pioneer juristic advocate of the prudent investment theory for man-made utilities, never, so far as I am able to discover, proposed its application to a natural gas case. On the other hand, dissenting in Commonwealth of Pennsylvania v. West Virginia, he reviewed the problems of gas supply and said, 'In no other field of public service regulation is the controlling body confronted with factors so baffling as in the natural gas industry, and in none is continuous supervision and control required in so high a degree.' 262 U.S. 553, 621, 43 S.Ct. 658, 674, 67 L.Ed. 1117, 32 A.L.R. 300. If natural gas rates are intelligently to be regulated we must fit our legal principles to the economy of the industry and not try to fit the industry to our books.

As our decisions stand the Commission was justified in believing that it was required to proceed by the rate base method even as to gas in the field. For this reason the Court may not merely wash its hands of the method and rationale of rate making. The fact is that this Court, with no discussion of its fitness, simply transferred the rate base method to the natural gas industry. It happened in Newark Natural Gas & Fuel Co. v. City of Newark, Ohio, 1917, 242 U.S. 405, 37 S.Ct. 156, 157, 61 L.Ed. 393, Ann.Cas.1917B, 1025, in which the company wanted 25 cents per m.c.f., and under the Fourteenth Amendment challenged the reduction to 18 cents by ordinance. This Court sustained the reduction because the court below 'gave careful consideration to the questions of the value of the property * * * at the time of the inquiry,' and whether the rate 'would be sufficient to provide a fair return on the value of the property.' The Court said this method was 'based upon principles thoroughly established by repeated secisions of this court,' citing many cases, not one of which involved natural gas or a comparable wasting natural resource. Then came issues as to state power to ***651** regulate as affected by the commerce clause. Public Utilities Commission v. Landon, 1919, 249 U.S. 236, 39 S.Ct. 268, 63 L.Ed. 577; Pennsylvania Gas Co. v. Public Service Commission, 1920, 252 U.S. 23, 40 S.Ct. 279, 64 L.Ed. 434. These questions settled, the Court again was called upon in natural gas cases to consider state rate-making claimed to be invalid under the Fourteenth Amendment. United Fuel Gas Co. v. Railroad Commission of Kentucky, 1929, 278 U.S. 300, 49 S.Ct. 150, 73 L.Ed. 390; United Fuel Gas Company v. Public Service Commission of West Virginia, 1929, 278 U.S. 322, 49 S.Ct. 157, 73 L.Ed. 402. Then, as now, the differences were 'due ****311** chiefly to the difference in value ascribed by each to the gas rights and leaseholds.' 278 U.S. 300, 311, 49 S.Ct. 150, 153, 73 L.Ed. 390. No one seems to have questioned that the rate base method must be pursued and the controversy was at what rate base must be used. Later the 'value' of gas in the field was questioned in determining the amount a regulated company should be allowed to pay an affiliate therefor—a state determination also reviewed under the Fourteenth Amendment. Dayton Power & Light Co. v. Public Utilities Commission of Ohio, 1934, 292 U.S. 290, 54 S.Ct. 647, 78 L.Ed. 1267; Columbus Gas & Fuel Co. v. Public Utilities Commission of Ohio, 1934, 292 U.S. 398, 54 S.Ct. 763, 78 L.Ed. 1327, 91 A.L.R. 1403. In both cases, one of which sustained, and one of which struck down a fixed rate the Court assumed the rate base method, as the legal way of testing reasonableness of natural gas prices fixed by public authority, without examining its real relevancy to the inquiry.

Under the weight of such precedents we cannot expect the Commission to initiate economically intelligent methods of fixing gas prices. But the Court now faces a new plan of federal regulation based on the power to fix the price at which gas shall be allowed to move in interstate commerce. I should now consider whether these rules devised under the Fourteenth Amendment are the exclusive tests of a just and reasonable rate under the federal statute, inviting reargument directed to that point ***652** if necessary. As I see it now I would be prepared to hold that these rules do not apply to a natural gas case arising under the Natural Gas Act.

Such a holding would leave the Commission to fix the price of gas in the field as one would fix maximum prices of oil or milk or coal, or any other commodity. Such a price is not calculated to produce a fair return on the synthetic value of a rate base of any individual producer, and would not undertake to assure a fair return to any producer. The emphasis would shift from the producer to the product, which would be regulated with an eye to average or typical producing conditions in the field.

Such a price fixing process on economic lines would offer little temptation to the judiciary to become back seat drivers of the price fixing machine. The unfortunate effect of judicial intervention in this field is to divert the attention of those engaged in the process from what is economically wise to what is legally permissible. It is probable that price reductions would reach economically unwise and self-defeating limits before they would reach constitutional ones. Any constitutional problems growing out of price fixing are quite different than those that have heretofore been considered to inhere in rate making. A producer would have difficulty showing the invalidity of such a fixed price so long as he voluntarily continued to sell his product in interstate commerce. Should he withdraw and other authority be invoked to compel him to part with his property, a different problem would be presented.

Allowance in a rate to compensate for gas removed from gas lands, whether fixed as of point of production or as of point of delivery, probably best can be measured by a functional test applied to the whole industry. For good or ill we depend upon private enterprise to exploit these natural resources for public consumption. The function which an allowance for gas in the field should perform ***653** for society in such circumstances is to be enough and no more than enough to induce private enterprise completely and efficiently to utilize gas resources, to acquire for public service any available gas or gas rights and to deliver gas at a rate and for uses which will be in the future as well as in the present public interest.

The Court fears that 'if we are now to tell the Commission to fix the rates so as to discourage particular uses, we would indeed be injecting into a rate case a 'novel' doctrine * * *.' With due deference I suggest that there is nothing novel in the idea that any change in price of a service or commodity reacts to encourage or discourage its use. The question is not whether such consequences will or will not follow; the question is whether effects must be suffered blindly or may be intelligently selected, whether price control shall have targets at which it deliberately aims or shall be handled like a gun in the hands of one who does not know it is loaded.

We should recognize 'price' for what it is—a tool, a means, an expedient. In public ****312** hands it has much the same economic effects as in private hands. Hope knew that a concession in industrial price would tend to build up its volume of sales. It used price as an expedient to that end. The Commission makes another cut in that same price but the Court thinks we should ignore the effect that it will have on exhaustion of supply. The fact is that in natural gas regulation price must be used to reconcile the private property right society has permitted to vest in an important natural resource with the claims of society upon it—price must draw a balance between wealth and welfare.

To carry this into techniques of inquiry is the task of the Commissioner rather than of the judge, and it certainly is no task to be solved by mere bookkeeping but requires the best economic talent available. There would doubtless be inquiry into the price gas is bringing in the ***654** field, how far that price is established by arms' length bargaining and how far it may be influenced by agreements in restraint of trade or monopolistic influences. What must Hope really pay to get and to replace gas it delivers under this order? If it should get more or less than that for its own, how much and why? How far are such prices influenced by pipe line access to markets and if the consumers pay returns on the pipe lines how far should the increment they cause go to gas producers? East Ohio is itself a producer in Ohio. [44] What do Ohio authorities require Ohio consumers to pay for gas in the field? Perhaps these are reasons why the Federal Government should put West Virginia gas at lower or at higher rates. If so what are they? Should East Ohio be required to exploit its half million acres of unoperated reserve in Ohio before West Virginia resources shall be supplied on a devalued basis of which that State complains and for which she threatens measures of self keep? What is gas worth in terms of other fuels it displaces?

A price cannot be fixed without considering its effect on the production of gas. Is it an incentive to continue to exploit vast unoperated reserves? Is it conducive to deep drilling tests the result of which we may know only after trial? Will it induce bringing gas from afar to supplement or even to substitute for Appalachian gas? [45] Can it be had from distant fields as cheap or cheaper? If so, that competitive potentiality is certainly a relevant consideration. Wise regulation must also consider, as a private buyer would, what alternatives the producer has ***655** if the price is not acceptable. Hope has intrastate business and domestic and industrial customers. What can it do by way of diverting its supply to intrastate sales? What can it do by way of disposing of its operated or reserve acreage to industrial concerns or other buyers? What can West Virginia do by way of conservation laws, severance or other taxation, if the regulated rate offends? It must be borne in mind that while West Virginia was prohibited from giving her own inhabitants a priority that discriminated against interstate commerce, we have never yet held that a good faith conservation act, applicable to her own, as well as to others, is not valid. In considering alternatives, it must be noted that federal regulation is very incomplete, expressly excluding regulation of 'production or gathering of natural gas,' and that the only present way to get the gas seems to be to call it forth by price inducements. It is plain that there is a downward economic limit on a safe and wise price.

But there is nothing in the law which compels a commission to fix a price at that 'value' which a company might give to its product by taking advantage of scarcity, or monopoly of supply. The very purpose of fixing maximum prices is to take away from the seller his opportunity to get all that otherwise the market would award him for his goods. This is a constitutional use of the power to fix maximum prices, ****313** Block v. Hirsh, 256 U.S. 135, 41 S.Ct. 458, 65 L.Ed. 865, 16 A.L.R. 165; Marcus Brown Holding Co. v. Feldman, 256 U.S. 170, 41 S.Ct. 465, 65 L.Ed. 877; International Harvester Co. v. Kentucky, 234 U.S. 216, 34 S.Ct. 853, 58 L.Ed. 1284; Highland v. Russell Car & Snow Plow Co., 279 U.S. 253, 49 S.Ct. 314, 73 L.Ed. 688, just as the fixing of minimum prices of goods in interstate commerce is constitutional although it takes away from the buyer the advantage in bargaining which market conditions would give him. United States v. Darby, 312 U.S. 100, 657, 61 S.Ct. 451, 85 L.Ed. 609, 132 A.L.R. 1430; Mulford v. Smith, 307 U.S. 38, 59 S.Ct. 648, 83 L.Ed. 1092; United States v. Rock Royal Co-operative, Inc., 307 U.S. 533, 59 S.Ct. 993, 83 L.Ed. 1446; Sunshine Anthracite Coal Co. v. Adkins, 310 U.S. 381, 60 S.Ct. 907, 84 L.Ed. 1263. The Commission has power to fix ***656** a price that will be both maximum and minimum and it has the incidental right, and I think the duty, to choose the economic consequences it will promote or retard in production and also more importantly in consumption, to which I now turn.

If we assume that the reduction in company revenues is warranted we then come to the question of translating the allowed return into rates for consumers or classes of consumers. Here the Commission fixed a single rate for all gas delivered irrespective of its use despite the fact that Hope has established what amounts to two rates—a high one for domestic use and a lower one for industrial contracts. [46] The Commission can fix two prices for interstate gas as readily as one—a price for resale to domestic users and another for resale to industrial users. This is the pattern Hope itself has established in the very contracts over which the Commission is expressly given jurisdiction. Certainly the Act is broad enough to permit two prices to be fixed instead of one, if the concept of the 'public interest' is not unduly narrowed.

The Commission's concept of the public interest in natural gas cases which is carried today into the Court's opinion was first announced in the opinion of the minority in the Pipeline case. It enumerated only two 'phases of the public interest: (1) the investor interest; (2) the consumer interest,' which it emphasized to the exclusion of all others. 315 U.S. 575, 606, 62 S.Ct. 736, 753, 86 L.Ed. 1037. This will do well enough in dealing with railroads or utilities supplying manufactured gas, electric, power, a communications service or transportation, where utilization of facilities does not impair their future usefulness. Limitation of supply, however, brings into a natural gas case another phase of the public interest that to my mind overrides both the owner ***657** and the consumer of that interest. Both producers and industrial consumers have served their immediate private interests at the expense of the long-range public interest. The public interest, of course, requires stopping unjust enrichment of the owner.

But it also requires stopping unjust impoverishment of future generations. The public interest in the use by Hope's half million domestic consumers is quite a different one from the public interest in use by a baker's dozen of industries.

Prudent price fixing it seems to me must at the very threshold determine whether any part of an allowed return shall be permitted to be realized from sales of gas for resale for industrial use. Such use does tend to level out daily and seasonal peaks of domestic demand and to some extent permits a lower charge for domestic service. But is that a wise way of making gas cheaper when, in comparison with any substitute, gas is already a cheap fuel? The interstate sales contracts provide that at times when demand is so great that there is not enough gas to go around domestic users shall first be served. Should the operation of this preference await the day of actual shortage? Since the propriety of a preference seems conceded, should it not operate to prevent the coming of a shortage as well as to mitigate its effects? Should industrial use jeopardize tomorrow's service to householders any more than today's? If, however, it is decided to cheapen domestic use by resort to industrial sales, should they be limited to the few uses ****314** for which gas has special values or extend also to those who use it only because it is cheaper than competitive fuels?[47] And how much cheaper should industrial ***658** gas sell than domestic gas, and how much advantage should it have over competitive fuels? If industrial gas is to contribute at all to lowering domestic rates, should it not be made to contribute the very maximum of which it is capable, that is, should not its price be the highest at which the desired volume of sales can be realized?

If I were to answer I should say that the household rate should be the lowest that can be fixed under commercial conditions that will conserve the supply for that use. The lowest probable rate for that purpose is not likely to speed exhaustion much, for it still will be high enough to induce economy, and use for that purpose has more nearly reached the saturation point. On the other hand the demand for industrial gas at present rates already appears to be increasing. To lower further the industrial rate is merely further to subsidize industrial consumption and speed depletion. The impact of the flat reduction ***659** of rates ordered here admittedly will be to increase the industrial advantages of gas over competing fuels and to increase its use. I think this is not, and there is no finding by the Commission that it is, in the public interest.

There is no justification in this record for the present discrimination against domestic users of gas in favor of industrial users. It is one of the evils against which the Natural Gas Act was aimed by Congress and one of the evils complained of here by Cleveland and Akron. If Hope's revenues should be cut by some $3,600,000 the whole reduction is owing to domestic users. If it be considered wise to raise part of Hope's revenues by industrial purpose sales, the utmost possible revenue should be raised from the least consumption of gas. If competitive relationships to other fuels will permit, the industrial price should be substantially advanced, not for the benefit of the Company, but the increased revenues from the advance should be applied to reduce domestic rates. For in my opinion the 'public interest' requires that the great volume of gas now being put to uneconomic industrial use should either be saved for its more important future domestic use or the present domestic user should have the full benefit of its exchange value in reducing his present rates.

Of course the Commission's power directly to regulate does not extend to the fixing of rates at which the local company shall sell to consumers. Nor is such power required to accomplish the purpose. As already pointed out, the very contract the Commission is altering classifies the gas according to the purposes for which it is to be resold and provides differentials between the two classifications. It would only be necessary for the Commission to order ****315** that all gas supplied under paragraph (a) of Hope's contract with the East Ohio Company shall be ***660** at a stated price fixed to give to domestic service the entire reduction herein and any further reductions that may prove possible by increasing industrial rates. It might further provide that gas delivered under paragraph (b) of the contract for industrial purposes to those industrial customers Hope has approved in writing shall be at such other figure as might be found consistent with the public interest as herein defined. It is too late in the day to contend that the authority of a regulatory commission does not extend to a consideration of public interests which it may not directly regulate and a conditioning of its orders for their protection. Interstate Commerce Commission v. Railway Labor Executives Ass'n, 315 U.S. 373, 62 S.Ct. 717, 86 L.Ed. 904; United States v. Lowden, 308 U.S. 225, 60 S.Ct. 248, 84 L.Ed. 208.

Whether the Commission will assert its apparently broad statutory authorization over prices and discriminations is, of course, its own affair, not ours. It is entitled to its own notion of the 'public interest' and its judgment of policy must prevail. However, where there is ground for thinking that views of this Court may have constrained the Commission to accept the rate-base method of decision and a particular single formula as 'all important' for a rate base, it is appropriate to make clear the reasons why I, at least, would not be so understood. The Commission is free to face up realistically to the nature and peculiarity of the resources in its control, to foster their duration in fixing price, and to consider future interests in addition to those of investors and present consumers. If we return this case it may accept or decline the proffered freedom. This problem presents the Commission an unprecedented opportunity if it will boldly make sound economic considerations, instead of legal and accounting theories, the foundation of federal policy. I would return the case to the Commission and thereby be clearly quit of what now may appear to be some responsibility for perpetrating a shortsighted pattern of natural gas regulation.

Parallel Citations

51 P.U.R.(NS) 193, 64 S.Ct. 281, 88 L.Ed. 333

Footnotes

1 Hope produces about one-third of its annual gas requirements and purchases the rest under some 300 contracts.

2 These five companies are the East Ohio Gas Co., the Peoples Natural Gas Co., the River Gas Co., the Fayette County Gas Co., and the Manufacturers Light & Heat Co. The first three of these companies are, like Hope, subsidiaries of Standard Oil Co. (N.J.). East Ohio and River distribute gas in Ohio, the other three in Pennsylvania. Hope's approximate sales in m.c.f. for 1940 may be classified as follows:

Local West Virginia sales	11,000,000
East Ohio	40,000,000
Peoples	10,000,000
River	400,000
Fayette	860,000
Manufacturers	2,000,000

Local West Virginia

Hope's natural gas is processed by Hope Construction & Refining Co., an affiliate, for the extraction of gasoline and butane. Domestic Coke Corp., another affiliate, sells coke-oven gas to Hope for boiler fuel.

3 These required minimum reductions of 7¢ per m.c.f. from the 36.5¢ and 35.5¢ rates previously charged East Ohio and Peoples, respectively, and 3¢ per m.c.f. from the 31.5¢ rate previously charged Fayette and Manufacturers.

4 The book reserve for interstate plant amounted at the end of 1938 to about $18,000,000 more than the amount determined by the Commission as the proper reserve requirement. The Commission also noted that ‘twice in the past the company has transferred amounts aggregating $7,500,000 from the depreciation and depletion reserve to surplus. When these latter adjustments are taken into account, the excess becomes $25,500,000, which has been exacted from the ratepayers over and above the amount required to cover the consumption of property in the service rendered and thus to keep the investment unimpaired.’ 44 P.U.R.,N.S., at page 22.

5 That contention was based on the fact that ‘every single dollar in the depreciation and depletion reserves' was taken ‘from gross operating revenues whose only source was the amounts charged customers in the past for natural gas. It is, therefore, a fact that the depreciation and depletion reserves have been contributed by the customers and do not represent any investment by Hope.’ Id., 44 P.U.R.,N.S., at page 40. And see Railroad Commission v. Cumberland Tel. & T. Co., 212 U.S. 414, 424, 425, 29 S.Ct. 357, 361, 362, 53 L.Ed. 577; 2 Bonbright, Valuation of Property (1937), p. 1139.

6 The Commission noted that the case was ‘free from the usual complexities involved in the estimate of gas reserves because the geologists for the company and the Commission presented estimates of the remaining recoverable gas reserves which were about one per cent apart.’ 44 P.U.R.,N.S., at pages 19, 20.

The Commission utilized the ‘straight-line-basis' for determining the depreciation and depletion reserve requirements. It used estimates of the average service lives of the property by classes based in part on an inspection of the physical condition of the property. And studies were made of Hope's retirement experience and maintenance policies over the years. The average service lives of the various classes of property were converted into depreciation rates and then applied to the cost of the property to ascertain the portion of the cost which had expired in rendering the service.

The record in the present case shows that Hope is on the lookout for new sources of supply of natural gas and is contemplating an extension of its pipe line into Louisiana for that purpose. The Commission recognized in fixing the rates of depreciation that much material may be used again when various present sources of gas supply are exhausted, thus giving that property more than scrap value at the end of its present use.

7 See Uniform System of Accounts prescribed for Natural Gas Companies effective January 1, 1940, Account No. 332.1.

8 Sec. 6 of the Act comes the closest to supplying any definite criteria for rate making. It provides in subsection (a) that, 'The Commission may investigate the ascertain the actual legitimate cost of the property of every natural-gas company, the depreciation therein, and, when found necessary for rate-making purposes, other facts which bear on the determination of such cost or depreciation and the fair value of such property.' Subsection (b) provides that every natural-gas company on request shall file with the Commission a statement of the 'original cost' of its property and shall keep the Commission informed regarding the 'cost' of all additions, etc.

9 We recently stated that the meaning of the word 'value' is to be gathered 'from the purpose for which a valuation is being made. Thus the question in a valuation for rate making is how much a utility will be allowed to earn. The basic question in a valuation for reorganization purposes is how much the enterprise in all probability can earn.' Institutional Investors v. Chicago, M., St. P. & P.R. Co., 318 U.S. 523, 540, 63 S.Ct. 727, 738.

10 Chief Justice Hughes said in that case (292 U.S. at pages 168, 169, 54 S.Ct. at page 665, 78 L.Ed. 1182): 'If the predictions of service life were entirely accurate and retirements were made when and as these predictions were precisely fulfilled, the depreciation reserve would represent the consumption of capital, on a cost basis, according to the method which spreads that loss over the respective service periods. But if the amounts charged to operating expenses and credited to the account for depreciation reserve are excessive, to that extent subscribers for the telephone service are required to provide, in effect, capital contributions, not to make good losses incurred by the utility in the service rendered and thus to keep its investment unimpaired, but to secure additional plant and equipment upon which the utility expects a return.'

11 See Mr. Justice Brandeis (dissenting) in United Railways & Electric Co. v. West, 280 U.S. 234, 259—288, 50 S.Ct. 123, 128—138, 74 L.Ed. 390, for an extended analysis of the problem.

12 It should be noted that the Act provides no specific rule governing depletion and depreciation. Sec. 9(a) merely states that the Commission 'may from time to time ascertain and determine, and by order fix, the proper and adequate rates of depreciation and amortization of the several classes of property of each natural-gas company used or useful in the production, transportation, or sale of natural gas.'

13 See Simonton, The Nature of the Interest of the Grantee Under an Oil and Gas Lease (1918), 25 W.Va.L.Quar. 295.

14 West Penn Power Co. v. Board of Review, 112 W.Va. 442, 164 S.E. 862.

15 W.Va.Rev.Code of 1943, ch. 11. Art. 13, ss 2a, 3a.

16 West Virginia suggests as a possible solution (1) that a 'going concern value' of the company's tangible assets be included in the rate base and (2) that the fair market value of gas delivered to customers be added to the outlay for operating expenses and taxes.

17 S.Doc. 92, Pt. 84-A, ch. XII, Final Report, Federal Trade Commission to the Senate pursuant to S.Res.No. 83, 70th Cong., 1st Sess.

18 S.Doc. 92, Pt. 84-A, chs. XII, XIII, op. cit., supra, note 17.

19 See Hearings on H.R. 11662, Subcommittee of House Committee on Interstate & Foreign Commerce, 74th Cong., 2d Sess.; Hearings on H.R. 4008, House Committee on Interstate & Foreign Commerce, 75th Cong., 1st Sess.

20 The power to investigate and ascertain the 'actual legitimate cost' of property (s 6), the requirement as to books and records (s 8), control over rates of depreciation (s 9), the requirements for periodic and special reports (s 10), the broad powers of investigation (s 14) are among the chief powers supporting the rate making function.

21 Apart from the grandfather clause contained in s 7(c), there is the provision of s 7(f) that a natural gas company may enlarge or extend its facilities with the 'service area' determined by the Commission without any further authorization.

22 See P.L. 117, approved July 7, 1943, 57 Stat. 383 containing an 'Interstate Compact to Conserve Oil and Gas' between Oklahoma, Texas, New Mexico, Illinois, Colorado, and Kansas.

23 As we have pointed out, s 7(c) was amended by the Act of February 7, 1942, 56 Stat. 83, so as to require certificates of public convenience and necessity not only where the extensions were being made to markets in which natural gas was already being sold by another company but to other situations as well. Considerations of conservation entered into the proposal to give the Act that broader scope. H.Rep.No. 1290, 77th Cong. 1st Sess., pp. 2, 3. And see Annual Report, Federal Power Commission (1940) pp. 79, 80; Baum, The Federal Power Commission and State Utility Regulation (1942), p. 261.

The bill amending s 7(c) originally contained a subsection (h) reading as follows: 'Nothing contained in this section shall be construed to affect the authority of a State within which natural gas is produced to authorize or require the construction or extension of facilities for the transportation and sale of such gas within such State: Provided, however, That the Commission, after a hearing upon complaint or upon its own motion, may by order forbid any intrastate construction or extension by any natural-gas company which it shall find will prevent such company from rendering adequate service to its customers in interstate or foreign commerce in territory already being served.' See Hearings on H.R. 5249, House

Committee on Interstate & Foreign Commerce, 77th Cong., 1st Sess., pp. 7, 11, 21, 29, 32, 33. In explanation of its deletion the House Committee Report stated, pp. 4, 5: 'The increasingly important problems raised by the desire of several States to regulate the use of the natural gas produced therein in the interest of consumers within such States, as against the Federal power to regulate interstate commerce in the interest of both interstate and intrastate consumers, are deemed by the committee to warrant further intensive study and probably a more retailed and comprehensive plan for the handling thereof than that which would have been provided by the stricken subsection.'

24 We have noted that in the annual operating expenses of some $16,000.000 the Commission included West Virginia and federal taxes. And in the net increase of $421,160 over 1940 operating expenses allowed by the Commission was some $80,000 for increased West Virginia property taxes. The adequacy of these amounts has not been challenged here.

25 The Commission included in the aggregate annual operating expenses which it allowed some $8,500,000 for gas purchased. It also allowed about $1,400,000 for natural gas production and about $600,000 for exploration and development.

It is suggested, however, that the Commission in ascertaining the cost of Hope's natural gas production plant proceeded contrary to s 1(b) which provides that the Act shall not apply to 'the production or gathering of natural gas'. But such valuation, like the provisions for operating expenses, is essential to the rate-making function as customarily performed in this country. Cf. Smith, The Control of Power Rates in the United States and England (1932), 159 The Annals 101. Indeed s 14(b) of the Act gives the Commission the power to 'determine the propriety and reasonableness of the inclusion in operating expenses, capital, or surplus of all delay rentals or other forms of rental or compensation for unoperated lands and leases.'

26 See note 25, supra.

27 The Commission has expressed doubts over its power to fix rates on 'direct sales to industries' from interstate pipelines as distinguished from 'sales for resale to the industrial customers of distributing companies.' Annual Report, Federal Power Commission (1940), p. 11.

28 Sec. 1(b) of the Act provides: 'The provisions of this Act shall apply to the transportation of natural gas in interstate commerce, to the sale in interstate commerce of natural gas for resale for ultimate public consumption for domestic, commercial, industrial, or any other use, and to natural-gas companies engaged in such transportation or sale, but shall not apply to any other transportation or sale of natural gas or to the local distribution of natural gas or to the facilities used for such distribution or to the production or gathering of natural gas.' And see s 2(6), defining a 'natural-gas company', and H.Rep.No. 709, supra, pp. 2, 3.

29 The wasting-asset characteristic of the industry was recognized prior to the Act as requiring the inclusion of a depletion allowance among operating expenses. See Columbus Gas & Fuel Co. v. Public Utilities Commission, 292 U.S. 398, 404, 405, 54 S.Ct. 763, 766, 767, 78 L.Ed. 1327, 91 A.L.R. 1403. But no such theory of rate-making for natural gas companies as is now suggested emerged from the cases arising during the earlier period of regulation.

30 The Commission has been alert to the problems of conservation in its administration of the Act. It has indeed suggested that it might be wise to restrict the use of natural gas 'by functions rather than by areas.' Annual Report (1940) p. 79.

The Commission stated in that connection that natural gas was particularly adapted to certain industrial uses. But it added that the general use of such gas 'under boilers for the production of steam' is 'under most circumstances of very questionable social economy.' Ibid.

31 The argument is that s 4(a) makes 'unlawful' the charging of any rate that is not just and reasonable. And s 14(a) gives the Commission power to investigate any matter 'which it may find necessary or proper in order to determine whether any person has violated' any provision of the Act. Moreover, s 5(b) gives the Commission power to investigate and determine the cost of production or transportation of natural gas in cases where it has 'no authority to establish a rate governing the transportation or sale of such natural gas.' And s 17(c) directs the Commission to 'make available to the several State commissions such information and reports as may be of assistance in State regulation of natural-gas companies.' For a discussion of these points by the Commission see 44 P.U.R.,N.S., at pages 34, 35.

1 Natural Gas Act, s 4(a), 52 Stat. 821, 822, 15 U.S.C. s 717c(a), 15 U.S.C.A. s 717c(a).

2 52 Stat. 821, 824, 15 U.S.C. s 717e, 15 U.S.C.A. s 717e:

'(a) The Commission may investigate and ascertain the actual legitimate cost of the property of every natural-gas company, the depreciation therein, and, when found necessary for rate-making purposes, other facts which bear on the determination of such cost or depreciation and the fair value of such property.

'(b) Every natural-gas company upon request shall file with the Commission an inventory of all or any part of its property and a statement of the original cost thereof, and shall keep the Commission informed regarding the cost of all additions, betterments, extensions, and new construction.'

3 'Reproduction cost' has been variously defined, but for rate making purposes the most useful sense seems to be, the minimum amount necessary to create at the time of the inquiry a modern plant capable of rendering equivalent service. See I Bonbright, Valuation of Property (1937) 152. Reproduction cost as the cost of building a replica of an obsolescent plant is not of real significance.

'Prudent investment' is not defined by the Court. It may mean the sum originally put in the enterprise, either with or without additional amounts from excess earnings reinvested in the business.

4 It is of no more than bookkeeping significance whether the Commission allows a rate of return commensurate with the risk of the original investment or the lower rate based on current risk and a capitalization reflecting the established earning power of a successful company and the probable cost of duplicating its services. Cf. American T. & T. Co. v. United States, 299 U.S. 232, 57 S.Ct. 170, 81 L.Ed. 142. But the latter is the traditional method.

1 315 U.S. 575, 62 S.Ct. 736, 86 L.Ed. 1037.

2 Judge Dobie, dissenting below, pointed out that the majority opinion in the Pipeline case 'contains no express discussion of the Prudent Investment Theory' and that the concurring opinion contained a clear one, and said, 'It is difficult for me to believe that the majority of the Supreme Court, believing otherwise, would leave such a statement unchallenged.' (134 F.2d 287, 312.) The fact that two other Justices had as matter of record in our books long opposed the reproduction cost theory of rate bases and had commented favorably on the prudent investment theory may have influenced that conclusion. See opinion of Mr. Justice Frankfurter in Driscoll v. Edison Light & Power Co., 307 U.S. 104, 122, 59 S.Ct. 715, 724, 83 L.Ed. 1134, and my brief as Solicitor General in that case. It should be noted, however, that these statements were made, not in a natural gas case, but in an electric power case—a very important distinction, as I shall try to make plain.

3 Natural gas from the Appalachian field averages about 1050 to 1150 B.T.U. content, while by-product manufactured gas is about 530 to 540. Moody's Manual of Public Utilities (1943) 1350; Youngberg, Natural Gas (1930) 7.

4 Sen.Rep. No. 1162, 75th Cong., 1st Sess., 2.

5 Arnold and Kemnitzer, Petroleum in the United States and Possessions (1931) 78.

6 Id. at 62-63.

7 Id. at 61.

8 At Fredonia, New York, in 1821, natural gas was conveyed from a shallow well to some thirty people. The lighthouse at Barcelona Harbor, near what is now Westfield, New York, was at about that time and for many years afterward lighted by gas that issued from a crevice. Report on Utility Corporations by Federal Trade Commission, Sen.Doc. 92, Pt. 84-A, 70th Cong., 1st Sess., 8-9.

9 In that year Pennsylvania enacted 'An Act to provide for the incorporation and regulation of natural gas companies.' Penn.Laws 1885, No. 32, 15 P.S. s 1981 et seq.

10 See Steptoe and Hoffheimer's Memorandum for Governor Cornwell of West Virginia (1917) 25 West Virginia Law Quarterly 257; see also Report on Utility Corporations by Federal Trade Commission, Sen.Doc. No. 92, Pt. 84-A, 70th Cong., 1st Sess.

11 Arnold and Kemnitzer, Petroleum in the United States and Possessions (1931) 73.

12 Id. at 63.

13 Id. at 64.

14 See Report on Utility Corporations by Federal Trade Commission, Sen.Doc. No. 92, Pt. 84-A, 70th Cong., 1st Sess.

15 Commonwealth of Pennsylvania v. West Virginia, 262 U.S. 553, 43 S.Ct. 658, 67 L.Ed. 1117, 32 A.L.R. 300. For conditions there which provoked this legislation, see 25 West Virginia Law Quarterly 257.

16 People ex rel. Pavilion Natural Gas Co. v. Public Service Commission, 188 App.Div. 36, 176 N.Y.S. 163.

17 Village of Falconer v. Pennsylvania Gas Company, 17 State Department Reports, N.Y., 407.

18 See, for example, Public Service Commission v. Iroquois Natural Gas Co., 108 Misc. 696, 178 N.Y.S. 24; Park Abbott Realty Co. v. Iroquois Natural Gas Co., 102 Misc. 266, 168 N.Y.S. 673; Public Service Commission v. Iroquois Natural Gas Co., 189 App.Div. 545, 179 N.Y.S. 230.

19 People ex rel. Pennsylvania Gas Co. v. Public Service Commission, 196 App.Div. 514, 189 N.Y.S. 478.

20 East Ohio Gas Co. v. Akron, 81 Ohio St. 33, 90 N.E. 40, 26 L.R.A., N.S., 92, 18 Ann.Cas. 332; Village of New-comerstown v. Consolidated Gas Co., 100 Ohio St. 494, 127 N.E. 414; Gress v. Village of Ft. Laramie, 100 Ohio St. 35, 125 N.E. 112, 8 A.L.R. 242; City of Jamestown v. Pennsylvania Gas Co., D.C., 263 F. 437; Id., D.C., 264 F. 1009. See, also, United Fuel Gas Co. v. Railroad Commission, 278 U.S. 300, 308, 49 S.Ct. 150, 152, 73 L.Ed. 390.

21 The New York Public Service Commission said: 'While the transportation of natural gas through pipe lines from one state to another state is interstate commerce * * *, Congress has not taken over the regulation of that particular industry. Indeed,

it has expressly excepted it from the operation of the Interstate Commerce Commissions Law (Interstate Commerce Commissions Law, section 1). It is quite clear, therefore, that this Commission can not require a Pennsylvania corporation producing gas in Pennsylvania to transport it and deliver it in the State of New York, and that the Interstate Commerce Commission is likewise powerless. If there exists such a power, and it seems that there does, it is a power vested in Congress and by it not yet exercised. There is no available source of supply for the Crystal City Company at present except through purchasing from the Porter Gas Company. It is possible that this Commission might fix a price at which the Potter Gas Company should sell if it sold at all, but as the Commission can not require it to supply gas in the State of New York, the exercise of such a power to fix the price, if such power exists, would merely say, sell at this price or keep out of the State.' Lane v. Crystal City Gas Co., 8 New York Public Service Comm.Reports, Second District, 210, 212.

22 Proclamation by the President of September 16, 1918; Rules and Regulations of H. A. Garfield, Fuel Administrator, September 24, 1918.

23 For example, the Iroquois Gas Corporation which formerly served Buffalo, New York, with natural gas ranging from 1050 to 1150 b.t.u. per cu. ft., now mixes a by-product gas of between 530 and 540 b.t.u. in proportions to provide a mixed gas of about 900 b.t.u. per cu. ft. For space heating or water heating its charges range from 65 cents for the first m.c.f. per month to 55 cents for all above 25 m.c.f. per month. Moody's Manual of Public Utilities (1943) 1350.

24 The United States Fuel Administration made the following cooking value comparisons, based on tests made in the Department of Home Economics of Ohio State University:

Natural gas at 1.12 per M. is equivalent to coal at $6.50 per ton.

Natural gas at 2.00 per M. is equivalent to gasoline at 27¢ per gal.

Natural gas at 2.20 per M. is equivalent to electricity at 3¢ per k.w.h.

Natural gas at 2.40 per M. is equivalent to coal oil at 15¢ per gal.

Use and Conservation of Natural Gas, issued by U.S. Fuel Administration (1918) 5.

25 See Brief on Behalf jof Legislation Imposing an Excise Tax on Natural Gas, submitted to N.R.A. by the United Mine Workers of America and the National Coal Association.

26 Brief of National Gas Association and United Mine Workers, supra, note 26, pp. 35, 36, compiled from Bureau of Mines Reports.

27 From the source quoted in the preceding note the spread elsewhere is shown to be:

State	Industrial	Domestic
Illinois...	29.2	1.678
Louisiana...	10.4	59.7
Oklahoma..	11.2	41.5
Texas..	13.1	59.7
Alabama..	17.8	1.227
Georgia...	22.9	1.043

28 In Corning, New York, rates were initiated by the Crystal City Gas Company as follows: 70¢ for the first 5,000 cu. ft. per month; 80¢ from 5,000 to 12,000; $1 for all over 12,000. The Public Service Commission rejected these rates and fixed a flat rate of 58¢ per m.c.f. Lane v. Crystal City Gas Co., 8 New York Public Service Comm. Reports, Second District, 210. The Pennsylvania Gas Company (National Fuel Gas Company group) also attempted a sliding scale rate for New York consumers, net per month as follows: First 5,000 feet, 35¢; second 5,000 feet, 45¢; third 5,000 feet, 50¢; all above 15,000, 55¢. This was eventually abandoned, however. The company's present scale in Pennsylvania appears to be reversed to the following net monthly rate; first 3 m.c.f., 75¢; next 4 m.c.f., 60¢; next 8 m.c.f., 55¢; over 15 m.c.f., 50¢ . Moody's Manual of Public Utilities (1943) 1350. In New York it now serves a mixed gas.

For a study of effect of sliding scale rates in reducing consumption see 11 Proceedings of Natural Gas Association of America (1919) 287.

29 See Report on Utility Corporations by Federal Trade Commission, Sen. Doc. 92, Pt. 84-A, 70th Cong., 1st Sess.

30 Four holding company systems control over 55 per cent of all natural gas transmission lines in the United States. They are Columbia Gas and Electric Corporation, Cities Service Co., Electric Bond and Share Co., and Standard Oil Co. of New Jersey. Columbia alone controls nearly 25 per cent, and fifteen companies account for over 80 per cent of the total. Report on Utility Corporations by Federal Trade Commission, Sen. Doc. 92, Pt. 84-A, 70th Cong., 1st Sess., 28.

In 1915, so it was reported to the Governor of West Virginia, 87 per cent of the total gas production of that state was under control of eight companies. Steptoe and Hoffheimer, Legislative Regulation of Natural Gas Supply in West Virginia, 17 West Virginia Law Quarterly 257, 260. Of these, three were subsidiaries of the Columbia system and others were subsidiaries of larger systems. In view of inter-system sales and interlocking interests it may be doubted whether there is much real competition among these companies.

31 This pattern with its effects on local regulatory efforts will be observed in our decisions. See United Fuel Gas Co. v. Railroad Commission, 278 U.S. 300, 49 S.Ct. 150, 73 L.Ed. 390; United Fuel Gas Co. v. Public Service Commission, 278 U.S. 322, 49 S.Ct. 157, 73 L.Ed. 402; Dayton Power & Light v. Public Utilities Commission, 292 U.S. 290, 54 S.Ct. 647, 78 L.Ed. 1267; Columbus Gas & Fuel Co. v. Public Utilities Commission, 292 U.S. 398, 54 S.Ct. 763, 78 L.Ed. 1327, 91 A.L.R. 1403, and the present case.

32 15 U.S.C. s 717(a), 15 U.S.C.A. s 717(a). (Italics supplied throughout this paragraph.)

33 s 7(c), 52 Stat. 825, 15 U.S.C.A. s 717f(c).

34 15 U.S.C. s 717f, 15 U.S.C.A. s 717f.

35 Id., s 717c(e).

36 Id., s 717c(b).

37 Id., s 717d(a).

38 Sen. Rep. No. 1162, 75th Cong., 1st Sess. 2.

39 The list of East Ohio Gas Company's special industrial contracts thus expressly under Hope's control and their demands are as follows:

40 To make a fetish of mere accounting is to shield from examination the deeper causes, forces, movements, and conditions which should govern rates. Even as a recording of current transactions, bookkeeping is hardly an exact science. As a representation of the condition and trend of a business, it uses symbols of certainty to express values that actually are in constant flux. It may be said that in commercial or investment banking or any business extending credit success depends on knowing what not to believe in accounting. Few concerns go into bankruptcy or reorganization whose books do not show them solvent and often even profitable. If one cannot rely on accountancy accurately to disclose past or current conditions of a business, the fallacy of using it as a sole guide to future price policy ought to be apparent. However, our quest for certitude is so ardent that we pay an irrational reverence to a technique which uses symbols of certainty, even though experience again and again warns us that they are delusive. Few writers have ventured to challenge this American idolatry, but see Hamilton, Cost as a standard for Price, 4 Law and Contemporary Problems 321, 323-25. He observes that ‘As the apostle would put it, accountancy is all things to all men. * * * Its purpose determines the character of a system of accounts.’ He analyzes the hypothetical character of accounting and says ‘It was no eternal mold for pecuniary verities handed down from on high. It was—like logic or algebra, or the device of analogy in the law—an ingenious contrivance of the human mind to serve a limited and practical purpose.’ ‘Accountancy is far from being a pecuniary expression of all that is industrial reality. It is an instrument, highly selective in its application, in the service of the institution of money making.’ As to capital account he observes ‘In an enterprise in lusty competition with others of its kind, survival is the thing and the system of accounts has its focus in solvency. * * * Accordingly depreciation, obsolescence, and other factors which carry no immediate threat are matters of lesser concern and the capital account is likely to be regarded as a secondary phenomenon. * * * But in an enterprise, such as a public utility, where continued survival seems assured, solvency is likely to be taken for granted. * * * A persistent and ingenious attention is likely to be directed not so much to securing the upkeep of the physical property as to making it certain that capitalization fails in not one whit to give full recognition to every item that should go into the account.’

41 See 2 Bonbright, Valuation of Property (1937) 1112.

42 Bonbright says, ‘* * * the vice of traditional law lies, not in its adoption of excessively rigid concepts of value and rules of valuation, but rather in its tendency to permit shifts in meaning that are inept, or else that are ill-defined because the judges that make them will not openly admit that they are doing so.’ Id., 1170.

43 ‘The attempt to regulate rates by reference to a periodic or occasional reappraisal of the properties has now been tested long enough to confirm the worst fears of its critics. Unless its place is taken by some more promising scheme of rate control, the days of private ownership under government regulation may be numbered.’ 2 Bonbright, Valuation of Property (1937) 1190.

44 East Ohio itself owns natural gas rights in 550,600 acres, 518,526 of which are reserved and 32,074 operated, by 375 wells. Moody's Manual of Public Utilities (1943) 5.

45 Hope has asked a certificate of convenience and necessity to lay 1140 miles of 22-inch pipeline from Hugoton gas fields in southwest Kansas to West Virginia to carry 285 million cu. ft. of natural gas per day. The cost was estimated at $51,000,000. Moody's Manual of Public Utilities (1943) 1760.

46 I find little information as to the rates for industries in the record and none at all in such usual sources as Moody's Manual.

47 The Federal Power Commission has touched upon the problem of conservation in connection with an application for a certificate permitting construction of a 1500-mile pipeline from southern Texas to New York City and says: ‘The Natural Gas Act as presently drafted does not enable the Commission to treat fully the serious implications of such a problem. The question should be raised as to whether the proposed use of natural gas would not result in displacing a less valuable fuel and create hardships in the industry already supplying the market, while at the same time rapidly depleting the country's natural-gas reserves. Although, for a period of perhaps 20 years, the natural gas could be so priced as to appear to offer an apparent saving in fuel costs, this would mean simply that social costs which must eventually be paid had been ignored. ‘Careful study of the entire problem may lead to the conclusion that use of natural gas should be restricted by functions rather than by areas. Thus, it is especially adapted to space and water heating in urban homes and other buildings and to the various industrial heat processes which require concentration of heat, flexibility of control, and uniformity of results. Industrial uses to which it appears particularly adapted include the treating and annealing of metals, the operation of kilns in the ceramic, cement, and lime industries, the manufacture of glass in its various forms, and use as a raw material in the chemical industry. General use of natural gas under boilers for the production of steam is, however, under most circumstances of very questionable social economy.’ Twentieth Annual Report of the Federal Power Commission (1940) 79.

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The Cost of Capital

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and Discounted Cash Flow (the techniques mentioned here are defined and evaluated in chapter 3). Since these methods look only at firms in a single "risk class," they do not require that the analyst estimate the entire risk-return line shown in figure 2.1; they focus directly on the vertical axis.

To use the second strategy, the analyst must examine (at least implicitly) both measures of the stock's risk and the current position of the market line. Methods that require explicit risk measurement include the Capital Asset Pricing Model and the Risk Positioning techniques. These methods first position the firm on the horizontal axis of figure 2.1, and then (again, at least implicitly) use an estimate of the risk-return line to find the proper level for the cost of capital on the vertical axis.

The advantages of one strategy are the disadvantages of the other. The first strategy avoids the need for an estimate of the market line but requires that the evidence used must be from investments of comparable risk. This immediately excludes data from other firms of differing risk. More subtly, it can exclude data on the firm whose cost of capital is now being estimated, if either its risk or the market line has changed since the evidence was collected.

This focus on estimation strategies may be premature. If the reader does not accept that the cost of capital as just defined is the right target for regulators, the general approaches to cost of capital estimation may be of little interest. The remainder of the chapter uses two approaches to develop the reasons that the cost of capital is indeed the appropriate allowed rate of return for a regulated company's investors.[7]

2. Why the Allowed Rate of Return Should Equal the Cost of Capital

Law

The United States Supreme Court has established that investors in companies subject to rate regulation must be allowed *an opportunity* to earn returns sufficient to attract capital and comparable to those they would expect in the unregulated sector for bearing the same degree of risk. The *Bluefield* and *Hope* cases provide the seminal decisions.[8]

The *Hope* test is the basic criterion for a legally sufficient rate of return on equity. The court stated:

> The rate-making process under the act, i.e., the fixing of "just and reasonable" rates, involves a balancing of the investor and the consumer interests. Thus we stated in the *Natural Gas Pipeline Co.* case that "regulation does not insure that the business shall produce net revenues." 315 U.S. p. 590. But such considerations aside, the investor interest has a legitimate concern with the financial integrity of the company whose rates are being regulated. From the investor or company point of view it is important that there be enough revenue not only for operating expenses but also for the capital costs of the business. These include service on the debt and dividends on the stock . . . . By that standard the return to the equity owner should be commensurate with returns on investments in other enterprises having corresponding risks. That return, moreover, should be sufficient to assure confidence in the financial integrity of the enterprise, so as to maintain its credit and to attract capital.[9]

Since by definition the cost of capital of a regulated firm represents precisely the expected return that investors could anticipate from other investments while bearing no more and no less risk, and since investors will not provide capital unless the investment is expected to yield its opportunity cost of capital, the correspondence of the definition of the cost of capital with the court's definition of legally required earnings appears clear. *Hope* refers to both "commensurate" earnings and the attraction of capital. These two approaches are harmonized when the allowed rate of return is set equal to the cost of capital.

Hope is sometimes cited for the proposition that some specific method of establishing the rate of return on equity is the only legally permissible technique. However, *Hope* states clearly that it is the "end result" of the regulation process that determines legality, not the specific technique used to calculate rate of return. All the standard cost-of-capital estimation techniques can meet the requisite legal tests; it is the way they are applied that is important.

Despite the obvious correspondence between the precepts of *Hope* and the financial concept of the cost of capital, public utility statutes and the applicable case law give no detailed prescription for what constitutes a "just and reasonable" rate of return on equity. In the absence of detailed guidelines from legislatures or the higher courts, various general judicial concepts about rate setting have been developed and applied by courts. The key concepts are:

1. Balance: the establishment of a just and reasonable rate involves a balancing of the investor and consumer interests.

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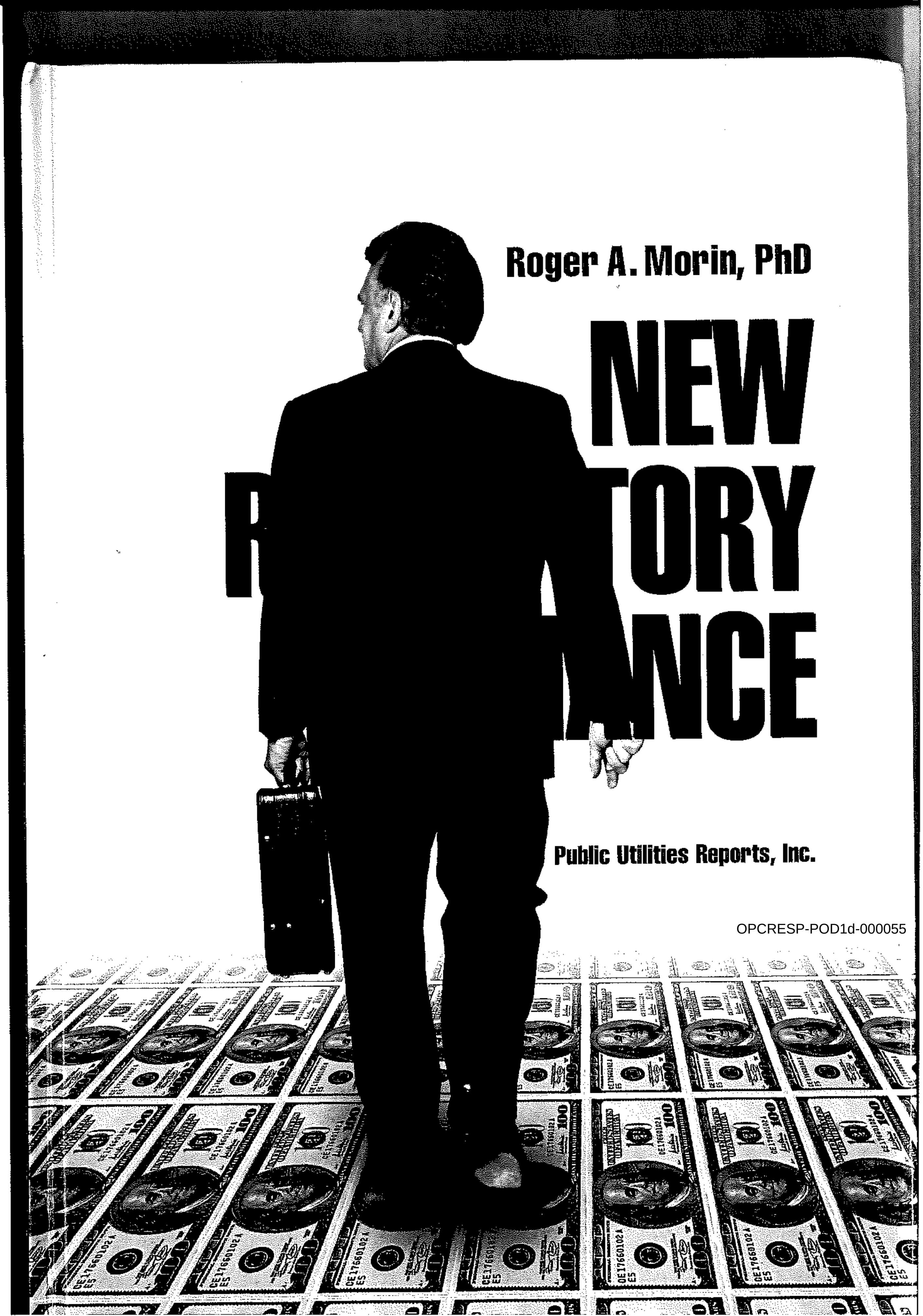

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supply side of capital markets, but also by reference to the demand side of the capital markets.

The demand side viewpoint recognizes that regulated utilities are private corporations with shareholders-owners, and that management's principal responsibility is to maximize their well-being, as measured by stock price. Thus, only those investment decisions that maximize the price of the stock should be undertaken. A utility company will continue to invest in real physical assets if the return on these investments exceeds or equals its cost of capital. The cost of capital is the minimum rate of return that must be earned on assets to justify their acquisition, and the regulator must set the allowed return so that optimal investment rates are obtained, and that no other investment rate would result in a higher share price.

In this context, the cost of capital is the expected earnings on the utility's investments that are required in order for the value of the previously invested capital to remain unchanged. If new capital does not earn its price or required rate of return, the value of existing equity has to make up the difference. If the new capital earns a return greater than its price, existing shareholders will participate in the difference. The converse is true as well. If earnings on the investment of capital meet the required rate of return, existing shareholders will neither gain nor lose.

$$\text{Cost of Capital} = \text{Required Rate of Return}$$
$$= \text{Required Earnings / Capital Invested}$$

1.8 The Allowed Rate of Return and Cost of Capital

The regulator should set the allowed rate of return equal to the cost of capital so that the utility can achieve the optimal rate of investment at the minimum price to the ratepayers. This can be demonstrated as follows.

In Example 1-2 shown earlier, a utility with a rate base of $900 million was considered, financed 60% by debt and 40% by equity. The cost of capital was estimated at 8.2%. Now, suppose the regulator sets the allowed return at 6% instead. To service the claims of both the bondholders and shareholders, earnings over costs should amount to $73.8 million, that is, 8.2% × $900 million.

If the utility is allowed a return of only 6% on a rate base of $900 million, earnings of only $54.0 million are produced. While the earnings are sufficient to cover the interest payments of $37.8 million ($900 × .60 × 7%) to the bondholders who have a prior claim on earnings, they are not enough to cover the claims of shareholders in the amount of $36 million ($900 × .40 ×

10%). The stock price has to fall to a level such that an investor who purchases the stock after the price reduction will just obtain his opportunity cost. If the utility nevertheless undertakes mandatory capital investments that are allowed to earn 6%, while the cost of the funds is 8.2%, the inevitable result is a reduction in stock price and a wealth transfer from shareholders to ratepayers.

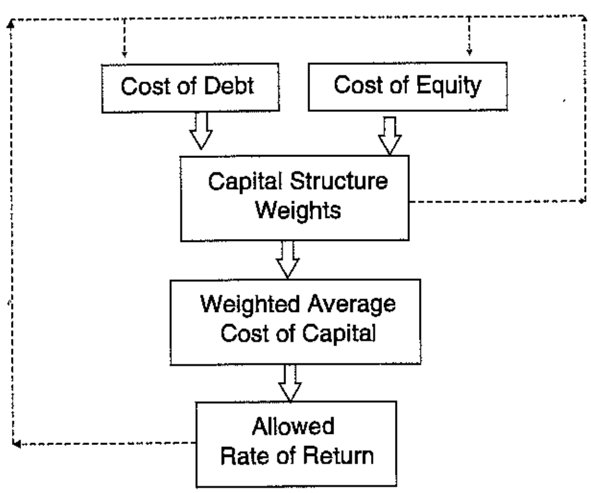
Conversely, if the allowed rate of return is greater than the cost of capital, capital investments are undertaken and investors' opportunity costs are more than achieved. Any excess earnings over and above those required to service debt capital accrue to the equity holders, and the stock price increases. In this case, the wealth transfer occurs from ratepayers to shareholders.

Investments are undertaken by the utility with no wealth transfer between ratepayers and shareholders only if the allowed rate of return is set equal to the cost of capital. In this case, the expected earnings generated from investments are just sufficient to service the claims of the debt and equity holders, no more no less. Setting the allowed return equal to the cost of capital is the only policy that will produce optimal investment rates at the minimum price to the ratepayer.

1.9 Determining the Cost of Capital

The general procedure that has evolved for determining the allowed rate of return is schematically depicted in Figure 1-6. The cost of debt and common equity are first determined separately, then weighted by the proportions of

FIGURE 1-6
SCHEMATIC REPRESENTATION OF COST OF CAPITAL DETERMINATION



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enunciated in the *Bluefield* and *Hope* cases. Some of the techniques treat risk explicitly and directly as a separate variable in the model; others treat risk implicitly and indirectly as somehow subsumed in security prices. These techniques are summarized in Figure 1-7.

1.10 The Use of Multiple Methods in Cost of Equity Determination

The court cases discussed previously indicated that there are no specific rules or infallible models for determining a fair rate of return. It is dangerous and inappropriate to rely on only one methodology in determining the cost of equity. The results from only one method are likely to contain a high degree of measurement error. The regulator's hands should not be bound to one methodology of estimating equity costs, nor should the regulator ignore relevant evidence and back itself into a corner. For instance, by relying solely on the DCF model at a time when the fundamental assumptions underlying the DCF model are tenuous, a regulatory body greatly limits its flexibility and increases the risk of authorizing unreasonable rates of return. The same is true for any one specific model.

There are four generic methodologies available to measure the cost of equity: DCF, Risk Premium, and CAPM, which are market-oriented, and Comparable Earnings, which is accounting-oriented. Each generic market-based methodology in turn contains several variants.

When measuring equity costs, which essentially deals with the measurement of investor expectations, no one single methodology provides a foolproof panacea. Each methodology requires the exercise of considerable judgment on the reasonableness of the assumptions underlying the methodology and on the reasonableness of the proxies used to validate the theory. It follows that more than one methodology should be employed in arriving at a judgment on the cost of equity and that these methodologies should be applied across a series of comparable risk companies. More on this issue in Chapter 15.

The concept of cost of capital described in this chapter can be succinctly summarized as follows: A regulated utility should be entitled to a return that allows it to raise the necessary capital to meet service demand without cost to existing shareholders. This return is the weighted average of the embedded cost of debt and preferred capital, and a return on the common equity capital equal to the currently required return on equity. The two principal problems in implementing the approach are the determination of the appropriate set of capital structure weights and the estimation of the required return on equity. The optimal capital structure issue is treated in Chapters 16, 17, and 18.

these two years excluded. It is clear from this example that a long time period is required to accurately estimate the equity risk premium. The shorter 30-year period places too much emphasis on the poor market performances of 1973–1974. In fact, the equity risk premium recovers significantly in more recent periods once the years 1973 and 1974 are truncated from the analysis, as seen in the rolling 20-year and 10-year Ibbotson data.

Some analysts employ a rolling average approach. For example, the analyst arbitrarily assumes a given time frame over which the equity risk premium should be calculated, say 30 years, and calculates a 30-year equity risk premium for all time periods from 1926 to the present. There is a premium for 1926–1955, 1927–1956, and so on to the present. The successive premiums are averaged to arrive at the eventual equity risk premium. This approach is highly suspect because it overweighs the middle years. In the example, the year 1926 appears in one 30-year average, 1927 in two 30-year averages, etc. Yet, the most current (and relevant) time period only appears once. The middle periods are given an inordinate amount of weight using this approach. The other fallacy of the approach is that it assumes that a 30-year period is an appropriate historical window over which to estimate the equity risk premium. This assumption is highly arbitrary.

While forward-looking risk premiums based on expected returns are preferable, historical return studies over long periods still provide a useful guide for the future. This is because over long periods, investors' expectations are eventually revised to match historical realizations, as market prices adjust to match anticipated and actual investment results. Otherwise, investors would never commit investment capital. In the long run, the difference between expected and realized risk premiums will decline because short-run periods during which investors earn a lower risk premium than they expect are offset by short-run periods during which investors earn a higher risk premium than they expect. Second, the investors' current expectations concerning the amount by which the return on equity will exceed the bond yield will be strongly influenced by historical differences in returns to bond and stock investors. For these reasons, we can estimate investors' current expected returns from an equity investment from knowledge of current bond yields and past differences between returns on stocks and bonds.

Computational Issues: Arithmetic vs Geometric Average

The second problem in relying on historical return results is the method of averaging historical returns, that is, whether to use the ordinary average (arithmetic mean) or the geometric mean return. Because valuation is forward-looking, the appropriate average is the one that most accurately approximates the expected future rate of return. The best estimate of expected returns over a given future holding period is the arithmetic average. Only arithmetic means

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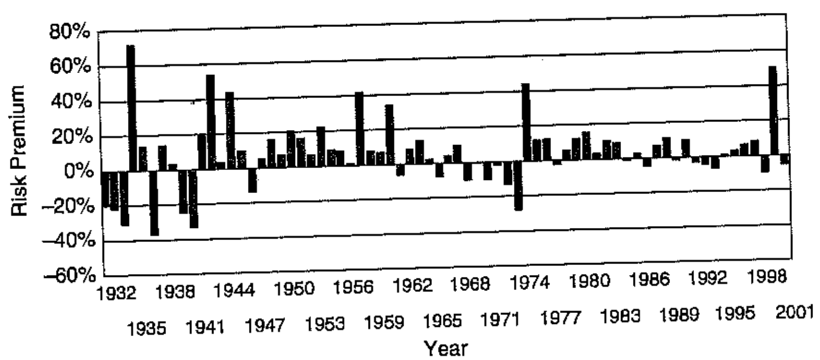
are correct for forecasting purposes and for estimating the cost of capital. There is no theoretical or empirical justification for the use of geometric mean rates of returns as a measure of the appropriate discount rate in computing the cost of capital or in computing present values. There is no dispute in academic circles as to whether the arithmetic or geometric average should be used for purposes of computing the cost of capital. The arithmetic mean should always be used in calculating the present value of a cash flow stream. Appendix A contains a comprehensive discussion of this issue, including the underlying theory, empirical evidence, and formal demonstrations.

Drawn from an actual rate case, the implementation of the historical Risk Premium approach is illustrated in Example 4-1 for the electric utility industry. Over the long term, realized utility equity risk premiums were 5.6% above Treasury bond yields for electric utilities.

FIGURE 4-2
EQUITY RISK PREMIUM
Electric Utilities 1931–2002



EXAMPLE 4-1

As a proxy for the risk premium applicable to the electric utility industry, a historical risk premium for the electric utility industry is estimated with an annual time series analysis applied to the industry as a whole, using *Moody's Electric Utility Index* as an industry proxy. The analysis is depicted in Figure 4-2. The risk premium is estimated by computing the actual return on equity capital for Moody's Index for each year, using the actual stock prices and dividends of the index, and then subtracting the long-term government bond return for that year. Dividend yields and stock prices on the index are obtained from *Moody's*

(continued next page)

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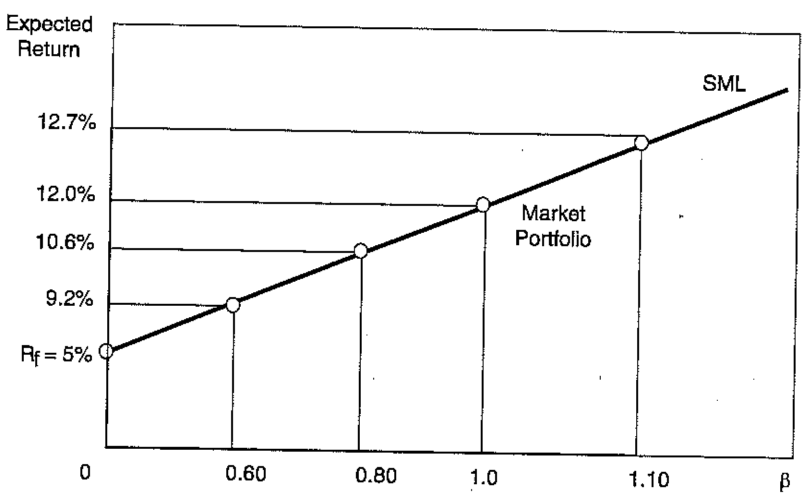
is therefore the return necessary to attract capital to investments of a given risk, taking into account the soundness criterion of *Bluefield.*

5.3 CAPM Application

At first glance, the CAPM appears simple in application. Numerical values of the CAPM's three input parameters, R_F, beta, and the market risk premium ($R_M - R_F$) are estimated and inserted into the CAPM formula to produce the cost of equity estimate, or used in reading the cost of equity directly from the SML. A numerical example is shown in Figure 5-3.

FIGURE 5-3
CAPM ESTIMATES OF UTILITY COST OF CAPITAL: ILLUSTRATION



Assuming a 5% risk-free rate, and a 12% market return, that is, a market risk premium of 7%, the cost of equity estimates for three companies are 9.2%, 10.6%, and 12.7%, respectively, corresponding to their respective betas of 0.60, 0.80, and 1.10.

Despite the CAPM's conceptual appeal and mechanistic simplicity, operationalizing the CAPM to estimate a fair return on equity presents several practical difficulties. From the start, the model itself is a prospective, forward-looking model. To stress this point, the following equation restates the CAPM formula with expectational operators attached to each input variable:

$$E(K) = E(R_F) + E(\beta) \times [E(R_M) - E(R_F)] \tag{5-2}$$

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historical risk premium approach assumes that the average realized return is an appropriate surrogate for expected return, or, in other words, that investor expectations are realized. However, realized returns can be substantially different from prospective returns anticipated by investors, especially when measured over short time periods.

The prospective (forecast) approach examines the returns expected from investments in common equities and bonds. The risk premium is simply the difference between the expected returns on stocks and bonds. The prospective approach is subject to the inevitable measurement errors involved in computing expected returns.

Therefore, a regulatory body should rely on the results of both historical and prospective studies in arriving at an appropriate risk premium, data permitting. Each proxy for the expected risk premium brings information to the judgment process from a different light. Neither proxy is without blemish, each has advantages and shortcomings. Historical risk premium data are available and verifiable, but may no longer be applicable if structural shifts have occurred. Prospective risk premium data may be more relevant since they encompass both history and current changes, but are nevertheless imperfect proxies. Giving equal weight to the historical risk premium and the prospective risk premium forecast represents a compromise between the certainty of the past and its possible irrelevance versus the greater relevance of the forecast and its possible estimation error.[13]

Faced with this myriad, and often conflicting, evidence on the magnitude of the risk premium, a regulator might very well be confused about the correct market risk premium. The author's opinion is that a range of 5% to 8% is reasonable for the United States with a slight preference for the upper end of the range.

As in the case of the beta estimate and risk-free rate estimate, a sensitivity analysis of possible CAPM cost of capital estimates should be conducted for a specified utility using a reasonable range of estimates for the market return. See Figure 5-6 for an illustration.

The range of cost of capital estimates obtained using a separate range for each of the three input variables to the CAPM, beta, risk-free rate, and market

[13] A survey of professional practices published in 1998 by Bruner, Eades, Harris, and Higgins (1998) found that 71% of textbooks/tradebooks used a historical arithmetic mean as the market risk premium and 60% of financial advisors used either a market risk premium of 7.0–7.4% (similar to the arithmetic mean) or a long-term arithmetic mean. For corporations, there was no single method that represented a consensus.

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1. That investors, in fact, evaluate common stocks in the classical valuation framework, and trade securities rationally at prices reflecting their perceptions of value. Given the universality and pervasiveness of the classical valuation framework in investment education and in the professional investment community, this assumption is plausible.
2. That investors discount the expected cash flows at the same rate K in every future period. In other words, a flat yield curve is assumed. If K varies over time, there is no single required return rate, and practical estimates of the required return must be considered as weighted averages of $\{K_1, K_2, K_3 \ldots K_n\}$. Since each of the 1-period return requirements can be thought of as an interest rate plus a risk premium, the required return to a multiple time horizon can be viewed as an average interest rate plus an average risk premium. More complex discounting models that incorporate these varying ''yield curve effects'' are available, but are of limited practical usefulness.
3. That the K obtained from the fundamental DCF equation corresponds to that specific stream of future cash flows alone, and no other. There may be alternate company policies (dividend payout, capital structure) that would generate the same future cash flows, but these policies may alter the risk of the cash flow stream, and hence modify the investor's required return, K.
4. That dividends, rather than earnings, constitute the source of value. The rationale for computing the value of common stock from dividends is that the only cash values ever received by investors are dividends. This does not mean that earnings are unimportant for they provide the basis for paying dividends.

Focusing on the present value of expected earnings can be misleading. It is earnings net of any investment required to produce the earnings that are of interest, and not earnings alone. For example, a company expects earnings per share of $1.00 per year; but to sustain the stream of future earnings, the company needs to invest in real assets at the rate of $1.00 per share each year. Since an amount equal to each year's earnings must be channeled into new asset investment, no sustainable dividend payout, hence value, is possible. In general, even for a non-dividend-paying company, earnings will eventually outrun the firm's need for additional asset investment, creating the capacity to pay dividends.

The finance literature has produced three general approaches to determine value, each involving discounting three different streams of money: (1) the present value of expected dividends, (2) the present value of expected earnings net of required investment, and (3) the present value of the cash flows produced by assets. All three approaches are equivalent, provided they are properly formulated.[2]

[2] The equivalence between the three approaches is demonstrated in several financial texts. See for example Morin (2002) and Francis (2000).

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8.3 The Standard DCF Model

The general common stock valuation model embodied in Equation 8-5 is not very operational, since it requires an estimation of an infinite stream of dividends. But by assigning a particular configuration to the dividend stream, a more practical formula can be derived. A formal derivation of the standard DCF model is provided in Appendix 8-A. Basically, assuming that dividends grow at a constant rate forever, that is,

$$D_t = D_0(1 + g)^t \tag{8-6}$$

Where g = expected dividend per share growth
D_0 = current dividend per share
D_1 = expected dividend per share one year from now

and substituting these values of future dividends per share into Equation 8-5, the familiar reduced form of the general dividend valuation model is obtained:

$$P_0 = \frac{D_1}{K - g} \tag{8-7}$$

In words, this fundamental equation states that the market price of a share of common stock is the value of next year's expected dividend discounted at the market's required return net of the effect of growth. Solving the equation for K, the cost of equity capital, the standard DCF formulation widely used in regulatory proceedings is obtained:

$$K = \frac{D_1}{P_0} + g \tag{8-8}$$

This formula states that under certain simplifying assumptions discussed below, which investors frequently make, the equity investor's expected return, K, can be envisaged as the sum of an expected dividend yield, (D_1/P_0), plus the expected growth rate of future dividends, g. Investors set the equity price so as to obtain an appropriate return consistent with the risk of the investnent and with the return forgone in investments of comparable risk. The basic idea of the standard DCF approach to estimating the cost of equity capital is to infer K from the observed share price and from an estimate of investors' expected future dividends. The principal appeal of the approach is its simplicity and its correspondence with the intuitive notion of dividends plus capital appreciation as a measure of investors' total expected return. The assumptions underlying the model are discussed in detail below. Essentially, a constant average growth trend for both dividends and earnings, a stable dividend payout and capital structure policy, and a discount rate in excess of the expected growth rate are assumed. A simple example will illustrate the standard DCF model, sometimes referred to as the "annual" or "single-period" DCF model.

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EXAMPLE 8-1

We have the following market data for Utility X:

current dividend per share = $1.62
current stock price = $25.00
expected dividend growth = 4%

From Equation 8-8, the standard DCF model produces a cost of equity of:

$$K = D_1 / P_0 + g$$
$$= D_0(1+g) / P_0 + g$$
$$= \$1.62\ (1.04) / \$25 + .04$$
$$= 6.7\% + 4.0\% = 10.7\%$$

Note that next year's expected dividend is the current spot dividend increased by the expected growth rate in dividends. In general, implementation of the approach requires finding D_0 and P_0 from readily available sources of market data; the growth rate, g, can be estimated using several techniques. One way is to rely on analysts' long-term growth forecasts. Chapter 9 will discuss the application of the DCF formulation in detail.

Standard DCF Model Assumptions

The assumptions underlying the standard DCF model have been the source of controversy, confusion, and misunderstanding in rate hearings. This section clarifies these assumptions.

Theories are simplifications of reality and the models articulated from theories are necessarily abstractions from and simplifications of the existing world so as to facilitate understanding and explanation of the real world. The DCF model is no exception to the rule. The assumptions of the standard DCF model are as follows:

Assumption #1. The four assumptions discussed earlier in conjunction with the general classical theory of security valuation still remain in force.

Assumption #2. The discount rate, K, must exceed the growth rate, g. In other words, the standard DCF model does not apply to growth stocks. In Equation 8-7, it is clear that as g approaches K, the denominator gets progressively smaller, and the price of the stock infinitely large. If g exceeds K, the price becomes negative, an implausible situation. In the derivation of the standard

DCF equation (8-7) from the general stock valuation equation (8-5), it was necessary to assume g is less than K in order for the series of terms to converge toward a finite number. With this assumption, the present value of steadily growing dividends becomes smaller as the discounting effect of K in the denominator more than offsets the effect of such growth in the numerator.

This assumption is realistic for most public utilities. Investors require a return commensurate with the amount of risk assumed, and this return likely exceeds the expected growth rate in dividends for most public utilities. Although it is possible that a firm could sustain very high growth rates for a few years, no firm could double or triple its earnings and dividends indefinitely.

Assumption #3. The dividend growth rate is constant in every year to infinity. This assumption is not as problematic as it appears. It is not necessary that g be constant year after year to make the model valid. The growth rate may vary randomly around some average expected value. Random variations around trend are perfectly acceptable, as long as the mean expected growth is constant. The growth rate must be "expectationally constant," to use formal statistical jargon. This assumption greatly simplifies the model without detracting from its usefulness.

If investors expect growth patterns to prevail in the future other than constant infinite growth, more complex DCF models are available. For example, investors may expect dividends to grow at a relatively modest pace for the first 5 years and to resume a higher normal steady-state course thereafter, or conversely. The general valuation framework of Equation 8-5 can handle such situations. The "non-constant growth" model presented later in the chapter is a popular version of the DCF model.

It should be pointed out that the standard DCF model does not require infinite holding periods to remain valid. It simply assumes that the stock will be yielding the same rate of return at the time of sale as it is currently yielding. Example 8-2 illustrates this point.

Another way of stating this assumption is that the DCF model assumes that market price grows at the same rate as dividends. Although g has been specified in the model to be the expected rate of growth in dividends, it is also implicitly the expected rate of increase in stock price (expected capital gain) as well as the expected growth rate in earnings per share. This can be seen from Equation 8-7, which in period 1 would give:

$$P_1 = D_2 / (K - g)$$

But $\quad D_2 = D_1(1 + g)$ and $P_0 = D_1 / (K - g)$

so that $\quad P_1 = D_1(1 + g)/(K - g) = P_0(1 + g)$

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yield must be adjusted for the flotation cost allowance by dividing it by (1 − f), where f is the flotation cost factor.[6]

$$K = D_1/P_0\,(1 - f) + g \qquad (9\text{-}4)$$

9.3 Growth Estimates: Historical Growth

The principal difficulty in calculating the required return by the DCF approach is in ascertaining the growth rate that investors are currently expecting. While there is no infallible method for assessing what the growth rate is precisely, an explicit assumption about its magnitude cannot be avoided. Estimating the growth component is the most difficult and controversial step in implementing DCF since it is a quantity that lies buried in the minds of investors. Three general approaches to estimating expected growth can be used, each with its own strengths and blemishes:

1. historical growth rates
2. analysts' forecasts
3. sustainable growth rates

This section describes the historical growth approach while the next two sections address the other two approaches.

Historical growth rates in dividends, earnings, and book value are often used as proxies for investor expectations in DCF analysis. Investors are certainly influenced to some extent by historical growth rates in formulating their future growth expectations. In addition, these historical indicators are widely used by analysts, investors, and expert witnesses in regulatory proceedings, at least as a starting point in their company analyses. Professional certified financial analysts are also well-versed in the use of historical growth indicators. To wit, the calculation of historical growth rates is normally one of the first steps in security analysis. Historical indicators are also used extensively in scholarly research. There exists a vast literature in empirical finance designed to evaluate the use of historical financial information as surrogates for expected values. This literature is discussed in the next section.

When using historical growth rates in a regulatory environment, a convenient starting point is to focus on the utility in question, and to assume that its growth profile is relatively stable and predictable. Under circumstances of stability, it is reasonable to examine past growth trends in earnings, dividends,

[6] The conceptual and empirical support for the flotation cost adjustment is fully discussed in Chapter 10.

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and book values as proxies for investor expectations. The fundamental assumption is made that investors arrive at their expected growth forecast by simply extrapolating past history. In other words, historical growth rates influence investor anticipations of long-run growth rates.

In using historical growth rates, three decisions must be made: (1) which historical data series is most relevant; (2) over what past period; and (3) which computational method is most appropriate.

Historical Series

DCF proponents have variously based their historical computations on earnings per share, dividends per share, and book value per share. Of the three possible growth rate measures, growth in dividends per share is likely to be preferable, at least conceptually. DCF theory states clearly that it is expected future cash flows in the form of dividends that constitute investment value.

However, since the ability to pay dividends stems from a company's ability to generate earnings, growth in earnings per share can be expected to strongly influence the market's dividend growth expectations. After all, dividend growth can only be sustained if there is growth in earnings. It is the expectation of earnings growth that is the principal driver of stock prices. On the down side, using earnings growth as a surrogate for expected dividend growth can be problematic since historical earnings per share are frequently more volatile than dividends per share. Past growth rates of earnings per share tend to be very volatile and can sometimes lead to unreasonable results, such as negative growth rates. For example, in the 1990s and early 2000s, electric and gas company earnings growth rates were unstable and volatile, and such growth rates could not reasonably be expected to continue. Historically based DCF estimates of the cost of equity were downward-biased by the anemic historical growth rates of earnings and dividends in those years of major restructuring efforts, writeoffs, mergers and acquisitions, and shrinking profitability in the passage from a regulated monopoly to a competitive industry.

The relative stability of earnings and dividends is displayed in Figure 9-1 for The Southern Company. Under normal circumstances, dividend growth rates are not nearly as affected by year-to-year inconsistencies in accounting procedures as are earnings growth rates, and they are not as likely to be distorted by an unusually poor or bad year. Dividend growth is more stable than earnings growth because dividends reflect normalized long-term earnings rather than transitory earnings, because investors value stable dividends, and because companies are reluctant to cut dividends because of the information effect of dividend payments.

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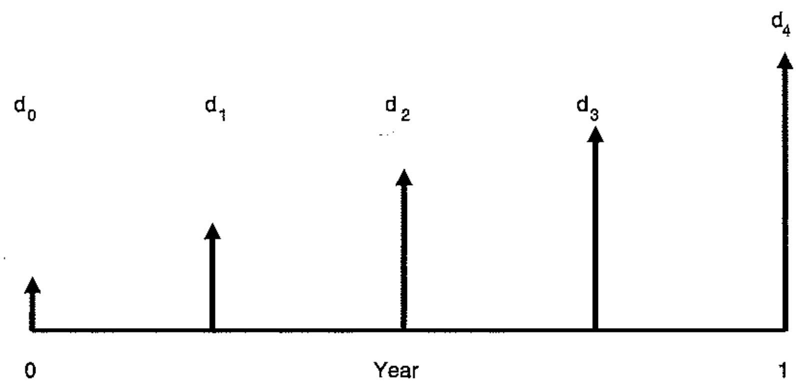
FIGURE 11-3
QUARTERLY DCF CONSTANT GROWTH



$$d_1 = d_0(1 + g)^{0.25}$$

$$d_2 = d_0(1 + g)^{0.50}$$

$$d_3 = d_0(1 + g)^{0.75}$$

$$d_4 = d_0(1 + g)^{1.00}$$

is computationally laborious. The following quarterly DCF model is a useful approximation and is far less laborious, although it does require the assumption that the company increases its dividend payments each quarter. The model assumes that each quarterly dividend differs from the previous one by $(1 + g)^{0.25}$, where g is the growth rate and the term 0.25 denotes one quarter of the year. Figure 11-3 shows the assumed dividend pattern. If it is assumed that dividends grow at a constant rate of g% every quarter starting from a base of d_0, the current quarterly rate, the company's stock price is given by:

$$P_0 = \sum_{n=1}^{4} \frac{d_0(1 + g)^{n/4}}{(1 + K)^{n/4}} + g$$

Which simplifies to:

$$P_0 = \frac{d_0(1 + g)^{1/4}}{(1 + K)^{1/4} - (1 + g)^{1/4}}$$

Solving the above equation for K, the simplified DCF formula for estimating the cost of equity under quarterly dividend payments emerges as Equation 11-4.

$$K = \left[\frac{d_0(1 + g)^{1/4}}{P_0} + (1 + g)^{1/4}\right] - 1 \qquad (11\text{-}4)$$

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volume of trading on public exchanges, and a ceiling on the amount of dividend cuts over a past period.

In defining a population of comparable-risk companies, care must be taken not to include other utilities in the sample, since the rate of return on other utilities depends on the allowed rate of return. The historical book return on equity for regulated firms is not determined by competitive forces but instead reflects the past actions of regulatory commissions. It would be circular to set a fair return based on the past actions of other regulators, much like observing a series of duplicate images in multiple mirrors. The rates of return earned by other regulated utilities may very well have been reasonable under historical conditions, but they are still subject to tests of reasonableness under current and prospective conditions.

Time Period

The cost of capital of a company refers to the expected long-run earnings level of other firms with similar risk. But a company's achieved earnings in any given year are likely to exceed or be less than their long-run average. Such deviations from expectations occur at the macroeconomic level as well. At the peak of the business cycle, firms generally earn more than their cost of capital, while at the trough the reverse is typical. Aggregating returns over a large number of comparable-risk unregulated firms averages the abnormally high and low rates of profitability in any given year. Furthermore, to dampen cyclical aberrations and remove the effects of cyclical peaks and troughs in profitability, an average over several time periods should be employed. The time period should include at least one full business cycle that is representative of prospective economic conditions for the next cycle. Such cyclical variations can be gauged by the official turning points in the U.S. business cycle, reported in *Business Conditions Digest.*

Averaging achieved returns over a full business cycle can serve as a reasonable compromise between the dual objectives of being representative of current economic conditions and of smoothing out cyclical fluctuations in earnings on unregulated firms. Some analysts confine their return study to the most recent time period. The most serious flaw of this approach is that historical returns on equity vary from year to year, responding to the cyclical forces of recession and expansion and to economic, industry-specific and company-specific trends. The most recent period is not likely to mirror expectations and be representative of prospective business conditions. Moreover, in the short run, reported book profitability frequently moves in the opposite direction to interest rates and to investors' required returns. For example, a period of disinflation and falling interest rates will increase company earnings and earned equity returns, while investors' return requirements are falling, and conversely.

Chapter 16
Weighted Average Cost of Capital

Traditionally, the allowed rate of return in regulatory hearings is calculated as the weighted average of the cost of each individual component of the capital structure weighted by its book value. This is illustrated in Table 16-1, where the capital structure, expressed as percent of book value, consists of 40% debt, 10% preferred stock, and 50% common stock, with individual cost rates of 8%, 6%, and 12%, respectively.

The estimated allowed rate of return of 9.8%, also known as the *weighted average cost of capital* ("WACC"), is then applied to the book value of the rate base to determine the total revenue requirements (costs of service) needed to service the capital employed by the utility.

Knowledge of the 9.8% allowed rate of return on total capital is not enough to determine the total cost of capital to the ratepayers, however, for it ignores the tax burden. Assuming a 50% tax rate, in order to provide a $1 return to the bondholders, the utility requires only $1 of revenue. But it takes $2 of pre-tax revenue to provide a $1 return to the preferred and common equity holders because the utility must pay corporate income taxes. Returning to the above example, if the rate base is $100 and the tax rate 50%, to provide a return of $3.20 on the bondholders' $40 investment, the utility requires $3.20 of pre-tax revenues. But to provide a return of $0.60 + $6.00 = $6.60 to the preferred and common equity holders' $60 investment, the regulatory commission must allow a profit of 2 x $6.60 = $13.20. From the ratepayers' viewpoint, the total cost of capital inclusive of taxes is $3.20 + $13.20 = $16.40, or 16.4%. The computation is shown on Table 16-2.

An alternate and equivalent computational procedure, shown in Table 16-3, is to express the cost of debt directly on an after-tax basis, and then compute the after-tax weighted average cost of capital ("ATWACC").

TABLE 16-1
ILLUSTRATIVE COST OF CAPITAL CALCULATION

Source	$$ Amount	% Weight	% Cost	Weighted Cost
Debt	$40	40%	8%	3.2%
Preferred	$10	10%	6%	0.6%
Equity	$50	50%	12%	6.0%
				9.8%

TABLE 16-2
ILLUSTRATIVE COST OF CAPITAL CALCULATION

Source	$$ Amount	% Weight	% Cost	Weighted Cost	Tax Factor	Capital Cost Including Tax
Debt	$40	40%	8%	3.2%	1.0	3.2%
Preferred	$10	10%	6%	0.6%	2.0	1.2%
Equity	$50	50%	12%	6.0%	2.0	12.0%
				9.8%		**16.4%**

Note: The tax factor is 1/(1 − tax rate): with a 50% corporate tax rate 1/(1 − 0.50) = 1/0.50 = 2.0

TABLE 16-3
ILLUSTRATIVE COST OF CAPITAL CALCULATION
Alternate Version

Source	% Weight	Return	After-tax Cost	Weighted Cost
Debt	40.0%	8.0%	4.0%	1.6%
Preferred	10.0%	6.0%	6.0%	0.6%
Equity	50.0%	12.0%	12.0%	6.0%
				8.2%

The resulting ATWACC is then multiplied by the tax factor to obtain directly the cost of capital inclusive of taxes. Going back to the above example, the after-tax cost of debt is 8% (1 − T) = 8% (1 − .50) = 4%, where T is the tax rate. The weighted cost of debt is then 1.6%, for a total WACC of 8.2%, instead of the 9.8% shown above. The pre-tax cost of capital is then simply the post-tax figure of 8.2% multiplied by the tax factor of 2, or 16.4%, the same figure obtained with the first procedure. Appendix 16-A shows that the dollar revenue requirement is the same whether the tax shield from debt financing is treated implicitly by multiplying the cost of debt by (1 − T) or explicitly as a separate line item in computing the revenue requirement.

More generally, if K_d and K_e are the costs of debt and equity, and W_d and W_e are, respectively, the weights of debt and equity to the total value of capital, the weighted average cost of capital, K, can be expressed as:

$$K = K_d W_d + K_e W_e \qquad (16\text{-}1)$$

Several issues regarding the WACC arise in regulatory proceedings, particularly with regard to the optimal set of weights W_d and W_e. Section 16.1

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When return distributions are normal, the characteristics of any investment can be measured with two variables—the expected return, which represents the opportunity in the investment, and the standard deviation or variance, which represents the danger. In this scenario, a rational investor, faced with a choice between two investments with the same standard deviation but different expected returns, will always pick the one with the higher expected return.

In the more general case, where distributions are neither symmetric nor normal, it is still conceivable that investors will choose between investments on the basis of only the expected return and the variance, if they possess utility functions that allow them to do so.[1] It is far more likely, however, that they prefer positive skewed distributions to negatively skewed ones, and distributions with a lower likelihood of jumps (lower kurtosis) over those with a higher likelihood of jumps (higher kurtosis). In this world, investors will trade off the good (higher expected returns and more positive skewness) against the bad (higher variance and kurtosis) in making investments.

In closing, it should be noted that the expected returns and variances that we run into in practice are almost always estimated using past returns rather than future returns. The assumption made when using historical variances is that past return distributions are good indicators of future return distributions. When this assumption is violated, as is the case when the asset's characteristics have changed significantly over time, the historical estimates may not be good measures of risk.

***optvar.xls:* This is a dataset on the Web that summarizes standard deviations and variances of stocks in various sectors in the United States.**

Diversifiable and Nondiversifiable Risk

Although there are many reasons why actual returns may differ from expected returns, we can group the reasons into two categories: firm-specific and marketwide. The risks that arise from firm-specific actions affect one or a few investments, while the risks arising from marketwide reasons affect many or all investments. This distinction is critical to the way we assess risk in finance.

Components of Risk When an investor buys stock or takes an equity position in a firm, he or she is exposed to many risks. Some risk may affect only one or a few firms, and this risk is categorized as firm-specific risk. Within this category, we would consider a wide range of risks, starting with the risk that a firm may have misjudged the demand for a product from its customers; we call this project risk. For instance, consider

[1]A utility function is a way of summarizing investor preferences into a generic term called "utility" on the basis of some choice variables. In this case, for instance, the investors' utility or satisfaction is stated as a function of wealth. By doing so, we effectively can answer questions such as, Will investors be twice as happy if they have twice as much wealth? Does each marginal increase in wealth lead to less additional utility than the prior marginal increase? In one specific form of this function, the quadratic utility function, the entire utility of an investor can be compressed into the expected wealth measure and the standard deviation in that wealth.

Boeing's investment in a Super Jumbo jet. This investment is based on the assumption that airlines want a larger airplane and are willing to pay a high price for it. If Boeing has misjudged this demand, it will clearly have an impact on Boeing's earnings and value, but it should not have a significant effect on other firms in the market. The risk could also arise from competitors proving to be stronger or weaker than anticipated, called competitive risk. For instance, assume that Boeing and Airbus are competing for an order from Qantas, the Australian airline. The possibility that Airbus may win the bid is a potential source of risk to Boeing and perhaps some of its suppliers, but again, few other firms will be affected by it. Similarly, Disney recently launched magazines aimed at teenage girls, hoping to capitalize on the success of its TV shows. Whether it succeeds is clearly important to Disney and its competitors, but it is unlikely to have an impact on the rest of the market. In fact, risk measures can be extended to include risks that may affect an entire sector but are restricted to that sector; we call this sector risk. For instance, a cut in the defense budget in the United States will adversely affect all firms in the defense business, including Boeing, but there should be no significant impact on other sectors. What is common across the three risks described—project, competitive, and sector risk—is that they affect only a small subset of firms.

There is another group of risks that is much more pervasive and affects many if not all investments. For instance, when interest rates increase, all investments are negatively affected, albeit to different degrees. Similarly, when the economy weakens, all firms feel the effects, though cyclical firms (such as automobiles, steel, and housing) may feel it more. We term this risk market risk.

Finally, there are risks that fall in a gray area, depending on how many assets they affect. For instance, when the dollar strengthens against other currencies, it has a significant impact on the earnings and values of firms with international operations. If most firms in the market have significant international operations, it could well be categorized as market risk. If only a few do, it would be closer to firm-specific risk. Figure 4.4 summarizes the spectrum of firm-specific and market risks.

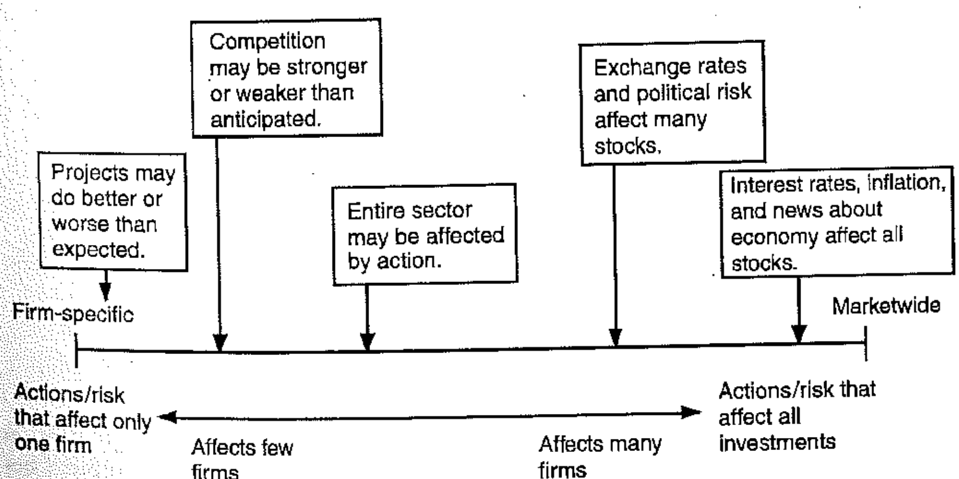

FIGURE 4.4 Breakdown of Risk

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Why Diversification Reduces or Eliminates Firm-Specific Risk: An Intuitive Explanation As an investor, you could invest all your portfolio in one asset. If you do so, you are exposed to both firm-specific and market risk. If, however, you expand your portfolio to include other assets or stocks, you are diversifying, and by doing so you can reduce your exposure to firm-specific risk. There are two reasons why diversification reduces or, at the limit, eliminates firm-specific risk. The first is that each investment in a diversified portfolio is a much smaller percentage of that portfolio than would be the case if you were not diversified. Any action that increases or decreases the value of only that investment or a small group of investments will have only a small impact on your overall portfolio, whereas undiversified investors are much more exposed to changes in the values of the investments in their portfolios. The second reason is that the effects of firm-specific actions on the prices of individual assets in a portfolio can be either positive or negative for each asset for any period. Thus, in very large portfolios this risk will average out to zero and will not affect the overall value of the portfolio.

In contrast, the effects of marketwide movements are likely to be in the same direction for most or all investments in a portfolio, though some assets may be affected more than others. For instance, other things being equal, an increase in interest rates will lower the values of most assets in a portfolio. Being more diversified does not eliminate this risk.

A Statistical Analysis of Diversification-Reducing Risk The effects of diversification on risk can be illustrated fairly dramatically by examining the effects of increasing the number of assets in a portfolio on portfolio variance. The variance in a portfolio is partially determined by the variances of the individual assets in the portfolio and partially by how they move together; the latter is measured statistically with a correlation coefficient or the covariance across investments in the portfolio. It is the covariance term that provides an insight into why diversification will reduce risk and by how much.

Consider a portfolio of two assets. Asset A has an expected return of μ_A and a variance in returns of σ^2_A, while asset B has an expected return of μ_B and a variance in returns of σ^2_B. The correlation in returns between the two assets, which measures how the assets move together, is ρ_{AB}. The expected returns and variances of a two-asset portfolio can be written as a function of these inputs and the proportion of the portfolio going to each asset.

$$\mu_{\text{portfolio}} = w_A\mu_A + (1 - w_A)\mu_B$$

$$\sigma^2_{\text{portfolio}} = w_A^2\sigma_A^2 + (1 - w_A)^2\sigma_B^2 + 2w_A(1 - w_A)\rho_{AB}\sigma_A\sigma_B$$

where w_A = Proportion of the portfolio in asset A

The last term in the variance formulation is sometimes written in terms of the covariance in returns between the two assets, which is:

$$\sigma_{AB} = \rho_{AB}\sigma_A\sigma_B$$

The savings that accrue from diversification are a function of the correlation coefficient. Other things remaining equal, the higher the correlation in returns between the two assets, the smaller are the potential benefits from diversification. It is rationale presented by those who use shorter periods is that the risk aversion of the average investor is likely to change over time, and that using a shorter time period provides a more updated estimate. This has to be offset against a cost associated with using shorter time periods, which is the greater noise in the risk premium estimate. In fact, given the annual standard deviation in stock prices[7] between 1926 and 2010 of 20 percent, the standard error[8] associated with the risk premium estimate can be estimated for different estimation periods in Table 7.2.

Note that to get reasonable standard errors, we need very long time periods of historical returns. Conversely, the standard errors from 10-year and 20-year estimates are likely to be almost as large as or larger than the actual risk premium estimated. This cost of using shorter time periods seems, in our view, to overwhelm any advantages associated with getting a more updated premium.

2. *Choice of risk-free security.* The Ibbotson database reports returns on both Treasury bills (T-bills) and Treasury bonds (T-bonds), and the risk premium for stocks can be estimated relative to each. Given that the yield curve in the United States has been upward-sloping for most of the past seven decades, the risk premium is larger when estimated relative to shorter-term government securities (such as Treasury bills). *The risk-free rate chosen in computing the premium has to be consistent with the risk-free rate used to compute expected returns.* Thus, if the Treasury bill rate is used as the risk-free rate, the premium has to be the premium earned by stocks over that rate. If the Treasury bond rate is used as the risk-free rate, the premium has to be estimated relative to that rate. For the most part, in corporate finance and valuation, the risk-free rate will be a long-term default-free Treasury (government) bond rate and not a Treasury bill rate. Thus, the risk premium used should be the premium earned by stocks over Treasury bonds.

3. *Arithmetic and geometric averages.* The final sticking point when it comes to estimating historical premiums relates to how the average returns on stocks, Treasury bonds, and Treasury bills are computed. The arithmetic average return measures the simple mean of the series of annual returns, whereas the geometric

TABLE 7.2 Standard Errors in Risk Premium Estimates

Estimation Period	Standard Error of Risk Premium Estimate
5 years	$20\%/\sqrt{5} = 8.94\%$
10 years	$20\%/\sqrt{10} = 6.32\%$
25 years	$20\%/\sqrt{25} = 4.00\%$
50 years	$20\%/\sqrt{50} = 2.83\%$

[7]For the historical data on stock returns, bond returns, and bill returns, check under "Updated Data" in www.stern.nyu.edu/~adamodar.

[8]These estimates of the standard error are probably understated, because they are based on the assumption that annual returns are uncorrelated over time. There is substantial empirical evidence that returns are correlated over time, which would make this standard error estimate much larger.

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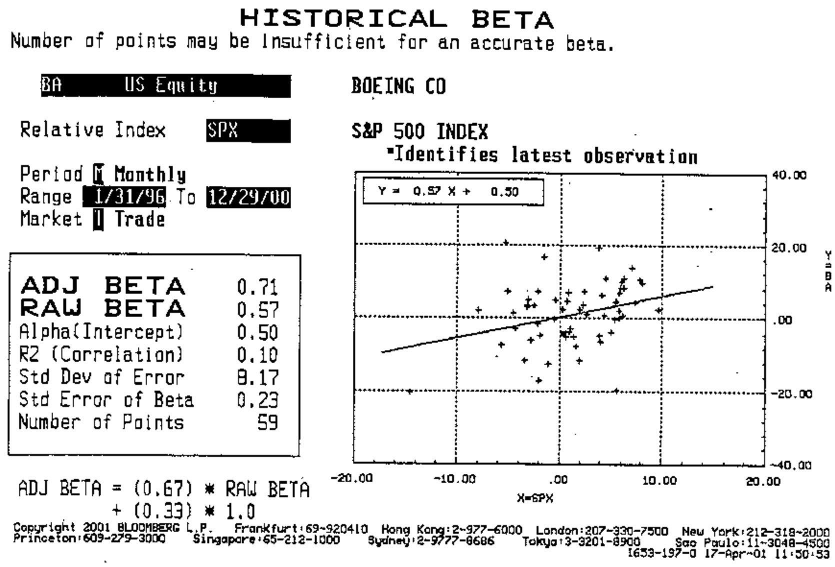

FIGURE 8.2 Beta Estimate for Boeing
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from Bloomberg for Boeing, using the same period as our regression (January 1996 to December 2000).

While the time period used is identical to the one used in our earlier regression, there are subtle differences between this regression and the one in Figure 8.1. First, Bloomberg uses price appreciation in the stock and the market index in estimating betas and ignores dividends.[4] The fact that dividends are ignored does not make much difference for a company like Boeing, but it could make a difference for a company that either pays no dividends or pays significantly higher dividends than the market. This explains the mild differences in the intercept (0.50% versus 0.54%) and the beta (0.57 versus 0.56).

Second, Bloomberg also computes what it calls an adjusted beta, which is estimated as follows:

$$\text{Adjusted beta} = \text{Raw beta}(0.67) + 1.00(0.33)$$

These weights (0.67 and 0.33) do not vary across stocks, and this process pushes all estimated betas toward 1. Most services employ similar procedures to adjust betas toward 1. In doing so, they are drawing on empirical evidence that suggests that the betas for most companies, over time, tend to move toward the average beta, which is 1. This may be explained by the fact that firms get more diversified in their product mix and client base as they get larger. While we agree with the notion that betas move toward 1 over time, the weighting process used by most services strikes us as arbitrary and not particularly useful.

[4]This is done purely for computational convenience.

The unlevered beta of a firm is determined by the nature of its products and services (cyclicality, discretionary nature) and its operating leverage. It is often also referred to as the asset beta, since it is determined by the assets owned by the firm. Thus, the levered beta, which is also the beta for an equity investment in a firm, is determined both by the riskiness of the business it operates in and by the amount of financial leverage risk it has taken on.

Since financial leverage multiplies the underlying business risk, it stands to reason that firms that have high business risk should be reluctant to take on financial leverage. It also stands to reason that firms that operate in stable businesses should be much more willing to take on financial leverage. Utilities, for instance, have historically had high debt ratios but have not had high betas, mostly because their underlying businesses have been stable and fairly predictable.

ILLUSTRATION 8.3: Effects of Leverage on Betas: Boeing

From the regression for the period from 1996 to 2000, Boeing had a historical beta of 0.56. Since this regression uses stock prices of Boeing over this period, we began by estimating the average debt-to-equity ratio between 1996 and 2000, using market values for debt and equity.

$$\text{Average debt-to-equity ratio between 1996 and 2000} = 15.56\%$$

The beta over the 1996–2000 period reflects this average leverage. To estimate the unlevered beta over the period, a marginal tax rate of 35% is used:

$$\begin{aligned}\text{Unlevered beta} &= \text{Current beta}/[1 + (1 - \text{Tax rate})(\text{Average debt/Equity})]\\ &= 0.56/[1 + (1 - 0.35)(0.1556)] = 0.51\end{aligned}$$

The unlevered beta for Boeing over the 1996–2000 period is 0.51. The levered beta at different levels of debt can then be estimated:

$$\text{Levered beta} = \text{Unlevered beta} \times [1 + (1 - \text{Tax rate})(\text{Debt/Equity})]$$

For instance, if Boeing were to increase its debt equity ratio to 10%, its equity beta will be:

$$\text{Levered beta (@10\% D/E)} = 0.51 \times [1 + (1 - 0.35)(0.10)] = 0.543$$

If the debt equity ratio were raised to 25%, the equity beta would be:

$$\text{Levered beta (@25\% D/E)} = 0.51 \times [1 + (1 - 0.35)(0.25)] = 0.59$$

The following table summarizes the beta estimates for different levels of financial leverage ranging from 0% to 90% debt.

Debt to Capital	*Debt/Equity Ratio*	*Beta*	*Effect of Leverage*
0%	0.00%	0.51	0.00
10%	11.11%	0.55	0.04
20%	25.00%	0.59	0.08
30%	42.86%	0.65	0.14
40%	66.67%	0.73	0.22
50%	100.00%	0.84	0.33
60%	150.00%	1.00	0.50
70%	233.33%	1.28	0.77
80%	400.00%	1.83	1.32
90%	900.00%	3.48	2.98

As Boeing's financial leverage increases, the beta increases concurrently.

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***levbeta.xls.* This spreadsheet allows you to estimate the unlevered beta for a firm and compute the betas as a function of the leverage of the firm.**

Bottom-Up Betas Breaking down betas into their business risk and financial leverage components provides us with an alternative way of estimating betas, in which we do not need past prices on an individual firm or asset to estimate its beta.

To develop this alternative approach, we need to introduce an additional property of betas that proves invaluable. The beta of two assets put together is a weighted average of the individual asset betas, with the weights based on market value. Consequently, the beta for a firm is a weighted average of the betas of all the different businesses it is in. We can estimate the beta for a firm in five steps:

Step 1: Identify the business or businesses the firm operates in.

Step 2: Find other publicly traded firms in each business and obtain their regression betas, which we use to compute an average beta for the firms.

Step 3: Estimate the average unlevered beta for the business by unlevering the average (or median) beta for the firms by their average (or median) debt-to-equity ratio. Alternatively, we could estimate the unlevered beta for each firm and then compute the average of the unlevered betas. The first approach is preferable because unlevering an erroneous regression beta is likely to compound the error.

$$\text{Unlevered beta}_{\text{business}} = \text{Beta}_{\text{comparable firms}}/[1 + (1 - t)(\text{D/E ratio}_{\text{comparable firms}})]$$

Step 4: Estimate an unlevered beta for the firm being analyzed, taking a weighted average of the unlevered betas for the businesses it operates in, using the proportion of firm value derived from each business as the weights. If values are not available, use operating income or revenues as weights. This weighted average is called the bottom-up unlevered beta.

$$\text{Unlevered beta}_{\text{firm}} = \sum_{j=1}^{j=k}\left(\text{Unlevered beta}_j \times \text{Value weight}_j\right)$$

where the firm is assumed to operating in k different businesses.

Step 5: Finally, estimate the current market values of debt and equity at the firm and use this debt-to-equity ratio to estimate a levered beta.

The betas estimated using this processs are called bottom-up betas.

The Case for Bottom-Up Betas At first sight, the use of bottom-up betas may seem to leave us exposed to all of the problems noted with regression betas. After

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series of correlations between growth rates in consecutive periods of different length, he frequently found negative correlations between growth rates in the two periods, and the average correlation across the two periods was close to zero (0.02).

If past growth is not a reliable indicator of future growth at many firms, it becomes even less so at smaller firms. The growth rates at smaller firms tend to be more volatile than growth rates at other firms in the market. The correlation between growth rates in earnings in consecutive time periods (five-year, three-year, and one-year) for firms in the United States, categorized by market value, is reported in Figure 11.2.

While the correlations tend to be higher across the board for one-year growth rates than for three-year or five-year growth rates in earnings, they are also consistently lower for smaller firms than they are for the rest of the market. This would suggest that you should be more cautious about using past growth, especially in earnings, for forecasting future growth at these firms.

Revenue Growth versus Earnings Growth In general, revenue growth tends to be more persistent and predictable than earnings growth. This is because accounting choices have a far smaller effect on revenues than they do on earnings. Figure 11.3 compares the correlations in revenue and earnings growth over five-year periods at U.S. firms. Revenue growth is consistently more correlated over time than earnings growth. The implication is that historical growth in revenues is a far more useful number when it comes to forecasting than historical growth in earnings.

Effects of Firm Size Since the growth rate is stated in percentage terms, the role of the size of the firm has to be weighed in the analysis. It is easier for a firm with $10 million in earnings to generate a 50 percent growth rate than it is for a firm with $500 million in earnings. Since it becomes harder for firms to sustain high growth rates as they become larger, past growth rates for firms that have grown dramatically in size may be difficult to sustain in the future. While this is a problem for all firms, it is a particular problem when analyzing small and growing firms.

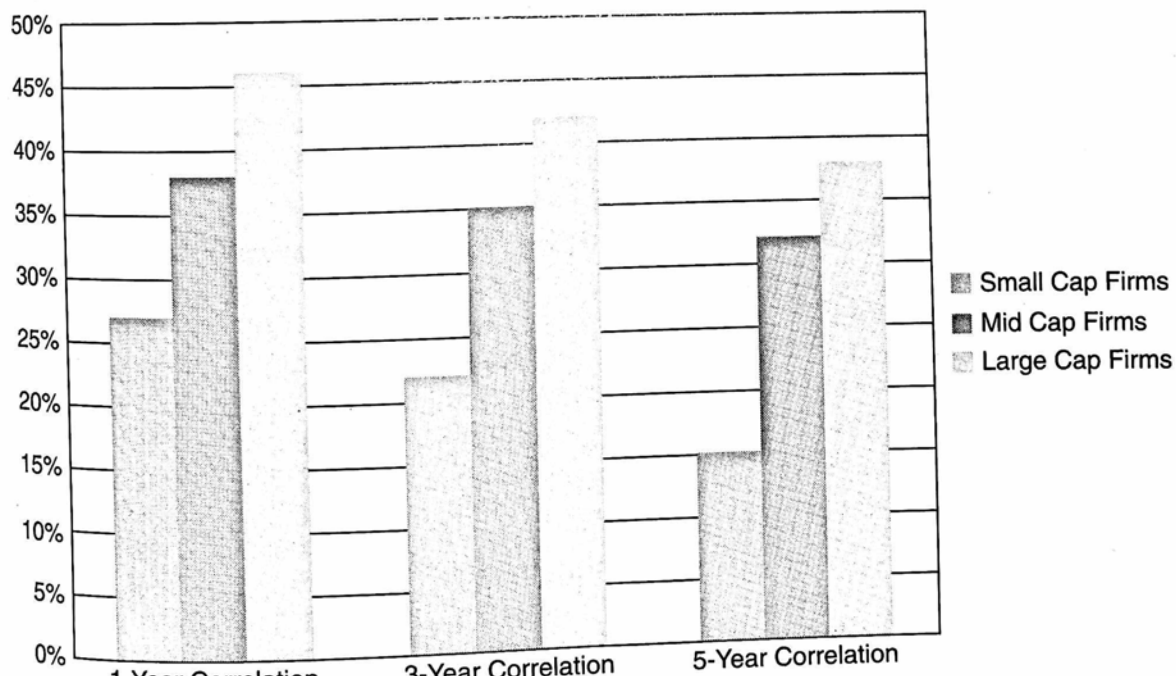

FIGURE 11.2 Correlation in Earnings: One-, Three-, and Five-Year Time Periods

3. *Extent of disagreement between analysts.* While consensus earnings growth rates are useful in valuation, the extent of disagreement between analysts measured by the standard deviation in growth predictions is also a useful measure of the reliability of the consensus forecasts. Givoly and Lakonsihok (1984) found that the dispersion of earnings is correlated with other measures of risk such as beta and is a good predictor of expected returns.

4. *Quality of analysts following the stock.* This is the hardest of the variables to quantify. One measure of quality is the size of the forecast error made by analysts following a stock, relative to models that use only historical data—the smaller this relative error, the larger the weight that should be attached to analyst forecasts. Another measure is the effect on stock prices of analyst revisions—the more informative the forecasts, the greater the effect on stock prices. There are some who argue that the focus on consensus forecasts misses the point that some analysts are better than others in predicting earnings, and that their forecasts should be isolated from the rest and weighted more.

Analyst forecasts may be useful in coming up with a predicted growth rate for a firm, but there is a danger to blindly following consensus forecasts. Analysts often make significant errors in forecasting earnings, partly because they depend on the same data sources (which might have been erroneous or misleading) and partly because they sometimes overlook significant shifts in the fundamental characteristics of the firm. The secret to successful valuation often lies in discovering inconsistencies between analysts' forecasts of growth and a firm's fundamentals. The next section examines this relationship in more detail.

FUNDAMENTAL DETERMINANTS OF GROWTH

With both historical and analyst estimates, growth is an exogenous variable that affects value but is divorced from the operating details of the firm. The soundest way of incorporating growth into value is to make it endogenous i.e., tie in more closely to the actions that a business takes to create and sustain that growth. This section begins by considering the relationship between fundamentals and growth in equity income, and then moves on to look at the determinants of growth in operating income.

Growth in Equity Earnings

When estimating cash flows to equity, we usually begin with estimates of net income, if we are valuing equity in the aggregate, or earnings per share, if we are valuing equity per share. This section begins by presenting the fundamentals that determine expected growth in earnings per share and then move on to consider a more expanded version of the model that looks at growth in net income.

Growth in Earnings per Share The simplest relationship determining growth is one based on the retention ratio (percentage of earnings retained in the firm) and the return on equity on its projects. Firms that have higher retention ratios and earn higher returns on equity should have much higher growth rates in earnings per share than firms that do not share these characteristics. To establish this, note that:

$$g_t = (NI_t - NI_{t-1})/NI_{t-1}$$

where g_t = Growth rate in net income
NI_t = Net income in year t

Also note that the ROE in period t can be written as NI in period t divided by the Book value of equity in period t – 1. Given the definition of return on equity, the net income in year t – 1 can be written as:

$$NI_{t-1} = \text{Book value of equity}_{t-2} \times ROE_{t-1}$$

where ROE_{t-1} = Return on equity in year t – 1

The net income in year t can be written as:

$$NI_t = (\text{Book value of equity}_{t-2} + \text{Retained earnings}_{t-1}) \times ROE_t$$

Assuming that the return on equity is unchanged (i.e., $ROE_t = ROE_{t-1} = ROE$):

$$\begin{aligned} g_t &= \text{Retained earnings}_{t-1}/NI_{t-1} \times ROE \\ &= \text{Retention ratio} \times ROE \\ &= b \times ROE \end{aligned}$$

where b is the retention ratio. Note that the firm is not being allowed to raise equity by issuing new shares. Consequently, the growth rate in net income and the growth rate in earnings per share are the same in this formulation.

ILLUSTRATION 11.5: Growth in Earnings per Share

This illustration considers the expected growth rate in earnings based on the retention ratio and return on equity for three firms—Consolidated Edison, a regulated utility that provides power to New York City and its environs; Procter & Gamble, a leading brand-name consumer product firm; and Intel, the technology giant—in 2010. The following table summarizes the returns on equity, retention ratios, and expected growth rates in earnings for the three firms in 2010:

	Return on Equity	*Retention Ratio*	*Expected Growth Rate*
Consolidated Edison	9.79%	36.00%	3.52%
Procter & Gamble	20.09%	50.26%	10.10%
Intel	32.00%	70.00%	22.40%

Intel has the highest expected growth rate in earnings per share, assuming that it can maintain its current return on equity and retention ratio. Procter & Gamble also can be expected to post a healthy growth rate, notwithstanding the fact that it pays out more than 50% of its earnings as dividends because of its high return on equity. Con Ed, on the other hand, has a very low expected growth rate because its return on equity and retention ratio are anemic.

Both measures—the reinvestment rate and return on capital—should be forward looking, and the return on capital should represent the expected return on capital on future investments. In the rest of this section, we consider how best to estimate the reinvestment rate and the return on capital.

Reinvestment Rate The reinvestment rate measures how much a firm is plowing back to generate future growth. The reinvestment rate is often measured using the most recent financial statements for the firm. Although this is a good place to start, it is not necessarily the best estimate of the future reinvestment rate. A firm's reinvestment rate can ebb and flow, especially in firms that invest in relatively few large projects or acquisitions. For these firms, looking at an average reinvestment rate over time may be a better measure of the future. In addition, as firms grow and mature, their reinvestment needs (and rates) tend to decrease. For firms that have expanded significantly over the last few years, the historical reinvestment rate is likely to be higher than the expected future reinvestment rate. For these firms, industry averages for reinvestment rates may provide a better indication of the future than using numbers from the past. Finally, it is important that we continue treating R&D expenses and operating lease expenses consistently. The R&D expenses, in particular, need to be categorized as part of capital expenditures for purposes of measuring the reinvestment rate.

Return on Capital The return on capital is often based on the firm's return on capital on existing investments, where the book value of capital is assumed to measure the capital invested in these investments. Implicitly, we assume that the current accounting return on capital is a good measure of the true returns earned on existing investments, and that this return is a good proxy for returns that will be made on future investments. This assumption, of course, is open to question for the following reasons:

- The book value of capital might not be a good measure of the capital invested in existing investments, since it reflects the historical cost of these assets and accounting decisions on depreciation. When the book value understates the capital invested, the return on capital will be overstated; when book value overstates the capital invested, the return on capital will be understated. This problem is exacerbated if the book value of capital is not adjusted to reflect the value of the research asset or the capital value of operating leases.
- The operating income, like the book value of capital, is an accounting measure of the earnings made by a firm during a period. All the problems in using unadjusted operating income described in Chapter 9 continue to apply.
- Even if the operating income and book value of capital are measured correctly, the return on capital on existing investments may not be equal to the marginal return on capital that the firm expects to make on new investments, especially as you go further into the future.

Given these concerns, we should consider not only a firm's current return on capital, but any trends in this return as well as the industry average return on capital. If the current return on capital for a firm is significantly higher than the industry average, the forecasted return on capital should be set lower than the current return to reflect the erosion that is likely to occur as competition responds.

Finally, any firm that earns a return on capital greater than its cost of capital is earning an excess return. The excess returns are the result of a firm's competitive advantages or barriers to entry into the industry. High excess returns locked in for very long periods imply that this firm has a permanent competitive advantage.

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ILLUSTRATION 11.9: Measuring the Reinvestment Rate, Return on Capital, and Expected Growth Rate: Tata Motors in 2010

In May 2010, we looked at Tata Motors, an Indian automobile company, which has been aggressive in its pursuit of growth through both internal investments and acquisitions over much of the past decade. Based upon its financial statements of 2009, we estimated a reinvestment rate of 116.83% and a return on capital of 11.81%:

$$\text{Reinvestment rate}_{\text{Tata Motors}} = \frac{\text{Net capital expendutures + Change in noncash WC}}{\text{EBIT }(1-t)}$$

$$= \frac{(\text{Rs } 40{,}291 - \text{Rs } 25{,}072) + \text{Rs } 957}{\text{Rs } 17{,}527(1-21)} = 116.83\%$$

$$\text{Return on capital} = \frac{\text{EBIT }(1-t)\text{ in 2009}}{(\text{BV of Equity + BV of Debt} - \text{cash})_{\text{End of 2008}}}$$

$$= \frac{\text{Rs } 17{,}527(1-.21)}{\text{Rs } 78.395 + \text{Rs } 62{,}805 - \text{Rs } 23{,}973} = 11.81\%$$

Note that the effective tax rate (21%) was used to compute the after-tax operating income for both the reinvestment rate and the return on capital. The capital invested was obtained by summing up the book value of debt and equity at the end of the 2008 fiscal year (the beginning of the 2009 fiscal year) and netting out the cash and marketable securities at that point in time.

If Tata Motors can maintain this return on capital and reinvestment rate going forward, its expected growth rate would be:

$$\text{Expected growth rate} = \text{Reinvestment rate} \times \text{Return on capital}$$
$$= 116.83\% \times 11.81\% = 13.80\%$$

As we will see in the next illustration, maintaining this reinvestment going forward may be very difficult to do.

ILLUSTRATION 11.10: Current and Historical Averages: Reinvestment Rate and Return on Capital for Tata Motors

Tata Motors has had a volatile history in terms of both reinvestment and returns on capital. Although the 2009 numbers were computed in the preceding illustration, those values have been in flux over the past five years. We summarize the numbers (in millions of rupees) for 2005 to 2009, with the aggregate in the last column:

	2005	*2006*	*2007*	*2008*	*2009*	*Aggregate*
EBIT(1 – t)	$12,197	$12,322	$25,203	$15,160	$13,846	$ 78,728
Capital Expenditures	$ 8,175	$11,235	$24,612	$44,113	$40,291	$120,251
Depreciation	$ 5,377	$ 6,274	$ 6,850	$ 7,826	$25,072	$ 46,022
Change in WC	$ 4,410	$23,191	$ 4,520	–$37,137	$ 957	–$ 8,469
Reinvestment	$ 7,208	$28,152	$22,282	–$ 850	$16,176	$ 65,760
Reinvestment Rate	59.10%	228.46%	88.41%	–5.61%	116.83%	83.53%

The reinvestment rate has swung between –5.61% and 228.46% over the period, but the aggregate reinvestment rate over the period was 83.53%.

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the market, the valuation becomes a relative valuation, rather than a discounted cash flow valuation. If the multiple is estimated using fundamentals, it converges on the stable growth model that is described in the next section.

All in all, using multiples to estimate terminal value, when those multiples are estimated from comparable firms, results in a dangerous mix of relative and discounted cash flow valuation. While there are advantages to relative valuation, and we consider these in a later chapter, a discounted cash flow valuation should provide you with an estimate of intrinsic value, not relative value. Consequently, the only consistent way of estimating terminal value in a discounted cash flow model is to use either a liquidation value or a stable growth model.

Stable Growth Model

In the liquidation value approach, you are assuming that your firm has a finite life and that it will be liquidated at the end of that life. Firms, however, can reinvest some of their cash flows back into new assets and extend their lives. If you assume that cash flows, beyond the terminal year, will grow at a constant rate forever, the terminal value can be estimated as follows:

$$\text{Terminal value}_t = \text{Cash flow}_{t+1}/(r - \text{Stable growth})$$

The cash flow and the discount rate used will depend on whether you are valuing the firm or valuing equity. If you are valuing equity, the terminal value of equity can be written as:

$$\text{Terminal value of equity}_n = \text{Cash flow to equity}_{n+1}/(\text{Cost of equity}_{n+1} - g_n)$$

The cash flow to equity can be defined strictly as dividends (in the dividend discount model) or as free cash flow to equity. If valuing a firm, the terminal value can be written as:

$$\text{Terminal value}_n = \text{Free cash flow to firm}_{n+1}/(\text{Cost of capital}_{n+1} - g_n)$$

where the cost of capital and the growth rate in the model are sustainable forever.

In this section, we will begin by considering how high a stable growth rate can be, how to best estimate when your firm will be a stable growth firm, and what inputs need to be adjusted as a firm approaches stable growth.

Constraints on Stable Growth Of all the inputs into a discounted cash flow valuation model, none creates as much angst as estimating the stable growth rate. Part of the reason for it is that small changes in the stable growth rate can change the terminal value significantly, and the effect gets larger as the growth rate approaches the discount rate used in the estimation.

The fact that a stable growth rate is constant forever, however, puts strong constraints on how high it can be. Since no firm can grow forever at a rate higher than the growth rate of the economy in which it operates, the constant growth rate cannot be greater than the overall growth rate of the economy. In making a judgment on what the limits on a stable growth rate are, we have to consider the following three questions:

1. *Is the company constrained to operate as a domestic company, or does it operate (or have the capacity to operate) multinationally?* If a firm is a purely domestic company, either because of internal constraints (such as those imposed by management) or external constraints (such as those imposed by a government), the growth rate in the domestic economy will be the limiting value. If the company is a multinational or has aspirations to be one, the growth rate in the global economy (or at least those parts of the globe that the firm operates in) will be the limiting value.
2. *Is the valuation being done in nominal or real terms?* If the valuation is a nominal valuation, the stable growth rate should also be a nominal growth rate (i.e., include an expected inflation component). If the valuation is a real valuation, the stable growth rate will be constrained to be lower. Using a U.S. company in 2011 as an example, the stable growth rate can be as high as 2.0 percent if the valuation is done in nominal U.S. dollars but only 1 percent if the valuation is done in real terms.
3. *What currency is being used to estimate cash flows and discount rates in the valuation?* The limits on stable growth will vary depending on what currency is used in the valuation. If a high-inflation currency is used to estimate cash flows and discount rates, the stable growth rate will be much higher, since the expected inflation rate is added on to real growth. If a low-inflation currency is used to estimate cash flows, the stable growth rate will be much lower. For instance, the stable growth rate that would be used to value Cemex, the Mexican cement company, will be much higher if the valuation is done in Mexican pesos than in U.S. dollars.

Although the stable growth rate cannot exceed the growth rate of the economy in which a firm operates, it can be lower. There is nothing that prevents us from assuming that mature firms will become a smaller part of the economy and it may, in fact, be the more reasonable assumption to make. Note that the growth rate of an economy reflects the contributions of both young, higher-growth firms and mature, stable-growth firms. If the former grow at a rate much higher than the growth rate of the economy, the latter have to grow at a rate that is lower.

Setting the stable growth rate to be less than or equal to the growth rate of the economy not only is the consistent thing to do but it also ensures that the growth rate will be less than the discount rate. This is because there is a link between the riskless rate that goes into the discount rate and the growth rate of the economy. Note that the riskless rate can be written as:

$$\text{Nominal riskless rate} = \text{Real riskless rate} + \text{Expected inflation rate}$$

In the long term, the real riskless rate will converge on the real growth rate of the economy, and the nominal riskless rate will approach the nominal growth rate of the economy. In fact, a simple rule of thumb on the stable growth rate is that it generally should not exceed the riskless rate used in the valuation.

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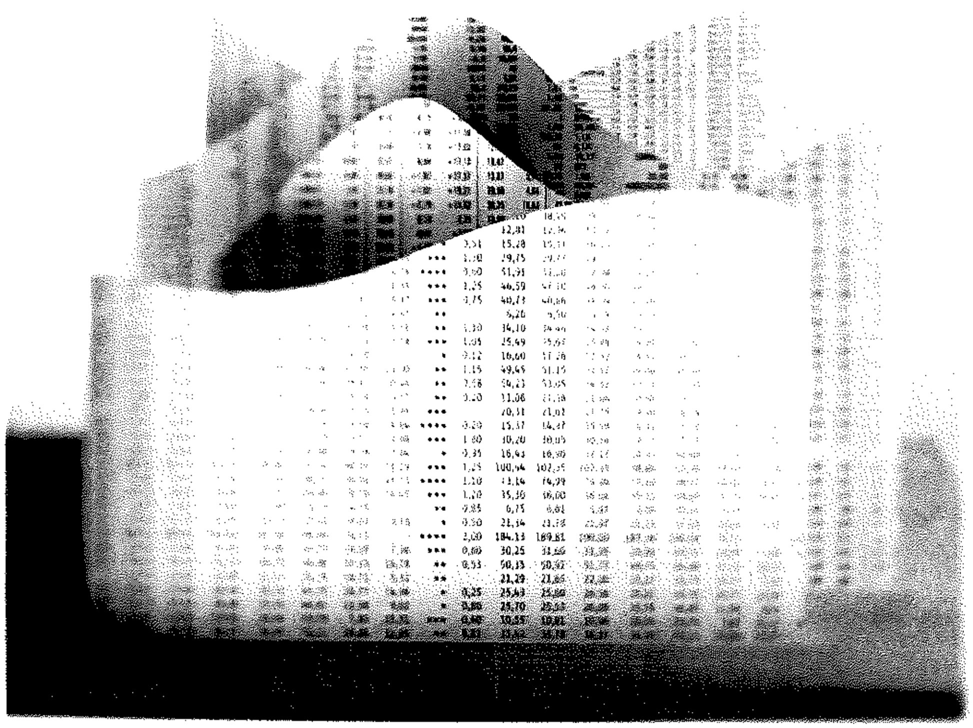
Essentials of Investments



Ninth Edition

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6.1 DIVERSIFICATION AND PORTFOLIO RISK

Suppose you have in your risky portfolio only one stock, say, Dell Computers. What are the sources of risk affecting this "portfolio"?

We can identify two broad sources of uncertainty. The first is the risk from general economic conditions, such as business cycles, inflation, interest rates, exchange rates, and so forth. None of these macroeconomic factors can be predicted with certainty, and all affect Dell stock. Then you must add firm-specific influences, such as Dell's success in R&D, its management style and philosophy, and so on. Firm-specific factors are those that affect Dell without noticeably affecting other firms.

Now consider adding another security to the risky portfolio. If you invest half of your risky portfolio in ExxonMobil, leaving the other half in Dell, what happens to portfolio risk? Because the firm-specific influences on the two stocks differ (statistically speaking, the influences are independent), this strategy should reduce portfolio risk. For example, when oil prices fall, hurting ExxonMobil, computer prices might rise, helping Dell. The two effects are offsetting, which stabilizes portfolio return.

But why stop at only two stocks? Diversifying into many more securities continues to reduce exposure to firm-specific factors, so portfolio volatility should continue to fall. Ultimately, however, there is no way to avoid all risk. To the extent that virtually all securities are affected by common (risky) macroeconomic factors, we cannot eliminate exposure to general economic risk, no matter how many stocks we hold.

Figure 6.1 illustrates these concepts. When all risk is firm-specific, as in Figure 6.1A, diversification can reduce risk to low levels. With all risk sources independent, and with investment spread across many securities, exposure to any particular source of risk is negligible. This is an application of the law of large numbers. The reduction of risk to very low levels because of independent risk sources is called the *insurance principle.*

When a common source of risk affects all firms, however, even extensive diversification cannot eliminate all risk. In Figure 6.1B, portfolio standard deviation falls as the number of securities increases, but it is not reduced to zero. The risk that remains even after diversification is called **market risk,** risk that is attributable to marketwide risk sources. Other terms are **systematic risk** or **nondiversifiable risk.** The risk that *can* be eliminated by diversification is called **unique risk, firm-specific risk, nonsystematic risk,** or **diversifiable risk.**

This analysis is borne out by empirical studies. Figure 6.2 shows the effect of portfolio diversification, using data on NYSE stocks. The figure shows the average standard deviations of equally weighted portfolios constructed by selecting stocks at random as a function of the number of stocks in the portfolio. On average, portfolio risk does fall with diversification, but

market risk, systematic risk, nondiversifiable risk

Risk factors common to the whole economy.

unique risk, firm-specific risk, nonsystematic risk, diversifiable risk

Risk that can be eliminated by diversification.

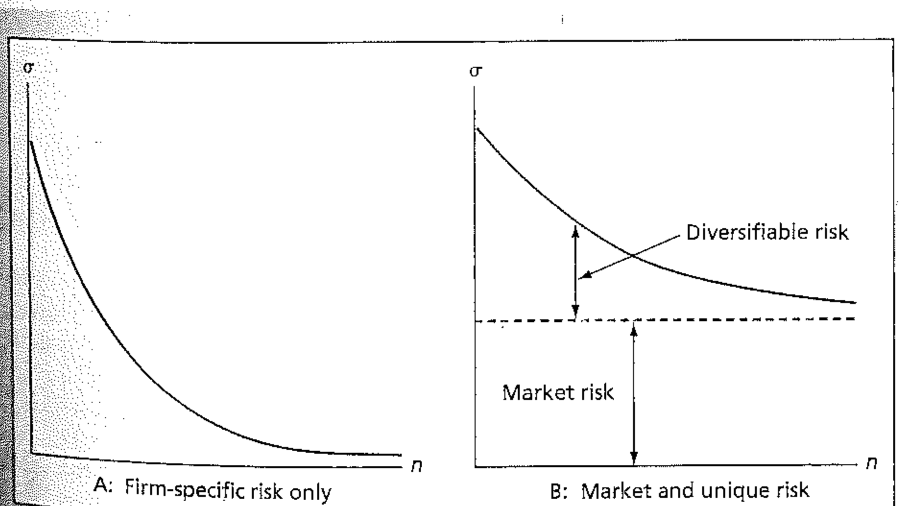


FIGURE 6.1

Portfolio risk as a function of the number of stocks in the portfolio the economy toward this goal. In contrast, supply-side policies treat the issue of the productive capacity of the economy. The goal is to create an environment in which workers and owners of capital have the maximum incentive and ability to produce and develop goods.

Supply-side economists also pay considerable attention to tax policy. While demand-siders look at the effect of taxes on consumption demand, supply-siders focus on incentives and marginal tax rates. They argue that lowering tax rates will elicit more investment and improve incentives to work, thereby enhancing economic growth. Some go so far as to claim that reductions in tax rates can lead to increases in tax revenues because the lower tax rates will cause the economy and the revenue tax base to grow by more than the tax rate is reduced.

CONCEPT check 12.3

Large tax cuts in 2001 were followed by relatively rapid growth in GDP. How would demand-side and supply-side economists differ in their interpretations of this phenomenon?

12.6 BUSINESS CYCLES

We've looked at the tools the government uses to fine-tune the economy, attempting to maintain low unemployment and low inflation. Despite these efforts, economies repeatedly seem to pass through good and bad times. One determinant of the broad asset allocation decision of many analysts is a forecast of whether the macroeconomy is improving or deteriorating. A forecast that differs from the market consensus can have a major impact on investment strategy.

The Business Cycle

business cycles
Recurring cycles of recession and recovery.

peak
The transition from the end of an expansion to the start of a contraction.

trough
The transition point between recession and recovery.

cyclical industries
Industries with above-average sensitivity to the state of the economy.

defensive industries
Industries with below-average sensitivity to the state of the economy.

The economy recurrently experiences periods of expansion and contraction, although the length and depth of these cycles can be irregular. These recurring patterns of recession and recovery are called **business cycles.** Figure 12.4 presents graphs of several measures of production and output. The production series all show clear variation around a generally rising trend. The bottom graph of capacity utilization also evidences a clear cyclical (although irregular) pattern.

The transition points across cycles are called peaks and troughs, identified by the boundaries of the shaded areas of the graph. A **peak** is the transition from the end of an expansion to the start of a contraction. A **trough** occurs at the bottom of a recession just as the economy enters a recovery. The shaded areas in Figure 12.4 all represent periods of recession.

As the economy passes through different stages of the business cycle, the relative profitability of different industry groups might be expected to vary. For example, at a trough, just before the economy begins to recover from a recession, one would expect that **cyclical industries**, those with above-average sensitivity to the state of the economy, would tend to outperform other industries. Examples of cyclical industries are producers of durable goods, such as automobiles or washing machines. Because purchases of these goods can be deferred during a recession, sales are particularly sensitive to macroeconomic conditions. Other cyclical industries are producers of capital goods, that is, goods used by other firms to produce their own products. When demand is slack, few companies will be expanding and purchasing capital goods. Therefore, the capital goods industry bears the brunt of a slowdown but does well in an expansion.

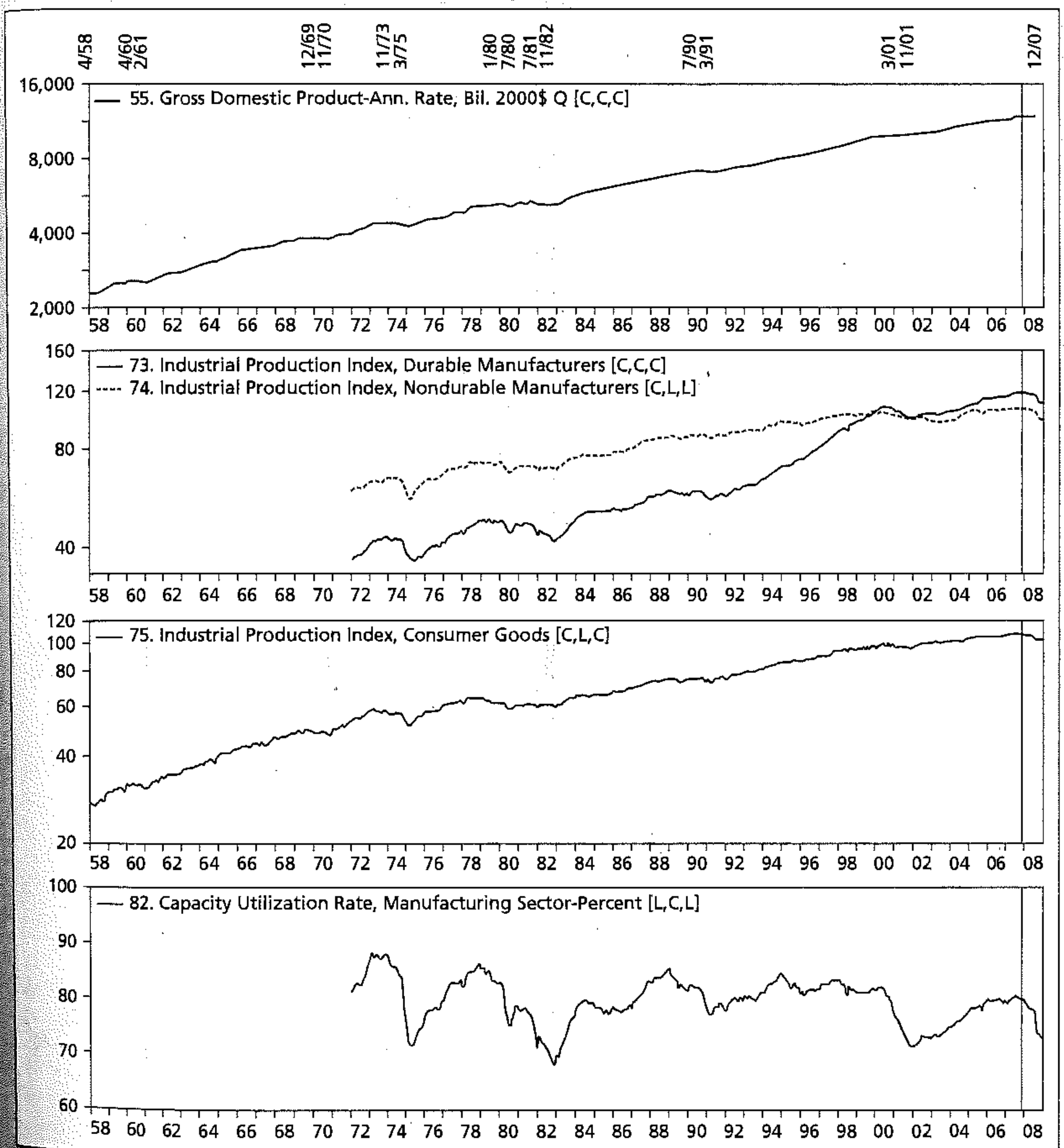
In contrast to cyclical firms, **defensive industries** have little sensitivity to the business cycle. These are industries that produce goods for which sales and profits are least sensitive to the state of the economy. Defensive industries include food producers and processors, pharmaceutical firms, and public utilities. These industries will outperform others when the economy enters a recession.

The cyclical/defensive classification corresponds well to the notion of systematic or market risk introduced in our discussion of portfolio theory. When perceptions about the health of the economy become more optimistic, for example, the prices of most stocks will increase as forecasts of profitability rise. Because the cyclical firms are most sensitive to such developments,

FIGURE 12.4

Cyclical indicators

Source: The Conference Board, *Business Cycle Indicators*, December 2008. Used with permission of The Conference Board, Inc.



their stock prices will rise the most. Thus, firms in cyclical industries will tend to have high-beta stocks. In general, then, stocks of cyclical firms will show the best results when economic news is positive, but they will also show the worst results when that news is bad. Conversely, defensive firms will have low betas and performance that is comparatively unaffected by overall market conditions.

If your assessments of the state of the business cycle were reliably more accurate than those of other investors, choosing between cyclical and defensive industries would be easy. You

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5. *Turnarounds.* These are firms that are in bankruptcy or soon might be. If they can recover from what might appear to be imminent disaster, they can offer tremendous investment returns. A good example of this type of firm would be Chrysler in 1982, when it required a government guarantee on its debt to avoid bankruptcy. The stock price rose 15-fold in the next five years.
6. *Asset plays.* These are firms that have valuable assets not currently reflected in the stock price. For example, a company may own or be located on valuable real estate that is worth as much or more than the company's business enterprises. Sometimes the hidden asset can be tax-loss carryforwards. Other times the assets may be intangible. For example, a cable company might have a valuable list of cable subscribers. These assets do not immediately generate cash flow and so may be more easily overlooked by other analysts attempting to value the firm.

Industry Structure and Performance

The maturation of an industry involves regular changes in the firm's competitive environment. As a final topic, we examine the relationship between industry structure, competitive strategy, and profitability. Michael Porter (1980, 1985) has highlighted these five determinants of competition: threat of entry from new competitors, rivalry between existing competitors, price pressure from substitute products, the bargaining power of buyers, and the bargaining power of suppliers.

Threat of entry New entrants to an industry put pressure on price and profits. Even if a firm has not yet entered an industry, the potential for it to do so places pressure on prices, since high prices and profit margins will encourage entry by new competitors. Therefore, barriers to entry can be a key determinant of industry profitability. Barriers can take many forms. For example, existing firms may already have secure distribution channels for their products based on long-standing relationships with customers or suppliers that would be costly for a new entrant to duplicate. Brand loyalty also makes it difficult for new entrants to penetrate a market and gives firms more pricing discretion. Proprietary knowledge or patent protection also may give firms advantages in serving a market. Finally, an existing firm's experience in a market may give it cost advantages due to the learning that takes place over time.

Rivalry between existing competitors When there are several competitors in an industry, there will generally be more price competition and lower profit margins as competitors seek to expand their share of the market. Slow industry growth contributes to this competition since expansion must come at the expense of a rival's market share. High fixed costs also create pressure to reduce prices since fixed costs put greater pressure on firms to operate near full capacity. Industries producing relatively homogeneous goods also are subject to considerable price pressure since firms cannot compete on the basis of product differentiation.

Pressure from substitute products Substitute products mean that the industry faces competition from firms in related industries. For example, sugar producers compete with corn syrup producers. Wool producers compete with synthetic fiber producers. The availability of substitutes limits the prices that can be charged to customers.

Bargaining power of buyers If a buyer purchases a large fraction of an industry's output, it will have considerable bargaining power and can demand price concessions. For example, auto producers can put pressure on suppliers of auto parts. This reduces the profitability of the auto parts industry.

Bargaining power of suppliers If a supplier of a key input has monopolistic control over the product, it can demand higher prices for the good and squeeze profits out of the industry. One special case of this issue pertains to organized labor as a supplier of a key

$$V_0 = \frac{D_1}{1+k} + \frac{D_2}{(1+k)^2} + \cdots + \frac{D_H + P_H}{(1+k)^H} \quad (13.2)$$

Note the similarity between this formula and the bond valuation formula developed in Chapter 10. Each relates price to the present value of a stream of payments (coupons in the case of bonds, dividends in the case of stocks) and a final payment (the face value of the bond or the sales price of the stock). The key differences in the case of stocks are the uncertainty of dividends, the lack of a fixed maturity date, and the unknown sales price at the horizon date. Indeed, one can continue to substitute for price indefinitely to conclude

$$V_0 = \frac{D_1}{1+k} + \frac{D_2}{(1+k)^2} + \frac{D_3}{(1+k)^3} + \cdots \quad (13.3)$$

dividend discount model (DDM)

A formula for the intrinsic value of a firm equal to the present value of all expected future dividends.

Equation 13.3 states the stock price should equal the present value of all expected future dividends into perpetuity. This formula is called the **dividend discount model (DDM)** of stock prices.

It is tempting, but incorrect, to conclude from Equation 13.3 that the DDM focuses exclusively on dividends and ignores capital gains as a motive for investing in stock. Indeed, we assume explicitly in Equation 13.1 that capital gains (as reflected in the expected sales price, P_1) are part of the stock's value. At the same time, the price at which you can sell a stock in the future depends on dividend forecasts at that time.

The reason only dividends appear in Equation 13.3 is not that investors ignore capital gains. It is instead that those capital gains will be determined by dividend forecasts at the time the stock is sold. That is why in Equation 13.2 we can write the stock price as the present value of dividends plus sales price for *any* horizon date. P_H is the present value at time H of all dividends expected to be paid after the horizon date. That value is then discounted back to today, time 0. The DDM asserts that stock prices are determined ultimately by the cash flows accruing to stockholders, and those are dividends.

The Constant-Growth DDM

Equation 13.3 as it stands is still not very useful in valuing a stock because it requires dividend forecasts for every year into the indefinite future. To make the DDM practical, we need to introduce some simplifying assumptions. A useful and common first pass at the problem is to assume that dividends are trending upward at a stable growth rate that we will call g. For example, if $g = .05$ and the most recently paid dividend was $D_0 = 3.81$, expected future dividends are

$$\begin{aligned} D_1 &= D_0(1+g) = 3.81 \times 1.05 = 4.00 \\ D_2 &= D_0(1+g)^2 = 3.81 \times (1.05)^2 = 4.20 \\ D_3 &= D_0(1+g)^3 = 3.81 \times (1.05)^3 = 4.41 \text{ etc.} \end{aligned}$$

Using these dividend forecasts in Equation 13.3, we solve for intrinsic value as

$$V_0 = \frac{D_0(1+g)}{1+k} + \frac{D_0(1+g)^2}{(1+k)^2} + \frac{D_0(1+g)^3}{(1+k)^3} + \cdots$$

This equation can be simplified to

$$V_0 = \frac{D_0(1+g)}{k-g} = \frac{D_1}{k-g} \quad (13.4)$$

Note in Equation 13.4 that we divide D_1 (not D_0) by $k - g$ to calculate intrinsic value. If the market capitalization rate for Steady State is 12%, we can use Equation 13.4 to show that the intrinsic value of a share of Steady State stock is

CONCEPT check 13.3

a. Calculate the price of a firm with a plowback ratio of .60 if its ROE is 20%. Current earnings, E_1, will be $5 per share, and $k = 12.5\%$.

b. What if ROE is 10%, which is less than the market capitalization rate? Compare the firm's price in this instance to that of a firm with the same ROE and E_1 but a plowback ratio of $b = 0$.

Life Cycles and Multistage Growth Models

As useful as the constant-growth DDM formula is, you need to remember that it is based on a simplifying assumption, namely, that the dividend growth rate will be constant forever. In fact, firms typically pass through life cycles with very different dividend profiles in different phases. In early years, there are ample opportunities for profitable reinvestment in the company. Payout ratios are low, and growth is correspondingly rapid. In later years, the firm matures, production capacity is sufficient to meet market demand, competitors enter the market, and attractive opportunities for reinvestment may become harder to find. In this mature phase, the firm may choose to increase the dividend payout ratio, rather than retain earnings. The dividend level increases, but thereafter it grows at a slower rate because the company has fewer growth opportunities.

Table 13.2 illustrates this profile. It gives Value Line's forecasts of return on capital, dividend payout ratio, and projected three-year growth rate in earnings per share of a sample of the firms included in the computer software and services industry versus those of East Coast electric utilities. (We compare return on capital rather than return on equity because the latter is affected by leverage, which tends to be far greater in the electric utility industry than in the software industry. Return on capital measures operating income per dollar of total long-term

TABLE 13.2 Financial ratios in two industries

	Return on Capital	Payout Ratio	Growth Rate 2012–2015
Computer Software			
Adobe Systems	13.0%	0.0%	15.4%
Cognizant	19.0	0.0	21.0
Compuware	16.5	0.0	18.6
Intuit	21.0	21.0	13.3
Microsoft	31.5	30.0	10.2
Oracle	20.0	14.0	10.3
Red Hat	14.0	0.0	17.9
Parametric Tech	15.5	0.0	9.6
SAP	18.5	28.0	6.7
Median	18.5%	0.0%	13.3%
Electric Utilities			
Central Hudson G&E	6.0%	67.0%	2.6%
Central Vermont	6.0	54.0	1.9
Consolidated Edison	6.0	63.0	2.7
Duke Energy	5.5	65.0	4.4
Northeast Utilities	6.5	47.0	6.3
NStar	9.0	60.0	8.4
Pennsylvania Power (PPL Corp.)	7.0	55.0	3.6
Public Services Enter.	6.5	45.0	8.4
United Illuminating	5.0	73.0	2.2
Median	6.0%	60.0%	3.6%

Source: From *Value Line Investment Survey*, November and December 2011. Reprinted with permission of Value Line Investment Survey © 2012 Value Line, Inc. All Rights Reserved Worldwide. "Value Line" is a registered trademark of Value Line Inc.

financing, regardless of whether the source of the capital supplied is debt or equity. We will return to this issue in the next chapter.)

By and large, software firms have attractive investment opportunities. The median return on capital of these firms is forecast to be 18.5%, and the firms have responded with quite high plowback ratios. Most of these firms pay no dividends at all. The high returns on capital and high plowback ratios result in rapid growth. The median growth rate of earnings per share in this group is projected at 13.3%.

In contrast, the electric utilities are more representative of mature firms. Their median return on capital is lower, 6%; dividend payout is higher, 60%; and average growth rate is lower, 3.6%. We conclude that the higher payouts of the electric utilities reflect their more limited opportunities to reinvest earnings at attractive rates of return.

To value companies with temporarily high growth, analysts use a multistage version of the dividend discount model. Dividends in the early high-growth period are forecast and their combined present value is calculated. Then, once the firm is projected to settle down to a steady growth phase, the constant-growth DDM is applied to value the remaining stream of dividends.

We can illustrate this with a real-life example using a **two-stage DDM.** Figure 13.2 is a *Value Line Investment Survey* report on Honda Motor Co. Some of Honda's relevant information of the end of 2011 is highlighted.

two-stage DDM

Dividend discount model in which dividend growth is assumed to level off only at some future date.

Honda's beta appears at the circled A, its recent stock price at the B, the per-share dividend payments at the C, the ROE (referred to as "return on shareholder equity") at the D, and the dividend payout ratio (referred to as "all dividends to net profits") at the E.[4] The rows ending at C, D, and E are historical time series. The boldfaced italicized entries under 2012 are estimates for that year. Similarly, the entries in the far right column (labeled 14–16) are forecasts for some time between 2014 and 2016, which we will take to be 2015.

Value Line provides explicit dividend forecasts over the relative short term, with dividends rising from \$.72 in 2012 to \$1 in 2015. We can obtain dividend inputs for this initial period by using the explicit forecasts for 2012–2015 and linear interpolation for the years between:

2012	\$ .72
2013	\$ .81
2014	\$ .90
2015	\$1.00

Now let us assume the dividend growth rate will be steady beyond 2015. What is a reasonable guess for that steady-state growth rate? Value Line forecasts a dividend payout ratio of .25 and an ROE of 10%, implying long-term growth will be

$$g = \text{ROE} \times b = 10\% \times (1 - .25) = 7.5\%$$

Our estimate of Honda's intrinsic value using an investment horizon of 2015 is therefore obtained from Equation 13.2, which we restate here:

$$V_{2011} = \frac{D_{2012}}{(1+k)} + \frac{D_{2013}}{(1+k)^2} + \frac{D_{2014}}{(1+k)^3} + \frac{D_{2015} + P_{2015}}{(1+k)^4}$$

$$= \frac{.72}{(1+k)} + \frac{.81}{(1+k)^2} + \frac{.90}{(1+k)^3} + \frac{1.00 + P_{2015}}{(1+k)^4}$$

Here, P_{2015} represents the forecast price at which we can sell our shares of Honda at the end of 2015, when dividends enter their constant-growth phase. That price, according to the constant-growth DDM, should be

[4] Because Honda is a Japanese firm, Americans would hold its shares via ADRs, or American Depository Receipts. ADRs are not shares of the firm but are *claims* to shares of the underlying foreign stock that are then traded in U.S. security markets. Value Line notes that each Honda ADR is a claim on one common share, but in other cases, each ADR may represent a claim to either multiple shares or even fractional shares.

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equal to $\overline{\sigma}^2$ and that, across any pair of stocks (say, stock i and stock j), the average covariance is $\overline{\sigma}_{ij}$. Then the portfolio variance equation can be written as shown at the bottom of Figure 5.7.

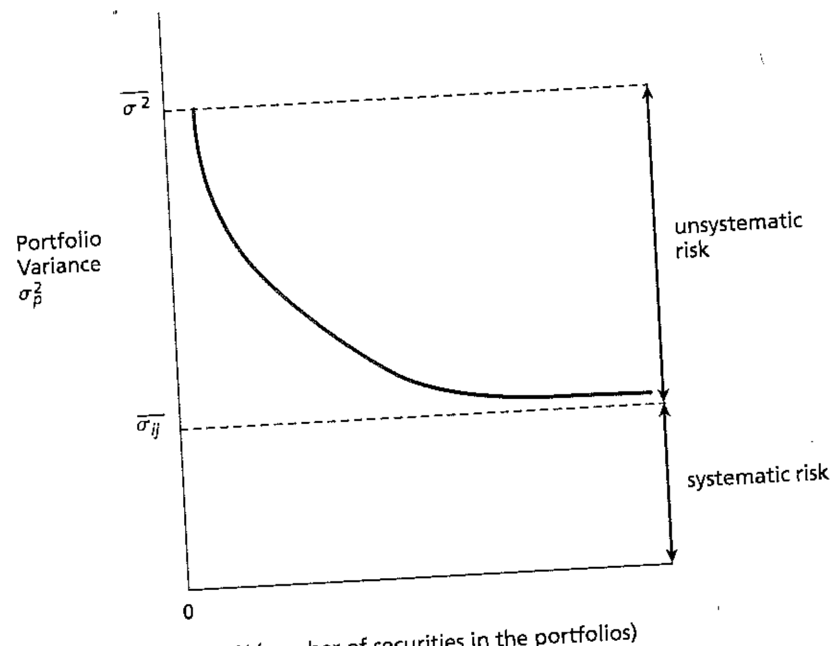
As the number N of stocks in the portfolio becomes very large, the variance term $N(1/N)^2\,\overline{\sigma}^2$ approaches zero. This means the average variance of individual stocks has no impact on portfolio variance. As N increases, the second term in the equation converges to $\overline{\sigma}_{ij}$, indicating that what really determines the risk of a large portfolio is the average covariance between all pairs of securities. A large portfolio consisting of securities that are, on average, only weakly correlated with each other will have a lower variance than a portfolio that consists of highly correlated securities.

Figure 5.8 plots the relationship between the number of securities in a portfolio and the portfolio's variance given by this equation. For investors, the figure contains both good and bad news. The good news is that as the number of securities in the portfolio increases, the portfolio's variance declines. Given the proliferation of low-cost mutual funds available today, investors can construct portfolios containing hundreds of securities, thereby reducing the variance of their investment portfolio to some degree. The bad news is that the marginal risk reduction benefit of adding more securities to the portfolio decreases as the number of securities in the portfolio increases. Not even a very well diversified portfolio can eliminate all risk.

FIGURE 5.8

Effect of Diversification on Portfolio Variance

Adding more securities to a portfolio lowers the portfolio's volatility, but the incremental benefit of adding more securities declines as the number of securities rises.



Because some risks systematically affect almost all securities, there is a limit to the risk reduction achievable by adding more securities to a portfolio. The average covariance term $\overline{\sigma}_{ij}$ represents this limit. No matter how diversified a portfolio becomes, its variance cannot fall below the average covariance of securities in the portfolio. Financial economists give this type of risk special names: undiversifiable risk, **systematic risk**, or market risk. Similarly, the risk that diversification eliminates is called diversifiable risk, **unsystematic risk**, idiosyncratic risk, or unique risk.

JOB INTERVIEW QUESTION

What is the difference between systematic and unsystematic risk?

In real-world terms, what exactly is systematic risk? This is a difficult question to answer, and we explore it in more depth in the next chapter. For now, we just say that systematic risks are those that are common across all types of securities. Fluctuations in gross domestic product, inflation, oil prices, or interest rates can be thought of as systematic risks, and so might certain political factors. For example, the legal system governing investors and markets in a given country can influence systematic risk because that system determines the level of protection given to minority shareholders, creditors, and ordinary investors. When investors perceive that the legal system protects their interests, their willingness to trade and invest in securities increases and so the returns they require for bearing risk decline.

Smart Ethics Video

Utpal Bhattacharya, Indiana University

"The cost of equity goes up if insider trading laws are not enforced."

See the entire interview at **SMARTFinance**

If investors can cheaply eliminate some risks through diversification, then we should not expect a security to earn higher returns for risks that can be eliminated through diversification. Investors can expect compensation only for bearing systematic risk (i.e., risk that cannot be diversified away). Refer back to the Example following Equation 5.4, which showed that the average return on Merck stock was about the same over 10 years as the average return on the S&P 500 *even though* Merck stock was much more volatile than the index. An undiversified investor who held only Merck stock had to bear twice as much volatility as an investor who owned the S&P 500, even though both investors earned the same reward. This is not to say that Merck was or is a bad investment. The point is that holding Merck (or any other individual company's stock) *in isolation* is a poor investment strategy. Undiversified portfolios are generally suboptimal because they expose investors to unsystematic risk without offering higher returns.

Measuring the Systematic Risk of an Individual Security

The previous section demonstrated two important facts. First, the formula for portfolio variance shows that each security contributes to a portfolio's risk through two channels: the security's own variance and its covariance with all other securities in the portfolio. In diversified portfolios, only the second channel matters. This implies that an individual stock's variance may be a poor measure of its risk. The variance of a stock captures its total volatility, some of which is unsystematic and some of which is systematic. Second, because diversification eliminates unsystematic risk, the market provides no reward for bearing it. As a consequence, though we still expect to see a positive relationship in the market between risk and return, we can no longer be confident that a positive relationship will exist between returns on an individual asset and its variance. Again, a stock's variance captures both its systematic and unsystematic fluctuations, but only the systematic component should be correlated with returns.

We need a new measure for an individual asset's risk, one that captures only the systematic component of its volatility. Remember, the primary contribution to portfolio risk from a single asset comes from its covariance with all the other assets in the portfolio. Imagine that an investor holds a fully diversified portfolio—literally, a portfolio containing every asset available in the economy. How would this investor determine the contribution of a single security to the portfolio's risk? One way to do that would be to measure the covariance between a single asset and the portfolio. Recall, though, the difficulty that nonstandard units cause for interpreting covariance calculations. A standardized measure would be preferable, and finance theory gives us just such a measure in the concept of *beta*:

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2} \qquad \text{(Eq. 5.1}$$

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The **beta** of asset i (β_i) equals the covariance of the asset's returns with the returns on the overall portfolio, divided by the portfolio's variance. As you will see in the next chapter, the portfolio we refer to here is known as "the market portfolio," a value-weighted portfolio of all available assets.[14] A security's beta gives us a standardized measure of its covariance with all other assets, or a measure of its systematic risk. If the market rewards only systematic risk and if beta captures the systematic risk of an individual asset, then we should observe a positive relationship between values of beta and returns in the market.

Observe that the formula for an asset's beta closely resembles that of the correlation coefficient:

$$\beta_i = \frac{\sigma_{im}}{(\sigma_m)(\sigma_m)};$$

$$\rho_i = \frac{\sigma_{im}}{(\sigma_i)(\sigma_m)}$$

The equations are identical except in one respect: The denominator of the correlation coefficient multiplies the standard deviations of the asset and the market, whereas the denominator of the beta formula squares the standard deviation of the market. This small adjustment to the denominator makes the interpretation of beta a little different from that of the correlation coefficient. First, unlike ρ, beta has no maximum or minimum value. Second, beta indicates how much the individual asset's return moves, on average, when the market moves by 1%. For example, if a stock has a beta of 1.5, then, when the market return increases by 1%, the stock return will (on average) increase by 1.5%.

EXAMPLE

Now that we understand the beta measure of a stock's risk, how does it compare to the measure we started with, standard deviation? Comparing the monthly returns on each of the four stocks listed in Table 5.3 to returns on the overall stock market, suppose you calculate the following statistics:

Stock	Covariance with Market
Mead	0.0031
Boise	0.0026
Nike	0.0011
Arrow	−0.0003

If the variance of market returns were 0.0028, then the betas of the four stocks would be as follows:

Mead 1.11 Boise 0.93 Nike 0.39 Arrow −0.11

These betas contain several surprises. First, based on comparison of the standard deviations of each stock in Table 5.3, we concluded that Nike was the riskiest security. Comparing the betas, however, suggests that Nike is less risky than either Mead or Boise Cascade. Recall

(continued)

[14]The modifier "value-weighted" means that the fraction invested in a particular security is equal to that security's total market value as a percentage of the market value of all securities. For example, if the total market value of all securities in the market is $10 trillion and if the total market value of a certain company's stock equals $100 billion, then the fraction of that stock in a value-weighted portfolio would be 0.01, or 1%.

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6.4 The Capital Asset Pricing Model (CAPM)

The Security Market Line

The basic CAPM was developed almost simultaneously during the mid-1960s by William Sharpe (1964), John Lintner, and Jan Mossin (1966); it was quickly embraced by academic researchers and, in time, by practitioners as well. The reason for the CAPM's widespread acceptance is not hard to understand—for the first time, researchers and practitioners had a model that generated specific predictions about the risk–return characteristics of individual assets, and this relation was driven by how each asset *covaries* with the market portfolio.

The formal development of the CAPM requires several assumptions about investors and markets. Rather than present a detailed list of these assumptions, we present the logic of the CAPM as it flows from the material we have covered so far.

1. Investors are risk averse and require higher returns on riskier investments.
2. Because investors can diversify, they care only about the systematic (or undiversifiable) risk of any investment.
3. The market offers no reward for bearing unsystematic risk because it can be diversified away.
4. Some portfolios are better than others. Portfolios that maximize expected return for any level of risk are efficient portfolios.
5. If investors can borrow and lend at the risk-free rate, then there exists a single risky portfolio that dominates all others. Only portfolios consisting of the risk-free asset and the optimal risky portfolio are efficient.
6. If investors have homogeneous expectations then they will agree on the composition of the optimal portfolio. In equilibrium, the optimal portfolio will be the market portfolio.
7. The central insight of the CAPM is that if all investors hold the market portfolio then—when evaluating the risk of any specific asset—they will be concerned with the covariance of that asset with the overall market. The implication is that any measure of an asset's systematic risk exposure must capture how it covaries with the rest of the market. An asset's beta provides a quantitative measure of this risk, and therefore the CAPM predicts a positive, linear relationship between expected return and beta. In the CAPM, beta risk (or market risk) is the only risk that is priced.

The **capital asset pricing model (CAPM)** indicates that the expected return on a specific asset, $E(R_i)$, equals the risk-free rate plus a premium that depends on the asset's beta, β_i, and the expected risk premium on the market portfolio, $E(R_m) - R_f$:

$$E(R_i) = R_f + \beta_i\left[E(R_m) - R_f\right] \quad \text{(Eq. 6.}$$

Recall that beta measures an asset's correlation with a broader portfolio—in this case, the market portfolio. The higher the beta of a security, the greater the security's exposure to systematic risk and the higher the expected return it must offer investors. Although there are three variables (R_f, β_i, and $E(R_m)$) on the right-hand side of the CAPM equation, only beta changes from one security to the next. For that reason, analysts classify the CAPM as a **single-factor model**, meaning that just one variable explains differences in returns across securities.

JOB INTERVIEW QUESTION

How would you estimate the expected return of a stock?

Figure 6.8 plots the CAPM equation on a diagram with the expected return on the y-axis and beta on the x-axis. The intercept of this line is R_f, and its slope is $E(R_m) - R_f$. According to the CAPM, the equilibrium expected returns of all securities must plot on

In the CAPM, you recall, the market portfolio is a value-weighted combination of all assets in the economy. At present, we are unaware of any market index that attempts to incorporate every type of asset. When using the CAPM, most practitioners and academics use the returns on a broad-based stock index as a proxy for the true market portfolio. Accordingly, rather than try to estimate the expected risk premium on the market portfolio, analysts usually focus on the expected equity risk premium: the difference in expected returns between a well-diversified portfolio of common stocks and a risk-free asset such as a U.S. Treasury bill.

Since 1900, the average real return on stocks outpaced the average real return on U.S. Treasury bills by about 5.4% per year. But in the CAPM, what matters is not the actual equity risk premium from the past but rather the expected equity risk premium looking forward. Though many analysts trust the historical evidence and simply plug in a figure close to 6% for the term $E(R_m - R_f)$, a naive reliance on long-run historical averages is not the only approach for estimating the expected risk premium. Using an unbiased estimate is important because an error in the risk premium translates directly into an error in the project's discount rate and thus in its NPV.

One variable that analysts can use to obtain a forward-looking estimate of the premium is the market's aggregate earnings yield, which is the reciprocal of the price-earnings ratios. For example, to calculate the earnings yield for the S&P 500, add up the earnings of all 500 companies and divide by the aggregate market value of these firms. Corporate earnings fluctuate with the business cycle, so analysts usually try to smooth out, or *normalize*, these temporary effects before using the earnings yield to estimate the risk premium. In the United States, the long-run average value of the earnings yield is about 7%, a little less than the average real return on stocks. It should come as no surprise that the earnings yield is closely related to the real return on stocks. After all, stocks represent a claim on corporate earnings.

A second forward-looking method for estimating the equity risk premium uses the dividend growth model. Recall that this model calculates the present value of a perpetual dividend stream growing at a constant rate, g:

$$P_0 = \frac{D_1}{r - g}$$

Rearranging this equation shows that the required return on the stock equals the sum of the dividend yield and the dividend growth rate:

$$r = \frac{D_1}{P_0} + g$$

To use this model when estimating the equity risk premium, we must think of the equation in aggregate, macroeconomic terms. In other words, r represents the required return on the stock market rather than the required return on a given stock. The ratio D_1/P_0 represents the aggregate dividend yield, and g represents the (real) growth rate of aggregate dividends. From 1872 to 1950, the expected equity premium derived from this model almost exactly matched the actual risk premium measured using average historical returns (a little more than 4%). From 1950 [illegible], however, the average real return on equities was much higher than predicted by the dividend growth model.[8]

[8]The opposite has been true since 2000: real equity returns have been *lower* on average than the dividend growth model would predict.

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efficiency is more important because efficient capital markets incorporate all relevant information into financial asset prices, which in turn helps ensure that promising investments receive funding.

The concept of efficient capital markets is one of the most influential contributions that financial economics has made to modern economic thought. The **efficient markets hypothesis (EMH)**, as formally presented by Eugene Fama in 1970, has revolutionized financial thought, practice, and regulation. The EMH asserts that financial asset prices fully reflect all available information. What do we mean by "all available information"? The answer to this question varies, and we discuss three distinct versions of the efficient markets hypothesis.

The Three Forms of Market Efficiency

The EMH presents three increasingly stringent definitions of efficiency based on the information that market prices reflect: weak-form, semistrong-form, and strong-form efficiency.

Weak-Form Efficiency In markets characterized by **weak-form efficiency**, asset prices incorporate all information from the historical record—that is, all information about price trends or repeating patterns that occurred in the past. This implies that trading strategies based on analyses of historical pricing trends or relationships cannot consistently yield market-beating returns.

Prices in a weak-form efficient market will be unpredictable and will change only in response to the arrival of new information. In technical terms, this means that prices follow a **random walk**: they wander aimlessly, with no connection to past price changes and no tendency to return to a mean value over time.

Semistrong-Form Efficiency The second form of market efficiency, **semistrong-form efficiency**, asserts that asset prices incorporate *all publicly available information*. The key point about this form of efficiency is that the prices need only reflect information from *public* sources (e.g., newspapers, press releases, computer databases).

There is both a "stock" and a "flow" aspect to the information processing capabilities of semistrong-form efficient markets: First, the *level* of asset prices should correctly reflect all pertinent historical, current, and predictable future information that investors can obtain from public sources. Second, asset prices should *change* fully and instantaneously in response to relevant new information.

Strong-Form Efficiency In markets characterized by **strong-form efficiency**, asset prices reflect *all* information, both public and private. This extreme form of market efficiency implies that important company-specific information will be fully incorporated in asset prices with the very first trade after the information is generated.

In strong-form efficient markets, most insider trading would be unprofitable and there would be no benefit to ferreting out information on publicly traded companies. Any data morsel so obtained would already be reflected in stock and bond prices.

Table 10.1 on page 358 describes the three forms of market efficiency and summarizes the key implications of each form.

Malcolm Baker, Harvard University

"There is increasing evidence that stock markets aren't perfectly efficient."

See the entire interview at **SMARTFinance**

Does Empirical Evidence Support Market Efficiency?

Ultimately, whether financial markets are efficient is an empirical question. For more than a quarter of a century, the efficient market hypothesis enjoyed overwhelming support among financial economists. However, in recent years a large body of empirical evidence challenging the EMH has caused many former "true believers" to take a fresh look at the efficiency question. It also seems likely that the paralysis and near-collapse of global

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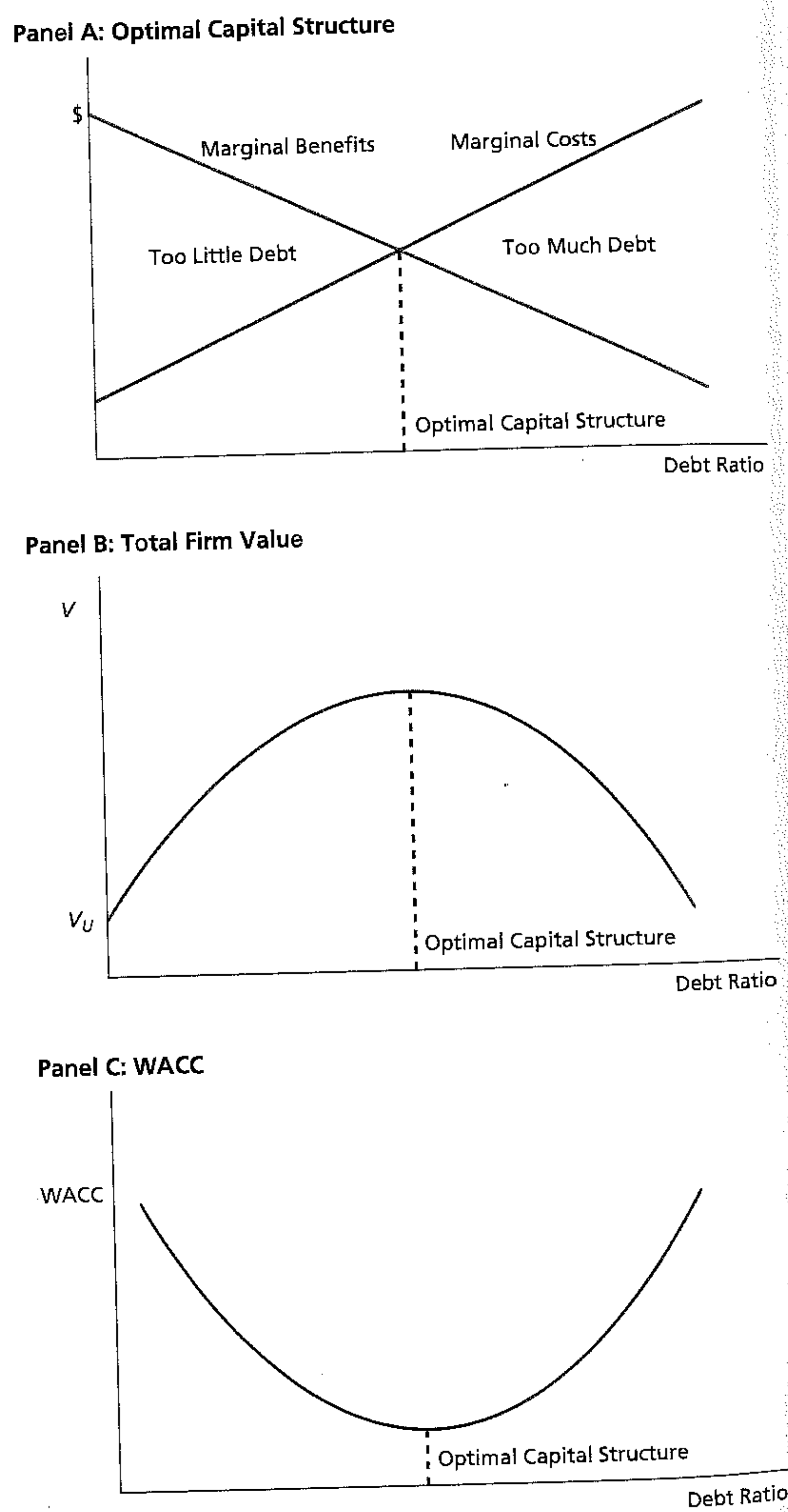
FIGURE 13.2

Weighing Debt's Benefits and Costs to Find Optimal Capital Structure

The optimal amount of debt occurs where the marginal cost and marginal benefit curves intersect. At that point, total firm value is at its peak, and the weighted average cost of capital (WACC) is at its minimum.



combination of equity and debt, with the debt interest sheltering cash flows from taxes. Even so, most firms do not finance their activities exclusively with debt. This suggests that managers see debt as having costs that at some point offset debt's tax advantages. Based on observing what companies actually do, the optimal capital structure for most firms is apparently one that contains some debt, but not too much.

Duke University

"... managers ... the benefits of debt ... of debt."

... entire interview at

Finance

How do managers trade-off the benefits and costs of debt to establish a target capital structure that maximizes firm value? Figure 13.2 offers a conceptual answer to this question. The blue line in Panel A shows that the marginal benefit of borrowing an additional dollar falls as the firm's overall debt ratio rises. The red line indicates that costs associated with using debt rise as leverage increases. We will explain in the next section why marginal benefits fall and marginal costs rise as debt increases, but for now you can just take the benefits and costs in Figure 13.2 as given. As in any cost–benefit analysis, the optimum occurs when marginal benefits and marginal costs are equal. Therefore, a manager facing these cost and benefit curves would choose a debt level where the two curves intersect. To the left of that point, the firm has too little debt in the sense that marginal benefits exceed marginal costs, so adding more debt would increase firm value. At higher debt levels, debt's marginal costs exceed its benefits, so adding leverage decreases firm value.

Panel B shows the relation between total firm value and leverage. If a firm has no debt, its value equals *Vu*. From that point, if the firm adds debt to its capital structure, its value begins to rise. At some point, firm value reaches a peak, and from that point, adding more debt decreases the value of the firm. The graph shows that, at the same point where the marginal benefit and cost curves in Panel A intersect, firm value reaches its peak. The point at which firm value begins to fall as leverage rises is exactly when debt's marginal costs first exceed its marginal benefits.

At the end of this chapter, we will demonstrate how to find the optimal debt ratio. But how much difference does finding the right capital structure really make in the overall value of the firm? In a recent study, van Binsbergen, Graham, and Yang (2008) estimate that, for the average firm, appropriate debt choices can increase firm value by about 5%. In some companies, like the one described at the beginning of Chapter 14, the increase in value may be 10% or more.

Panel C of Figure 13.2 demonstrates how a firm's weighted-average-cost of capital (WACC) changes as leverage rises. Here, the relation is U-shaped. A firm with no leverage can reduce its WACC by substituting debt for equity, but, eventually, the firm reaches a point where further increases in debt cause the WACC to increase. Naturally, managers want to find the debt ratio that minimizes the cost of capital because doing so maximizes firm value. Therefore, the optimum point in Panel C is the same optimum debt ratio in Panels A and B.

In the next section, we explore in more detail why debt's marginal benefits fall and its marginal costs rise as a firm uses more debt in its capital structure. To begin, we revisit the tax advantage of debt, taking into account some important features of the tax code that we have neglected thus far.

Concept Review Questions

1. How large would the costs of debt have to be in order to justify a firm's decision to operate with 100% equity?
2. If a firm is operating well below its optimum debt level, then what market forces might prompt it to use more debt?

Co-head of M&A
Sachs in the

at the
industry and
markets."

entire interview at
Finance

The Investment Banker's Role in Equity Issues

We now turn to the services that investment banks provide to issuing companies, with particular attention to U.S. practices.[1] We focus on common stock offerings, though the procedures for selling bonds and preferred stocks are similar. Investment banks play several different roles throughout the securities offering process, and this section describes the evolution of these roles over the course of an issue. We also describe how issuers compensate IBs for the services they provide.

Although firms can issue stock without the assistance of investment bankers, in practice almost all firms hire IBs to help issue equity. Firms can choose an investment banker in one of two ways. The most common approach is a **negotiated offer**, where the issuing firm negotiates the terms of the offer directly with one investment bank. Alternatively, in a **competitively bid offer**, the firm announces the terms of its intended equity sale and then investment banks bid for the business. Intuition suggests that competitive bidding should be cheaper, but the empirical evidence is mixed. One clear sign that competitive offers are not better and cheaper is that the vast majority of equity sales are negotiated. If the costs of negotiated deals were truly higher, then why would so many firms choose that approach?

Firms issuing securities often hire more than one investment bank. In these cases, one of the banks is usually named the **lead underwriter**, or *book-runner*, while the other leading banks are called **co-managers**. Chen and Ritter (2000) argue that firms often prefer to issue securities with several co-managers because doing so increases the number of stock analysts that will follow the firm after the offering. Firms believe that a larger analyst following leads to greater liquidity and higher stock values. Cliff and Denis (2004) verify the importance of attracting the coverage of top-rated analysts by showing that issuing firms willingly allow their IPO share price to be set low enough to attract excess demand and high trading volume, since this will indirectly compensate the underwriters' star analysts.

Investment bankers sell equity under two types of contracts. In a **best-efforts offering**, the investment bank merely promises to give its best effort to sell the firm's securities at the agreed-upon price but makes no guarantee about the ultimate success of the offering. If there is insufficient demand, the firm withdraws the issue from the market. Best-efforts offerings are most commonly used for small, high-risk companies, and the IB receives a commission based on the number of shares sold.

In contrast, in a **firm-commitment offering** the investment bank agrees to underwrite the issue, meaning that the bank guarantees (**underwrites**) the offering price. The IB actually purchases the shares from the firm and resells them to investors. This arrangement requires the investment bank to bear the risk of inadequate demand for the issuer's shares, but banks mitigate this risk in two ways. First, the lead underwriter forms an **underwriting syndicate** consisting of many investment banks. These banks collectively purchase the firm's shares and market them, thus spreading the risk across the syndicate. Second, underwriters go to great lengths to determine the demand for a new issue before it comes to market, and they generally set the issue's *offer price* and take possession of the securities no more than a day or two before the issue date. These steps help ensure that the investment bank faces only a small risk of being unable to sell the shares that it underwrites.

In firm-commitment offerings, investment banks receive compensation for their services via the **underwriting spread**, the difference between the price at which the banks purchase shares from firms (the **net price**) and the price at which they sell the shares to institutional and individual investors (the **offer price**). In some offerings, underwriters receive additional compensation in the form of warrants that grant the right to buy shares

[1]Ljungqvist, Jenkinson, and Wilhelm (2003) and DeGeorge, Derrien, and Womack (2007) document an increasing tendency for security issues around the world to conform to U.S. standards.

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SESSION TOPIC: STOCK MARKET PRICE BEHAVIOR

SESSION CHAIRMAN: BURTON G. MALKIEL

EFFICIENT CAPITAL MARKETS: A REVIEW OF THEORY AND EMPIRICAL WORK*

EUGENE F. FAMA**

I. INTRODUCTION

THE PRIMARY ROLE of the capital market is allocation of ownership of the economy's capital stock. In general terms, the ideal is a market in which prices provide accurate signals for resource allocation: that is, a market in which firms can make production-investment decisions, and investors can choose among the securities that represent ownership of firms' activities under the assumption that security prices at any time "fully reflect" all available information. A market in which prices always "fully reflect" available information is called "efficient."

This paper reviews the theoretical and empirical literature on the efficient markets model. After a discussion of the theory, empirical work concerned with the adjustment of security prices to three relevant information subsets is considered. First, *weak form* tests, in which the information set is just historical prices, are discussed. Then *semi-strong form* tests, in which the concern is whether prices efficiently adjust to other information that is obviously publicly available (e.g., announcements of annual earnings, stock splits, etc.) are considered. Finally, *strong form* tests concerned with whether given investors or groups have monopolistic access to any information relevant for price formation are reviewed.[1] We shall conclude that, with but a few exceptions, the efficient markets model stands up well.

Though we proceed from theory to empirical work, to keep the proper historical perspective we should note to a large extent the empirical work in this area preceded the development of the theory. The theory is presented first here in order to more easily judge which of the empirical results are most relevant from the viewpoint of the theory. The empirical work itself, however, will then be reviewed in more or less historical sequence.

Finally, the perceptive reader will surely recognize instances in this paper where relevant studies are not specifically discussed. In such cases my apologies should be taken for granted. The area is so bountiful that some such injustices are unavoidable. But the primary goal here will have been accomplished if a coherent picture of the main lines of the work on efficient markets is presented, along with an accurate picture of the current state of the arts.

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1. The distinction between weak and strong form tests was first suggested by Harry Roberts.

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II. The Theory of Efficient Markets

A. *Expected Return or "Fair Game" Models*

The definitional statement that in an efficient market prices "fully reflect" available information is so general that it has no empirically testable implications. To make the model testable, the process of price formation must be specified in more detail. In essence we must define somewhat more exactly what is meant by the term "fully reflect."

One possibility would be to posit that equilibrium prices (or expected returns) on securities are generated as in the "two parameter" Sharpe [40]-Lintner [24, 25] world. In general, however, the theoretical models and especially the empirical tests of capital market efficiency have not been this specific. Most of the available work is based only on the assumption that the conditions of market equilibrium can (somehow) be stated in terms of expected returns. In general terms, like the two parameter model such theories would posit that conditional on some relevant information set, the equilibrium expected return on a security is a function of its "risk." And different theories would differ primarily in how "risk" is defined.

All members of the class of such "expected return theories" can, however, be described notationally as follows:

$$E(\tilde{p}_{j,t+1}|\Phi_t) = [1 + E(\tilde{r}_{j,t+1}|\Phi_t)]p_{jt}, \tag{1}$$

where E is the expected value operator; p_{jt} is the price of security j at time t; $p_{j,t+1}$ is its price at $t + 1$ (with reinvestment of any intermediate cash income from the security); $r_{j,t+1}$ is the one-period percentage return $(p_{j,t+1} - p_{jt})/p_{jt}$; Φ_t is a general symbol for whatever set of information is assumed to be "fully reflected" in the price at t; and the tildes indicate that $p_{j,t+1}$ and $r_{j,t+1}$ are random variables at t.

The value of the equilibrium expected return $E(\tilde{r}_{j,t+1}|\Phi_t)$ projected on the basis of the information Φ_t would be determined from the particular expected return theory at hand. The conditional expectation notation of (1) is meant to imply, however, that whatever expected return model is assumed to apply, the information in Φ_t is fully utilized in determining equilibrium expected returns. And this is the sense in which Φ_t is "fully reflected" in the formation of the price p_{jt}.

But we should note right off that, simple as it is, the assumption that the conditions of market equilibrium can be stated in terms of expected returns elevates the purely mathematical concept of expected value to a status not necessarily implied by the general notion of market efficiency. The expected value is just one of many possible summary measures of a distribution of returns, and market efficiency per se (i.e., the general notion that prices "fully reflect" available information) does not imbue it with any special importance. Thus, the results of tests based on this assumption depend to some extent on its validity as well as on the efficiency of the market. But some such assumption is the unavoidable price one must pay to give the theory of efficient markets empirical content.

The assumptions that the conditions of market equilibrium can be stated in terms of expected returns and that equilibrium expected returns are formed on the basis of (and thus "fully reflect") the information set Φ_t have a major empirical implication—they rule out the possibility of trading systems based only on information in Φ_t that have expected profits or returns in excess of equilibrium expected profits or returns. Thus let

$$x_{j,t+1} = p_{j,t+1} - E(p_{j,t+1}|\Phi_t). \tag{2}$$

Then

$$E(\tilde{x}_{j,t+1}|\Phi_t) = 0 \tag{3}$$

which, *by definition,* says that the sequence $\{x_{jt}\}$ is a "fair game" with respect to the information sequence $\{\phi_t\}$. Or, equivalently, let

$$z_{j,t+1} = r_{j,t+1} - E(\tilde{r}_{j,t+1}|\Phi_t), \tag{4}$$

then

$$E(\tilde{z}_{j,t+1}|\Phi_t) = 0, \tag{5}$$

so that the sequence $\{z_{jt}\}$ is also a "fair game" with respect to the information sequence $\{\Phi\}$.

In economic terms, $x_{j,t+1}$ is the excess market value of security j at time $t + 1$: it is the difference between the observed price and the expected value of the price that was projected at t on the basis of the information Φ_t. And similarly, $z_{j,t+1}$ is the return at $t + 1$ in excess of the equilibrium expected return projected at t. Let

$$\alpha(\Phi_t) = [\alpha_1(\Phi_t), \alpha_2(\Phi_t), \ldots, \alpha_n(\Phi_t)]$$

be any trading system based on Φ_t which tells the investor the amounts $\alpha_j(\Phi_t)$ of funds available at t that are to be invested in each of the n available securities. The total excess market value at $t + 1$ that will be generated by such a system is

$$V_{t+1} = \sum_{j=1}^{n} \alpha_j(\Phi_t)[r_{j,t+1} - E(\tilde{r}_{j,t+1}|\Phi_t)],$$

which, from the "fair game" property of (5) has expectation,

$$E(\tilde{V}_{t+1}|\Phi_t) = \sum_{j=1}^{n} \alpha_j(\Phi_t)E(\tilde{z}_{j,t+1}|\Phi_t) = 0.$$

The expected return or "fair game" efficient markets model[2] has other important testable implications, but these are better saved for the later discussion of the empirical work. Now we turn to two special cases of the model, the submartingale and the random walk, that (as we shall see later) play an important role in the empirical literature.

2. Though we shall sometimes refer to the model summarized by (1) as the "fair game" model, keep in mind that the "fair game" properties of the model are *implications* of the assumptions that (i) the conditions of market equilibrium can be stated in terms of expected returns, and (ii) the information Φ_t is fully utilized by the market in forming equilibrium expected returns and thus current prices.

The role of "fair game" models in the theory of efficient markets was first recognized and studied rigorously by Mandelbrot [27] and Samuelson [38]. Their work will be discussed in more detail later.

B. *The Submartingale Model*

Suppose we assume in (1) that for all t and Φ_t

$$E(\tilde{p}_{j,t+1}|\Phi_t) \geqslant p_{jt}, \quad \text{or equivalently,} \quad E(\tilde{r}_{j,t+1}|\Phi_t) \geqslant 0. \tag{6}$$

This is a statement that the price sequence $\{p_{jt}\}$ for security j follows a submartingale with respect to the information sequence $\{\Phi_t\}$, which is to say nothing more than that the expected value of next period's price, as projected on the basis of the information Φ_t, is equal to or greater than the current price. If (6) holds as an equality (so that expected returns and price changes are zero), then the price sequence follows a martingale.

A submartingale in prices has one important empirical implication. Consider the set of "one security and cash" mechanical trading rules by which we mean systems that concentrate on individual securities and that define the conditions under which the investor would hold a given security, sell it short, or simply hold cash at any time t. Then the assumption of (6) that expected returns conditional on Φ_t are non-negative directly implies that such trading rules based only on the information in Φ_t cannot have greater expected profits than a policy of always buying-and-holding the security during the future period in question. Tests of such rules will be an important part of the empirical evidence on the efficient markets model.[3]

C. *The Random Walk Model*

In the early treatments of the efficient markets model, the statement that the current price of a security "fully reflects" available information was assumed to imply that successive price changes (or more usually, successive one-period returns) are independent. In addition, it was usually assumed that successive changes (or returns) are identically distributed. Together the two hypotheses constitute the random walk model. Formally, the model says

$$f(r_{j,t+1}|\Phi_t) = f(r_{j,t+1}), \tag{7}$$

which is the usual statement that the conditional and marginal probability distributions of an independent random variable are identical. In addition, the density function f must be the same for all t.[4]

3. Note that the expected profitability of "one security and cash" trading systems vis-à-vis buy-and-hold is not ruled out by the general expected return or "fair game" efficient markets model. The latter rules out systems with expected profits in excess of equilibrium expected returns, but since in principle it allows equilibrium expected returns to be negative, holding cash (which always has zero actual and thus expected return) may have higher expected return than holding some security.

And negative equilibrium expected returns for some securities are quite possible. For example, in the Sharpe [40]-Lintner [24, 25] model (which is in turn a natural extension of the portfolio models of Markowitz [30] and Tobin [43]) the equilibrium expected return on a security depends on the extent to which the dispersion in the security's return distribution is related to dispersion in the returns on all other securities. A security whose returns on average move opposite to the general market is particularly valuable in reducing dispersion of portfolio returns, and so its equilibrium expected return may well be negative.

4. The terminology is loose. Prices will only follow a random walk if price changes are independent, identically distributed; and even then we should say "random walk with drift" since expected price changes can be non-zero. If one-period returns are independent, identically distributed, prices will not follow a random walk since the distribution of price changes will depend

Expression (7) of course says much more than the general expected return model summarized by (1). For example, if we restrict (1) by assuming that the expected return on security j is constant over time, then we have

$$E(\tilde{r}_{j,t+1}|\Phi_t) = E(\tilde{r}_{j,t+1}). \tag{8}$$

This says that the mean of the distribution of $r_{j,t+1}$ is independent of the information available at t, Φ_t, whereas the random walk model of (7) in addition says that the entire distribution is independent of Φ_t.[5]

We argue later that it is best to regard the random walk model as an extension of the general expected return or "fair game" efficient markets model in the sense of making a more detailed statement about the economic environment. The "fair game" model just says that the conditions of market equilibrium can be stated in terms of expected returns, and thus it says little about the details of the stochastic process generating returns. A random walk arises within the context of such a model when the environment is (fortuitously) such that the evolution of investor tastes and the process generating new information combine to produce equilibria in which return distributions repeat themselves through time.

Thus it is not surprising that empirical tests of the "random walk" model that are in fact tests of "fair game" properties are more strongly in support of the model than tests of the additional (and, from the viewpoint of expected return market efficiency, superfluous) pure independence assumption. (But it is perhaps equally surprising that, as we shall soon see, the evidence against the independence of returns over time is as weak as it is.)

D. *Market Conditions Consistent with Efficiency*

Before turning to the empirical work, however, a few words about the market conditions that might help or hinder efficient adjustment of prices to information are in order. First, it is easy to determine *sufficient* conditions for capital market efficiency. For example, consider a market in which (i) there are no transactions costs in trading securities, (ii) all available information is costlessly available to all market participants, and (iii) all agree on the implications of current information for the current price and distributions of future prices of each security. In such a market, the current price of a security obviously "fully reflects" all available information.

But a frictionless market in which all information is freely available and investors agree on its implications is, of course, not descriptive of markets met in practice. Fortunately, these conditions are sufficient for market efficiency, but not necessary. For example, as long as transactors take account of all

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on the price level. But though rigorous terminology is usually desirable, our loose use of terms should not cause confusion; and our usage follows that of the efficient markets literature.

Note also that in the random walk literature, the information set Φ_t in (7) is usually assumed to include only the past return history, $r_{j,t}, r_{j,t-1}, \ldots$

5. The random walk model does not say, however, that past information is of no value in *assessing* distributions of future returns. Indeed since return distributions are assumed to be stationary through time, past returns are the best source of such information. The random walk model does say, however, that the *sequence* (or the order) of the past returns is of no consequence in assessing distributions of future returns.

available information, even large transactions costs that inhibit the flow of transactions do not in themselves imply that when transactions do take place, prices will not "fully reflect" available information. Similarly (and speaking, as above, somewhat loosely), the market may be efficient if "sufficient numbers" of investors have ready access to available information. And disagreement among investors about the implications of given information does not in itself imply market inefficiency unless there are investors who can consistently make better evaluations of available information than are implicit in market prices.

But though transactions costs, information that is not freely available to all investors, and disagreement among investors about the implications of given information are not necessarily sources of market inefficiency, they are potential sources. And all three exist to some extent in real world markets. Measuring their effects on the process of price formation is, of course, the major goal of empirical work in this area.

III. The Evidence

All the empirical research on the theory of efficient markets has been concerned with whether prices "fully reflect" particular subsets of available information. Historically, the empirical work evolved more or less as follows. The initial studies were concerned with what we call *weak form* tests in which the information subset of interest is just past price (or return) histories. Most of the results here come from the random walk literature. When extensive tests seemed to support the efficiency hypothesis at this level, attention was turned to *semi-strong form* tests in which the concern is the speed of price adjustment to other obviously publicly available information (e.g., announcements of stock splits, annual reports, new security issues, etc.). Finally, *strong form* tests in which the concern is whether any investor or groups (e.g., managements of mutual funds) have monopolistic access to any information relevant for the formation of prices have recently appeared. We review the empirical research in more or less this historical sequence.

First, however, we should note that what we have called *the* efficient markets model in the discussions of earlier sections is the hypothesis that security prices at any point in time "fully reflect" *all* available information. Though we shall argue that the model stands up rather well to the data, it is obviously an extreme null hypothesis. And, like any other extreme null hyposthesis, we do not expect it to be literally true. The categorization of the tests into weak, semi-strong, and strong form will serve the useful purpose of allowing us to pinpoint the level of information at which the hypothesis breaks down. And we shall contend that there is no important evidence against the hypothesis in the weak and semi-strong form tests (i.e., prices seem to efficiently adjust to obviously publicly available information), and only limited evidence against the hypothesis in the strong form tests (i.e., monopolistic access to information about prices does not seem to be a prevalent phenomenon in the investment community).

A. *Weak Form Tests of the Efficient Markets Model*

1. Random Walks and Fair Games: A Little Historical Background

As noted earlier, all of the empirical work on efficient markets can be considered within the context of the general expected return or "fair game" model, and much of the evidence bears directly on the special submartingale expected return model of (6). Indeed, in the early literature, discussions of the efficient markets model were phrased in terms of the even more special random walk model, though we shall argue that most of the early authors were in fact concerned with more general versions of the "fair game" model.

Some of the confusion in the early random walk writings is understandable. Research on security prices did not begin with the development of a theory of price formation which was then subjected to empirical tests. Rather, the impetus for the development of a theory came from the accumulation of evidence in the middle 1950's and early 1960's that the behavior of common stock and other speculative prices could be well approximated by a random walk. Faced with the evidence, economists felt compelled to offer some rationalization. What resulted was a theory of efficient markets stated in terms of random walks, but usually implying some more general "fair game" model.

It was not until the work of Samuelson [38] and Mandelbrot [27] in 1965 and 1966 that the role of "fair game" expected return models in the theory of efficient markets and the relationships between these models and the theory of random walks were rigorously studied.[6] And these papers came somewhat after the major empirical work on random walks. In the earlier work, "theoretical" discussions, though usually intuitively appealing, were always lacking in rigor and often either vague or *ad hoc*. In short, until the Mandelbrot-Samuelson models appeared, there existed a large body of empirical results in search of a rigorous theory.

Thus, though his contributions were ignored for sixty years, the first statement and test of the random walk model was that of Bachelier [3] in 1900. But his "fundamental principle" for the behavior of prices was that speculation should be a "fair game"; in particular, the expected profits to the speculator should be zero. With the benefit of the modern theory of stochastic processes, we know now that the process implied by this fundamental principle is a martingale.

After Bachelier, research on the behavior of security prices lagged until the

6. Basing their analyses on futures contracts in commodity markets, Mandelbrot and Samuelson show that if the price of such a contract at time t is the expected value at t (given information Φ_t) of the spot price at the termination of the contract, then the futures price will follow a martingale with respect to the information sequence $\{\Phi_t\}$; that is, the expected price change from period to period will be zero, and the price changes will be a "fair game." If the equilibrium expected return is not assumed to be zero, our more general "fair game" model, summarized by (1), is obtained.

But though the Mandelbrot-Samuelson approach certainly illuminates the process of price formation in commodity markets, we have seen that "fair game" expected return models can be derived in much simpler fashion. In particular, (1) is just a formalization of the assumptions that the conditions of market equilibrium can be stated in terms of expected returns and that the information Φ_t is used in forming market prices at t.

coming of the computer. In 1953 Kendall [21] examined the behavior of weekly changes in nineteen indices of British industrial share prices and in spot prices for cotton (New York) and wheat (Chicago). After extensive analysis of serial correlations, he suggests, in quite graphic terms:

> The series looks like a wandering one, almost as if once a week the Demon of Chance drew a random number from a symetrical population of fixed dispersion and added it to the current price to determine the next week's price [21, p. 13].

Kendall's conclusion had in fact been suggested earlier by Working [47], though his suggestion lacked the force provided by Kendall's empirical results. And the implications of the conclusion for stock market research and financial analysis were later underlined by Roberts [36].

But the suggestion by Kendall, Working, and Roberts that series of speculative prices may be well described by random walks was based on observation. None of these authors attempted to provide much economic rationale for the hypothesis, and, indeed, Kendall felt that economists would generally reject it. Osborne [33] suggested market conditions, similar to those assumed by Bachelier, that would lead to a random walk. But in his model, independence of successive price changes derives from the assumption that the decisions of investors in an individual security are independent from transaction to transaction—which is little in the way of an economic model.

Whenever economists (prior to Mandelbrot and Samuelson) tried to provide economic justification for the random walk, their arguments usually implied a "fair game." For example, Alexander [8, p. 200] states:

> If one were to start out with the assumption that a stock or commodity speculation is a "fair game" with equal expectation of gain or loss or, more accurately, with an expectation of zero gain, one would be well on the way to picturing the behavior of speculative prices as a random walk.

There is an awareness here that the "fair game" assumption is not sufficient to lead to a random walk, but Alexander never expands on the comment. Similarly, Cootner [8, p. 232] states:

> If any substantial group of buyers thought prices were too low, their buying would force up the prices. The reverse would be true for sellers. Except for appreciation due to earnings retention, the conditional expectation of tomorrow's price, given today's price, is today's price.
>
> In such a world, the only price changes that would occur are those that result from new information. Since there is no reason to expect that information to be non-random in appearance, the period-to-period price changes of a stock should be random movements, statistically independent of one another.

Though somewhat imprecise, the last sentence of the first paragraph seems to point to a "fair game" model rather than a random walk.[7] In this light, the second paragraph can be viewed as an attempt to describe environmental conditions that would reduce a "fair game" to a random walk. But the specification imposed on the information generating process is insufficient for this purpose; one would, for example, also have to say something about investor

7. The appropriate conditioning statement would be "Given the sequence of historical prices."

tastes. Finally, lest I be accused of criticizing others too severely for ambiguity, lack of rigor and incorrect conclusions,

> By contrast, the stock market trader has a much more practical criterion for judging what constitutes important dependence in successive price changes. For his purposes the random walk model is valid as long as knowledge of the past behavior of the series of price changes cannot be used to increase expected gains. More specifically, the independence assumption is an adequate description of reality as long as the actual degree of dependence in the series of price changes is not sufficient to allow the past history of the series to be used to predict the future in a way which makes expected profits greater than they would be under a naive buy-and hold model [10, p 35].

We know now, of course, that this last condition hardly requires a random walk. It will in fact be met by the submartingale model of (6).

But one should not be too hard on the theoretical efforts of the early empirical random walk literature. The arguments were usually appealing; where they fell short was in awareness of developments in the theory of stochastic processes. Moreover, we shall now see that most of the empirical evidence in the random walk literature can easily be interpreted as tests of more general expected return or "fair game" models.[8]

2. Tests of Market Efficiency in the Random Walk Literature

As discussed earlier, "fair game" models imply the "impossibility" of various sorts of trading systems. Some of the random walk literature has been concerned with testing the profitability of such systems. More of the literature has, however, been concerned with tests of serial covariances of returns. We shall now show that, like a random walk, the serial covariances of a "fair game" are zero, so that these tests are also relevant for the expected return models.

If x_t is a "fair game," its unconditional expectation is zero and its serial covariance can be written in general form as:

$$E(\tilde{x}_{t+\tau}\,\tilde{x}_t) = \int_{x_t} x_t E(\tilde{x}_{t+\tau}|x_t)\,f(x_t)\,dx_t,$$

where f indicates a density function. But if x_t is a "fair game,"

$$E(\tilde{x}_{t+\tau}|x_t) = 0.^{9}$$

8. Our brief historical review is meant only to provide perspective, and it is, of course, somewhat incomplete. For example, we have ignored the important contributions to the early random walk literature in studies of warrants and other options by Sprenkle, Kruizenga, Boness, and others. Much of this early work on options is summarized in [8].

9. More generally, if the sequence $\{x_t\}$ is a fair game with respect to the information sequence $\{\Phi_t\}$, (i.e., $E(\tilde{x}_{t+1}|\Phi_t) = 0$ for all Φ_t); then x_t is a fair game with respect to any Φ'_t that is a subset of Φ_t (i.e., $E(\tilde{x}_{t+1}|\Phi'_t) = 0$ for all Φ'_t). To show this, let $\Phi_t = (\Phi'_t, \ \Phi''_t)$. Then, using Stieltjes integrals and the symbol F to denote cumulative distinction functions, the conditional expectation

$$E(\tilde{x}_{t+1}|\Phi'_t) = \int_{\Phi''_t}\int_{x_{t+1}} x_{t+1}\,dF(x_{t+1}, \Phi''_t|\Phi'_t) \quad = \int_{\Phi''_t}\left[\int_{x_{t+1}} x_{t+1}\,dF(x_{t+1}|\Phi'_t,\Phi''_t)\right] dF(\Phi''_t|\Phi'_t).$$

From this it follows that for all lags, the serial covariances between lagged values of a "fair game" variable are zero. Thus, observations of a "fair game" variable are linearly independent.[10]

But the "fair game" model does not necessarily imply that the serial covariances of *one-period returns* are zero. In the weak form tests of this model the "fair game" variable is

$$z_{j,t} = r_{j,t} - E(\tilde{r}_{j,t}|r_{j,t-1}, r_{j,t-2}, \ldots). \quad \text{(Cf. fn. 9)} \tag{9}$$

But the covariance between, for example, r_{jt} and $r_{j,t+1}$ is

$$E([\tilde{r}_{j,t+1} - E(\tilde{r}_{j,t+1})]\,[\tilde{r}_{jt} - E(\tilde{r}_{jt})])$$
$$= \int_{r_{jt}} [r_{jt} - E(\tilde{r}_{jt})]\,[E(\tilde{r}_{j,t+1}|r_{jt}) - E(\tilde{r}_{j,t+1})]\,f(r_{jt})dr_{jt},$$

and (9) does not imply that $E(\tilde{r}_{j,t+1}|r_{jt}) = E(\tilde{r}_{j,t+1})$: In the "fair game" efficient markets model, the deviation of the return for $t + 1$ from its conditional expectation is a "fair game" variable, but the conditional expectation itself can depend on the return observed for t.[11]

In the random walk literature, this problem is not recognized, since it is assumed that the expected return (and indeed the entire distribution of returns) is stationary through time. In practice, this implies estimating serial covariances by taking cross products of deviations of observed returns from the overall sample mean return. It is somewhat fortuitous, then, that this procedure, which represents a rather gross approximation from the viewpoint of the general expected return efficient markets model, does not seem to greatly affect the results of the covariance tests, at least for common stocks.[12]

But the integral in brackets is just $E(\tilde{x}_{t+1}|\Phi_t)$ which by the "fair game" assumption is 0, so that

$$E(x_{t+1}|\Phi'_t) = 0 \text{ for all } \Phi'_t \subset \Phi_t.$$

10. But though zero serial covariances are consistent with a "fair game," they do not imply such a process. A "fair game" also rules out many types of non linear dependence. Thus using arguments similar to those above, it can be shown that if x is a "fair game," $E(\tilde{x}_t\tilde{x}_{t+1} \ldots \tilde{x}_{t+\tau}) = 0$ for all τ, which is not implied by $E(\tilde{x}_t\tilde{x}_{t+\tau}) = 0$ for all τ. For example, consider a three-period case where x must be either $\pm$ 1. Suppose the process is $x_{t+2} = \text{sign}\ (x_t x_{t+1})$, i.e.,

x_t	x_{t+1}	$\rightarrow$	x_{t+2}
+	+	→	+
+	−	→	−
−	+	→	−
−	−	→	+.

If probabilities are uniformly distributed across events,

$$E(\tilde{x}_{t+2}|x_{t+1}) = E(\tilde{x}_{t+2}|x_t) = E(\tilde{x}_{t+1}|x_t) = E(\tilde{x}_{t+2}) = E(\tilde{x}_{t+1}) = E(\tilde{x}_t) = 0,$$

so that all pairwise serial covariances are zero. But the process is not a "fair game," since $E(\tilde{x}_{t+2}|x_{t+1}, x_t) \neq 0$, and knowledge of (x_{t+1}, x_t) can be used as the basis of a simple "system" with positive expected profit.

11. For example, suppose the level of one-period returns follows a martingale so that

$$E(\tilde{r}_{j,t+1}|r_{jt}, r_{j,t-1} \ldots) = r_{jt}.$$

Then covariances between successive returns will be nonzero (though in this special case first differences of returns will be uncorrelated).

12. The reason is probably that for stocks, changes in equilibrium expected returns for the

TABLE 1 (from [10])
First-order Serial Correlation Coefficients for One-, Four-, Nine-, and Sixteen-Day Changes in Log_e Price

	Differencing Interval (Days)			
Stock	One	Four	Nine	Sixteen
Allied Chemical	.017	.029	−.091	−.118
Alcoa	.118*	.095	−.112	−.044
American Can	−.087*	−.124*	−.060	.031
A. T. & T.	−.039	−.010	−.009	−.003
American Tobacco	.111*	−.175*	.033	.007
Anaconda	.067*	−.068	−.125	.202
Bethlehem Steel	.013	−.122	−.148	.112
Chrysler	.012	.060	−.026	.040
Du Pont	.013	.069	−.043	−.055
Eastman Kodak	.025	−.006	−.053	−.023
General Electric	.011	.020	−.004	.000
General Foods	.061*	−.005	−.140	−.098
General Motors	−.004	−.128*	.009	−.028
Goodyear	−.123*	.001	−.037	.033
International Harvester	−.017	−.068	−.244*	.116
International Nickel	.096*	.038	.124	.041
International Paper	.046	.060	−.004	−.010
Johns Manville	.006	−.068	−.002	.002
Owens Illinois	−.021	−.006	.003	−.022
Procter & Gamble	.099*	−.006	.098	.076
Sears	.097*	−.070	−.113	.041
Standard Oil (Calif.)	.025	−.143*	−.046	.040
Standard Oil (N.J.)	.008	−.109	−.082	−.121
Swift & Co.	−.004	−.072	.118	−.197
Texaco	.094*	−.053	−.047	−.178
Union Carbide	.107*	.049	−.101	.124
United Aircraft	.014	−.190*	−.192*	−.040
U.S. Steel	.040	−.006	−.056	.236*
Westinghouse	−.027	−.097	−.137	.067
Woolworth	.028	−.033	−.112	.040

* Coefficient is twice its computed standard error.

For example, Table 1 (taken from [10]) shows the serial correlations between successive changes in the natural log of price for each of the thirty stocks of the Dow Jones Industrial Average, for time periods that vary slightly from stock to stock, but usually run from about the end of 1957 to September 26, 1962. The serial correlations of successive changes in $\log_e$ price are shown for differencing intervals of one, four, nine, and sixteen days.[13]

common differencing intervals of a day, a week, or a month, are trivial relative to other sources of variation in returns. Later, when we consider Roll's work [37], we shall see that this is not true for one week returns on U.S. Government Treasury Bills.

13. The use of changes in $\log_e$ price as the measure of return is common in the random walk literature. It can be justified in several ways. But for current purposes, it is sufficient to note that for price changes less than fifteen per cent, the change in $\log_e$ price is approximately the percentage price change or one-period return. And for differencing intervals shorter than one month, returns in excess of fifteen per cent are unusual. Thus [10] reports that for the data of Table 1, tests carried out on percentage or one-period returns yielded results essentially identical to the tests based on changes in $\log_e$ price.

The results in Table 1 are typical of those reported by others for tests based on serial covariances. (Cf. Kendall [21], Moore [31], Alexander [1], and the results of Granger and Morgenstern [17] and Godfrey, Granger and Morgenstern [16] obtained by means of spectral analysis.) Specifically, there is no evidence of substantial linear dependence between lagged price changes or returns. In absolute terms the measured serial correlations are always close to zero.

Looking hard, though, one can probably find evidence of statistically "significant" linear dependence in Table 1 (and again this is true of results reported by others). For the daily returns eleven of the serial correlations are more than twice their computed standard errors, and twenty-two out of thirty are positive. On the other hand, twenty-one and twenty-four of the coefficients for the four and nine day differences are negative. But with samples of the size underlying Table 1 ($N = 1200$-1700 observations per stock on a daily basis) statistically "significant" deviations from zero covariance are not necessarily a basis for rejecting the efficient markets model. For the results in Table 1, the standard errors of the serial correlations were approximated as $(1/(N\text{-}1))^{1/2}$, which for the daily data implies that a correlation as small as .06 is more than twice its standard error. But a coefficient this size implies that a linear relationship with the lagged price change can be used to explain about .36% of the variation in the current price change, which is probably insignificant from an economic viewpoint. In particular, it is unlikely that the small absolute levels of serial correlation that are always observed can be used as the basis of substantially profitable trading systems.[14]

It is, of course, difficult to judge what degree of serial correlation would imply the existence of trading rules with substantial expected profits. (And indeed we shall soon have to be a little more precise about what is implied by "substantial" profits.) Moreover, zero serial covariances are consistent with a "fair game" model, but as noted earlier (fn. 10), there are types of nonlinear dependence that imply the existence of profitable trading systems, and yet do not imply nonzero serial covariances. Thus, for many reasons it is desirable to directly test the profitability of various trading rules.

The first major evidence on trading rules was Alexander's [1, 2]. He tests a variety of systems, but the most thoroughly examined can be decribed as follows: If the price of a security moves up at least y%, buy and hold the security until its price moves down at least y% from a subsequent high, at which time simultaneously sell and go short. The short position is maintained until the price rises at least y% above a subsequent low, at which time one covers the short position and buys. Moves less than y% in either direction are

14. Given the evidence of Kendall [21], Mandelbrot [28], Fama [10] and others that large price changes occur much more frequently than would be expected if the generating process were Gaussian, the expression $(1/(N\text{-}1))^{1/2}$ understates the sampling dispersion of the serial correlation coefficient, and thus leads to an overstatement of significance levels. In addition, the fact that sample serial correlations are predominantly of one sign or the other is not in itself evidence of linear dependence. If, as the work of King [23] and Blume [7] indicates, there is a market factor whose behavior affects the returns on all securities, the sample behavior of this market factor may lead to a predominance of signs of one type in the serial correlations for individual securities, even though the population serial correlations for both the market factor and the returns on individual securities are zero. For a more extensive analysis of these issues see [10].

ignored. Such a system is called a y% filter. It is obviously a "one security and cash" trading rule, so that the results it produces are relevant for the submartingale expected return model of (6).

After extensive tests using daily data on price indices from 1897 to 1959 and filters from one to fifty per cent, and after correcting some incorrect presumptions in the initial results of [1] (see fn. 25), in his final paper on the subject, Alexander concludes:

> In fact, at this point I should advise any reader who is interested only in practical results, and who is not a floor trader and so must pay commissions, to turn to other sources on how to beat buy and hold. The rest of this article is devoted principally to a theoretical consideration of whether the observed results are consistent with a random walk hypothesis [8], p. 351).

Later in the paper Alexander concludes that there is some evidence in his results against the independence assumption of the random walk model. But market efficiency does not require a random walk, and from the viewpoint of the submartingale model of (6), the conclusion that the filters cannot beat buy-and-hold is support for the efficient markets hypothesis. Further support is provided by Fama and Blume [13] who compare the profitability of various filters to buy-and-hold for the individual stocks of the Dow-Jones Industrial Average. (The data are those underlying Table 1.)

But again, looking hard one can find evidence in the filter tests of both Alexander and Fama-Blume that is inconsistent with the submartingale efficient markets model, if that model is interpreted in a strict sense. In particular, the results for very small filters (1 per cent in Alexander's tests and .5, 1.0, and 1.5 per cent in the tests of Fama-Blume) indicate that it is possible to devise trading schemes based on very short-term (preferably intra-day but at most daily) price swings that will on average outperform buy-and-hold. The average profits on individual transactions from such schemes are miniscule, but they generate transactions so frequently that over longer periods and ignoring commissions they outperform buy-and-hold by a substantial margin. These results are evidence of persistence or positive dependence in very short-term price movements. And, interestingly, this is consistent with the evidence for slight positive linear dependence in successive daily price changes produced by the serial correlations.[15]

15. Though strictly speaking, such tests of pure independence are not directly relevant for expected return models, it is interesting that the conclusion that very short-term swings in prices persist slightly longer than would be expected under the martingale hypothesis is also supported by the results of non-parametric runs tests applied to the daily data of Table 1. (See [10], Tables 12-15.) For the daily price changes, the actual number of runs of price changes of the same sign is less than the expected number for 26 out of 30 stocks. Moreover, of the eight stocks for which the actual number of runs is more than two standard errors less than the expected number, five of the same stocks have positive daily, first order serial correlations in Table 1 that are more than twice their standard errors. But in both cases the statistical "significance" of the results is largely a reflection of the large sample sizes. Just as the serial correlations are small in absolute terms (the average is .026), the differences between the expected and actual number of runs on average are only three per cent of the total expected number.

On the other hand, it is also interesting that the runs tests do not support the suggestion of slight negative dependence in four and nine day changes that appeared in the serial correlations. In the runs tests such negative dependence would appear as a tendency for the actual number of runs to exceed the expected number. In fact, for the four and nine day price changes, for 17 and

But when one takes account of even the minimum trading costs that would be generated by small filters, their advantage over buy-and-hold disappears. For example, even a floor trader (i.e., a person who owns a seat) on the New York Stock Exchange must pay clearinghouse fees on his trades that amount to about .1 per cent per turnaround transaction (i.e., sales plus purchase). Fama-Blume show that because small filters produce such frequent trades, these minimum trading costs are sufficient to wipe out their advantage over buy-and-hold.

Thus the filter tests, like the serial correlations, produce empirically noticeable departures from the strict implications of the efficient markets model. But, in spite of any statistical significance they might have, from an economic viewpoint the departures are so small that it seems hardly justified to use them to declare the market inefficient.

3. Other Tests of Independence in the Random Walk Literature

It is probably best to regard the random walk model as a special case of the more general expected return model in the sense of making a more detailed specification of the economic environment. That is, the basic model of market equilibrium is the "fair game" expected return model, with a random walk arising when additional environmental conditions are such that distributions of one-period returns repeat themselves through time. From this viewpoint violations of the pure independence assumption of the random walk model are to be expected. But when judged relative to the benchmark provided by the random walk model, these violations can provide insights into the nature of the market environment.

For example, one departure from the pure independence assumption of the random walk model has been noted by Osborne [34], Fama ([10], Table 17 and Figure 8), and others. In particular, large daily price changes tend to be followed by large daily changes. The signs of the successor changes are apparently random, however, which indicates that the phenomenon represents a denial of the random walk model but not of the market efficiency hypothesis. Nevertheless, it is interesting to speculate why the phenomenon might arise. It may be that when important new information comes into the market it cannot always be immediately evaluated precisely. Thus, sometimes the initial price will overadjust to the information, and other times it will underadjust. But since the evidence indicates that the price changes on days following the initial large change are random in sign, the initial large change at least represents an unbiased adjustment to the ultimate price effects of the information, and this is sufficient for the expected return efficient markets model.

Niederhoffer and Osborne [32] document two departures from complete randomness in common stock price changes from transaction to transaction. First, their data indicate that reversals (pairs of consecutive price changes of opposite sign) are from two to three times as likely as continuations (pairs of consecutive price changes of the same sign). Second, a continuation is

18 of the 30 stocks in Table 1 the actual number of runs is less than the expected number. Indeed, runs tests in general show no consistent evidence of dependence for any differencing interval longer than a day, which seems especially pertinent in light of the comments in footnote 14.

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slightly more frequent after a preceding continuation than after a reversal. That is, let (+|++) indicate the occurrence of a positive price change, given two preceding positive changes. Then the events (+|++) and (−|−−) are slightly more frequent than (+|+−) or (−|−+).[16]

Niederhoffer and Osborne offer explanations for these phenomena based on the market structure of the New York Stock Exchange (N.Y.S.E.). In particular, there are three major types of orders that an investor might place in a given stock: (a) buy limit (buy at a specified price or lower), (b) sell limit (sell at a specified price or higher), and (c) buy or sell at market (at the lowest selling or highest buying price of another investor). A book of unexecuted limit orders in a given stock is kept by the specialist in that stock on the floor of the exchange. Unexecuted sell limit orders are, of course, at higher prices than unexecuted buy limit orders. On both exchanges, the smallest non-zero price change allowed is ⅛ point.

Suppose now that there is more than one unexecuted sell limit order at the lowest price of any such order. A transaction at this price (initiated by an order to buy at market[17]) can only be followed either by a transaction at the same price (if the next market order is to buy) or by a transaction at a lower price (if the next market order is to sell). Consecutive price increases can usually only occur when consecutive market orders to buy exhaust the sell limit orders at a given price.[18] In short, the excessive tendency toward reversal for consecutive non-zero price changes could result from bunching of unexecuted buy and sell limit orders.

The tendency for the events (+|++) and (−|−−) to occur slightly more frequently than (+|+−) and (−|−+) requires a more involved explanation which we shall not attempt to reproduce in full here. In brief, Niederhoffer and Osborne contend that the higher frequency of (+|++) relative to (+|+−) arises from a tendency for limit orders "to be concentrated at integers (26, 43), halves (26½, 43½), quarters and odd eighths in descending order of preference."[19] The frequency of the event (+|++), which usually requires that sell limit orders be exhausted at at least two consecutively higher prices (the last of which is relatively more frequently at an odd eighth), more heavily reflects the absence of sell limit orders at odd eighths than the event (+|+−), which usually implies that sell limit orders at only one price have been exhausted and so more or less reflects the average bunching of limit orders at all eighths.

But though Niederhoffer and Osborne present convincing evidence of sta-

16. On a transaction to transaction basis, positive and negative price changes are about equally likely. Thus, under the assumption that price changes are random, any pair of non-zero changes should be as likely as any other, and likewise for triplets of consecutive non-zero changes.

17. A buy limit order for a price equal to or greater than the lowest available sell limit price is effectively an order to buy at market, and is treated as such by the broker.

18. The exception is when there is a gap of more than ⅛ between the highest unexecuted buy limit and the lowest unexecuted sell limit order, so that market orders (and new limit orders) can be crossed at intermediate prices.

19. Their empirical documentation for this claim is a few samples of specialists' books for selected days, plus the observation [34] that actual trading prices, at least for volatile high priced stocks, seem to be concentrated at integers, halves, quarters and odd eighths in descending order.

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tistically significant departures from independence in price changes from transaction to transaction, and though their analysis of their findings presents interesting insights into the process of market making on the major exchanges, the types of dependence uncovered do not imply market inefficiency. The best documented source of dependence, the tendency toward excessive reversals in pairs of non-zero price changes, seems to be a direct result of the ability of investors to place limit orders as well as orders at market, and this negative dependence in itself does not imply the existence of profitable trading rules. Similarly, the apparent tendency for observed transactions (and, by implication, limit orders) to be concentrated at integers, halves, even eighths and odd eighths in descending order is an interesting fact about investor behavior, but in itself is not a basis on which to conclude that the market is inefficient.[20]

The Niederhoffer-Osborne analysis of market making does, however, point clearly to the existence of market inefficiency, but with respect to strong form tests of the efficient markets model. In particular, the list of unexecuted buy and sell limit orders in the specialist's book is important information about the likely future behavior of prices, and this information is only available to the specialist. When the specialist is asked for a quote, he gives the prices and can give the quantities of the highest buy limit and lowest sell limit orders on his book, but he is prevented by law from divulging the book's full contents. The interested reader can easily imagine situations where the structure of limit orders in the book could be used as the basis of a profitable trading rule.[21] But the record seems to speak for itself:

> It should not be assumed that these transactions undertaken by the specialist, and in which he is involved as buyer or seller in 24 per cent of all market volume, are necessarily a burden to him. Typically, the specialist sells above his last purchase on 83 per cent of all his sales, and buys below his last sale on 81 per cent of all his purchases ([32], p. 908).

Thus it seems that the specialist has monopoly power over an important block of information, and, not unexpectedly, uses his monopoly to turn a profit. And this, of course, is evidence of market inefficiency in the strong form sense. The important economic question, of course, is whether the market making

20. Niederhoffer and Osborne offer little to refute this conclusion. For example ([32], p. 914):

> Although the specific properties reported in this study have a significance from a statistical point of view, the reader may well ask whether or not they are helpful in a practical sense. Certain trading rules emerge as a result of our analysis. One is that limit and stop orders should be placed at odd eights, preferably at ⅞ for sell orders and at ⅛ for buy orders. Another is to buy when a stock advances through a barrier and to sell when it sinks through a barrier.

The first "trading rule" tells the investor to resist his innate inclination to place orders at integers, but rather to place sell orders ⅛ below an integer and buy orders ⅛ above. Successful execution of the orders is then more likely, since the congestion of orders that occur at integers is avoided. But the cost of this success is apparent. The second "trading rule" seems no more promising, if indeed it can even be translated into a concrete prescription for action.

21. See, for example, ([32], p. 908). But it is unlikely that anyone but the specialist could earn substantial profits from knowledge of the structure of unexecuted limit orders on the book. The specialist makes trading profits by engaging in many transactions, each of which has a small average profit; but for any other trader, including those with seats on the exchange, these profits would be eaten up by commissions to the specialist.

function of the specialist could be fulfilled more economically by some non-monopolistic mechanism.[22]

4. Distributional Evidence

At this date the weight of the empirical evidence is such that economists would generally agree that whatever dependence exists in series of historical returns cannot be used to make profitable predictions of the future. Indeed, for returns that cover periods of a day or longer, there is little in the evidence that would cause rejection of the stronger random walk model, at least as a good first approximation.

Rather, the last burning issue of the random walk literature has centered on the nature of the distribution of price changes (which, we should note immediately, is an important issue for the efficient markets hypothesis since the nature of the distribution affects both the types of statistical tools relevant for testing the hypothesis and the interpretation of any results obtained). A model implying normally distributed price changes was first proposed by Bachelier [3], who assumed that price changes from transaction to transaction are independent, identically distributed random variables with finite variances. If transactions are fairly uniformly spread across time, and if the number of transactions per day, week, or month is very large, then the Central Limit Theorem leads us to expect that these price changes will have normal or Gaussian distributions.

Osborne [33], Moore [31], and Kendall [21] all thought their empirical evidence supported the normality hypothesis, but all observed high tails (i.e., higher proportions of large observations) in their data distributions vis-à-vis what would be expected if the distributions were normal. Drawing on these findings and some empirical work of his own, Mandelbrot [28] then suggested that these departures from normality could be explained by a more general form of the Bachelier model. In particular, if one does not assume that distributions of price changes from transaction to transaction necessarily have finite variances, then the limiting distributions for price changes over longer differencing intervals could be any member of the stable class, which includes the normal as a special case. Non-normal stable distributions have higher tails than the normal, and so can account for this empirically observed feature of distributions of price changes. After extensive testing (involving the data from the stocks in Table 1), Fama [10] concludes that non-normal stable distributions are a better description of distributions of daily returns on common stocks than the normal. This conclusion is also supported by the empirical work of Blume [7] on common stocks, and it has been extended to U.S. Government Treasury Bills by Roll [37].

Economists have, however, been reluctant to accept these results,[23] primar-

22. With modern computers, it is hard to believe that a more competitive and economical system would not be feasible. It does not seem technologically impossible to replace the entire floor of the N.Y.S.E. with a computer, fed by many remote consoles, that kept all the books now kept by the specialists, that could easily make the entire book on any stock available to anybody (so that interested individuals could then compete to "make a market" in a stock) and that carried out transactions automatically.

23. Some have suggested that the long-tailed empirical distributions might result from processes

ily because of the wealth of statistical techniques available for dealing with normal variables and the relative paucity of such techniques for non-normal stable variables. But perhaps the biggest contribution of Mandelbrot's work has been to stimulate research on stable distributions and estimation procedures to be applied to stable variables. (See, for example, Wise [46], Fama and Roll [15], and Blattberg and Sargent [6], among others.) The advance of statistical sophistication (and the importance of examining distributional assumptions in testing the efficient markets model) is well illustrated in Roll [37], as compared, for example, with the early empirical work of Mandelbrot [28] and Fama [10].

5. "Fair Game" Models in the Treasury Bill Market

Roll's work is novel in other respects as well. Coming after the efficient markets models of Mandelbrot [27] and Samuelson [38], it is the first weak form empirical work that is consciously in the "fair game" rather than the random walk tradition.

More important, as we saw earlier, the "fair game" properties of the general expected return models apply to

$$z_{jt} = r_{jt} - E(\tilde{r}_{jt}|\Phi_{t-1}). \quad (10)$$

For data on common stocks, tests of "fair game" (and random walk) properties seem to go well when the conditional expected return is estimated as the average return for the sample of data at hand. Apparently the variation in common stock returns about their expected values is so large relative to any changes in the expected values that the latter can safely be ignored. But, as Roll demonstrates, this result does not hold for Treasury Bills. Thus, to test the "fair game" model on Treasury Bills requires explicit economic theory for the evolution of expected returns through time.

Roll uses three existing theories of the term structure (the pure expectations hypothesis of Lutz [26] and two market segmentation hypotheses, one of which is the familiar "liquidity preference" hypothesis of Hicks [18] and Kessel [22]) for this purpose.[24] In his models r_{jt} is the rate observed from the term structure at period t for one week loans to commence at $t + j - 1$, and can be thought of as a "futures" rate. Thus $r_{j+1,\, t-1}$ is likewise the rate on

that are mixtures of normal distributions with different variances. Press [35], for example, suggests a Poisson mixture of normals in which the resulting distributions of price changes have long tails but finite variances. On the other hand, Mandelbrot and Taylor [29] show that other mixtures of normals can still lead to non-normal stable distributions of price changes for finite differencing intervals.

If, as Press' model would imply, distributions of price changes are long-tailed but have finite variances, then distributions of price changes over longer and longer differencing intervals should be progressively closer to the normal. No such convergence to normality was observed in [10] (though admittedly the techniques used were somewhat rough). Rather, except for origin and scale, the distributions for longer differencing intervals seem to have the same "high-tailed" characteristics as distributins for shorter differencing intervals, which is as would be expected if the distributions are non-normal stable.

24. As noted early in our discussions, all available tests of market efficiency are implicitly also tests of expected return models of market equilibrium. But Roll formulates explicitly the economic models underlying his estimates of expected returns, and emphasizes that he is simultaneously testing economic models of the term structure as well as market efficiency.

one week loans to commence at $t + j - 1$, but observed in this case at $t - 1$. Similarly, L_{jt} is the so-called "liquidity premium" in r_{jt}; that is

$$r_{jt} = E(\tilde{r}_{0,t+j-1}|\Phi_t) + L_{jt}.$$

In words, the one-week "futures" rate for period $t + j - 1$ observed from the term structure at t is the expectation at t of the "spot" rate for $t + j - 1$ plus a "liquidity premium" (which could, however, be positive or negative).

In all three theories of the term structure considered by Roll, the conditional expectation required in (10) is of the form

$$E(\tilde{r}_{j,t}|\Phi_{t-1}) = r_{j+1,t-1} + E(\tilde{L}_{jt}|\Phi_{t-1}) - L_{j+1,t-1}.$$

The three theories differ only in the values assigned to the "liquidity premiums." For example, in the "liquidity preference" hypothesis, investors must always be paid a positive premium for bearing interest rate uncertainty, so that the L_{jt} are always positive. By contrast, in the "pure expectations" hypothesis, all liquidity premiums are assumed to be zero, so that

$$E(\tilde{r}_{jt}|\Phi_{t-1}) = r_{j+1,t-1}.$$

After extensive testing, Roll concludes (i) that the two market segmentation hypotheses fit the data better than the pure expectations hypothesis, with perhaps a slight advantage for the "liquidity preference" hypothesis, and (ii) that as far as his tests are concerned, the market for Treasury Bills is effcient. Indeed, it is interesting that when the best fitting term structure model is used to estimate the conditional expected "futures" rate in (10), the resulting variable z_{jt} seems to be serially independent! It is also interesting that if he simply assumed that his data distributions were normal, Roll's results would not be so strongly in support of the efficient markets model. In this case taking account of the observed high tails of the data distributions substantially affected the interpretation of the results.[25]

6. Tests of a Multiple Security Expected Return Model

Though the weak form tests support the "fair game" efficient markets model, all of the evidence examined so far consists of what we might call "single security tests." That is, the price or return histories of individual securities are examined for evidence of dependence that might be used as the basis of a trading system for *that* security. We have not discussed tests of whether securities are "appropriately priced" vis-à-vis one another.

But to judge whether differences between average returns are "appropriate" an economic theory of equilibrium expected returns is required. At the moment, the only fully developed theory is that of Sharpe [40] and Lintner [24,

25. The importance of distributional assumptions is also illustrated in Alexander's work on trading rules. In his initial tests of filter systems [1], Alexander assumed that purchases could always be executed exactly (rather than at least) y% above lows and sales exactly y% below highs. Mandelbrot [28] pointed out, however, that though this assumption would do little harm with normally distributed price changes (since price series are then essentially continuous), with non-normal stable distributions it would introduce substantial positive bias into the filter profits (since with such distributions price series will show many discontinuities). In his later tests [2], Alexander does indeed find that taking account of the discontinuities (i.e., the presence of large price changes) in his data substantially lowers the profitability of the filters.

25] referred to earlier. In this model (which is a direct outgrowth of the mean-standard deviation portfolio models of investor equilibrium of Markowitz [30] and Tobin [43]), the expected return on security j from time t to t + 1 is

$$E(\tilde{r}_{j,t+1}|\Phi_t) = r_{f,t+1} + \left[\frac{E(\tilde{r}_{m,t+1}|\Phi_t) - r_{f,t+1}}{\sigma(\tilde{r}_{m,t+1}|\Phi_t)}\right] \frac{\text{cov}\,(\tilde{r}_{j,t+1}, \tilde{r}_{m,t+1}|\Phi_t)}{\sigma(\tilde{r}_{m,t+1}|\Phi_t)}, \tag{11}$$

where $r_{f,t+1}$ is the return from t to t + 1 on an asset that is riskless in money terms; $r_{m,t+1}$ is the return on the "market portfolio" m (a portfolio of all investment assets with each weighted in proportion to the total market value of all its outstanding units); $\sigma^2(\tilde{r}_{m,t+1}|\Phi_t)$ is the variance of the return on m; cov $(\tilde{r}_{j,t+1}, \tilde{r}_{m,t+1}|\Phi_t)$ is the covariance between the returns on j and m; and the appearance of Φ_t indicates that the various expected returns, variance and covariance, could in principle depend on Φ_t. Though Sharpe and Lintner derive (11) as a one-period model, the result is given a multiperiod justification and interpretation in [11]. The model has also been extended in (12) to the case where the one-period returns could have stable distributions with infinite variances.

In words, (11) says that the expected one-period return on a security is the one-period riskless rate of interest $r_{f,t+1}$ plus a "risk premium" that is proportional to $\text{cov}(\tilde{r}_{j,t+1}, \tilde{r}_{m,t+1}|\Phi_t)/\sigma(\tilde{r}_{m,t+1}|\Phi_t)$. In the Sharpe-Lintner model each investor holds some combination of the riskless asset and the market portfolio, so that, given a mean-standard deviation framework, the risk of an individual asset can be measured by its contribution to the standard deviation of the return on the market portfolio. This contribution is in fact cov $(\tilde{r}_{j,t+1}, \tilde{r}_{m,t+1}|\Phi_t)/\sigma(\tilde{r}_{m,t+1}|\Phi_t)$.[26] The factor

$$[E(\tilde{r}_{m,t+1}|\Phi_t) - r_{f,t+1}]/\sigma(\tilde{r}_{m,t+1}|\Phi_t),$$

which is the same for all securities, is then regarded as the market price of risk.

Published empirical tests of the Sharpe-Lintner model are not yet available, though much work is in progress. There is some published work, however, which, though not directed at the Sharpe-Lintner model, is at least consistent with some of its implications. The stated goal of this work has been to determine the extent to which the returns on a given security are related to the returns on other securities. It started (again) with Kendall's [21] finding that though common stock price changes do not seem to be serially correlated, there is a high degree of cross-correlation between the *simultaneous* returns of different securities. This line of attack was continued by King [23] who (using factor analysis of a sample of monthly returns on sixty N.Y.S.E. stocks for the period 1926-60) found that on average about 50% of the variance of an individual stock's returns could be accounted for by a "market factor" which affects the returns on all stocks, with "industry factors" accounting for at most an additional 10% of the variance.

26. That is,

$$\sum_j \text{cov}\,(\tilde{r}_{j,t+1}, \tilde{r}_{m,t+1}|\Phi_t)/\sigma(\tilde{r}_{m,t+1}|\Phi_t) = \sigma(\tilde{r}_{m,t+1}|\Phi_t).$$

For our purposes, however, the work of Fama, Fisher, Jensen, and Roll [14] (henceforth FFJR) and the more extensive work of Blume [7] on monthly return data is more relevant. They test the following "market model," originally suggested by Markowitz [30]:

$$\tilde{r}_{j,t+1} = \alpha_j + \beta_j \tilde{r}_{M,t+1} + \tilde{u}_{j,t+1} \tag{12}$$

where $r_{j,t+1}$ is the rate of return on security j for month t, $r_{M,t+1}$ is the corresponding return on a market index M, α_j and β_j are parameters that can vary from security to security, and $u_{j,t+1}$ is a random disturbance. The tests of FFJR and subsequently those of Blume indicate that (12) is well specified as a linear regression model in that (i) the estimated parameters $\hat{\alpha}_j$ and $\hat{\beta}_j$ remain fairly constant over long periods of time (e.g., the entire post-World War II period in the case of Blume), (ii) $r_{M,t+1}$ and the estimated $\hat{u}_{j,t+1}$, are close to serially independent, and (iii) the $\hat{u}_{j,t+1}$ seem to be independent of $r_{M,t+1}$.

Thus the observed properties of the "market model" are consistent with the expected return efficient markets model, and, in addition, the "market model" tells us something about the process generating expected returns from security to security. In particular,

$$E(\tilde{r}_{j,t+1}) = \alpha_j + \beta_j E(\tilde{r}_{M,t+1}). \tag{13}$$

The question now is to what extent (13) is consistent with the Sharpe-Lintner expected return model summarized by (11). Rearranging (11) we obtain

$$E(\tilde{r}_{j,t+1}|\Phi_t) = \alpha_j(\Phi_t) + \beta_j(\Phi_t)E(\tilde{r}_{m,t+1}|\Phi_t), \tag{14}$$

where, noting that the riskless rate $r_{f,t+1}$ is itself part of the information set Φ_t, we have

$$\alpha_j(\Phi_t) = r_{f,t+1}[1 - \beta_j(\Phi_t)], \tag{15}$$

and

$$\beta_j(\Phi_t) = \frac{\text{cov}\,(\tilde{r}_{j,t+1}, \tilde{r}_{m,t+1}|\Phi_t)}{\sigma^2(\tilde{r}_{m,t+1}|\Phi_t)}. \tag{16}$$

With some simplifying assumptions, (14) can be reduced to (13). In particular, if the covariance and variance that determine $\beta_j(\Phi_t)$ in (16) are the same for all t and Φ_t, then $\beta_j(\Phi_t)$ in (16) corresponds to β_j in (12) and (13), and the least squares *estimate* of β_j in (12) is in fact just the ratio of the sample values of the covariance and variance in (16). If we also assume that $r_{f,t+1}$ is the same for all t, and that the behavior of the returns on the market portfolio m are closely approximated by the returns on some representative index M, we will have come a long way toward equating (13) and (11). Indeed, the only missing link is whether in the estimated parameters of (12)

$$\hat{\alpha}_j \cong r_f(1 - \hat{\beta}_j). \tag{17}$$

Neither FFJR nor Blume attack this question directly, though some of Blume's evidence is at least promising. In particular, the magnitudes of the estimated $\hat{\alpha}_j$ are roughly consistent with (17) in the sense that the estimates are always close to zero (as they should be with monthly return data).[27]

In a sense, though, in establishing the apparent empirical validity of the "market model" of (12), both too much and too little have been shown *vis-à-vis* the Sharpe-Lintner expected return model of (11). We know that during the post-World War II period one-month interest rates on riskless assets (e.g., government bills with one month to maturity) have not been constant. Thus, if expected security returns were generated by a version of the "market model" that is fully consistent with the Sharpe-Lintner model, we would, according to (15), expect to observe some non-stationarity in the estimates of α_j. On a monthly basis, however, variation through time in one-period riskless interest rates is probably trivial relative to variation in other factors affecting monthly common stock returns, so that more powerful statistical methods would be necessary to study the effects of changes in the riskless rate.

In any case, since the work of FFJR and Blume on the "market model" was not concerned with relating this model to the Sharpe-Lintner model, we can only say that the results for the former are somewhat consistent with the implications of the latter. But the results for the "market model" are, after all, just a statistical description of the return generating process, and they are probably somewhat consistent with other models of equilibrium expected returns. Thus the only way to generate strong empirical conclusions about the Sharpe-Lintner model is to test it directly. On the other hand, any alternative model of equilibrium expected returns must be somewhat consistent with the "market model," given the evidence in its support.

B. *Tests of Martingale Models of the Semi-strong Form*

In general, semi-strong form tests of efficient markets models are concerned with whether current prices "fully reflect" all obviously publicly available information. Each individual test, however, is concerned with the adjustment of security prices to one kind of information generating event (e.g., stock splits, announcements of financial reports by firms, new security issues, etc.). Thus each test only brings supporting evidence for the model, with the idea that by accumulating such evidence the validity of the model will be "established."

In fact, however, though the available evidence is in support of the efficient markets model, it is limited to a few major types of information generating events. The initial major work is apparently the study of stock splits by Fama,

27. With least squares applied to monthly return data, the estimate of α_j in (12) is

$$\hat{\alpha}_j = \bar{r}_{j,t} - \hat{\beta}_j \bar{r}_{M,t},$$

where the bars indicate sample mean returns. But, in fact, Blume applies the market model to the wealth relatives $R_{jt} = 1 + r_{jt}$ and $R_{Mt} = 1 + r_{Mt}$. This yields precisely the same estimate of β_j as least squares applied to (12), but the intercept is now

$$\hat{\alpha}'_j = \bar{R}_{jt} - \hat{\beta}_j \bar{R}_{Mt} = 1 + \bar{r}_{jt} - \hat{\beta}_j(1 + \bar{r}_{Mt}) = 1 - \hat{\beta}_j + \hat{\alpha}_j.$$

Thus what Blume in fact finds is that for almost all securities, $\hat{\alpha}'_j + \hat{\beta}_j \cong 1$, which implies that $\hat{\alpha}_j$ is close to 0.

Fisher, Jensen, and Roll (FFJR) [14], and all the subsequent studies summarized here are adaptations and extensions of the techniques developed in FFJR. Thus, this paper will first be reviewed in some detail, and then the other studies will be considered.

1. Splits and the Adjustment of Stock Prices to New Information

Since the only apparent result of a stock split is to multiply the number of shares per shareholder without increasing claims to real assets, splits in themselves are not necessarily sources of new information. The presumption of FFJR is that splits may often be associated with the appearance of more fundamentally important information. The idea is to examine security returns around split dates to see first if there is any "unusual" behavior, and, if so, to what extent it can be accounted for by relationships between splits and other more fundamental variables.

The approach of FFJR to the problem relies heavily on the "market model" of (12). In this model if a stock split is associated with abnormal behavior, this would be reflected in the estimated regression residuals for the months surrounding the split. For a given split, define month O as the month in which the effective date of a split occurs, month 1 as the month immediately following the split month, month -1 as the month preceding, etc. Now define the average residual over all split securities for month m (where for each security m is measured relative to the split month) as

$$u_m = \sum_{j=1}^{N} \frac{\hat{u}_{jm}}{N},$$

where $\hat{u}_{jm}$ is the sample regression residual for security j in month m and N is the number of splits. Next, define the cumulative average residual U_m as

$$U_m = \sum_{k=-29}^{m} u_k.$$

The average residual u_m can be interpreted as the average deviation (in month m relative to split months) of the returns of split stocks from their normal relationships with the market. Similarly, U_m can be interpreted as the cumulative deviation (from month -29 to month m). Finally, define u_m^+, u_m^-, U_m^+, and U_m^- as the average and cumulative average residuals for splits followed by "increased" (+) and "decreased" (—) dividends. An "increase" is a case where the percentage change in dividends on the split share in the year after the split is greater than the percentage change for the N.Y.S.E. as a whole, while a "decrease" is a case of relative dividend decline.

The essence of the results of FFJR are then summarized in Figure 1, which shows the cumulative average residuals U_m U_m^+, and U_m^- for $-29 \leq m \leq 30$. The sample includes all 940 stock splits on the N.Y.S.E. from 1927-59, where the exchange was at least five new shares for four old, and where the security was listed for at least twelve months before and after the split.

For all three dividend categories the cumulative average residuals rise in the 29 months prior to the split, and in fact the average residuals (not shown here) are uniformly positive. This cannot be attributed to the splitting process, since in only about ten per cent of the cases is the time between the announcement and effective dates of a split greater than four months. Rather, it seems that firms tend to split their shares during "abnormally" good times—that is, during periods when the prices of their shares have increased more than would

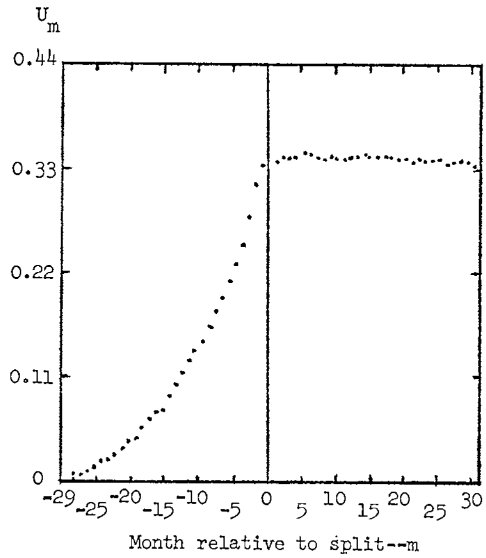

FIGURE 1a
Cumulative average residuals—all splits.

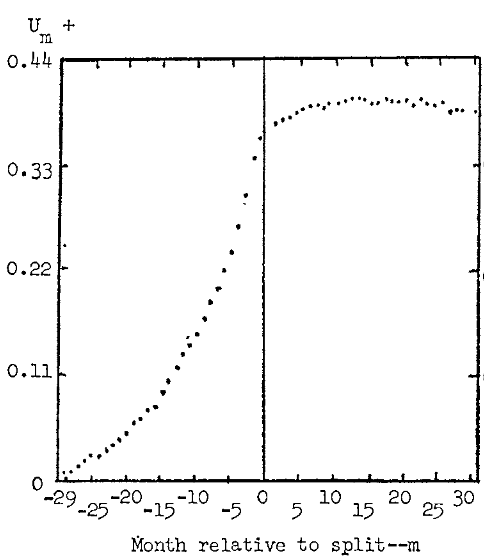

FIGURE 1b
Cumulative average residuals for dividend "increases."

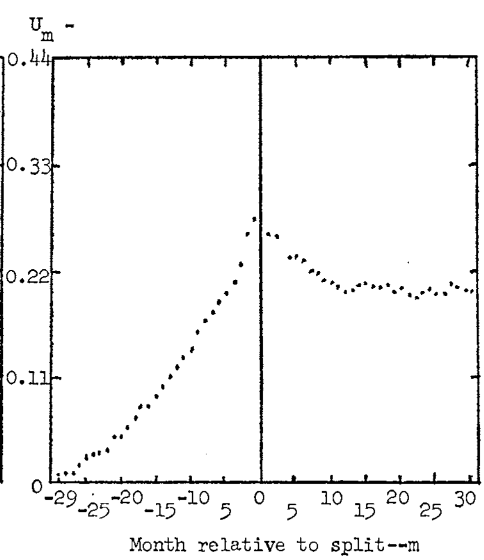

FIGURE 1c
Cumulative average residuals for dividend "decreases."

be implied by their normal relationships with general market prices, which itself probably reflects a sharp improvement, relative to the market, in the earnings prospects of these firms sometime during the years immediately preceding a split.[28]

After the split month there is almost no further movement in U_m, the cumulative average residual for all splits. This is striking, since 71.5 per cent (672 out of 940) of all splits experienced greater percentage dividend increases in the year after the split than the average for all securities on the N.Y.S.E. In light of this, FFJR suggest that when a split is announced the market interprets this (and correctly so) as a signal that the company's directors are probably confident that future earnings will be sufficient to maintain dividend payments at a higher level. Thus the large price increases in the months immediately preceding a split may be due to an alteration in expectations concerning the future earning potential of the firm, rather than to any intrinsic effects of the split itself.

If this hypothesis is correct, return behavior subsequent to splits should be substantially different for the cases where the dividend increase materializes than for the cases where it does not. FFJR argue that in fact the differences are in the directions that would be predicted. The fact that the cumulative average residuals for the "increased" dividends (Figure 1b) drift upward but only slightly in the year *after* the split is consistent with the hypothesis that when the split is *declared*, there is a price adjustment in anticipation of future dividend increases. But the behavior of the residuals for stock splits associated with "decreased" dividends offers even stronger evidence for the split hypothesis. The cumulative average residuals for these stocks (Figure 1c) rise in the few months before the split, but then fall dramatically in the few months after the split when the anticipated dividend increase is not forthcoming. When a year has passed after the split, the cumulative average residual has fallen to about where it was five months prior to the split, which is about the earliest time reliable information about a split is likely to reach the market. Thus by the time it becomes clear that the anticipated dividend increase is not forthcoming, the apparent effects of the split seem to have been wiped away, and the stock's returns have reverted to their normal relationship with market returns.

Finally, and most important, although the behavior of post-split returns will be very different depending on whether or not dividend "increases" occur, and in spite of the fact that a large majority of split securities do experience dividend "increases," when all splits are examined together (Figure 1a), subsequent to the split there is no net movement up or down in the cumulative

28. It is important to note, however, that as FFJR indicate, the persistent upward drift of the cumulative average residuals in the months preceding the split is not a phenomenon that could be used to increase expected trading profits. The reason is that the behavior of the average residuals is not representative of the behavior of the residuals for individual securities. In months prior to the split, successive sample residuals for individual securities seem to be independent. But in most cases, there are a few months in which the residuals are abnormally large and positive. The months of large residuals differ from security to security, however, and these differences in timing explain why the signs of the average residuals are uniformly positive for many months preceding the split.

average residuals. Thus, apparently the market makes unbiased forecasts of the implications of a split for future dividends, and these forecasts are fully reflected in the prices of the security by the end of the split month. After considerably more data analysis than can be summarized here, FFJR conclude that their results lend considerable support to the conclusion that the stock market is efficient, at least with respect to its abiliy to adjust to the information implicit in a split.

2. Other Studies of Public Announcements

Variants of the method of residual analysis developed in [14] have been used by others to study the effects of different kinds of public announcements, and all of these also support the efficient markets hypothesis.

Thus using data on 261 major firms for the period 1946-66, Ball and Brown [4] apply the method to study the effects of annual earnings announcements. They use the residuals from a time series regression of the annual earnings of a firm on the average earnings of all their firms to classify the firm's earnings for a given year as having "increased" or "decreased" relative to the market. Residuals from regressions of monthly common stock returns on an index of returns (i.e., the market model of (12)) are then used to compute cumulative average return residuals separately for the earnings that "increased," and those that "decreased." The cumulative average return residuals rise throughout the year in advance of the announcement for the earnings "increased" category, and fall for the earnings "decreased" category.[29] Ball and Brown [4, p. 175] conclude that in fact no more than about ten to fifteen percent of the information in the annual earnings announcement has not been anticipated by the month of the announcement.

On the macro level, Waud [45] has used the method of residual analysis to examine the effects of announcements of discount rate changes by Federal Reserve Banks. In this case the residuals are essentially just the deviations of the daily returns on the Standard and Poor's 500 Index from the average daily return. He finds evidence of a statistically significant "announcement effect" on stock returns for the first trading day following an announcement, but the magnitude of the adjustment is small, never exceeding .5%. More interesting from the viewpoint of the efficient markets hypothesis is his conclusion that, if anything, the market anticipates the announcements (or information is somehow leaked in advance). This conclusion is based on the non-random patterns of the signs of average return residuals on the days immediately preceding the announcement.

Further evidence in support of the efficient markets hypothesis is provided in the work of Scholes [39] on large secondary offerings of common stock (ie., large underwritten sales of existing common stocks by individuals and institutions) and on new issues of stock. He finds that on average secondary issues are associated with a decline of between one and two per cent in the cumulative average residual returns for the corresponding common stocks. Since the magnitude of the price adjustment is unrelated to the size of the

29. But the comment of footnote 28 is again relevant here.

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issue, Scholes concludes that the adjustment is not due to "selling pressure" (as is commonly believed), but rather results from negative information implicit in the fact that somebody is trying to sell a large block of a firm's stock. Moreover, he presents evidence that the value of the information in a secondary depends to some extent on the vendor; somewhat as would be expected, by far the largest negative cumulative average residuals occur where the vendor is the corporation itself or one of its officers, with investment companies a distant second. But the identity of the vendor is not generally known at the time of the secondary, and corporate insiders need only report their transactions in their own company's stock to the S.E.C. within six days after a sale. By this time the market on average has fully adjusted to the information in the secondary, as indicated by the fact that the average residuals behave randomly thereafter.

Note, however, that though this is evidence that prices adjust efficiently to public information, it is also evidence that corporate insiders at least sometimes have important information about their firm that is not yet publicly known. Thus Scholes' evidence for secondary distributions provides support for the efficient markets model in the semi-strong form sense, but also some strong-form evidence against the model.

Though his results here are only preliminary, Scholes also reports on an application of the method of residual analysis to a sample of 696 new issues of common stock during the period 1926-66. As in the FFJR study of splits, the cumulative average residuals rise in the months preceding the new security offering (suggesting that new issues tend to come after favorable recent events)[30] but behave randomly in the months following the offering (indicating that whatever information is contained in the new issue is on average fully reflected in the price of the month of the offering).

In short, the available semi-strong form evidence on the effect of various sorts of public announcements on common stock returns is all consistent with the efficient markets model. The strong point of the evidence, however, is its consistency rather than its quantity; in fact, few different types of public information have been examined, though those treated are among the obviously most important. Moreover, as we shall now see, the amount of semi-strong form evidence is voluminous compared to the strong form tests that are available.

C. *Strong Form Tests of the Efficient Markets Models*

The strong form tests of the efficient markets model are concerned with whether all available information is fully reflected in prices in the sense that no individual has higher expected trading profits than others because he has monopolistic access to some information. We would not, of course, expect this model to be an exact description of reality, and indeed, the preceding discussions have already indicated the existence of contradictory evidence. In particular, Niederhoffer and Osborne [32] have pointed out that specialists on the N.Y.S.E. apparently use their monopolistic access to information concern-

30. Footnote 28 is again relevant here.

ing unfilled limit orders to generate monopoly profits, and Scholes' evidence [39] indicates that officers of corporations sometimes have monopolistic access to information about their firms.

Since we already have enough evidence to determine that the model is not strictly valid, we can now turn to other interesting questions. Specifically, how far down through the investment community do deviations from the model permeate? Does it pay for the average investor (or the average economist) to expend resources searching out little known information? Are such activities even generally profitable for various groups of market "professionals"? More generally, who are the people in the investment community that have access to "special information"?

Though this is a fascinating problem, only one group has been studied in any depth—the managements of open end mutual funds. Several studies are available (e.g., Sharpe [41, 42] and Treynor [44]), but the most thorough are Jensen's [19, 20], and our comments will be limited to his work. We shall first present the theoretical model underlying his tests, and then go on to his empirical results.

1. Theoretical Framework

In studying the performance of mutual funds the major goals are to determine (a) whether in general fund managers seem to have access to special information which allows them to generate "abnormal" expected returns, and (b) whether some funds are better at uncovering such special information than others. Since the criterion will simply be the ability of funds to produce higher returns than some norm with no attempt to determine what is responsible for the high returns, the "special information" that leads to high performance could be either keener insight into the implications of publicly available information than is implicit in market prices or monopolistic access to specific information. Thus the tests of the performance of the mutual fund industry are not strictly strong form tests of the efficient markets model.

The major theoretical (and practical) problem in using the mutual fund industry to test the efficient markets model is developing a "norm" against which performance can be judged. The norm must represent the results of an investment policy based on the assumption that prices fully reflect all available information. And if one believes that investors are generally risk averse and so on average must be compensated for any risks undertaken, then one has the problem of finding appropriate definitions of risk and evaluating each fund relative to a norm with its chosen level of risk.

Jensen uses the Sharpe [40]-Lintner [24, 25] model of equilibrium expected returns discussed above to derive a norm consistent with these goals. From (14)-(16), in this model the expected return on an asset or portfolio j from t to t + 1 is

$$E(\tilde{r}_{j,t+1}|\Phi_t) = r_{f,t+1}\,[1 - \beta_j(\Phi_t)] + E(\tilde{r}_{m,t+1}|\Phi_t)\beta_j(\Phi_t), \qquad (18)$$

where the various symbols are as defined in Section III. A. 6. But (18) is an *ex ante* relationship, and to evaluate performance an *ex post* norm is needed.

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One way the latter can be obtained is to substitute the realized return on the market portfolio for the expected return in (18) with the result[31]

$$E(\tilde{r}_{j,t+1}|\Phi_t, r_{m,t+1}) = r_{f,t+1}\,[1 - \beta_j(\Phi_t)] + r_{m,t+1}\beta_j(\Phi_t). \tag{19}$$

Geometrically, (19) says that within the context of the Sharpe-Lintner model, the expected return on j (given information Φ_t and the return $r_{m,t+1}$ on the market portfolio) is a linear function of its risk

$$\beta_j(\Phi_t) = \text{cov}\,(\tilde{r}_{j,t+1}, \tilde{r}_{m,t+1}|\Phi_t)/\sigma^2(\tilde{r}_{m,t+1}|\Phi_t),$$

as indicated in Figure 2. Assuming that the value of $\beta_j(\Phi_t)$ is somehow known, or can be reliably estimated, if j is a mutual fund, its *ex post* performance from t to t + 1 might now be evaluated by plotting its combination of realized return $r_{j,t+1}$ and risk in Figure 2. If (as for the point a) the combination falls above the expected return line (or, as it is more commonly called, the "market line"), it has done better than would be expected given its level of risk, while if (as for the point b) it falls below the line it has done worse.

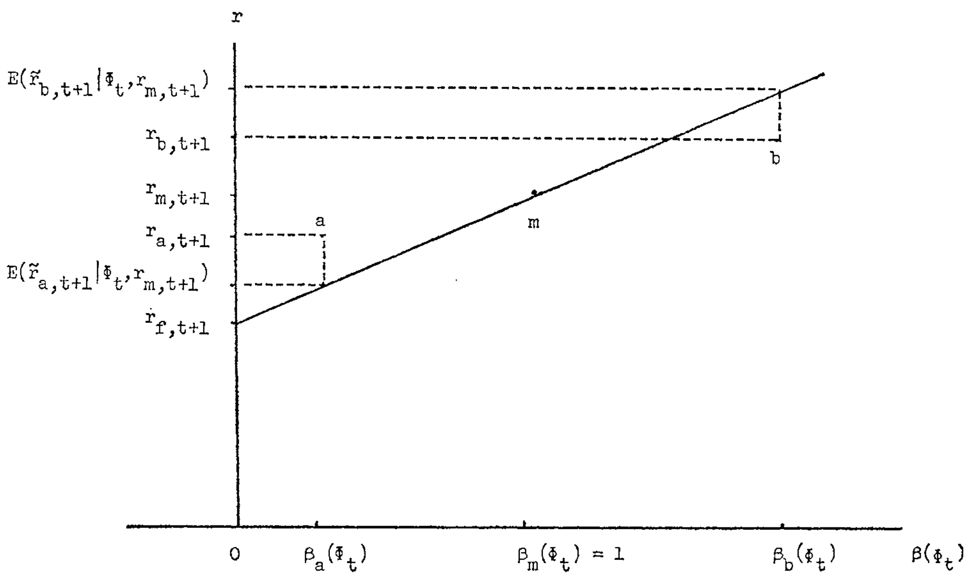

FIGURE 2
Performance Evaluation Graph

Alternatively, the market line shows the combinations of return and risk provided by portfolios that are simple mixtures of the riskless asset and the market portfolio m. The returns and risks for such portfolios (call them c) are

$$r_{c,t+1} = \alpha r_{f,t+1} + (1 - \alpha) r_{m,t+1}$$

$$\beta_c(\Phi_t) = \frac{\text{cov}\,(\tilde{r}_{c,t+1}, \tilde{r}_{m,t+1}|\Phi_t)}{\sigma^2(\tilde{r}_{m,t+1}|\Phi_t)} = \frac{\text{cov}\,((1-\alpha)\tilde{r}_{m,t+1}, \tilde{r}_{m,t+1}|\Phi_t)}{\sigma^2(\tilde{r}_{m,t+1}|\Phi_t)} = 1 - \alpha,$$

31. The assumption here is that the return $\tilde{r}_{j,t+1}$ is generated according to

$$\tilde{r}_{j,t+1} = r_{f,t+1}[1 - \beta_j(\Phi_t)] + r_{m,t+1}\beta_j(\Phi_t) + \tilde{u}_{j,t+1},$$

and

$$E(\tilde{u}_{j,t+1}|r_{m,t+1}) = 0 \text{ for all } r_{m,t+1}.$$

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where α is the proportion of portfolio funds invested in the riskless asset. Thus, when $1 \geqslant \alpha \geqslant 0$ we obtain the combinations of return and risk along the market line from $r_{f,t+1}$ to m in Figure 2, while when $\alpha < 0$ (and under the assumption that investors can borrow at the same rate that they lend) we obtain the combinations of return and risk along the extension of the line through m. In this interpretation, the market line represents the results of a naive investment strategy, which the investor who thinks prices reflect all available information might follow. The performance of a mutual fund is then measured relative to this naive strategy.

2. Empirical Results

Jensen uses this risk-return framework to evaluate the performance of 115 mutual funds over the ten year period 1955-64. He argues at length for measuring return as the nominal ten year rate with continuous compounding (i.e., the natural log of the ratio of terminal wealth after ten years to initial wealth) and for using historical data on nominal one-year rates with continuous compounding to estimate risk. The Standard and Poor Index of 500 major common stocks is used as the proxy for the market portfolio.

The general question to be answered is whether mutual fund managements have any special insights or information which allows them to earn returns above the norm. But Jensen attacks the question on several levels. First, can the funds in general do well enough to compensate investors for loading charges, management fees, and other costs that might be avoided by simply choosing the combination of the riskless asset f and the market portfolio m with risk level comparable to that of the fund's actual portfolio? The answer seems to be an emphatic no. As far as net returns to investors are concerned, in 89 out of 115 cases, the fund's risk-return combination for the ten year period is below the market line for the period, and the average over all funds of the deviations of ten year returns from the market time is -14.6%. That is, on average the consumer's wealth after ten years of holding mutual funds is about fifteen per cent less than if he held the corresponding portfolios along the market line.

But the loading charge that an investor pays in buying into a fund is usually a pure salesman's commission that the fund itself never gets to invest. Thus one might ask whether, ignoring loading charges (i.e., assuming no such charges were paid by the investor), in general fund managements can earn returns sufficiently above the norm to cover all other expenses that are presumably more directly related to the management of the fund portfolios. Again, the answer seems to be no. Even when loading charges are ignored in computing returns, the risk-return combinations for 72 out of 115 funds are below the market line, and the average deviation of ten year returns from the market line is -8.9%.

Finally, as a somewhat stronger test of the efficient markets model, one would like to know if, ignoring all expenses, fund managements in general showed any ability to pick securities that outperformed the norm. Unfortunately, this question cannot be answered with precision for individual funds since, curiously, data on brokerage commissions are not published regularly.

But Jensen suggests the available evidence indicates that the answer to the question is again probably negative. Specifically, adding back all other published expenses of funds to their returns, the risk-return combinations for 58 out of 115 funds were below the market line, and the average deviation of ten year return from the line was -2.5%. But part of this result is due to the absence of a correction for brokerage commissions. Estimating these commissions from average portfolio turnover rates for all funds for the period 1953-58, and adding them back to returns for all funds increases the average deviation from the market line from -2.5% to $.09\%$, which still is not indicative of the existence of special information among mutual fund managers.

But though mutual fund managers in general do not seem to have access to information not already fully reflected in prices, perhaps there are individual funds that consistently do better than the norm, and so provide at least some strong form evidence against the efficient markets model. If there are such funds, however, they escape Jensen's search. For example, for individual funds, returns above the norm in one subperiod do not seem to be associated with performance above the norm in other subperiods. And regardless of how returns are measured (i.e., net or gross of loading charges and other expenses), the number of funds with large positive deviations of returns from the market line of Figure 2 is less than the number that would be expected by chance with 115 funds under the assumption that fund managements have no special talents in predicting returns.[32]

Jensen argues that though his results apply to only one segment of the investment community, they are nevertheless striking evidence in favor of the efficient markets model:

> Although these results certainly do not imply that the strong form of the martingale hypothesis holds for all investors and for all time, they provide strong evidence in support of that hypothesis. One must realize that these analysts are extremely well endowed. Moreover, they operate in the securities markets every day and have wide-ranging contacts and associations in both the business and financial communities. Thus, the fact that they are apparently unable to forecast returns accurately enough to recover their research and transactions costs is a striking piece of evidence in favor of the strong form of the martingale hypothesis—at least as far as the extensive subset of information available to these analysts is concerned [20, p. 170].

IV. Summary and Conclusions

The preceding (rather lengthy) analysis can be summarized as follows. In general terms, the theory of efficient markets is concerned with whether prices at any point in time "fully reflect" available information. The theory only has empirical content, however, within the context of a more specific model of

32. On the other hand, there is some suggestion in Scholes' [39] work on secondary issues that mutual funds may occassionally have access to "special information." After corporate insiders, the next largest negative price changes occur when the secondary seller is an investment company (including mutual funds), though on average the price changes are much smaller (i.e., closer to 0) than when the seller is a corporate insider.

Moreover, Jensen's evidence itself, though not indicative of the existence of special information among mutual fund managers, is not sufficiently precise to conclude that such information never exists. This stronger conclusion would require exact data on unavoidable expenses (including brokerage commissions) of portfolio management incurred by funds.

market equilibrium, that is, a model that specifies the nature of market equilibrium when prices "fully reflect" available information. We have seen that all of the available empirical literature is implicitly or explicitly based on the assumption that the conditions of market equilibrium can be stated in terms of expected returns. This assumption is the basis of the expected return or "fair game" efficient markets models.

The empirical work itself can be divided into three categories depending on the nature of the information subset of interest. *Strong-form* tests are concerned with whether individual investors or groups have monopolistic access to any information relevant for price formation. One would not expect such an extreme model to be an exact description of the world, and it is probably best viewed as a benchmark against which the importance of deviations from market efficiency can be judged. In the less restrictive *semi-strong-form* tests the information subset of interest includes all obviously publicly available information, while in the *weak form* tests the information subset is just historical price or return sequences.

Weak form tests of the efficient market model are the most voluminous, and it seems fair to say that the results are strongly in support. Though statistically significant evidence for dependence in successive price changes or returns has been found, some of this is consistent with the "fair game" model and the rest does not appear to be sufficient to declare the market inefficient. Indeed, at least for price changes or returns covering a day or longer, there isn't much evidence against the "fair game" model's more ambitious offspring, the random walk.

Thus, there is consistent evidence of positive dependence in day-to-day price changes and returns on common stocks, and the dependence is of a form that can be used as the basis of marginally profitable trading rules. In Fama's data [10] the dependence shows up as serial correlations that are consistently positive but also consistently close to zero, and as a slight tendency for observed numbers of runs of positive and negative price changes to be less than the numbers that would be expected from a purely random process. More important, the dependence also shows up in the filter tests of Alexander [1, 2] and those of Fama and Blume [13] as a tendency for very small filters to produce profits in excess of buy-and-hold. But any systems (like the filters) that attempt to turn short-term dependence into trading profits of necessity generate so many transactions that their expected profits would be absorbed by even the minimum commissions (security handling fees) that floor traders on major exchanges must pay. Thus, using a less than completely strict interpretation of market efficiency, this positive dependence does not seem of sufficient importance to warrant rejection of the efficient markets model.

Evidence in contradiction of the "fair game" efficient markets model for price changes or returns covering periods longer than a single day is more difficult to find. Cootner [9], and Moore [31] report preponderantly negative (but again small) serial correlations in weekly common stock returns, and this result appears also in the four day returns analyzed by Fama [10]. But it does not appear in runs tests of [10], where, if anything, there is some slight indication of positive dependence, but actually not much evidence of any dependence at all. In any case, there is no indication that whatever dependence exists in weekly returns can be used as the basis of profitable trading rules.

Other existing evidence of dependence in returns provides interesting insights into the process of price formation in the stock market, but it is not relevant for testing the efficient markets model. For example, Fama [10] shows that large daily price changes tend to be folowed by large changes, but of unpredictable sign. This suggests that important information cannot be completely evaluated immediately, but that the initial first day's adjustment of prices to the information is unbiased, which is sufficient for the martingale model. More interesting and important, however, is the Niederhoffer-Osborne [32] finding of a tendency toward excessive reversals in common stock price changes from transaction to transaction. They explain this as a logical result of the mechanism whereby orders to buy and sell at market are matched against existing limit orders on the books of the specialist. Given the way this tendency toward excessive reversals arises, however, there seems to be no way it can be used as the basis of a profitable trading rule. As they rightly claim, their results are a strong refutation of the theory of random walks, at least as applied to price changes from transaction to transaction, but they do not constitute refutation of the economically more relevant "fair game" efficient markets model.

Semi-strong form tests, in which prices are assumed to fully reflect all obviously publicly available information, have also supported the efficient markets hypothesis. Thus Fama, Fisher, Jensen, and Roll [14] find that the information in stock splits concerning the firm's future dividend payments is on average fully reflected in the price of a split share at the time of the split. Ball and Brown [4] and Scholes [39] come to similar conclusions with respect to the information contained in (i) annual earning announcements by firms and (ii) new issues and large block secondary issues of common stock. Though only a few different types of information generating events are represented here, they are among the more important, and the results are probably indicative of what can be expected in future studies.

As noted earlier, the strong-form efficient markets model, in which prices are assumed to fully reflect all available information, is probably best viewed as a benchmark against which deviations from market efficiency (interpreted in its strictest sense) can be judged. Two such deviations have in fact been observed. First, Niederhoffer and Osborne [32] point out that specialists on major security exchanges have monopolistic access to information on unexecuted limit orders and they use this information to generate trading profits. This raises the question of whether the "market making" function of the specialist (if indeed this is a meaningful economic function) could not as effectively be carried out by some other mechanism that did not imply monopolistic access to information. Second, Scholes [39] finds that, not unexpectedly, corporate insiders often have monopolistic access to information about their firms.

At the moment, however, corporate insiders and specialists are the only two groups whose monopolistic access to information has been documented. There is no evidence that deviations from the strong form of the efficient markets model permeate down any further through the investment community. For the purposes of most investors the efficient markets model seems a good first (and second) approximation to reality.

In short, the evidence in support of the efficient markets model is extensive, and (somewhat uniquely in economics) contradictory evidence is sparse. Nevertheless, we certainly do not want to leave the impression that all issues are closed. The old saw, "much remains to be done," is relevant here as elsewhere. Indeed, as is often the case in successful scientific research, now that we know we've been in the past, we are able to pose and (hopefully) to answer an even more interesting set of questions for the future. In this case the most pressing field of future endeavor is the development and testing of models of market equilibrium under uncertainty. When the process generating equilibrium expected returns is better understood (and assuming that some expected return model turns out to be relevant), we will have a more substantial framework for more sophisticated intersecurity tests of market efficiency.

REFERENCES

1. Sidney S. Alexander. "Price Movements in Speculative Markets: Trends or Random Walks." *Industrial Management Review,* 2 (May 1961), 7-26. Also reprinted in [8], 199-218.
2. ——————. "Price Movements in Speculative Markets: Trends or Random Walks. No. 2," in [8], 338-72.
3. Louis Bachelier. *Théorie de la Speculation* (Paris: Gauthier-Villars, 1900), and reprinted in English in [8], 17-78.
4. Ray Ball and Phillip Brown. "An Empirical Evaluation of Accounting Income Numbers." *Journal of Accounting Research,* 6 (Autumn, 1968), 159-78.
5. William Beaver. "The Information Content of Annual Earnings Announcements." *Empirical Research in Accounting: Selected Studies, 1968,* supplement to Vol. 7 of the *Journal of Accounting Research,* 67-92.
6. Robert Blattberg and Thomas Sargent. "Regression with Non-Gaussian Disturbances: Some Sampling Results," forthcoming in Econometrica.
7. Marshall Blume. "The Assessment of Portfolio Performance." Unpublished Ph.D. thesis, University of Chicago, 1968. A paper summarizing much of this work will appear in the April, 1970, Journal of Business.
8. Paul Cootner (ed.). *The Random Character of Stock Market Prices.* Cambridge: M.I.T., 1964.
9. ——————. "Stock Prices: Random vs. Systematic Changes." *Industrial Management Review,* 3 (Spring 1962), 24-45. Also reprinted in [8], 231-52.
10. Eugene F. Fama. "The Behavior of Stock Market Prices." *Journal of Business,* 38 (January, 1965), 34-105.
11. ——————. "Multiperiod Consumption-Investment Decisions." *American Economic Review,* (March, 1970).
12. ——————. "Risk, Return and Equilibrium." Report No. 6831, University of Chicago, Center for Math. Studies in Business and Economics, June, 1968.
13. —————— and Marshall Blume. "Filter Rules and Stock Market Trading Profits." *Journal of Business,* 39 (Special Supplement, January, 1966), 226-41.
14. ——————, Lawrence Fisher, Michael Jensen and Richard Roll. "The Adjustment of Stock Prices to New Information." *International Economic Review,* X (February, 1969), 1-21.
15. —————— and Richard Roll. "Some Properties of Symmetric Stable Distributions." *Journal of the American Statistical Association,* 63 (September, 1968), 817-36.
16. Michael D. Godfrey, C. W. J. Granger and O. Morgenstern. "The Random Walk Hypothesis of Stock Market Behavior." *Kyklos,* 17 (1964), 1-30.
17. C. W. J. Granger and O. Morgenstern. "Spectral Analysis of New York Stock Market Prices," *Kyklos,* 16 (1963), 1-27. Also reprinted in [8], 162-88.
18. John R. Hicks. *Value and Capital.* Oxford: The Clarendon Press, 1946.
19. Michael Jensen. "The Performance of Mutual Funds in the Period 1945-64," *Journal of Finance,* 23 (May, 1968), 389-416.

OPCRESP-POD1d-000140

20. ——————. "Risk, the Pricing of Capital Assets, and the Evaluation of Investment Portfolios," *Journal of Business*, 42 (April, 1969), 167-247.
21. Maurice G. Kendall. "The Analysis of Economic Time-Series, Part I: Prices," *Journal of the Royal Statistical Society*, 96 (Part I, 1953), 11-25.
22. Ruben A. Kessel. "The Cyclical Behavior of the Term Structure of Interest Rates," National Bureau of Economic Research Occasional Paper No. 91. New York: Columbia University Press, 1965.
23. Benjamin F. King. "Market and Industry Factors in Stock Price Behavior," *Journal of Business*, 39 (Special Supplement January, 1966), 139-90.
24. John Lintner. "Security Prices, Risk, and Maximal Gains from Diversification," *Journal of Finance*, 20 (December, 1965), 587-615.
25. ——————. "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," *Review of Economics and Statistics*, 47 (February, 1965), 13-37.
26. Fredrich A. Lutz. "The Structure of Interest Rates," *Quarterly Journal of Economics*, 40 (1940-41).
27. Benoit Mandelbrot. "Forecasts of Future Prices, Unbiased Markets, and Martingale Models," *Journal of Business*, 39 (Special Supplement, January, 1966), 242-55.
28. ——————. "The Variation of Certain Speculative Prices." *Journal of Business*, 36 (October, 1963), 394-419.
29. —————— and Howard M. Taylor. "On the Distribution of Stock Price Differences." *Operations Research*, 15 (November-December, 1967), 1057-62.
30. Harry Markowitz. *Portfolio Selection:Efficient Diversification of Investment*. New York: John Wiley & Sons, 1959.
31. Arnold Moore. "A Statistical Analysis of Common Stock Prices. Unpublished Ph.D. thesis, Graduate School of Business, University of Chicago, 1962.
32. Victor Niederhoffer and M. F. M. Osborne. "Market Making and Reversal on the Stock Exchange." *Journal of the American Statistical Association*, 61 (December, 1966), 897-916.
33. M. F. M. Osborne. "Brownian Motion in the Stock Market," *Operations Research*, 7 (March-April, 1959), 145-73. Also reprinted in [8], 100-28.
34. ——————. "Periodic Structure in the Brownian Motion of Stock Prices." *Operations Research*, 10 (May-June, 1962), 345-79. Also reprinted in [8], 262-96.
35. S. James Press. "A compound Events Model for Security Prices." *Journal of Business*, 40 (July, 1968), 317-35.
36. Harry V. Roberts. "Stock Market 'Patterns' and Financial Analysis: Methodological Suggestions." *Journal of Finance*, 14 (March, 1959), 1-10.
37. Richard Roll. "The Efficient Market Model Applied to U.S. Treasury Bill Rates." Unpublished Ph.D. thesis, Graduate School of Business, University of Chicago, 1968.
38. Paul A. Samuelson. "Proof That Properly Anticipated Prices Fluctuate Randomly." *Industrial Management Review*, 6 (Spring, 1965), 41-9.
39. Myron Scholes. "A Test of the Competitive Hypothesis: The Market for New Issues and Secondary Offerings." Unpublished PH.D. thesis, Graduate School of Business, University of Chicago, 1969.
40. William F. Sharpe. "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk." *Journal of Finance*, 19 (September, 1964), 425-42.
41. ——————. "Mutual Fund Performance." *Journal of Business*, 39 (Special Supplement, January, 1966), 119-38.
42. ——————. "Risk Aversion in the Stock Market." *Journal of Finance*, 20 (September, 1965), 416-22.
43. James Tobin. "Liquidity Preference as Behavior Towards Risk," *Review of Economic Studies*, 25 (February, 1958), 65-85.
44. Jack L. Treynor. "How to Rate Management of Investment Funds." *Harvard Business Review*, 43 (January-February, 1965), 63-75.
45. Roger N. Waud. "Public Interpretation of Discount Rate Changes: Evidence on the 'Announcement Effect.'" forthcoming in *Econometrica*.
46. John Wise. "Linear Estimators for Linear Regression Systems Having Infinite Variances." Unpublished paper presented at the Berkeley-Stanford Mathematical Economics Seminar, October, 1963.
47. Holbrook Working. "A Random Difference Series for Use in the Analysis of Time Series." *Journal of the American Statistical Association*, 29 (March, 1934), 11-24.

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A SIMPLIFIED MODEL FOR PORTFOLIO ANALYSIS*

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This paper describes the advantages of using a particular model of the relationships among securities for practical applications of the Markowitz portfolio analysis technique. A computer program has been developed to take full advantage of the model: 2,000 securities can be analyzed at an extremely low cost—as little as 2% of that associated with standard quadratic programming codes. Moreover, preliminary evidence suggests that the relatively few parameters used by the model can lead to very nearly the same results obtained with much larger sets of relationships among securities. The possibility of low-cost analysis, coupled with a likelihood that a relatively small amount of information need be sacrificed make the model an attractive candidate for initial practical applications of the Markowitz technique.

1. Introduction

Markowitz has suggested that the process of portfolio selection be approached by (1) making probabilistic estimates of the future performances of securities, (2) analyzing those estimates to determine an *efficient set* of portfolios and (3) selecting from that set the portfolios best suited to the investor's preferences [1, 2, 3]. This paper extends Markowitz' work on the second of these three stages —*portfolio analysis*. The preliminary sections state the problem in its general form and describe Markowitz' solution technique. The remainder of the paper presents a simplified model of the relationships among securities, indicates the manner in which it allows the portfolio analysis problem to be simplified, and provides evidence on the costs as well as the desirability of using the model for practical applications of the Markowitz technique.

2. The Portfolio Analysis Problem

A security analyst has provided the following predictions concerning the future returns from each of N securities:

$E_i \equiv$ the expected value of R_i (the return from security i)
C_{i1} through C_{in} ; C_{ij} represents the covariance between R_i and R_j (as usual, when $i = j$ the figure is the variance of R_i)

* Received December 1961.

† The author wishes to express his appreciation for the cooperation of the staffs of both the Western Data Processing Center at UCLA and the Pacific Northwest Research Computer Laboratory at the University of Washington where the program was tested. His greatest debt, however, is to Dr. Harry M. Markowitz of the RAND Corporation, with whom he was privileged to have a number of stimulating conversations during the past year. It is no longer possible to segregate the ideas in this paper into those which were his, those which were the author's, and those which were developed jointly. Suffice it to say that the only accomplishments which are unquestionably the property of the author are those of authorship—first of the computer program and then of this article.

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The portfolio analysis problem is as follows. Given such a set of predictions, determine the set of *efficient portfolios*; a portfolio is efficient if none other gives either (a) a higher expected return and the same variance of return or (b) a lower variance of return and the same expected return.

Let X_i represent the proportion of a portfolio invested in security i. Then the expected return (E) and variance of return (V) of any portfolio can be expressed in terms of (a) the basic data (E_i-values and C_{ij}-values) and (b) the amounts invested in various securities:

$$E = \sum_i X_i E_i$$

$$V = \sum_i \sum_j X_i X_j C_{ij} .$$

Consider an objective function of the form:

$$\begin{aligned}\phi &= \lambda E - V \\ &= \lambda \sum_i X_i E_i - \sum_i \sum_j X_i X_j C_{ij} .\end{aligned}$$

Given a set of values for the parameters (λ, E_i's and C_{ij}'s), the value of ϕ can be changed by varying the X_i values as desired, as long as two basic restrictions are observed:

1. The entire portfolio must be invested:[1]

$$\sum_i X_i = 1$$

and 2. no security may be held in negative quantities:[2]

$$X_i \geqq 0 \quad \text{for all} \quad i.$$

A portfolio is described by the proportions invested in various securities—in our notation by the values of X_i. For each set of admissable values of the X_i variables there is a corresponding predicted combination of E and V and thus of ϕ. Figure 1 illustrates this relationship for a particular value of λ. The line ϕ_1 shows the combinations of E and V which give $\phi = \phi_1$, where $\phi = \lambda_k E - V$; the other lines refer to larger values of ϕ ($\phi_3 > \phi_2 > \phi_1$). Of all possible portfolios, one will maximize the value of ϕ;[3] in figure 1 it is portfolio C. The relationship between this solution and the portfolio analysis problem is obvious. The E, V combination obtained will be on the boundary of the set of attainable combinations; moreover, the objective function will be tangent to the set at that point. Since this function is of the form

$$\phi = \lambda E - V$$

[1] Since cash can be included as one of the securities (explicitly or implicitly) this assumption need cause no lack of realism.

[2] This is the standard formulation. Cases in which short sales are allowed require a different approach.

[3] This fact is crucial to the critical line computing procedure described in the next section.

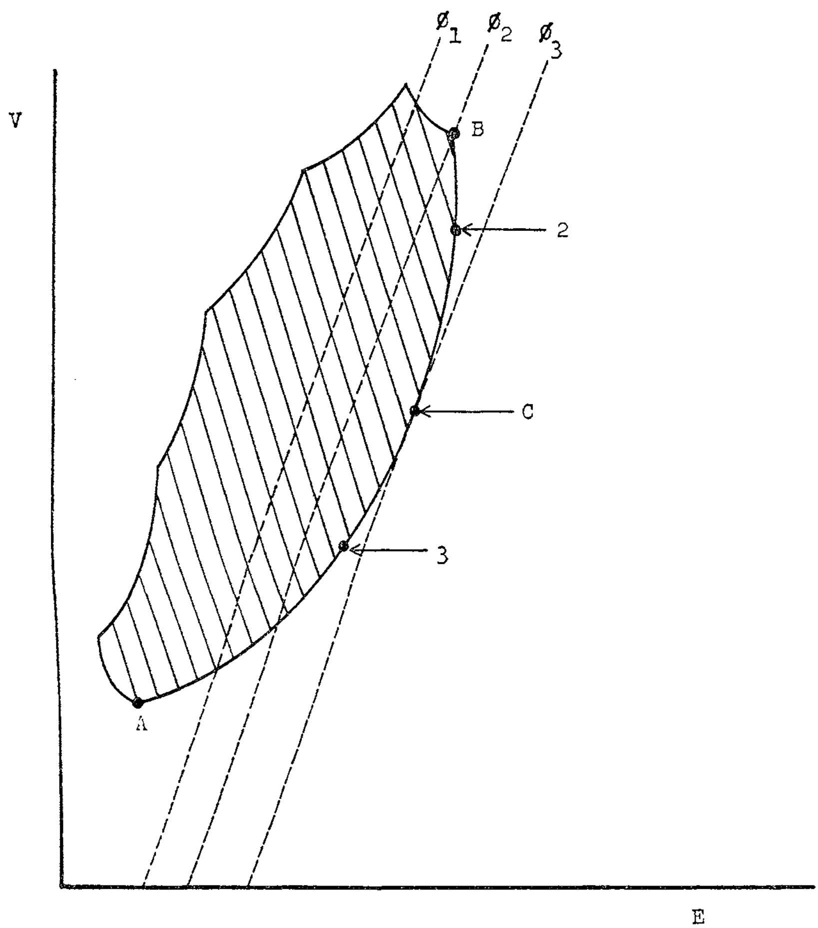

FIGURE 1

the slope of the boundary at the point must be λ; thus, by varying λ from $+\infty$ to 0, every solution of the portfolio analysis problem can be obtained.

For any given value of λ the problem described in this section requires the maximization of a quadratic function, ϕ (which is a function of X_i, X_i^2, and X_iX_j terms) subject to a linear constraint ($\sum_i X_i = 1$), with the variables restricted to non-negative values. A number of techniques have been developed to solve such *quadratic programming problems*. The critical line method, developed by Markowitz in conjunction with his work on portfolio analysis, is particularly suited to this problem and was used in the program described in this paper.

3. The Critical Line Method

Two important characteristics of the set of efficient portfolios make systematic solution of the portfolio analysis problem relatively straightforward. The first concerns the relationships among portfolios. Any set of efficient portfolios can be described in terms of a smaller set of *corner portfolios*. Any point on the E, V curve (other than the points associated with corner portfolios) can be obtained with a portfolio constructed by dividing the total investment between the two adjacent corner portfolios. For example, the portfolio which gives E, V combination C in Figure 1 might be some linear combination of the two corner portfolios with E, V combinations shown by points 2 and 3. This characteristic allows the analyst to restrict his attention to corner portfolios rather than the complete set of efficient portfolios; the latter can be readily derived from the former.

The second characteristic of the solution concerns the relationships among corner portfolios. Two corner portfolios which are adjacent on the E, V curve are related in the following manner: one portfolio will contain either (1) all the securities which appear in the other, plus one additional security or (2) all but one of the securities which appear in the other. Thus in moving down the E, V curve from one corner portfolio to the next, the quantities of the securities in efficient portfolios will vary until either one drops out of the portfolio or another enters. The point at which a change takes place marks a new corner portfolio.

The major steps in the critical line method for solving the portfolio analysis problem are:

1. The corner portfolio with $\lambda = \infty$ is determined. It is composed entirely of the one security with the highest expected return.[4]
2. Relationships between (a) the amounts of the various securities contained in efficient portfolios and (b) the value of λ are computed. It is possible to derive such relationships for any section of the E, V curve between adjacent corner portfolios. The relationships which apply to one section of the curve will not, however, apply to any other section.
3. Using the relationships computed in (2), each security is examined to determine the value of λ at which a change in the securities included in the portfolio would come about:
   a. securities presently in the portfolio are examined to determine the value of λ at which they would drop out, and
   b. securities not presently in the portfolio are examined to determine the value of λ at which they would enter the portfolio.
4. The next largest value of λ at which a security either enters or drops out of the portfolio is determined. This indicates the location of the next corner portfolio.
5. The composition of the new corner portfolio is computed, using the relationships derived in (2). However, since these relationships held only for the section of the curve between this corner portfolio and the preceding one, the solution process can only continue if new relationships are derived. The method thus returns to step (2) unless $\lambda = 0$, in which case the analysis is complete.

The amount of computation required to complete a portfolio analysis using

[4] In the event that two or more of the securities have the same (highest) expected return, the first efficient portfolio is the combination of such securities with the lowest variance.

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this method is related to the following factors:

1. The number of securities analyzed
   This will affect the extent of the computation in step (2) and the number of computations in step (3).
2. The number of corner portfolios
   Steps (2) through (5) must be repeated once to find each corner portfolio.
3. The complexity of the variance-covariance matrix
   Step (2) requires a matrix be inverted and must be repeated once for each corner portfolio.

The amount of computer memory space required to perform a portfolio analysis will depend primarily on the size of the variance-covariance matrix. In the standard case, if N securities are analyzed this matrix will have $\frac{1}{2}(N^2 + N)$ elements.

4. The Diagonal Model

Portfolio analysis requires a large number of comparisons; obviously the practical application of the technique can be greatly facilitated by a set of assumptions which reduces the computational task involved in such comparisons. One such set of assumptions (to be called the diagonal model) is described in this article. This model has two virtues: it is one of the simplest which can be constructed without assuming away the existence of interrelationships among securities and there is considerable evidence that it can capture a large part of such interrelationships.

The major characteristic of the diagonal model is the assumption that the returns of various securities are related only through common relationships with some basic underlying factor. The return from any security is determined solely by random factors and this single outside element; more explicitly:

$$R_i = A_i + B_i I + C_i$$

where A_i and B_i are parameters, C_i is a random variable with an expected value of zero and variance Q_i, and I is the level of some index. The index, I, may be the level of the stock market as a whole, the Gross National Product, some price index or any other factor thought to be the most important single influence on the returns from securities. The future level of I is determined in part by random factors:

$$I = A_{n+1} + C_{n+1}$$

where A_{n+1} is a parameter and C_{n+1} is a random variable with an expected value of zero and a variance of Q_{n+1}. It is assumed that the covariance between C_i and C_j is zero for all values of i and j ($i \neq j$).

Figure 2 provides a graphical representation of the model. A_i and B_i serve to locate the line which relates the expected value of R_i to the level of I. Q_i indicates the variance of R_i around the expected relationship (this variance is assumed to be the same at each point along the line). Finally, A_{n+1} indicates the expected value of I and Q_{n+1} the variance around that expected value.

The diagonal model requires the following predictions from a security analyst:

1) values of A_i, B_i and Q_i for each of N securities
2) values of A_{n+1} and Q_{n+1} for the index I.

The number of estimates required from the analyst is thus greatly reduced: from 5,150 to 302 for an analysis of 100 securities and from 2,003,000 to 6,002 for an analysis of 2,000 securities.

Once the parameters of the diagonal model have been specified all the inputs required for the standard portfolio analysis problem can be derived. The relationships are:

$$E_i = A_i + B_i(A_{n+1})$$

$$V_i = (B_i)^2(Q_{n+1}) + Q_i$$

$$C = (B_i)(B_j)(Q_{n+1})$$

A portfolio analysis could be performed by obtaining the values required by the diagonal model, calculating from them the full set of data required for the standard portfolio analysis problem and then performing the analysis with the derived values. However, additional advantages can be obtained if the portfolio analysis problem is restated directly in terms of the parameters of the diagonal model. The following section describes the manner in which such a restatement can be performed.

5. The Analogue

The return from a portfolio is the weighted average of the returns from its component securities:

$$R_p = \sum_{i=1}^{N} X_i R_i$$

The contribution of each security to the total return of a portfolio is simply $X_i R_i$ or, under the assumptions of the diagonal model:

$$X_i(A_i + B_i I + C_i).$$

The total contribution of a security to the return of the portfolio can be broken into two components: (1) an investment in the "basic characteristics" of the security in question and (2) an "investment" in the index:

$$\begin{aligned} &(1) \qquad X_i(A_i + B_i I + C_i) = X_i(A_i + C_i) \\ &(2) \qquad\qquad\qquad\qquad\qquad\quad + X_i B_i I \end{aligned}$$

The return of a portfolio can be considered to be the result of (1) a series of investments in N "basic securities" and (2) an investment in the index:

$$R_p = \sum_{i=1}^{N} X_i(A_i + C_i) + \left[\sum_{i=1}^{N} X_i B_i\right] I$$

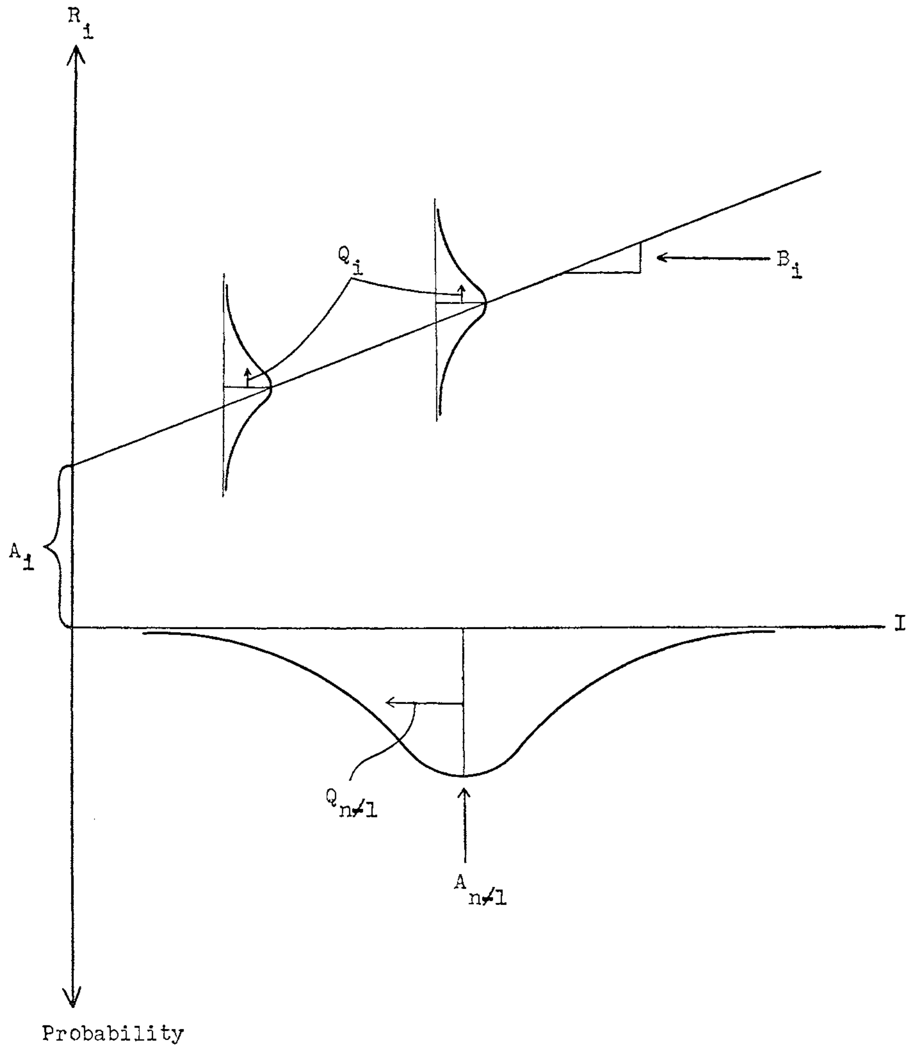

FIGURE 2

Defining X_{n+1} as the weighted average responsiveness of R_p to the level of I:

$$X_{n+1} \equiv \sum_{i=1}^{N} X_i B_i$$

and substituting this variable and the formula for the determinants of I, we obtain:

$$\begin{aligned} R_p &= \sum_{i=1}^{N} X_i(A_i + C_i) + X_{n+1}(A_{n+1} + C_{n+1}) \\ &= \sum_{i=1}^{N+1} X_i(A_i + C_i). \end{aligned}$$

The expected return of a portfolio is thus:

$$E = \sum_{i=1}^{N+1} X_i A_i$$

while the variance is:[5]

$$V = \sum_{i=1}^{N+1} X_i^2 Q_i$$

This formulation indicates the reason for the use of the parameters A_{n+1} and Q_{n+1} to describe the expected value and variance of the future value of I. It also indicates the reason for calling this the "diagonal model". The variance-covariance matrix, which is full when N securities are considered, can be expressed as a matrix with non-zero elements only along the diagonal by including an $(n + 1)$st security defined as indicated. This vastly reduces the number of computations required to solve the portfolio analysis problem (primarily in step 2 of the critical line method, when the variance-covariance matrix must be inverted) and allows the problem to be stated directly in terms of the basic parameters of the diagonal model:

$$\text{Maximize:} \quad \lambda E - V$$

$$\text{Where:} \quad E = \sum_{i=1}^{N+1} X_i A_i$$

$$V = \sum_{i=1}^{N+1} X_i^2 Q_i$$

$$\text{Subject to:} \quad X_i \geqq 0 \text{ for all } i \text{ from 1 to } N$$

$$\sum_{i=1}^{N} X_i = 1$$

$$\sum_{i=1}^{N} X_i B_i = X_{n+1} .$$

6. The Diagonal Model Portfolio Analysis Code

As indicated in the previous section, if the portfolio analysis problem is expressed in terms of the basic parameters of the diagonal model, computing time and memory space required for solution can be greatly reduced. This section describes a machine code, written in the FØRTRAN language, which takes full advantage of the characteristics of the diagonal model. It uses the critical line method to solve the problem stated in the previous section.

The computing time required by the diagonal code is considerably smaller than that required by standard quadratic programming codes. The RAND QP

[5] Recall that the diagonal model assumes $\text{cov}(C_i, C_j) = 0$ for all i and j $(i \neq j)$.

code[6] required 33 minutes to solve a 100-security example on an IBM 7090 computer; the same problem was solved in 30 seconds with the diagonal code. Moreover, the reduced storage requirements allow many more securities to be analyzed: with the IBM 709 or 7090 the RAND QP code can be used for no more than 249 securities, while the diagonal code can analyze up to 2,000 securities.

Although the diagonal code allows the total computing time to be greatly reduced, the cost of a large analysis is still far from insignificant. Thus there is every incentive to limit the computations to those essential for the final selection of a portfolio. By taking into account the possibilities of borrowing and lending money, the diagonal code restricts the computations to those absolutely necessary for determination of the final set of efficient portfolios. The importance of these alternatives, their effect on the portfolio analysis problem and the manner in which they are taken into account in the diagonal code are described in the remainder of this section.

A. The "lending portfolio"

There is some interest rate (r_l) at which money can be lent with virtual assurance that both principal and interest will be returned; at the least, money can be buried in the ground ($r_l = 0$). Such an alternative could be included as one possible security ($A_i = 1 + r_l$, $B_i = 0$, $Q_i = 0$) but this would necessitate some needless computation.[7] In order to minimize computing time, lending at some pure interest rate is taken into account explicitly in the diagonal code.

The relationship between lending and efficient portfolios can best be seen in terms of an E, σ curve showing the combinations of expected return and standard deviation of return ($= \sqrt{V}$) associated with efficient portfolios. Such a curve is shown in Figure 3 (FBCG); point A indicates the E, σ combination attained if all funds are lent. The relationship between lending money and purchasing portfolios can be illustrated with the portfolio which has the E, σ combination shown by point Z. Consider a portfolio with X_z invested in portfolio Z and the remainder $(1 - X_z)$ lent at the rate r_l. The expected return from such a portfolio would be:

$$E = X_z E_z + (1 - X_z)(1 + r_l)$$

and the variance of return would be:

$$V = X_z^2 V_z + (1 - X_z)^2 V_l + 2X_z(1 - X_z)(\text{cov}_{zl})$$

[6] The program is described in [4]. Several alternative quadratic programming codes are available. A recent code, developed by IBM, which uses the critical line method is likely to prove considerably more efficient for the portfolio analysis problem. The RAND code is used for comparison since it is the only standard program with which the author has had experience.

[7] Actually, the diagonal code cannot accept non-positive values of Q_i; thus if the lending alternative is to be included as simply another security, it must be assigned a very small value of Q_i. This procedure will give virtually the correct solution but is inefficient.

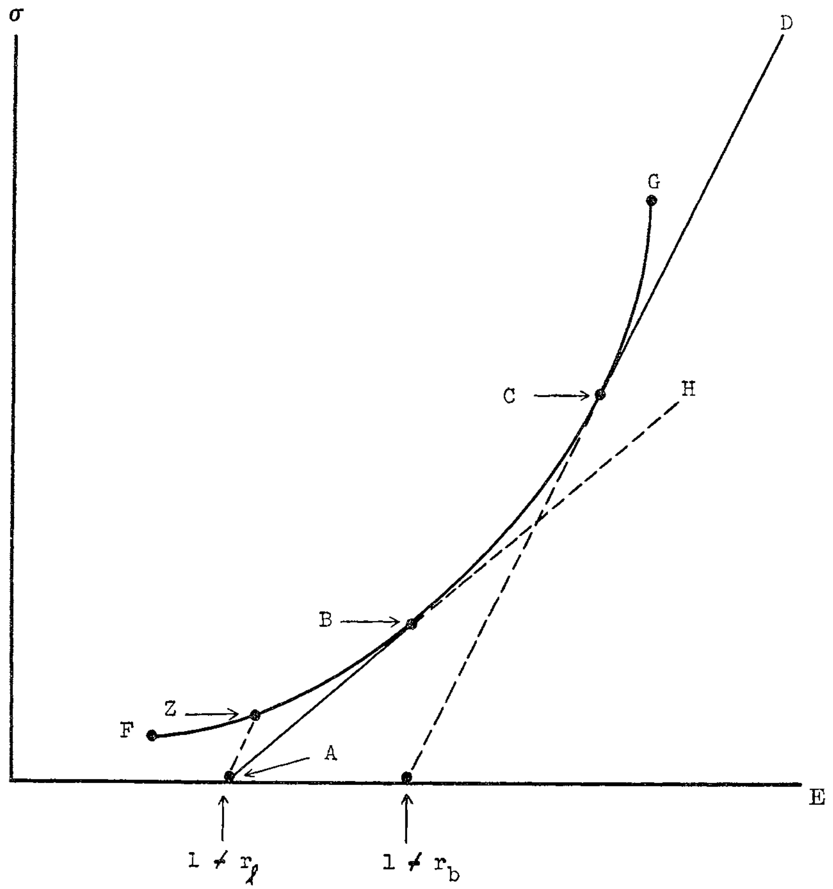

FIGURE 3

But, since V_l and cov_{zl} are both zero:

$$V = X_z^2 V_z$$

and the standard deviation of return is:

$$\sigma = X_z \sigma_z \, .$$

Since both E and σ are linear functions of X_z, the E, σ combinations of all portfolios made up of portfolio Z plus lending must lie on a straight line connecting points Z and A. In general, by splitting his investment between a portfolio and lending, an investor can attain any E, σ combination on the line connecting the E, σ combinations of the two components.

Many portfolios which are efficient in the absence of the lending alternative becomes inefficient when it is introduced. In Figure 3, for example, the possibility of attaining E, σ combinations along the line AB makes all portfolios along the original E, σ curve from point F to point B inefficient. For any desired level of

E below that associated with portfolio B, the most efficient portfolio will be some combination of portfolio B and lending. Portfolio B can be termed the "lending portfolio" since it is the appropriate portfolio whenever some of the investor's funds are to be lent at the rate r_l. This portfolio can be found readily once the E, σ curve is known. It lies at the point on the curve at which a ray from ($E = 1 + r_l$, $\sigma = 0$) is tangent to the curve. If the E, σ curve is not known in its entirety it is still possible to determine whether or not a particular portfolio is the lending portfolio by computing the rate of interest which *would* make the portfolio in question the lending portfolio. For example, the rate of interest associated in this manner with portfolio C is r_b, found by extending a tangent to the curve down to the E-axis. The diagonal code computes such a rate of interest for each corner portfolio as the analysis proceeds; when it falls below the previously stated lending rate the code computes the composition of the lending portfolio and terminates the analysis.

B. The "borrowing portfolio"

In some cases an investor may be able to borrow funds in order to purchase even greater amounts of a portfolio than his own funds will allow. If the appropriate rate for such borrowing were r_b, illustrated in figure 3, the E, σ combinations attainable by purchasing portfolio C with both the investor's funds and with borrowed funds would lie along the line CD, depending on the amount borrowed. Inclusion of the borrowing alternative makes certain portfolios inefficient which are efficient in the absence of the alternative; in this case the affected portfolios are those with E, σ combinations along the segment of the original E, σ curve from C to G. Just as there is a single appropriate portfolio if any lending is contemplated, there is a single appropriate portfolio if borrowing is contemplated. This "borrowing portfolio" is related to the rate of interest at which funds can be borrowed in exactly the same manner as the "lending portfolio" is related to the rate at which funds can be lent.

The diagonal code does not take account of the borrowing alternative in the manner used for the lending alternative since it is necessary to compute all previous corner portfolios in order to derive the portion of the E, σ curve below the borrowing portfolio. For this reason all computations required to derive the full E, σ curve above the lending portfolio must be made. However, the code does allow the user to specify the rate of interest at which funds can be borrowed. If this alternative is chosen, none of the corner portfolios which will be inefficient when borrowing is considered will be printed. Since as much as 65% of the total computer time can be spent recording (on tape) the results of the analysis this is not an insignificant saving.

7. The Cost of Portfolio Analysis with the Diagonal Code

The total time (and thus cost) required to perform a portfolio analysis with the diagonal code will depend upon the number of securities analyzed, the number of corner portfolios and, to some extent, the composition of the corner portfolios. A formula which gives quite an accurate estimate of the time required to perform an analysis on an IBM 709 computer was obtained by analyzing a series of runs during which the time required to complete each major segment of the program was recorded. The approximate time required for the analysis will be:[8]

$$\begin{aligned}\text{Number of seconds} = {} & .6 \\ & + .114 \times \text{number of securities analyzed} \\ & + .54 \times \text{number of corner portfolios} \\ & + .0024 \times \text{number of securities analyzed} \times \text{number of corner portfolios.}\end{aligned}$$

Unfortunately only the number of securities analyzed is known before the analysis is begun. In order to estimate the cost of portfolio analysis before it is performed, some relationship between the number of corner portfolios and the number of securities analyzed must be assumed. Since no theoretical relationship can be derived and since the total number of corner portfolios could be several times the number of securities analysed, it seemed desirable to obtain some crude notion of the typical relationship when "reasonable" inputs are used. To accomplish this, a series of portfolio analyses was performed using inputs generated by a Monte Carlo model.

Data were gathered on the annual returns during the period 1940–1951 for 96 industrial common stocks chosen randomly from the New York Stock Exchange. The returns of each security were then related to the level of a stock market index and estimates of the parameters of the diagonal model obtained. These parameters were assumed to be samples from a population of A_i, B_i and Q_i triplets related as follows:

$$A_i = \bar{A} + r_1$$

$$B_i = \bar{B} + \psi A_i + r_2$$

$$Q_i = \bar{Q} + \theta A_i + \gamma B_i + r_3$$

where r_1, r_2 and r_3 are random variables with zero means. Estimates for the parameters of these three equations were obtained by regression analysis and estimates of the variances of the random variables determined.[9] With this information the characteristics of any desired number of securities could be generated. A random number generator was used to select a value for A_i; this value, together with an additional random number determined the value of B_i; the value of Q_i was then determined with a third random number and the previously obtained values of A_i and B_i.

Figure 4 shows the relationship between the number of securities analyzed

[8] The computations in this section are based on the assumption that no corner portfolios prior to the lending portfolio are printed. If the analyst chooses to print all preceding portfolios, the estimates given in this section should be multiplied by 2.9; intermediate cases can be estimated by interpolation.

[9] The random variables were considered normally distributed; in one case, to better approximate the data, two variances were used for the distribution—one for the portion above the mean and another for the portion below the mean.

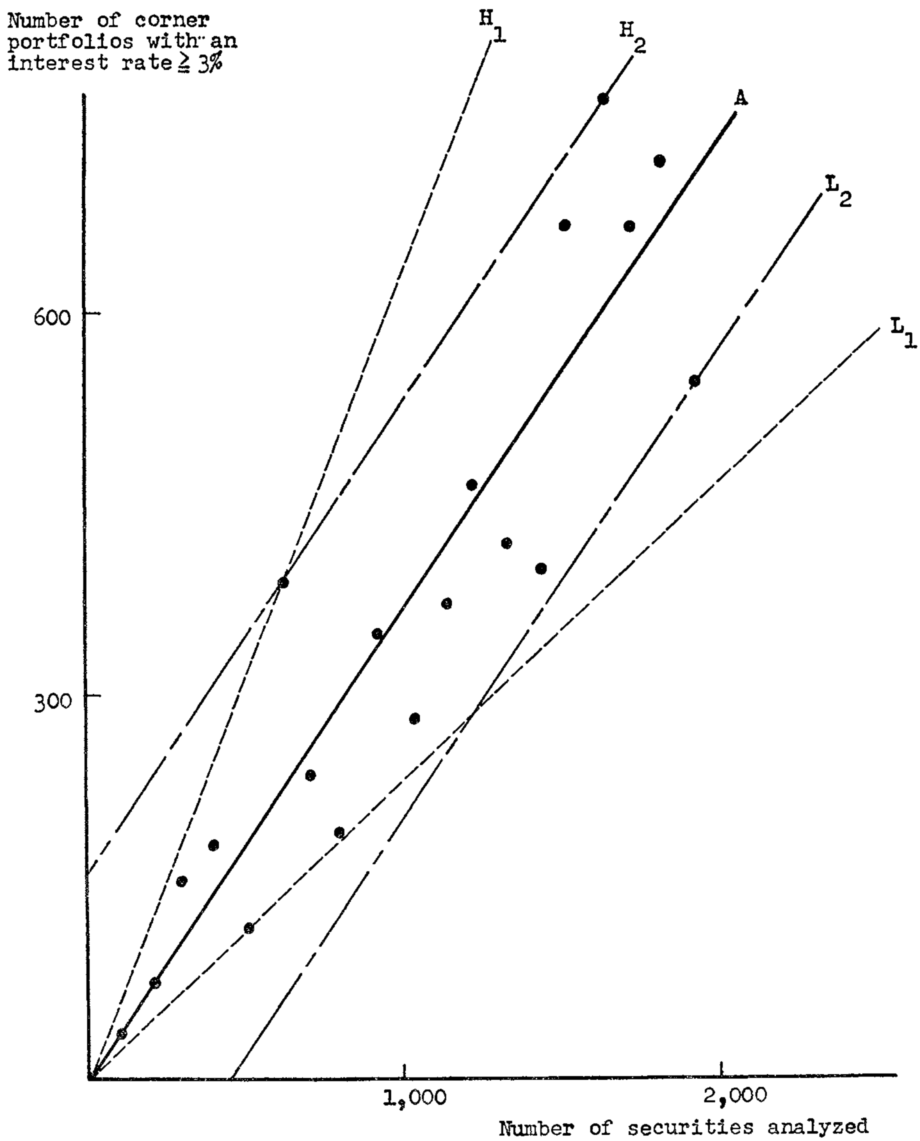

FIGURE 4

and the number of corner portfolios with interest rates greater than 3% (an approximation to the "lending rate"). Rather than perform a sophisticated analysis of these data, several lines have been used to bracket the results in various ways. These will be used subsequently as extreme cases, on the presumption that most practical cases will lie within these extremes (but with no presumption that these limits will never be exceeded). Curve A indicates the average relationship between the number of portfolios and the number of securities:

average $(N_p/N_s) = .37$. Curve H_1 indicates the highest such relationship: maximum $(N_p/N_s) = .63$; the line L_1 indicates the lowest: minimum $(N_p/N_s) = .24$. The other two curves, H_2 and L_2, indicate respectively the maximum deviation above (155) and below (173) the number of corner portfolios indicated by the average relationship $N_p = .37\ N_s$.

In Figure 5 the total time required for a portfolio analysis is related to the number of securities analyzed under various assumptions about the relationship

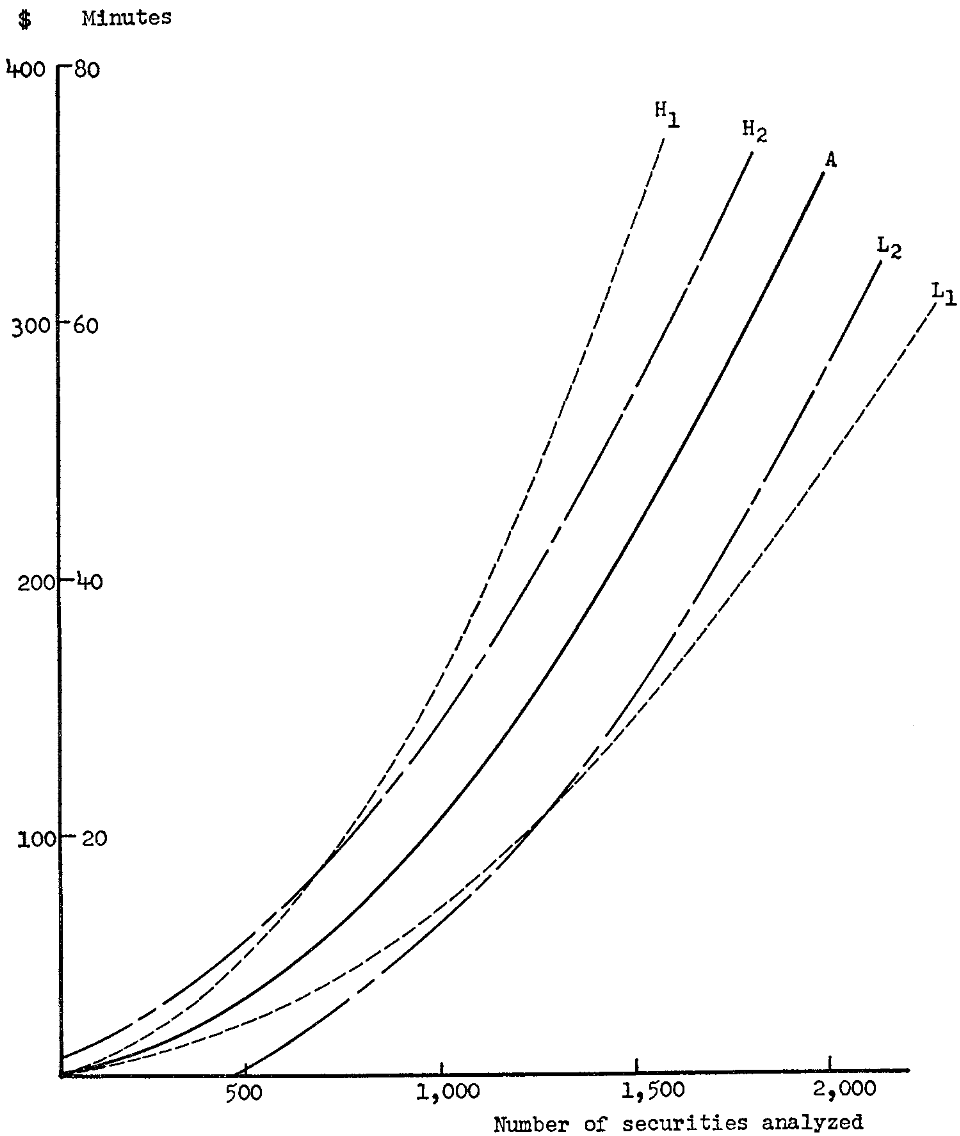

FIGURE 5

between the number of corner portfolios and the number of securities analyzed. Each of the curves shown in Figure 5 is based on the corresponding curve in Figure 4; for example, curve A in Figure 5 indicates the relationship between total time and number of securities analyzed on the assumption that the relationship between the number of corner portfolios and the number of securities is that shown by curve A in Figure 4. For convenience a second scale has been provided in Figure 5, showing the total cost of the analysis on the assumption that an IBM 709 computer can be obtained at a cost of $300 per hour.

8. The Value of Portfolio Analysis Based on the Diagonal Model

The assumptions of the diagonal model lie near one end of the spectrum of possible assumptions about the relationships among securities. The model's extreme simplicity enables the investigator to perform a portfolio analysis at a very small cost, as we have shown. However, it is entirely possible that this simplicity so restricts the security analyst in making his predictions that the value of the resulting portfolio analysis is also very small.

In order to estimate the ability of the diagonal model to summarize information concerning the performance of securities a simple test was performed. Twenty securities were chosen randomly from the New York Stock Exchange and their performance during the period 1940–1951 used to obtain two sets of

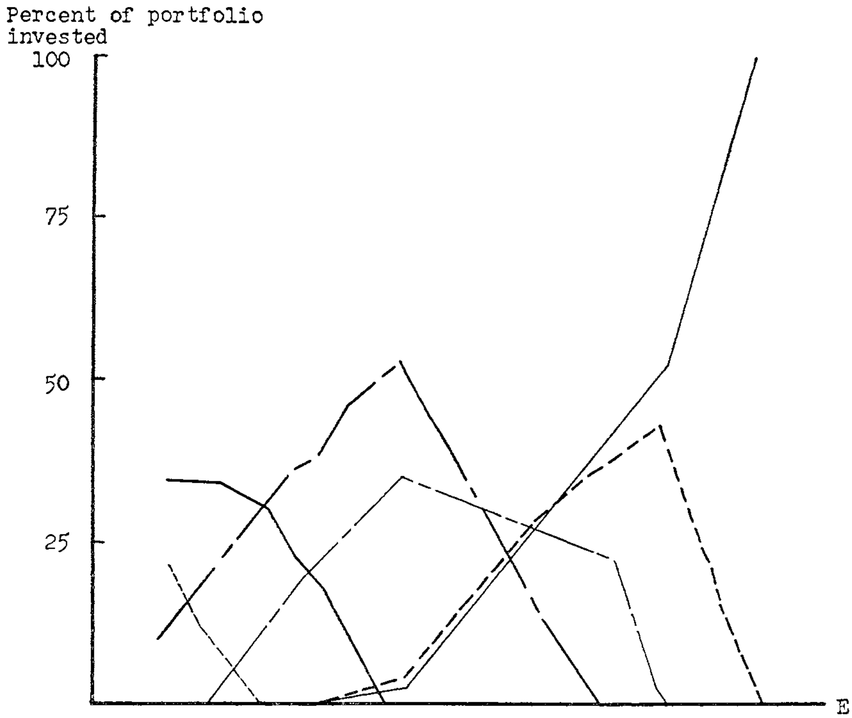

FIG. 6a. Composition of efficient portfolios derived from the analysis of the parameters of the diagonal model.

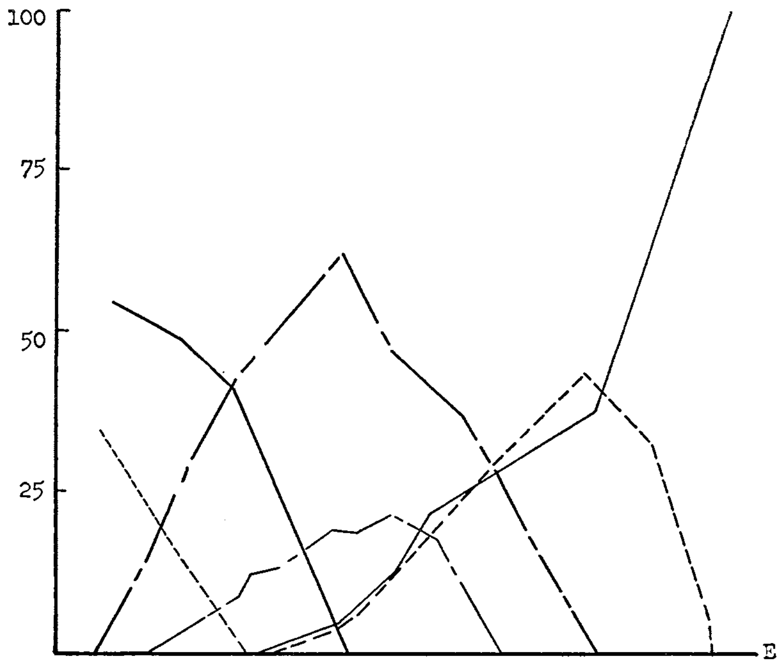

FIG. 6b. Composition of efficient portfolios derived from the analysis of historical data

data: (1) the actual mean returns, variances of returns and covariances of returns during the period and (2) the parameters of the diagonal model, estimated by regression techniques from the performance of the securities during the period. A portfolio analysis was then performed on each set of data. The results are summarized in Figures 6a and 6b. Each security which entered any of the efficient portfolios in significant amounts is represented by a particular type of line; the height of each line above any given value of E indicates the percentage of the efficient portfolio with that particular E composed of the security in question. The two figures thus indicate the compositions of all the efficient portfolios chosen from the analysis of the historical data (Figure 6b) and the compositions of all the portfolios chosen from the analysis of the parameters of the diagonal model (Figure 6a). The similarity of the two figures indicates that the 62 parameters of the diagonal model were able to capture a great deal of the information contained in the complete set of 230 historical relationships. An additional test, using a second set of 20 securities, gave similar results.

These results are, of course, far too fragmentary to be considered conclusive but they do suggest that the diagonal model may be able to represent the relationships among securities rather well and thus that the value of portfolio analyses based on the model will exceed their rather nominal cost. For these reasons it appears to be an excellent choice for the initial practical applications of the Markowitz technique.

References

1. MARKOWITZ, HARRY M., *Portfolio Selection, Efficient Diversification of Investments*, New York, John Wiley and Sons, Inc., 1959.
2. MARKOWITZ, HARRY M., "Portfolio Selection", *The Journal of Finance*, Vol. 12, (March 1952), 77–91.
3. MARKOWITZ, HARRY M., "The Optimization of a Quadratic Function Subject to Linear Constraints," *Naval Research Logistics Quarterly*, Vol. 3, (March and June, 1956), 111–133.
4. WOLFE, PHILIP, "The Simplex Method for Quadratic Programming," *Econometrica*, Vol. 27, (July, 1959), 382–398.

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Andrew J. Cueter | October 02, 2012

In finance, the Beta of a security (or portfolio) is used as an indicator of its historical volatility in regards to a benchmark, generally the New York Stock Exchange (NYSE) Composite Index or the S&P 500 Index. At Value Line, we derive the Beta coefficient from a regression analysis of the relationship between weekly percentage changes in the price of a stock and weekly percentage changes in the NYSE Composite Index over a period of five years. In the case of shorter price histories, a shorter time period is used, but two years is the minimum. Value Line then adjusts these Betas to account for their long-term tendency to converge toward 1.00. (Though the scope of this convergence is beyond our purposes here, readers can refer to M. Blume, "On the Assessment of Risk," Journal of Finance, March 1971 for further details.)

Now that we have our Beta number, what does it mean? If an equity mirrors the benchmark, then it carries a Beta of 1.00. If Stock X has a Beta of 2.00, it is expected to rise (or fall) twice as much as the movement of the benchmark. For example, if the NYSE Composite Index rises (falls) 10%, Stock X will likely rise (fall) 20%. (For a more detailed overview, see **Understanding Beta**). Beta can also be negative (infrequent but possible), which would mean that the equity's return tends to move in the opposite direction from the market's move. Moreover, there is no upper or lower bound to Beta, although it typically does not stray too far from 1.00. Finally, a Beta of zero does not mean the asset is risk-free, just that the correlation of that asset's return to the market's return is zero.

Now that we know what Beta is and its implications, how can we use it? If we were able to predict the movements of the overall market, we would simply buy high Beta stocks while the market rises, and low Beta stocks while the market is falling. However, no one is capable of timing the market over the long term. So, what should we do?

If we define a high risk asset in terms of the movement of its price, we can look towards Beta as one indicator of this riskiness. Though Beta by itself does not give a perfect indication of volatility, it does imply the direction and magnitude of movements. Using Beta as a measure of risk, we can relate this to a basic tenet of finance theory, which states that investors demand a return in exchange for assuming risk. Therefore, high-risk (or high-Beta) investments should provide a higher payout, and conversely, low-risk (or low-Beta) investments should provide a lower payout. This proposition seems reasonable and intuitive, but it may not always hold.

In a paper entitled "Re-Thinking Risk: What the Beta Puzzle Tells Us about Investing," written by David Cowan and Sam Wilderman of GMO LLC, they show just the opposite. For the paper, Beta was measured using 250-day returns of a universe of 1,000 stocks, regressed against 250-day returns of that universe. Low- and high-Beta Portfolios were then formed monthly and weighted by market capitalization, with the universe used as the benchmark. Their results present data starting in December, 1969 and show that high-Beta stocks have significantly underperformed the market (average annualized return of 7.2% vs. 10.6% for low-Beta and 9.8% for the universe), and done so with substantially higher annualized volatility (24.5% vs. 12.5% and 16.0%, respectively) and larger drawdown (-84.4% vs. -39.5% and -50.3%, respectively).

Though low-Beta may trump high-Beta over longer periods, there are some problems with solely relying on the Beta coefficient. It is a backward looking metric, and therefore may not be an accurate predictor of the future. The markets change all the time and just because a relationship held in the past does not mean it is certain to continue into the future. Also, since it is solely a statistical measure, it fails to consider underlying business fundamentals or economic developments. Consider **Altria Group** (**MO**). This stock has a Beta of 0.55 and the company primarily sells cigarettes. Due to the low Beta, we may say this is a low-risk stock. However, if for some reason cigarettes were deemed illegal to sell, this company would probably not stick around very long and any investment in the stock will likely become worthless. Solely looking at a stock's Beta will not uncover this risk.

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So, back to our question posed earlier; what should we do? We propose Beta should be used as one factor in the equity analysis framework. Investors should also look at our Safety rank and Price Stability score when making investment decisions. Considered in conjunction with Value Line's fundamental research and valuation ratios, we believe investors can create a portfolio that may provide superior risk-adjusted returns over the long haul.

At the time of this article's writing, the author did not have positions in any of the companies mentioned.

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Time-Series Processes of Utility Betas: Implications for Forecasting Systematic Risk

Michael J. Gombola and Douglas R. Kahl

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■ Brigham and Crum [5] describe difficulties with the Capital Asset Pricing Model (CAPM) in estimating utility cost of capital. This controversial article elicited six comments [7, 15, 17, 21, 22, 24], a reply [6], and one extension [11]. Examining the dividend omission by Consolidated Edison (Con Ed), Brigham and Crum note that this information release could confound estimation of Con Ed's beta. Although the Ordinary Least Squares (OLS) beta estimate decreased concurrent with the dividend omission, Brigham and Crum contend that Con Ed's risk had not decreased.

An OLS estimate of beta requires an estimation period during which the relationship between stock return and market return is stable. Without this stability, the forecaster needs alternatives for forecasting a time-varying relationship, such as the general Bayesian adjustment process [25] or its specific variations employed by Merrill Lynch [18]. The appropriateness of a given procedure depends on the particular time-series properties of the beta being forecast.

Information on the time-series properties of utility betas, including the variability of beta and the tendency of utility betas to auto-regress toward an underlying mean, is presented here. The degree of difficulty in forecasting beta depends on both of these properties. Since the basis of Bayesian adjustment lies in beta's tendency to return to an underlying mean, if betas follow a random walk process then Bayesian adjustment will be fruitless.

Collins, Ledolter, and Rayburn [10] explain that random variation in beta leads to severe forecasting difficulties, unlike variability due to auto-regression in beta. To the extent that beta instability is auto-correlated, an unstable beta can be forecasted accurately. Estimating that about 25% of beta variability in their sample is due to auto-correlated beta changes, Collins,

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Ledolter, and Rayburn suggest that recognition of auto-correlation can improve forecasting accuracy by 15%.

Auto-correlated beta changes allow use of beta adjustment models to improve beta forecasts. A general Bayesian adjustment model would adjust the short-term (transient) beta estimate towards a long-term underlying mean. An example of such an application is the Merrill Lynch [18] adjustment process:

$$B_t = 0.65(B_{t-1}) + 0.35(1.0)\,. \qquad (1)$$

Here, the transient beta estimate obtained by OLS is presumed to return to an underlying mean of 1.0 slowly, since more weight is placed on the transient beta than on the underlying mean.

Studying the time-series properties of utility betas—including their tendency to return to an underlying mean, the speed of this return, and the underlying mean itself—should prove helpful in formulating Bayesian adjustments of beta forecasts. Carleton [7] suggests that Bayesian-adjusted beta forecasts have been applied, often inappropriately, to beta forecasts in regulatory proceedings. This study strives to determine whether such Bayesian adjustment processes are appropriate at all.

I. Beta Coefficient Instability and the Rate-Setting Process

Cooley [12] points out the widespread, albeit controversial, use of the Capital Asset Pricing Model in estimating required return for utility equity. Exchanges published by two journals dealing with the CAPM for rate setting ([7, 15, 17, 21, 22, 24] and [4, 19, 20]) center not on the validity of the theory but on the reliability and usefulness of beta estimates.

Concern over empirical estimates of systematic risk is based on a substantial body of empirical literature pointing to beta instability. From the early descriptive work of Blume [2] through later tests by Fabozzi and Francis [13] and Collins, Ledolter, and Rayburn [10], the evidence supports instability in security betas. Studying specifically the behavior of utility betas, Bey [1], Chen [8], and Pettway [23] all demonstrate instability.

Although the size of beta instability has been extensively investigated, comparatively little attention has been focused on the form of that instability, particularly for utilities. Beta instability does not necessarily preclude application of the CAPM unless combined with a random walk process for beta.

The simplest case, a constant coefficient process for beta, may be expressed as:

$$B_{it} = B_{i,t-1} = B_i^m \text{ for all } t\,. \qquad (2)$$

In Equation (2), the beta at any point in time remains equal to the previous beta and also to a constant underlying mean beta, B_i^m. This constant coefficient process is assumed in OLS estimation of a beta and serves as the null hypothesis in tests of beta variability [3, 13].

When the transient beta for a particular company (B_{it}) is distributed around an underlying mean beta for that company B_i^m, the resulting time-series process may be described as:

$$B_{it} = B_i^m + u_{it}\,. \qquad (3)$$

Equation (3) describes the random coefficient model tested by Fabozzi and Francis [13] and assumed in a beta forecasting model by Chen and Keown [9]. Since the deviations of beta from its underlying mean (u_{it}) are limited to a single period and are serially uncorrelated, the transient beta (B_{it}) tends to return quickly to the underlying mean.

If the transient beta takes more than one period to return to its underlying mean, then an auto-regressive process describes the time-series behavior of beta:

$$B_{it} = a_i B_{i,t-1} + (1 - a_i)B_i^m + u_{it}\,. \qquad (4)$$

This process is very similar to the random coefficient process, except for the strength of the tendency for mean-reversion. A value of 0.9 for $1 - a_i$ would cause the process to be classified as auto-regressive, whereas a value of 1.0 would label it random coefficient. Otherwise, there is little difference.

The auto-regressive model described in Equation (4) is the same one studied by Bos and Newbold [3] and Collins, Ledolter, and Rayburn [10]. The process considers a tendency to return to an underlying mean beta, where the tendency is measured by $1 - a_i$. The Merrill Lynch adjustment process [18] describes a special case in which the underlying mean beta (B_i^m) is 1.0 and the adjustment factor to the mean, also called the regression rate ($1 - a_i$), is 0.35. Vasicek's adjustment model [25] is a less restrictive case in which the underlying mean beta is unity and no restriction is made on the adjustment rate toward the underlying mean.

If all beta variation is random, then there will be no tendency for beta to return to an underlying mean, resulting in a random walk process:

$$B_{it} = B_{i,t-1} + u_{it}. \tag{5}$$

This model has been suggested as a time-varying model for beta in a stability test described by Garbade and Rentzler [14]. Since there are no bounds on the value that beta can assume, the process is difficult to forecast, especially in the long run. If beta follows a random walk process then the best long-term forecast is the short-term beta, and a Bayesian adjustment process will not improve the forecast. Notably, Brigham and Crum's [6] original criticism of the CAPM was based on unadjusted OLS estimates of Con Ed's beta, which implicitly assumes that an unstable beta follows a random walk.

II. The Beta Coefficient as an Auto-Regressive Variable

Any of the four beta-generating processes can be represented as a special case of a general auto-regressive process. The general model has a measurement equation,

$$R_{it} = B_{it} R_{mt} + e_{it}, \tag{6}$$

and state equation,

$$B_{it} = a_i B_{i,t-1} + (1 - a_i) B_i^m + u_{it}, \tag{6'}$$

where R_{it} is the excess return on the ith security during time t, R_{mt} is the return on the market index during time t, B_i^m is the underlying mean beta for the ith stock, and B_{it} is the transient beta for the ith stock at time t.

Equation (6') specifies a first-order auto-regressive process for beta. If the value for $1 - a_i$ is 0.0, then (6') reverts to the random walk process described in Equation (5). If the value for $1 - a_i$ is 1.0, then (6') reverts to the random coefficient process described in Equation (3). If the residual variance is 0.0, then $1 - a_i$ becomes 0.0 and the underlying mean and error terms in Equation (6') drop out, leaving the constant beta process in Equation (2).

III. Estimating Parameters of the Model

The parameters of the model in Equations (6) and (6') were estimated using monthly stock return data from the Compustat PDE file for 109 utility companies. 61 electric and 48 electric and gas. The 15-year sample period is from January 1967–December 1981. The period contains both the dividend omission by Consolidated Edison [5] and the Three Mile Island incident.

The model in Equations (6) can be expressed in matrix format as:

$$R_{it} = \underline{h}_t \underline{B}_{mt} + \underline{e}_{it}, \tag{7}$$

$$\underline{B}_{it} = \underline{A}_i \underline{B}_{i,t-1} + \underline{U}_{it}, \tag{7'}$$

where

$\underline{h}_t = (R_{mt}, 0)$;
$\underline{B}'_{it} = (B_{it}, B_i^m)$;
$\underline{U}'_{it} = (u_{it}, 0)$ and is distributed as $N(0, W_i S_i^2)$,

$$\underline{W} = \begin{bmatrix} w_i & 0 \\ 0 & 0 \end{bmatrix}, \tag{8}$$

$$\underline{A} = \begin{bmatrix} a_i & 1 - a_i \\ 0 & 1 \end{bmatrix}. \tag{9}$$

The recursive Kalman filtering approach described by Kahl and Ledolter [16] is used to estimate simultaneously the three parameters of the market model in Equations (6). These parameters are: the underlying mean beta (B_i^m), the regression rate toward the underlying mean ($1 - a_i$), and the variance of beta over time.

Simultaneous estimation of three parameters requires considerable data and computer resources which might explain why studies using broad samples and large numbers of stocks formulate the problem somewhat differently. Bos and Newbold estimated a Kalman filtering model with a two-pass process. Decreasing the number of parameters from three to two reduces the computation time to only a fraction of that required for a full model. Collins, Ledolter, and Rayburn [10] suggest that the procedure followed by Bos and Newbold [3] creates a downward bias in the estimate of beta's regression rate. They were able to eliminate the estimate of the underlying mean beta in the model and focus on beta regression tendencies.

The model used in this study produces independent variance estimates like the model used by Collins, Ledolter, and Rayburn. In addition, this model estimates the underlying mean beta. Maximum likelihood estimates of elements in the transition matrix (a_i), the variance ratio (w_i), and the variance of the measurement equa-

Exhibit 1. Maximum Likelihood Estimates of Model Parameters

Regression Rate	Standard Deviation of Beta										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0			2[a]	3[a]	4[a]	6[a]	12[a]	5[a]	3[a]		
0.1				1	2	5	1				
0.2					1	7	2	5	2		
0.3				1	1	2	5	1	3		
0.4				1	2	1	3	1			
0.5											
0.6						1					
0.7						1					
0.8											
0.9		1	1								
1.0	6[b]	17[c]									

[a]These firms display characteristics of firms whose betas follow a random coefficient process.
[b]These firms display characteristics of firms whose betas are constant.
[c]These firms display characteristics of firms whose betas follow a random walk process.

tion (S_i^2), were all concurrently estimated using a grid search procedure.

IV. Results

The particular time-series process followed by a beta can be indicated by two parameters: the standard deviation of this beta over time, u_{it} in Equation (6'); and its adjustment rate to the mean, $(1 - a_i)$ in Equation (6'). Consequently, the cross-tabulation of these two parameters in Exhibit 1 is also a tabulation of the process followed by the beta. The most common process shown in Exhibit 1 is the auto-regressive process. Nearly half of the companies in the sample, 51 out of 109, show a nonzero standard deviation of beta together with a value for the regression rate between zero and unity.

The next most common process is the random coefficient process, indicated by a nonzero value for the standard deviation of beta together with an estimate of 1.0 for $1 - a_i$. These estimates are shown by 35 of the sample companies. The firms with auto-regressive betas and those with very similar random coefficient betas jointly comprise 86 of the 109 sample firms.

A nonzero estimate of the standard deviation of beta combined with a regression rate of zero indicates a beta following a random walk process. Parameter estimates consistent with a random walk process are shown for only 17 companies.

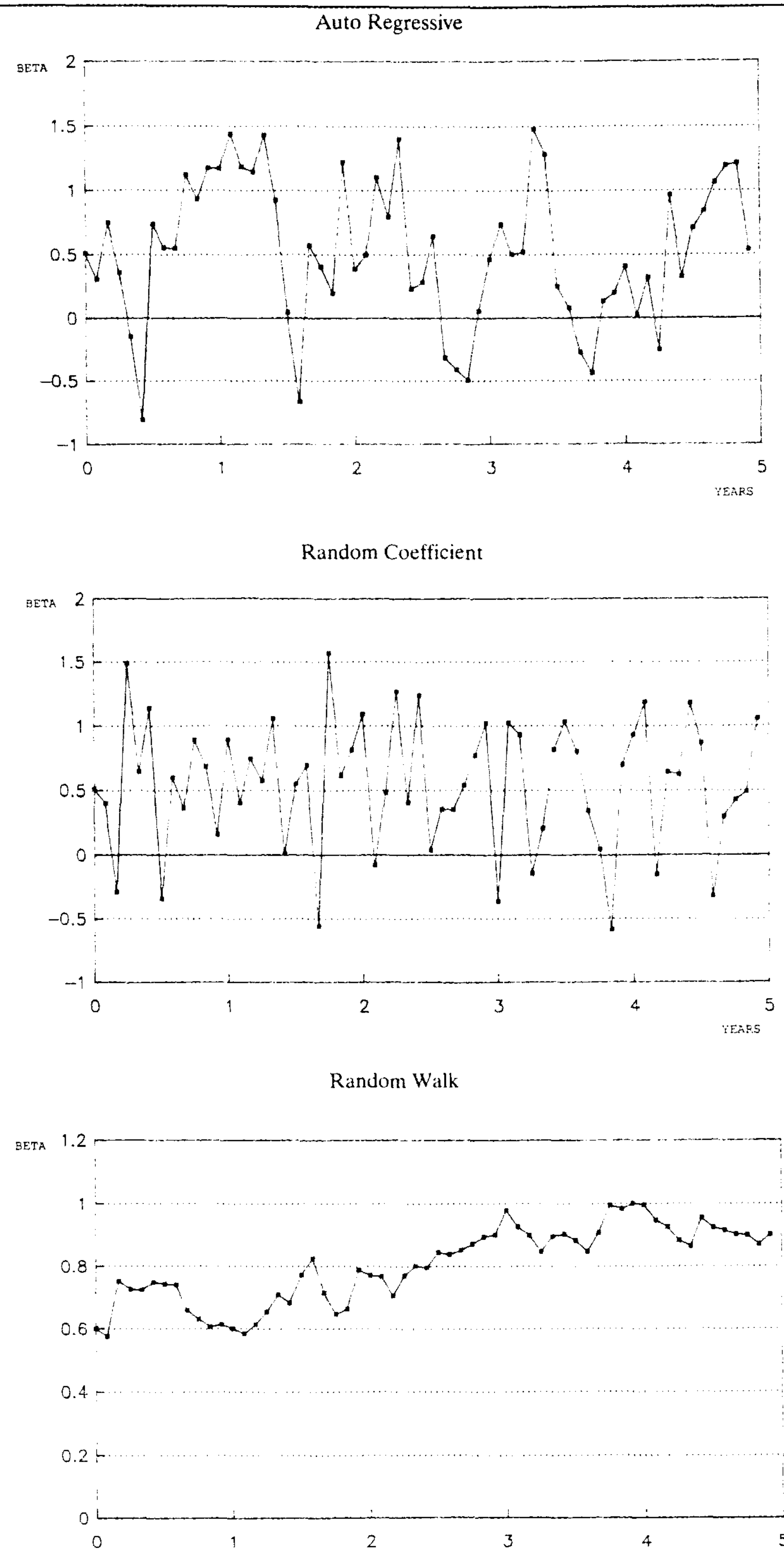
The least common process indicated by companies in the sample is the constant coefficient process, shown by only 6 companies. A constant beta coefficient is indicated by a zero estimate for the standard deviation of beta.

Since the estimation period covers 15 years (180 months), many companies could not maintain a constant beta coefficient. The long estimation period allows management, regulators, and the markets to react to any exogenous changes affecting systematic risk so as to bring risk back to reasonable levels. Such reaction is consistent with a beta that follows an auto-regressive process. Consequently, the preponderance of companies with auto-regressive betas in Exhibit 1 conforms to expected long-term behavior of management and markets.

Internal consistency of parameter estimates in Exhibit 1 is just as important as reasonableness. All companies having a zero estimate for the standard deviation of beta also show a value of 0.0 for the adjustment rate estimate. Any other estimate would be ambiguous for classifying the process. A positive association between the estimate of the standard deviation of beta and the estimate of $1 - a_i$ further points to the lack of ambiguity and helps in interpreting the process for all of the sample companies.

The positive association between beta variability and the regression rate is also consistent with boundaries upon beta values. Companies with high beta variability tend to have betas that return quickly to an underlying mean. Companies with low or zero return rates have low beta variability. High variability to-

Exhibit 2. Three Time-Series Processes for Beta



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gether with a low or zero return rate would lead to extreme beta instability and preclude application of the CAPM. The results show no evidence of this type of beta instability.

A. Behavior of Transient Betas

To illustrate the implications of different processes and parameters, plots of betas following an auto-regressive process, a random coefficient process, and a random walk process are presented in Exhibit 2. Each of these processes behaves according to average coefficient values of companies with that process in Exhibit 1. For the auto-regressive process, the coefficients are an underlying mean of 0.51, a standard deviation of transient beta of 0.50, and a return rate toward the underlying mean of 0.52. For the random coefficient process, the underlying mean is 0.52 and its standard deviation is 0.53. For the random walk process the standard deviation of beta is 0.05.

The auto-regressive beta depicted in Exhibit 2 shows considerable variability and ranges between a minimum value of -0.8 and a maximum value of 1.50. Although the variability in the short run is rather large, the beta at no time takes longer than 9 months to return to its underlying mean, usually returning in three or four months. However, upon returning to its underlying mean it often strays on the opposite side, requiring several additional months to return.

Over the 60-month period shown for the auto-regressive process in Exhibit 2, only 36 of the transient beta values fall between a low of 0.0 and a high of 1.0. These bounds might be considered reasonable for a utility. Nine of the 60 beta observations lie below 0.0. The presence of such outliers might frustrate, but not obviate, application of OLS techniques for beta estimation. Although Exhibit 2 indicates that extreme beta values, such as those discussed by Brigham and Crum [5], might be common in the short run, the forecaster should not be deterred by the presence of short-run instability. In the long run, beta will return to its mean.

The similarity between the auto-regressive process and the random coefficient process, also shown in Exhibit 2, is obvious. Even if rather extreme values are encountered, the random coefficient beta reverts back to the mean within the next two observations. The upper and lower bounds on beta as well as the proportion of betas less than zero are very similar for the two processes.

Exhibit 2 also contains a plot of the time-series behavior of a beta following a random walk process. Although the beta behavior for the random walk process seems more stable than the auto-regressive or random coefficient process, such apparent short-run stability is misleading. Over the 60 months depicted in Exhibit 2, the beta wanders from a value of 0.6 to a value of about 0.9. Over the next 60 months, the beta could potentially rise by another 0.3, fall back to 0.6, or be anywhere in between. In the longer run, the beta becomes even more difficult to forecast, due to the lack of any tendency to revert to an underlying mean.

B. Focusing on the Consolidated Edison Dividend Omission

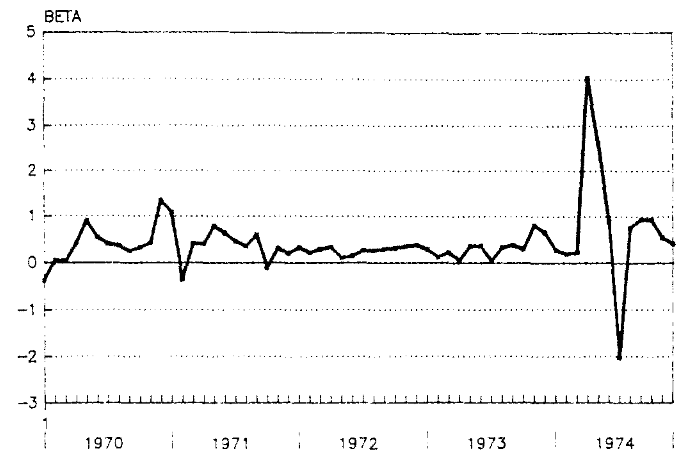
A plot during the period from January 1970–December 1984 of the behavior of the transient beta for Consolidated Edison is presented in Exhibit 3. The transient beta behaves much like the typical beta for any utility with an auto-regressive beta, except for the period immediately following the dividend omission. During this period, the transient beta becomes very erratic for about 9 months. Once it settles down, it continues to behave like any other utility with a typical auto-regressive beta. The plot of the transient beta for Con Ed over the last 60 months, if placed on the same scale as Exhibit 2, would be visually indistinguishable from the auto-regressive process depicted in that exhibit.

The plot of Con Ed's transient beta shown in Exhibit 3 depicts the transitory effect of economic disturbances on beta estimates. Even in this dramatic case of a dividend omission, the relationship between the stock and the market returned to normal within less than one year. This strong tendency to return to the mean beta gives empirical support to forecaster-supplied prior values in Bayesian adjustment models that place more weight on the underlying mean beta and less weight on the transient beta than the Merrill Lynch model would imply.

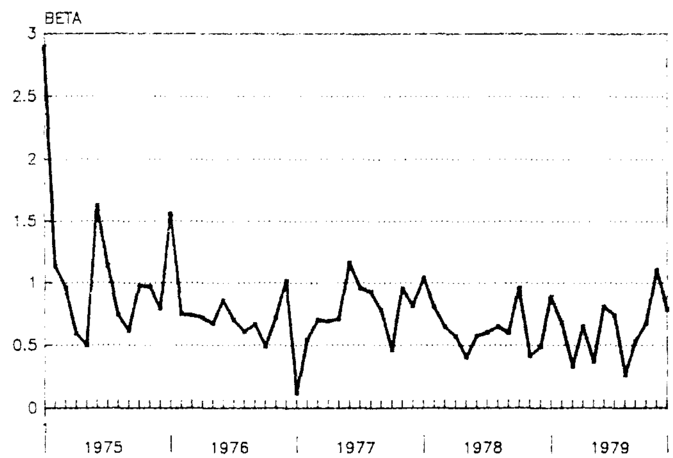
Some additional information on the behavior of Con Ed's beta is presented in Exhibit 4. During the overall period, which extends from January 1970–June 1984, its OLS beta estimate was 0.61 and the estimate of its underlying mean beta was 0.58. Since this overall period contains the dividend omission, a null hypothesis of a constant coefficient process for beta can be easily rejected. The regression rate of 0.70 toward the underlying mean indicates a strong mean-reversion tendency.

Exhibit 3. Transient Beta for Consolidated Edison, 1970–1984

1970–1974



1975–1979



1980–1984

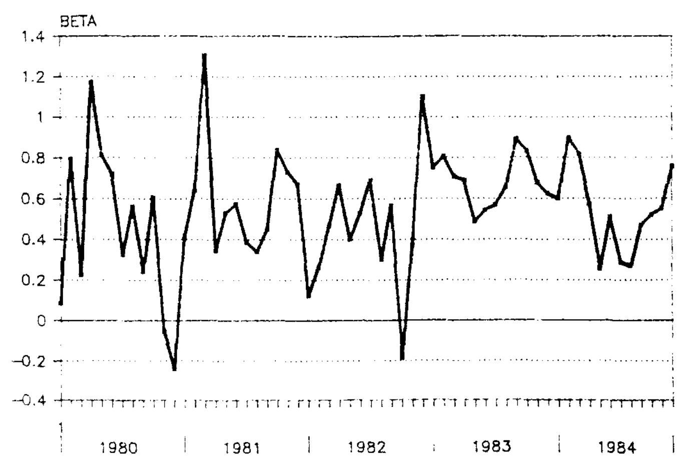


Exhibit 4. Parameter Estimates for Consolidated Edison Beta

Parameter	Overall Period 1970–1984	Before Dividend Omission 1970–1973	After Dividend Omission 1978–1981
Ordinary Least Squares Beta	0.61	0.39	0.62
Standard Error of OLS Beta	0.08	0.04	0.05
$K - F$ Underlying Mean Beta	0.58	0.34	0.47
$K - F$ Regression Rate to Mean	0.70	1.00	1.00
$K - F$ Standard Deviation of Beta	0.74	0.62	0.78
$K - F$ Residual Error in Market Model	0.05	0.03	0.04
$K - F$ Beta Stability Test	58.80*	20.30*	7.00*

*Significant at the 0.05 level.

Exhibit 4 also contains Kalman filtering and OLS estimates of beta for both a four-year period prior to the divided omission and a four-year period after the dividend omission. Forty-eight monthly observations is not sufficient to estimate reliably the underlying mean beta, since by nature this parameter reveals itself only over the long run. Likewise, the estimate of $1 - a_i$ may also be unreliable when estimated by only a few observations over a short time period. However, the subperiods do depict the variability that is characteristic of short-term estimates, whether those estimates are obtained by OLS or by Kalman filtering.

Although these short-term estimates should be approached with caution, some effects of the dividend omission on Con Ed's risk might be inferred. First, estimates for the long-term period or either of the short-term periods do not appear contaminated by the dividend omission but appear quite reasonable for a utility. Second, no indication of a decline in the beta estimate due to inclusion of the dividend omission period is evident. The indication is to the contrary. The estimate of the underlying mean beta for the overall period is higher than either the four- year period prior to the omission or the four years following the omission.

V. Implications for Beta Forecasting and Rate Setting

A partial resolution to the beta measurement problem is outlined by Peseau and Zepp [22], who show that the effect of the dividend omission was transitory and could be diagnosed from examination of OLS statistics. Although the dividend omission produces beta estimation problems for Consolidated Edison, subsequent estimates using data after the omission become much more reasonable.

The primary difference between the Brigham and Crum [5] forecast using an OLS beta and the Peseau and Zepp comment lies in the assumption of the time-series process followed by beta. The OLS estimate for five years of return data is only a good beta forecast if beta follows a constant coefficient process. This assumption is untenable for an estimation period containing a major information release.

When beta is time-varying, a short-term unadjusted OLS estimate may not be the best estimate of beta. Instead, the forecaster, taking advantage of auto-regressive properties of beta, should adjust that short-term estimate toward an underlying mean beta. When beta is unstable but reverts to an underlying mean, beta instability would not preclude application of the CAPM, but might preclude use of an OLS beta.

Reliance on a short-term beta forecast, whether from an OLS estimate or the transient beta estimate in the Kalman filtering model, is appropriate only if the firm's beta follows a random walk process. This research shows little evidence suggesting the typical utility beta follows a random walk and no evidence that, specifically, Con Ed's beta follows a random walk.

Due to the preponderance of auto-regressive or random coefficient betas, the results of this study strongly support the use of Bayesian-type adjustment processes such as the one employed by Merrill Lynch. The results also suggest that the behavior of utility betas may differ from the behavior of large diversified samples of stocks. For example, since Blume [2] finds an underlying mean beta of 1.0 for a large sample of stocks, many Bayesian models will adjust the OLS beta estimate toward 1.0. The results of this study, however, indicate that 1.0 is too high an underlying mean for most utilities. Instead, they should be adjusted toward a value that is less than one. For Consolidated Edison, an underlying mean of 0.7 would be more appropriate.

VI. Conclusions

Understanding beta behavior requires more information than whether or not betas are stable. Development of statistical procedures admitting a continuously time-varying beta now allows forecasters to understand how beta may behave over the short run and how that short-run behavior can differ from long-run behavior. Measuring continuously time-varying betas also frees the forecaster from the limitations imposed by assuming a constant coefficient beta. Instead, like most economic variables, beta can be modeled as a coefficient that is always changing. From the time series process followed by betas, the forecaster also gains an understanding of the difficult problem of forecasting beta. The beta for the majority of utility companies in this sample follows either an auto-regressive process or a constant coefficient process. Very few appear to follow a random walk process, which would produce betas that are not only unstable but very difficult to forecast. On the other hand, with an auto-regressive process, a patient forecaster using relatively simple diagnostic procedures should be able to obtain a reasonable long-run estimate of systematic risk. A reasonable forecast of beta then admits application of the CAPM for utilities even if beta is time varying.

The strong evidence of auto-regressive tendencies in utility betas lends support to the application of adjustment procedures such as the Bayesian adjustment procedure presented by Vasicek [25]. This procedure depends upon beta following an auto-regressive process. In addition, the Kalman filtering methodology also provides objective prior estimates of the underlying mean beta and the adjustment rate toward that underlying mean.

Typical adjustment models use a prior estimate of about 0.35 for the adjustment rate toward the underlying mean and a prior estimate of 1.0 as the underlying mean. The results of this study indicate that an underlying mean of 1.0 is too high for most utilities and an adjustment rate of 0.35 is too low.

Although considerable variability in adjustment rates and underlying mean betas can be observed in the sample, it may not be necessary for a forecaster to apply the Kalman filtering approach in order to obtain these estimates. A reasonable estimate of the underlying mean may be obtained by OLS if applied to a very long time period. The prior estimate of the adjustment rate toward the mean can be obtained by considering the positive relationship between the adjustment rate and beta variability. Estimates of the prior adjustments in the Bayesian adjustment models could be applied without relying blindly on large-sample estimates that may not be applicable to utilities.

References

1. R. Bey, "Market Model Stationarity of Individual Public Utilities," *Journal of Financial and Quantitative Analysis* (March 1983), pp. 67–86.
2. M.E. Blume, "On the Assessment of Risk," *Journal of Finance* (March 1971), pp. 1–10.
3. T. Bos and P. Newbold, "An Empirical Investigation of the Possibility of Stochastic Systematic Risk in the Market Model," *Journal of Business* (January 1984), pp. 35–41.
4. W.J. Breen and E.M. Lerner, "On the Use of Beta in Regulatory Proceedings," *Bell Journal of Economics and Management Science* (Autumn 1972), pp. 612–621.
5. E.F. Brigham and R.L. Crum, "On the Use of the CAPM in Public Utility Rate Cases," *Financial Management* (Summer 1977), pp. 7–15.
6. ———, "Reply to Comments on 'On the Use of the CAPM in Public Utility Rate Cases'," *Financial Management* (Autumn 1978), pp. 72–76.
7. W.T. Carleton, "A Highly Personal Comment on 'On the Use of the CAPM in Public Utility Rate Cases'," *Financial Management* (Autumn 1978), pp. 57–59.
8. C.R. Chen, "Time-Series Analysis of Beta Stationarity and its Determinants: A Case of Public Utilities," *Financial Management* (Autumn 1982), pp. 64–71.
9. S.N. Chen and A.J. Keown, "Pure Residual and Market Risk: A Note," *Journal of Finance* (December 1981), pp. 1,203–1,210.
10. D.W. Collins, J. Ledolter, and J. Rayburn, "Some Further Evidence on the Stochastic Properties of Systematic Risk," *Journal of Business* (July 1987), pp. 425–448.
11. T.E. Conine, Jr. and M. Tamarkin, "Implications of Skewness in Returns for Utilities' Cost of Equity Capital," *Financial Management* (Winter 1985), pp. 66–71.
12. P.L. Cooley, "A Review of the Use of Beta in Regulatory Proceedings," *Financial Management* (Winter 1981), pp. 75–81.
13. F.J. Fabozzi and J.C. Francis, "Beta as a Random Coefficient," *Journal of Financial and Quantitative Analysis* (March 1978), pp. 101–116.
14. K. Garbade and J. Rentzler, "Testing the Hypothesis of Beta Stationarity," *International Economic Review* (October 1981), pp. 578–587.
15. J.E. Gilster, Jr. and C. M.Linke, "More on the Estimation of Beta for Public Utilities: Biases Resulting from Structural Shifts in True Beta," *Financial Management* (Autumn 1978), pp. 60–65.
16. D.R. Kahl and J. Ledolter, "A Recursive Kalman Filter Forecasting Approach," *Management Science* (November 1983), pp. 1,325–1,333.
17. R. McEnally, "On the Use of the CAPM in Public Utility Rate Cases: Comment," *Financial Management* (Autumn 1978), pp. 69–70.
18. Merrill, Lynch, Pierce, Fenner, and Smith, *Security Risk Evaluation*, New York, Merrill, Lynch, Pierce, Fenner, and Smith, 1983.

19. S.C. Myers, "The Application of Finance Theory to Public Utility Rate Cases," *Bell Journal of Economics and Management Science* (Spring 1972), pp. 58–97.
20. ———, "On the Use of Beta in Regulatory Proceedings: A Comment," *Bell Journal of Economics and Management Science* (Autumn 1972), pp. 622–627.
21. ———, "On the Use of Modern Portfolio Theory in Public Utility Rate Cases: Comment," *Financial Management* (Autumn 1978), pp. 66–68.
22. D.E. Peseau and T.M. Zepp, "On the Use of the CAPM in Public Utility Rate Cases: Comment," *Financial Management* (Autumn 1978), pp. 52–56.
23. R.H. Pettway, "On the Use of Beta in Regulatory Proceedings: An Empirical Examination," *Bell Journal of Economics and Management Science* (Spring 1978), pp. 239–248.
24. W.F. Sharpe, "On the Use of the CAPM in Public Utility Rate Cases: Comment," *Financial Management* (Autumn 1978), pp. 71–72.
25. O.A. Vasicek, "A Note on Using Cross-Sectional Information in Bayesian Estimation of Security Betas," *Journal of Finance* (December 1973), pp. 1,233–1,239.

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ON THE ASSESSMENT OF RISK

MARSHALL E. BLUME*

INTRODUCTION

THE CONCEPT OF RISK has so permeated the financial community that no one needs to be convinced of the necessity of including risk in investment analysis. Still of controversy is what constitutes risk and how it should be measured. This paper examines the statistical properties of one measure of risk which has had wide acceptance in the academic community: namely the coefficient of non-diversifiable risk or more simply the beta coefficient in the market model.

The next section defines this beta coefficient and presents a brief non-rigorous justification of its use as a measure of risk. After discussing the sample and its basic properties in Section III, Section IV examines the stationarity of this beta coefficient over time and proposes a method of obtaining improved assessments of this measure of risk.

II. THE RATIONALE OF BETA AS A MEASURE OF RISK

The interpretation of the beta coefficient as a measure of risk rests upon the empirical validity of the market model. This model asserts that the return from time (t-1) to t on asset i, $\tilde{R}_{it}$,[1] is a linear function of a market factor common to all assets $\tilde{M}_t$, and independent factors unique to asset i, $\tilde{\varepsilon}_{it}$.

Symbolically, this relationship takes the form

$$\tilde{R}_{it} = \alpha_i + \beta_i \tilde{M}_t + \tilde{\varepsilon}_{it}, \tag{1}$$

where the tilde indicates a random variable, α_i is a parameter whose value is such that the expected value of $\tilde{\varepsilon}_{it}$ is zero, and β_i is a parameter appropriate to asset i.[2] That the random variables $\tilde{\varepsilon}_{it}$ are assumed to be independent and

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1. In this paper, return will be measured as the ratio of the value of the investment at time t with dividends reinvested to the value of the investment at time (t-1). Dividends are assumed reinvested at time t.

2. The parameter β_i is defined as Cov $(\tilde{R}_i, \tilde{M})$/Var $(\tilde{M})$.

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unique to asset i implies that Cov $(\tilde{\varepsilon}_{it}, \tilde{M}_t)$ is zero and that Cov $(\tilde{\varepsilon}_{it}, \tilde{\varepsilon}_{jt})$, $i \neq j$, are zero. This last conclusion is tantamount to assuming the absence of industry effects.

The empirical validity of the market model as it applies to common stocks listed on the NYSE has been examined extensively in the literature.[3] The principal conclusions are: (1) The linearity assumption of the model is adequate.[4] (2) The variables $\tilde{\varepsilon}_{it}$ cannot be assumed independent between securities because of the existence of industry effects. However, these industry effects, as documented by King,[5] probably account for only about ten percent of the variation in returns, so that as a first approximation they can be ignored. (3) The unique factors $\tilde{\varepsilon}_{it}$ correspond more closely to non-normal stable variates than to normal ones. This conclusion means that variances and covariances of the unique factors do not exist. Nonetheless, this paper will make the more common assumption of the existence of these statistics in justifying the beta coefficient as a measure of risk since Fama[6] and Jensen[7] have shown that this coefficient can still be interpreted as a measure of risk under the assumption that the $\tilde{\varepsilon}_{it}$'s are non-normal stable variates.

That the beta coefficient, β_i, in the market model can be interpreted as a measure of risk will be justified in two different ways: the portfolio approach and the equilibrium approach.

A. *The Portfolio Approach*

The important assumption underlying the portfolio approach is that individuals evaluate the risk of a portfolio as a whole rather than the risk of each asset individually. An example will illustrate the meaning of this statement. Consider two assets, each of which by itself is extremely risky. If, however, it is always the case that when one of the assets has a high return, the other has a low return, the return on a combination of these two assets in a portfolio may be constant. Thus, the return on the portfolio may be risk free whereas each of the assets has a highly uncertain return. The discussion of such an

3. See Marshall E. Blume, "Portfolio Theory: A Step Towards Its Practical Application," forthcoming *Journal of Business;* Eugene F. Fama, "The Behavior of Stock Market Prices," *Journal of Business* (1965), 34-105; Eugene F. Fama, Lawrence Fisher, Michael Jensen, and Richard Roll, "The Adjustment of Stock Prices to New Information," *International Economic Review* (1969), 1-21; Michael Jensen, "Risk, the Pricing of Capital Assets, and the Evaluation of Investment Portfolios," *Journal of Business* (1969), 167-247; Benjamin F. King, "Market and Industry Factors in Stock Price Behavior," *Journal of Business* (1966), 139-90; and William F. Sharpe, "Mutual Fund Performance," *Journal of Business* (1966), 119-38.

4. The linearity assumption of the model should not be confused with the equilibrium requirement of William F. Sharpe, "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk," *Journal of Finance* (1964), 425-42, which states that $\alpha_i = (1 - \beta_i)\ R_F$, where R_F is the risk free rate. It is quite possible that this equality does not hold and at the same time that the market model is linear.

5. King, *op. cit.*

6. Eugene F. Fama, "Risk, Return, and Equilibrium" (Report No. 6831, University of Chicago, Center for Mathematical Studies in Business and Economics, June, 1968).

7. Jensen, *op. cit.*

obvious point may seem unwarranted, but there is very little empirical work which indicates that people do in fact behave according to it.

Now if an individual is willing to judge the risk inherent in a portfolio solely in terms of the variance of the future aggregate returns, the risk of a portfolio of n securities with an equal amount invested in each, according to the market model, will be given by

$$\text{Var}(\tilde{W}_t) = \left(\sum_{i=1}^{n} \frac{1}{n}\beta_i\right)^2 \text{Var}(\tilde{M}_t) + \sum_{i=1}^{n} \left(\frac{1}{n}\right)^2 \text{Var}(\tilde{\varepsilon}_{it}) \qquad (2)$$

where $\tilde{W}_t$ is the return on the portfolio. Equation (2) can be rewritten as

$$\text{Var}(\tilde{W}_t) = \bar{\beta}^2 \text{Var}(\tilde{M}_t) + \frac{\overline{\text{Var}(\tilde{\varepsilon})}}{n} \qquad (3)$$

where the bar indicates an average. As one diversifies by increasing the number of securities n, the last term in equation (3) will decrease. Evans and Archer[8] have shown empirically that this process of diversification proceeds quite rapidly, and with ten or more securities most of the effect of diversification has taken place. For a well diversified portfolio, Var $(\tilde{W}_t)$ will approximate $\bar{\beta}^2$ Var $(\tilde{M}_t)$. Since Var $(\tilde{M}_t)$ is the same for all securities, $\bar{\beta}$ becomes a measure of risk for a portfolio and thus β_i, as it contributes to the value of $\bar{\beta}$, is a measure of risk for a security. The larger the value of β_i, the more risk the security will contribute to a portfolio.[9]

B. *The Equilibrium Approach*

Using the market model, Sharpe[10] and Lintner,[11] as clarified by Fama,[12] have developed a theory of equilibrium in the capital markets. This theory relates the risk premium for an individual security, $E(\tilde{R}_{it}) - R_F$, where R_F is the risk free rate, to the risk premium of the market, $E(\tilde{M}_t) - R_F$, by the formula

$$E(\tilde{R}_{it}) - R_F = \beta_i[E(\tilde{M}_t) - R_F]. \qquad (4)$$

The risk premium for an individual security is proportional to the risk premium for the market. The constant of proportionality β_i can therefore be interpreted as a measure of risk for individual securities.

8. John L. Evans and Stephan H. Archer, "Diversification and the Reduction of Dispersion: An Empirical Analysis," *Journal of Finance* (1968), 761-68.

9. This argument has been extended to a non-Gaussian, symmetric stable world by E. F. Fama, "Portfolio Analysis in a Stable Paretian Market," *Management Science* (1965), 404-19; and P. A. Samuelson, "Efficient Portfolio Selection for Pareto-Levy Investments," *Journal of Financial and Quantitative Analysis* (1967), 107-22.

10. Sharpe, "Capital Asset Prices," *op. cit.*

11. John Lintner, "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," *Review of Economics and Statistics* (1965), 13-37.

12. Eugene F. Fama, "Risk, Return, and Equilibrium: Some Clarifying Comments," *Journal of Finance* (1968), 29-40.

This theory of equilibrium, although theoretically sound, is based upon numerous assumptions which obviously do not hold in the real world. A theoretical model, however, should not be judged by the accuracy of its assumptions but rather by the accuracy of its predictions. The empirical work of Friend and Blume[13] suggests that the predictions of this model are seriously biased and that this bias is primarily attributable to the inaccuracy of one key assumption, namely that the borrowing and lending rates are equal and the same for all investors. Therefore, although Sharpe's and Lintner's theory of equilibrium can be used as a justification for β_i as measure of risk, it is a weaker and considerably less robust justification than that provided by the portfolio approach.

III. The Sample and Its Properties

The sample was taken from the updated Price Relative File of the Center for Research in Security Prices at the Graduate School of Business, University of Chicago. This file contains the monthly investment relatives, adjusted for dividends and capital changes of all common stocks listed on the New York Stock Exchange during any part of the period from January 1926 through June 1968, for the months in which they were listed. Six equal time periods beginning in July 1926 and ending in June 1968 were examined. Table 1 lists these six periods and the number of companies in each for which there was a complete history of monthly return data. This number ranged from 415 to 890.

The investment relatives for a particular security and a particular period were regressed[14] upon the corresponding combination market link relatives, which were originally prepared by Fisher[15] as a measure of the market factor. This process was repeated for each security and each period, yielding, for instance, in the July 1926 through June 1933 period, 415 separate regressions. The average coefficient of determination of these 415 regressions was 0.51. The corresponding average coefficients of determination for the next five periods were, respectively, 0.49, 0.36, 0.32, 0.25, and 0.28. These figures are consistent with King's findings[16] in that the proportion of the variance of returns explained by the market declined steadily until 1960 when his sample terminated. Since 1960, the importance of the market factor has increased slightly according to these figures.

Table 1, besides giving the number of companies analyzed, summarizes the distributions of the estimated beta coefficients in terms of the means, standard deviations, and various fractiles of these distributions. In addition, the number of estimated betas which were less than zero is given. In three of the periods,

13. Irwin Friend and Marshall Blume, "Measurement of Portfolio Performance Under Uncertainty," *American Economic Review* (1970), 561-75.

14. John Wise, "Linear Estimators for Linear Regression Systems Having Infinite Variances," (Berkeley-Stanford Mathematics-Economics Seminar, October, 1963) has given some justification for the use of least squares in estimating coefficients of regressions in which the disturbances are non-normal symmetric stable variates.

15. Lawrence Fisher, "Some New Stock-Market Indexes," *Journal of Business* (1966), 191-225.

16. King, *op. cit.*

TABLE 1

DESCRIPTIVE SUMMARY OF ESTIMATED BETA COEFFICIENTS

Period	Number of Companies	Mean	Standard Deviation	Number of BETAS less than Zero	Fractiles				
					.10	.25	.50	.75	.90
7/26-6/33	415	1.051	0.462	1	0.498	0.711	1.023	1.352	1.616
7/33-6/40	604	1.036	0.474	0	0.436	0.701	1.015	1.349	1.581
7/40-6/47	731	0.990	0.504	0	0.500	0.643	0.872	1.186	1.606
7/47-6/54	870	1.010	0.409	2	0.473	0.727	0.996	1.263	1.565
7/54-6/61	890	0.998	0.423	0	0.458	0.678	0.984	1.250	1.558
7/61-6/68	847	0.962	0.390	4	0.475	0.681	0.934	1.199	1.491

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none of the estimated betas was negative. Of the 4357 betas estimated in all six periods, only seven or 0.16 per cent were negative. This means that although the inclusion of a stock which moves counter to the market can reduce the risk of a portfolio substantially, there are virtually no opportunities to do this. Nearly every stock appears to move with the market.[17]

IV. The Stationarity of Beta over Time

No economic variable including the beta coefficient is constant over time. Yet for some purposes, an individual might be willing to act *as if* the values of beta for individual securities were constant or stationary over time. For example, a person who wishes to assess the future risk of a well diversified portfolio is really interested in the behavior of averages of the β_i's over time and not directly in the values for individual securities. For the purposes of evaluating a portfolio, it may be sufficient that the historical values of β_i be unbiased estimates of the future values for an individual to act *as if* the values of the β_i's for individual securities are stationary over time. This is because the errors in the assessment of an average will tend to be less than those of the components of the average providing that the errors in the assessments of the components are independent of each other.[18] Yet, a statistician or a person who wishes to assess the risk of an individual security may have completely different standards in determining whether he would act as if the β_i's are constant over time. The remainder of the paper examines the stationarity of the β_i's from the point of view of a person who wishes to analyze a portfolio.

A. *Correlations*

To examine the empirical behavior of the risk measures for portfolios over time, arbitrary portfolios of n securities were selected as follows: The estimates of β_i were derived using data from the first period, July 1926 through June 1933, and were then ranked in ascending order.[19] The first portfolio of n securities consisted of those securities with the n smallest estimates of β_i. The second portfolio consisted of those securities with the next n smallest estimates of β_i, and so on until the number of securities remaining was less than n. The number of securities n was allowed to vary over 1, 2, 4, 7, 10, 20, 35, 50, 75, and 100. This process was repeated for each of the next four periods.

Table 2 presents the product moment and rank order correlation coefficients between the risk measures for portfolios of n securities assuming an equal investment in each security estimated in one period and the corresponding risk

17. The use of considerably less than seven years of monthly data such as two or three years to estimate the beta coefficient results in a larger proportion of negative estimates. This larger proportion is probably due to sampling errors which, as documented in Richard Roll, "The Efficient Market Model Applied to U. S. Treasury Bill Rates," (Unpublished Ph.D. thesis, Graduate School of Business, University of Chicago, 1968) may be quite large for models with non-normal symmetric stable disturbances.

18. This property of averages does not hold for all distributions (*cf.* Eugene F. Fama, "Portfolio Analysis in a Stable Paretian Market"), but for the distributions associated with stock market returns it almost certainly holds.

19. Only securities which also had complete data in the next seven year period were included in this ranking.

measure for the same portfolio estimated in the next period.[20] The risk measure calculated using the earlier data might be regarded as an individual's assessment of the future risk, and the measure calculated using the later data can be regarded as the realized risk. Thus, these correlation coefficients can be interpreted as a measure of the accuracy of one's assessments, which in this case are simple extrapolations of historical data.

TABLE 2

PRODUCT MOMENT AND RANK ORDER CORRELATION COEFFICIENTS OF BETAS FOR PORTFOLIOS OF N SECURITIES

Number of Securities per Portfolio	7/26-6/33 and 7/33-6/40		7/33-6/40 and 7/40-6/47		7/40-6/47 and 7/47-6/54		7/47-6/54 and 7/54-6/61		7/54-6/61 and 7/61-6/68	
	P.M.	Rank	P.M.	Rank	P.M.	Rank	P.M.	Rank	P.M.	Rank
1	0.63	0.69	0.62	0.73	0.59	0.65	0.65	0.67	0.60	0.62
2	0.71	0.75	0.76	0.83	0.72	0.79	0.76	0.76	0.73	0.74
4	0.80	0.84	0.85	0.90	0.81	0.89	0.84	0.84	0.84	0.85
7	0.86	0.90	0.91	0.93	0.88	0.93	0.87	0.88	0.88	0.89
10	0.89	0.93	0.94	0.95	0.90	0.95	0.92	0.93	0.92	0.93
20	0.93	0.99	0.97	0.98	0.95	0.98	0.95	0.96	0.97	0.98
35	0.96	1.00	0.98	0.99	0.95	0.99	0.97	0.98	0.97	0.97
50	0.98	1.00	0.99	0.98	0.98	0.99	0.98	0.98	0.98	0.97

The values of these correlation coefficients are striking. For the assessments based upon the data from July 1926 through June 1933 and evaluated using data from July 1933 through June 1940, the product moment correlations varied from 0.63 for single securities to 0.98 for portfolios of 50 securities. The high value of the latter coefficient indicates that substantially all of the variation in the risk among portfolios of 50 securities can be explained by assessments based upon previous data. The former correlation suggests that assessments for individual securities derived from historical data can explain roughly 36 per cent of the variation in the future estimated values, leaving about 64 per cent unexplained.[21]

These results, which are typical of the other periods, suggest that at least as measured by the correlation coefficients, naively extrapolated assessments of future risk for larger portfolios are remarkably accurate, whereas extrapolated assessments of future risk for individual securities and smaller portfolios are of some, but limited value in forecasting the future.

B. *A Closer Examination*

Table 3 presents the actual estimates of the risk parameters for portfolios of 100 securities for successive periods. For all five different sets of portfolios, the rank order correlations between the successive estimates are one, but there is obviously some tendency for the estimated values of the risk parameter to

20. Because of the small number of portfolios of 100 securities, correlations are not presented in Table 2 for these portfolios.

21. This large magnitude of unexplained variation may make the beta coefficient an inadequate measure of risk for analyzing the cost of equity for an individual firm although it may be adequate for cross-section analyses of cost of equity.

TABLE 3

ESTIMATED BETA COEFFICIENTS FOR PORTFOLIOS OF 100 SECURITIES IN TWO SUCCESSIVE PERIODS

Portfolio	7/26-6/33	7/33-6/40	7/33-6/40	7/40-6/47	7/40-6/47	7/47-6/54	7/47-6/54	7/54-6/61	7/54-6/61	7/61-6/68
1	0.528	0.610	0.394	0.573	0.442	0.593	0.385	0.553	0.393	0.620
2	0.898	1.004	0.708	0.784	0.615	0.776	0.654	0.748	0.612	0.707
3	1.225	1.296	0.925	0.902	0.746	0.887	0.832	0.971	0.810	0.861
4			1.177	1.145	0.876	1.008	0.967	1.010	0.987	0.914
5			1.403	1.354	1.037	1.124	1.093	1.095	1.138	0.995
6					1.282	1.251	1.245	1.243	1.337	1.169

change gradually over time. This tendency is most pronounced in the lowest risk portfolios, for which the estimated risk in the second period is invariably higher than that estimated in the first period. There is some tendency for the high risk portfolios to have lower estimated risk coefficients in the second period than in those estimated in the first. Therefore, the estimated values of the risk coefficients in one period are biased assessments of the future values, and furthermore the values of the risk coefficients as measured by the estimates of β_1 tend to regress towards the means with this tendency stronger for the lower risk portfolios than the higher risk portfolios.

C. *A Method of Correction*

In so far as the rate of regression towards the mean is stationary over time, one can in principle correct for this tendency in forming one's assessments. An obvious method is to regress the estimated values of β_1 in one period on the values estimated in a previous period and to use this estimated relationship to modify one's assessments of the future.

Table 4 presents these regressions for five successive periods of time for individual securities.[22] The slope coefficients are all less than one in agreement with the regression tendency, observed above. The coefficients themselves do change over time, so that the use of the historical rate of regression to correct

TABLE 4

MEASUREMENT OF REGRESSION TENDENCY OF ESTIMATED BETA COEFFICIENTS FOR INDIVIDUAL SECURITIES

Regression Tendency Implied Between Periods	$\beta_2 = a + b\beta_1$
7/33-6/40 and 7/26-6/33	$\beta_2 = 0.320 + 0.714\beta_1$
7/40-6/47 and 7/33-6/40	$\beta_2 = 0.265 + 0.750\beta_1$
7/47-6/54 and 7/40-6/47	$\beta_2 = 0.526 + 0.489\beta_1$
7/54-6/61 and 7/47-6/54	$\beta_2 = 0.343 + 0.677\beta_1$
7/61-6/68 and 7/54-6/61	$\beta_2 = 0.399 + 0.546\beta_1$

22. The reader should not think of these regressions as a test of the stationarity of the risk of securities over time but rather merely as a test of the accuracy of the assessments of future risk which happen to be derived as historical estimates. In this test of accuracy, the independent variable in these regressions is measured without error, so that the estimated coefficients are unbiased. In the test of the stationarity of the risk measures over time, the independent variable would be measured with error, so that the coefficients in Table 4 would be biased.

for the future rate will not perfectly adjust the assessments and may even overcorrect by introducing larger errors into the assessments than were present in the unadjusted data.

To examine the efficacy of using historical rates of regression to correct one's assessments, the estimated risk coefficients for the individual securities for the period from July 1933 through June 1940 were modified using the first equation in Table 4 to obtain adjusted risk coefficients under the assumption that the future rate of regression will be the same as the past. This process was repeated for each of the next three periods using respectively the next three equations in Table 4 to estimate the rate of regression.

Table 5 compares these adjusted assessments with the unadjusted assessments which were used in Tables 2 and 3. For the portfolios selected previously using the data from July 1933 through June 1940, both the unadjusted

TABLE 5
MEAN SQUARE ERRORS BETWEEN ASSESSMENTS AND FUTURE ESTIMATED VALUES

	Assessments Based Upon							
Number of Sec./ Port.	7/33-6/40		7/40-6/47		7/47-6/54		7/54-6/61	
	unadjusted	adjusted	unadjusted	adjusted	unadjusted	adjusted	unadjusted	adjusted
1	0.1929	0.1808	0.1747	0.1261	0.1203	0.1087	0.1305	0.1013
2	0.0915	0.0813	0.1218	0.0736	0.0729	0.0614	0.0827	0.0535
4	0.0538	0.0453	0.0958	0.0483	0.0495	0.0381	0.0587	0.0296
7	0.0323	0.0247	0.0631	0.0276	0.0387	0.0281	0.0523	0.0231
10	0.0243	0.0174	0.0535	0.0220	0.0305	0.0189	0.0430	0.0169
20	0.0160	0.0090	0.0328	0.0106	0.0258	0.0139	0.0291	0.0089
35	0.0120	0.0055	0.0266	0.0080	0.0197	0.0101	0.0302	0.0089
50	0.0096	0.0046	0.0192	0.0046	0.0122	0.0097	0.0237	0.0064
75	0.0081	0.0035	0.0269	0.0067	0.0112	0.0078	0.0193	0.0056
100	0.0084	0.0020	0.0157	0.0035	0.0114	0.0084	0.0195	0.0056

and adjusted assessments of future risk were obtained. The accuracy of these two alternative methods of assessment were compared through the mean squared errors of the assessments versus the estimated risk coefficients in the next period, July 1940 through June 1947.[23] This process was repeated for each of the next three periods.

For individual securities as well as portfolios of two or more securities, the assessments adjusted for the historical rate of regression are more accurate than the unadjusted or naive assessments. Thus, an improvement in the accuracy of one's assessments of risk can be obtained by adjusting for the historical rate of regression even though the rate of regression over time is not strictly stationary.

23. The mean square error was calculated by $\frac{\Sigma(\beta_1 - \beta_2)^2}{n}$ where β_1 is the assessed value of the future risk, β_2 is the estimated value of the risk, and n is the number of portfolios. In using an estimate of beta rather than the actual value, the mean square error will be biased upwards, but the effect of this bias will be the same for both the adjusted and unadjusted assessments.

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V. Conclusion

This paper examined the empirical behavior of one measure of risk over time. There was some tendency for the estimated values of these risk measures to regress towards the mean over time. Correcting for this regression tendency resulted in considerably more accurate assessments of the future values of risk.

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A NOTE ON USING CROSS-SECTIONAL INFORMATION IN BAYESIAN ESTIMATION OF SECURITY BETAS

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Bayesian decision theory provides formal procedures which utilize information available prior to sampling, together with the sample information, to construct estimates which are optimal with respect to the minimization of the expected loss. This paper presents a method for generating Bayesian estimates of the regression coefficient of rates of return of a security against those of a market index. The distribution of the regression coefficients across securities is used as the prior distribution in the analysis. Explicit formulas are given for the estimates. The Bayesian approach is discussed in comparison with the current practice of sampling-theory procedures.

I. Introduction

The Capital Asset Pricing Model of Treynor [7], Sharpe [6], and Lintner [4] states that the expected rate of return on a security in excess of the risk-free rate is proportional to the slope coefficient of the regression of that security's rates of return on a market index. The slope coefficient, or beta, is for this reason one of the basic concepts of modern capital market theory, and considerable attention has been devoted to its measurement.

Customarily, beta is estimated from past data by least-squares regression procedures. The least-squares technique consists of fitting a linear relationship between the rates of return on a security and the rates of return on a market index so that the sum of squared differences between the security's actual returns and those implied by the relationship is minimized.

If y_t, $t = 1,2, \ldots, T$ and x_t, $t = 1,2, \ldots, T$ are the series of rates of return on a security and on a market index, respectively, the least-squares estimates of the parameters β, α, σ^2 in the simple linear regression process

$$y_t = \alpha + \beta x_t + e_t, \quad t = 1,2,\ldots,T \tag{1}$$

$$Ee_t = 0, \quad Ee_te_s = 0 \quad \text{for} \quad t \neq s, \quad Ee_t^2 = \sigma^2$$

are given as

$$b = \Sigma(y_t - \bar{y})(x_t - \bar{x})/\Sigma(x_t - \bar{x})^2 \tag{2}$$

$$a = \bar{y} - b\bar{x} \tag{3}$$

$$s^2 = \frac{1}{T-2}\Sigma(y_t - a - bx_t)^2, \tag{4}$$

respectively, and the variance of b is estimated as

* Wells Fargo Bank, N.A. This paper is a minor revision of the author's unpublished memorandum "Bayesian Estimates of Beta," Wells Fargo Bank, August 1971.

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$$s_b^2 = s^2/\Sigma(x_t - \bar{x})^2. \tag{5}$$

These are the best unbiased estimates of the parameters in the sense that the expected value of each of the estimates is equal to the corresponding parameter and the expected quadratic error attains the minimal value. In particular, when the beta coefficient of a stock is estimated by b, the following holds:

$$E(b|\beta) = \beta. \tag{6}$$

$$\mathrm{Var}(b|\beta) = \text{minimum over all estimates of } \beta \text{ satisfying (6)}. \tag{7}$$

For these reasons, the sampling-theory estimation procedures are commonly applied to the estimation of the beta of a security. Yet, the criteria as represented by Equations (6) and (7) do not satisfactorily reflect the desired properties of a beta estimator. Equation (6) describes an aspect of the distribution of the estimate assuming that the true value of the parameter is given. The actual situation is just the reverse: it is the sample coefficient that is known, and on the basis of this (and any prior or additional) information we want to infer about the distribution of the parameter.

To illustrate this point, assume that the estimated beta of a stock traded on the New York Stock Exchange is b = .2. In the absence of any additional information, this value is taken by sampling theory as being the best estimate of the true beta *because* any given true beta is equally likely to be overestimated as underestimated by the sample b. This, however, does not imply that given the sample estimate b, the true parameter is equally likely to be below or above the value .2. In fact, it is known from previous measurements that betas of stocks traded on the New York Stock Exchange are concentrated around unity, and most of them range in value between .5 and 1.5. Thus, an observed beta as low as 0.2 is more likely to be a result of underestimation than overestimation. The question of whether the estimate b is equally likely to lie below or above the true beta is irrelevant, since the true beta is not known. What is desired is an estimate such that given the sample information (which is available), the true beta will with equal probability lie below or above it.

To pursue this example further, assume that there are 1000 stocks under consideration, the betas of which are known to be distributed approximately normally around 1.0 with standard deviation of .5. Each of these true betas is equally likely to be underestimated or overestimated by b. Therefore, there are 500 stocks with true beta higher than the observed estimate, and 500 with true beta lower than the estimate. If an estimate of b = .2 is observed, the stock might be any of the approximately 500 × .945 = 473 stocks with β larger than .2 and underestimated, or any of the approximately 500 × .055 = 27 stocks with β smaller than .2 and overestimated. Apparently, given the sample and our prior knowledge of beta distribution, the former is much more likely, and thus, it is not correct to take .2 for an unbiased estimate.

This has been recognized before in the special situation where portfolios were formed by ranking of sample estimates (cf. Wagner and Vasicek [8]). The knowledge of the cross-sectional distribution of betas, however, can be used as a prior information whenever a beta of a security is estimated. Also, as a referee pointed out to the author, a similar problem has been recently addressed by Bogue [1]. Following is a Bayesian analysis of the simple normal regression process with the cross-sectional prior information. For information about the principles and techniques of Bayesian statistical theory, the reader is referred to Raiffa and Schlaifer [5].

II. Bayesian Estimates

For computational convenience, reparametrize the regression process (1) as follows:

$$y_t = \eta + \beta(x_t - \bar{x}) + e_t, \quad t = 1,2,\ldots,T \tag{8}$$

where

$$\eta = \alpha + \beta\bar{x}.$$

Assuming normal distribution of the disturbances, the kernel $k(b,\bar{y},s|v,\beta,\eta,\sigma)$ of the likelihood is proportional to (see [5], p. 335)

$$\sigma^{-T}\exp[-(T-2)s^2/(2\sigma^2)].\exp\left[-\frac{1}{2\sigma^2}(T(\bar{y}-\eta)^2 + v(b-\beta)^2)\right] \tag{9}$$

where b, s^2 is given by Equations (2), (4),

$$\bar{y} = \frac{1}{T}\Sigma y_t,$$

and

$$v = \Sigma(x_t - \bar{x})^2.$$

Let the information available prior to sampling consist of knowledge of the cross-sectional distribution of betas. Assuming that the distribution is approximately normal with parameters b', s'_b, the marginal prior density of β is

$$f'(\beta) \propto \exp[-(\beta - b')^2/(2s'^2_b)]. \tag{10}$$

(In accordance with practice, the prior distributions and parameters are denoted by primed letters, the posterior by letters with double primes, and the sample information without superscripts.)

Unless some prior information is available on η, σ, it is assumed that the prior density of these parameters is assessed as

$$f'(\eta,\sigma) \propto \sigma^{-1} \tag{11}$$

and independent of $f'(\beta)$. The density (11) is an improper density function corresponding to the limiting case where the prior information on η, σ is totally negligible. The joint prior density of the parameters β, η, σ is then

$$f'(\beta,\eta,\sigma) \propto \sigma^{-1}.\exp[-(\beta - b')^2/(2s'^2_b)]. \tag{12}$$

Note that the prior distribution (12) is not of the natural conjugate form (the bivariate normal-gamma distribution for the simple normal regression process). The reason why the natural conjugate density is not suitable here is that the conjugate prior expresses prior information in the form as if it were results of previous sampling from the same process, and it is not rich enough to give a good representation of the case when the prior information involves a cross-sectional relationship among several regression processes.

Given the prior density (12), the posterior density f'' of the parameters β, η, σ is evaluated using Bayes' theorem:

$$f''(\beta, \eta, \sigma | v, b, \bar{y}, s) \tag{13}$$
$$= f'(\beta, \eta, \sigma) k(b, \bar{y}, s | v, \beta, \eta, \sigma) N(b, \bar{y}, s)$$

where

$$N^{-1}(b, \bar{y}, s) = \int f'(\beta, \eta, \sigma) k(b, \bar{y}, s | v, \beta, \eta, \sigma) d\beta\, d\eta\, d\sigma.$$

The marginal posterior density of β is evaluated as

$$f''(\beta | v, b, \bar{y}, s) = \int f''(\beta, \eta, \sigma | v, b, \bar{y}, s) d\eta\, d\sigma.$$

After substitution, this yields

$$f''(\beta | v, b, \bar{y}, s) \propto \exp[-(\beta - b')^2/(2s'_b{}^2)] \cdot \left[T - 2 + \frac{v(\beta - b)^2}{s^2}\right]^{-\frac{1}{2}(T-1)}. \tag{14}$$

When T is larger than 20, the posterior distribution of β is approximately normal with mean b'' and variance $s''_b{}^2$, where

$$b'' = \frac{b'/s'_b{}^2 + b/s_b{}^2}{1/s'_b{}^2 + 1/s_b{}^2} \tag{15}$$

$$s''_b{}^2 = \frac{1}{1/s'_b{}^2 + 1/s_b{}^2}. \tag{16}$$

Here

$$s_b{}^2 = s^2/v$$

is the estimated variance of b as given by Equation (5). (In sampling-theory terminology, s_b is usually called the standard error of the estimate b.)

The marginal posterior density of β describes the knowledge about the distribution of the estimated parameter, given the information from the sample and the prior information. The choice of a point estimate of β depends on this posterior distribution as well as the utility function on the space of decisions (estimates). Under a quadratic terminal loss function (which is a Bayesian analogue to the sampling-theory concept of minimum variance estimates) the optimal estimate of β is the mean of the posterior distribution (14). For $T > 20$, the error of approximating the posterior mean by b'' does not exceed .01 and decreases approximately linearly with $1/T$. Since this error is small in comparison with the dispersion s''_b of the posterior distribution, no material loss is incurred when b'' is taken for the estimate that minimizes the expected quadratic opportunity loss.

III. Discussion and Conclusions

The Bayesian estimate b'' as given by Equation (15) can be interpreted as an adjustment of the sample estimate b toward the best prior estimate b', the degree of adjustment being proportionate to the precision $h = 1/s_b^2$, $h' = 1/s'^2_b$ of the sample estimate and the prior distribution, respectively. Equation (16) can be interpreted as stating that the precision $h'' = 1/s''^2_b$ of the posterior distribution is the sum of the precision of b and that of the prior distribution.

The choice of the parameters b', s'_b of the prior density $f'(\beta)$ depends on the prior information available. If nothing is known about a stock prior to sampling except that it comes from a certain population of stocks (for instance, from the population of all stocks traded on the New York Stock Exchange), an appropriate choice of the prior density is the cross-sectional distribution of betas observed for that population. For the New York Stock Exchange population, the prior parameters might be approximately $b' = 1$, $s'_b = .5$. In this case, the regression coefficient estimated from the sample is linearly adjusted toward unity, the degree of the adjustment depending upon the standard error s_b of the estimate.

A somewhat similar procedure is used in the Security Risk Evaluation service by Merrill Lynch, Pierce, Fenner & Smith, Inc. Their simplified method utilizes a formula of the form

$$b'' = 1 + k(b - 1) \tag{17}$$

where k is a constant common for all stocks. This constant can be interpreted as the slope of the cross-sectional regression of beta estimates on those obtained over a prior non-overlapping period. Comparison of Equation (17) with Equation (15) shows that this method assumes that the variance s_b^2 of the sample regression coefficient is the same for all securities. The effect of this procedure is thus to overadjust more accurate estimates and underadjust the less accurate ones.

In some cases, more can be known about a stock than that it comes from a certain population. Assume, for instance, that a stock is selected on the basis of an instrumental variable, which may be related to the true betas but not to the estimation error of the sample estimates b. In this case, a proper choice of the prior distribution is the distribution of betas implied by the knowledge of the instrumental variable. Thus, if a utility stock is considered, and t is known from previous measurements that betas of utilities are centered around .8 with a dispersion of .3, the estimate b is adjusted toward .8 by the formula (15) with $b' = .8$, $s'_b = .3$. In general, the degree and direction of the adjustment depend on the prior distribution $f'(\beta)$ as characterizing the information pertaining to β that is contained in the instrumental variable.

When estimating beta of a portfolio composed of N stocks, the sample estimate b is again adjusted through the formula (15). In this case, however, the value used for s'_b is the cross-sectional dispersion of betas of portfolios of size N.

In most instances, a good approximation for this dispersion is obtained by assuming cross-sectional independence of the regression residuals (as in the diagonal model) and consequently using the cross-sectional dispersion of individual securities' betas reduces by the factor of $1/\sqrt{N}$.

In some cases, the prior information may contain information of another sample from the same process (as, regression results over a previous period) but the two samples cannot be pooled. This situation arises, for example, when a portfolio is formed by ranking securities on the basis of their estimated betas and then the portfolio's beta is estimated over the next period. In such cases, the estimation proceeds in two steps. First, the posterior distribution based on the first sample and the cross-sectional prior is obtained. Next, this posterior distribution is used as the prior density to utilize the information of the second sample. Thus, the sample estimate from the second sample is adjusted toward the adjusted first sample estimate.

In summary, the estimate of a security's beta that minimizes the expected squared estimation error is given by Equation (15), where the parameters b', s'_b of the prior distribution are chosen to reflect *all* the information on beta available prior to sampling. The mean squared estimation error s''^2_b is given by Equation (16).

The relative merit of this Bayesian estimation method as contrasted to procedures of sampling theory will now be briefly discussed. The main objection to the Bayesian estimation method is that the estimate b'' is not an unbiased estimate of β (in the sampling-theory sense), while b is unbiased,

$$E(b''|\beta) \neq \beta,$$
$$E(b|\beta) = \beta. \tag{18}$$

To discuss this objection, it is useful to ask why unbiasedness in the sense of Equation (18) is desirable. One can identify two reasons, the first of which is that, in virtue of the law of large numbers, an unbiased estimate converges in probability to the estimated parameter as the sample size increases,

$$\operatorname*{Plim}_{T\to\infty} b = \beta.$$

The same, however, is true for the estimate b'',

$$\operatorname*{Plim}_{T\to\infty} b'' = \beta,$$

since with increasing sample size $s_b^2 \to 0$ and the degree of the adjustment decreases.

The second reason for requiring an unbiased estimate is that the mean quadratic error

$$E((\hat{\beta} - \beta)^2|\beta) \tag{19}$$

is minimized in a class of estimates $\hat{\beta}$ of the same variance by an unbiased estimate. The expected value (19) is taken with respect to the conditional likelihood (9) of the sample. This, however, is not justified. Rather than minimizing the squared *sampling* error, what should be done is to minimize the squared *estimation* error. That is, minimize

$$E''(\hat{\beta} - \beta)^2, \tag{20}$$

the expectation being taken with respect to the posterior distribution of β. The estimate b'', not b, is the estimate $\hat{\beta}$ to minimize (20).

This is more than a mere philosophical point. If two persons, one using the estimate b and the other b'', were penalized proportionally to the squared difference of their respective estimates from the true parameter value β (or, for that matter, from the next-period sample estimate), the former would go broke first.

In conclusion, Bayesian estimates (15) are preferred to the classical sampling-theory estimates (2) for the following reasons: First, Bayesian procedures provide estimates that minimize the loss due to misestimation, while sampling-theory estimates minimize the error of sampling. This is because Bayesian theory deals with the distribution of the parameters given the available information, while sampling theory deals with the properties of sample statistics given the true value of the parameters. Secondly, Bayesian theory weights the expected losses by a prior distribution of the parameters, thus incorporating knowledge which is available in addition to the sample information. This is particularly important in the case of estimating betas of stocks, where the prior information is usually sizeable.

REFERENCES

1. Marcus C. Bogue. "The Estimation and Behavior of Systematic Risk," unpublished dissertation, Graduate School of Business Administration, Stanford University (1972).
2. Basil A. Kalymon. "Estimation Risk in the Portfolio Selection Model," *Journal of Financial and Quantitative Analysis* (January 1971).
3. Michael Kantor. "Market Sensitivities," *Financial Analysts Journal* (January 1971).
4. John Lintner. "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," *Review of Economics and Statistics* (February 1965).
5. Howard Raiffa and Robert Schlaifer. *Applied Statistical Decision Theory* (Harvard University, Boston, 1961).
6. William F. Sharpe. "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk," *Journal of Finance* (September 1964).
7. Jack L. Treynor. "Toward A Theory of Market Value of Risky Assets," unpublished memorandum (1961).
8. Wayne H. Wagner and Oldrich A. Vasicek. "The Effect of Estimation Error of Beta on the Risk of Passive Portfolios," unpublished memorandum, Wells Fargo Bank (March 1971).

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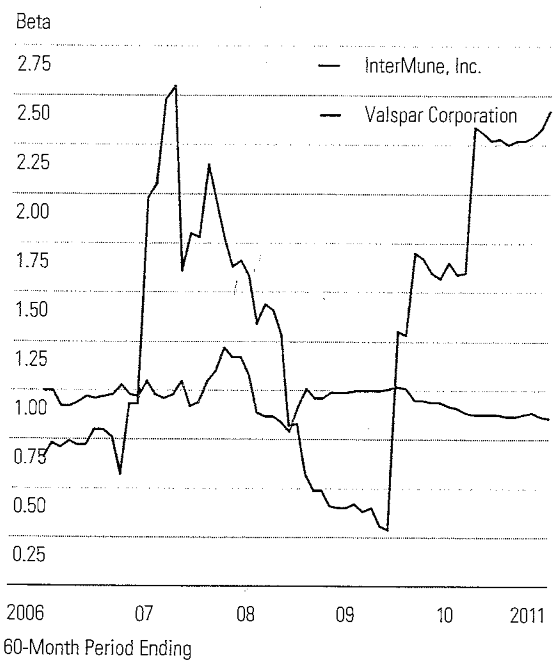
with a high standard error. An example of two companies is presented in Graph 6-3. The current beta of 0.86 for Valspar Corp. has a low standard error of 0.13. On the other hand, InterMune, Inc. has a current beta of 2.42 with a high standard error of 1.00. The chart shows the beta of each company on a rolling 60-month basis, meaning that the beta is calculated over the 60-month period October 31, 2001 through September 30, 2006, then the calculation is carried forward for each consecutive 60-month period through September 30, 2011. The betas of the company with the low standard error, Valspar Corp., exhibit remarkable stability when calculated over different time periods. On the other hand, the betas of the company with the high standard error, InterMune, Inc., display considerable variation depending on the period of the regression.

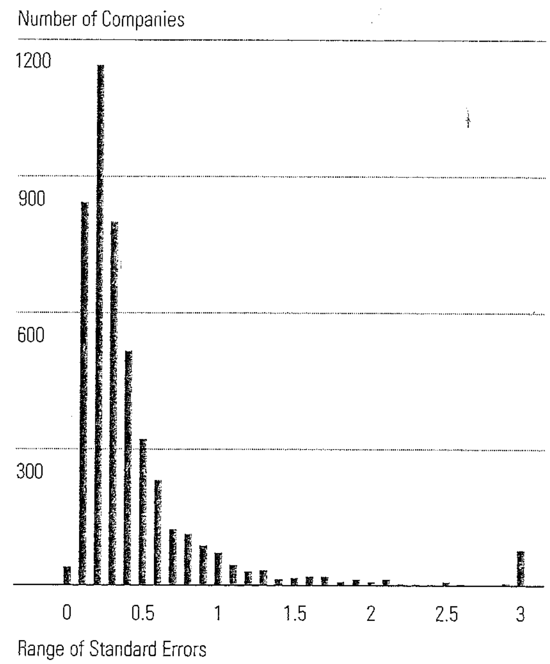
Graph 6-3: 60-Month Rolling Betas with High versus Low Standard Error



Data from October 2001–September 2011.
*Betas calculated using five years of monthly excess return data.

The standard error provides another statistical clue to help determine the reliability of the beta produced by the regression. To better illustrate the typical range of standard errors with respect to beta estimation, Graph 6-4 shows the distribution of standard error statistics across the population of companies in the *Ibbotson® Company Beta Reports*.

Graph 6-4: Standard Error Distribution



Data from October 2006–September 2011.

Beta Adjustment Methodologies

In calculating beta estimates for cost of capital projections, we are seeking a forward-looking or prospective beta. What we have measured using historical data in the beta regression is a historical beta. In this section we will examine two of the most common techniques used to adjust betas from historical to prospective or forward-looking.

Blume

One of the first academics to study whether historical betas are reliable estimates of future systematic risk was Marshall Blume.[2] What Blume found is that betas tend to revert toward their mean value, or the market beta of one. This means that high historical betas (those in excess of one) tend to over-estimate betas in future time periods, and low historical betas (those under one) tend to underestimate betas in future time periods.

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Blume's analysis regressed estimates of beta in one period against estimates in the previous period. By performing this analysis over different time periods, Blume was able to develop a convergence tendency that could be measured by the following formula:

$$\beta_1 = 0.371 + 0.635\,\beta_0$$

where:

β_1 = prospective beta; and
β_0 = historical beta.

The formula tells us that the forecast of next year's beta is equal to 0.635 times this year's historical beta plus 0.371.

Stated another way, betas will trend toward the market average of one (the market beta) times 0.371 plus 0.635 times the historical beta.

What are the practical implications of Blume's analysis? The Blume equation has the impact of lowering high historical betas and increasing low historical betas. A historical beta of 1.40 becomes an adjusted beta of 1.26 under the Blume methodology. Similarly, a historical beta of 0.80 becomes an adjusted beta of 0.88.

In short, Blume suggests that all betas using historical regression techniques should be adjusted in this fashion. The closer a historical beta is to 1.0, the less the magnitude of the adjustment. The Blume equation is often referred to as the 1/3 + 2/3 adjustment. When simplified, the adjustment procedure takes 1/3 plus 2/3(β_0). Use of this type of adjustment procedure is common and will be discussed further in the commercial beta section.

Vasicek

Vasicek has proposed another beta adjustment technique that considers the statistical accuracy of the beta calculation.[3] The Vasicek adjustment seeks to overcome one weakness of the Blume model by not applying the same adjustment to every security; rather, a security-specific adjustment is made depending on the statistical quality of the regression.

The Vasicek adjustment process focuses on the standard error of the beta estimate—the higher the standard error, the lower the statistical significance of the beta estimate. Therefore, a company beta with a high standard error should have a greater adjustment than a company beta with a low standard error. The Vasicek formula is as follows:

$$\beta_{s1} = \frac{\sigma^2_{\beta s0}}{\sigma^2_{\beta 0} + \sigma^2_{\beta s0}}\beta_0 + \frac{\sigma^2_{\beta 0}}{\sigma^2_{\beta 0} + \sigma^2_{\beta s0}}\beta_{s0}$$

where:

β_{s1} = the Vasicek adjusted beta for security **s**;
β_{s0} = the historical beta for security **s**;
β_0 = the beta of the market, industry, or peer group;
$\sigma^2_{\beta 0}$ = the variance of betas in the market, industry, or peer group; and
$\sigma^2_{\beta s0}$ = the square of the standard error of the historical beta for security **s**.

While the Vasicek formula looks intimidating, it is really quite simple. The adjusted beta for a company is a weighted average of the company's historical beta and the beta of the market, industry, or peer group. How much weight is given to the company and historical beta depends on the statistical significance of the company beta statistic. If a company beta has a low standard error, then it will have higher weighting in the Vasicek formula. If a company beta has a high standard error, then it will have lower weighting in the Vasicek formula. In all cases, the Vasicek weights will sum to one.

An advantage of this adjustment methodology is that it does not force an adjustment to the market as a whole. Instead, the adjustment can be toward an industry or some other peer group. This is most useful in looking at companies in industries that on average have high or low betas. If evaluating the beta for a company in the petroleum refining industry, which traditionally has had betas below one, it may be more desirable to adjust the beta of that company toward the industry average rather than toward the market average of one.

Because this method varies by company and allows for adjustment toward industry averages, we have selected the Vasicek adjustment technique for our beta calculations.

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Triumph of the Optimists

101 YEARS OF GLOBAL INVESTMENT RETURNS

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countries, over the whole of the 101-year period from 1900–2000. We also have century-long evidence on the small-firm and value/growth phenomena. We have put significant effort into compiling complete financial market histories, so that we can present consistent and comparable records for different countries. But *Triumph of the Optimists* is about much more than just data, since it has description and analysis at its core.

There is an obvious need for a reliable and truly international dataset for the investment industry as it continues relentlessly toward full market globalization. One of the many changes taking place in the investment business is the increasing demand for locally sourced research placed in a global context. Another innovation is the growing number of truly global mandates being given to fund managers. Globalization may be a cliché, but for portfolio managers it is fast becoming a reality. Access to a properly constituted and rigorously maintained international database is a sine qua non for the start of any investment process.

The period since spring 2000 has come as a shock to those who had become used to the bull market conditions of previous years. The bursting of the technology bubble, the rapid decline in economic growth rates, especially in the United States, and the advent of international terrorism raised questions about what we can expect for the future. We assert in this book that the single most important variable for making investment decisions is the equity risk premium, and we argue that high long-term returns on equities, relative to bonds, are unlikely to persist. Even after the setbacks of 2000–01, it is necessary to justify the relatively high rating of today's stock markets in terms of a historically low forward-looking equity risk premium. For the investment strategist this raises the most fundamental question of all: Do investors realize that returns are likely to revert to more normal levels, or do current valuations embody exaggerated expectations based on an imperfect understanding of history?

Good data is the key to understanding history. With this as our guiding principle, assembling the data for this book was a major task. For the United Kingdom, ABN AMRO supported us in compiling an authoritative record of UK equity market performance over the last 101 years. We did this because we were not satisfied with the data that previously existed, and there was anyway no comprehensive record of equity returns extending back to 1900. To construct our UK indexes, we devoted intensive efforts to financial archaeology. This involved transcribing original source data from dusty newspaper archives and ancient reference books into our database. A resulting benefit is that we have not simply assembled an index, but we also have the underlying stock-by-stock data, so we can now study the performance of segments of the market, such as industry sectors and market-capitalization bands. We also compiled a series of UK government bond indexes especially for this study.

For the other fifteen countries covered in this book, we have linked together the best quality indexes and returns data available from previous studies and other sources, a number of which are previously unpublished, and some of which are still work in progress. In addition to the United Kingdom, we cover two North American markets, the United States and Canada; ten other European markets, namely, Belgium, Denmark, France, Germany, Ireland, Italy, The Netherlands, Spain, Sweden and Switzerland; two Asia-Pacific markets, Australia

Chapter 3 Measuring long-term returns

Good measures of long-run returns should accurately reflect the outcome of an implementable investment strategy. The strategy should be one that could have been set up in advance, and followed in real life, and which is representative of the asset class and country in question. It is only too easy for researchers to fail to meet these criteria.

This chapter begins in section 3.1 by setting out the principles that need to be followed in constructing long-run return indexes. These provide a benchmark for assessing previous studies, and have been the guiding framework for this book. Given that our data goes back to the beginning of the last century and covers sixteen countries, we have not always been able to adhere to every principle, especially in the earliest years. Nevertheless, these standards have guided our choices, and we indicate where compromises have been necessary.

Next, in section 3.2 we take a closer look at equity index construction and at a bias that has afflicted some previous studies. When an index is compiled retrospectively, a crucial issue is how to avoid tilting its composition toward companies that, with hindsight, are known to have survived and/or to have been successful. In section 3.3, we review other issues that arise in index design, such as dividend reinvestment, index coverage, and index weighting.

In section 3.4 we consider how best to assemble a sample of international indexes. We show that reliance on data that is easy to acquire, such as indexes that start after the end of a war, tends to result in overstated performance. Both success bias and easy-data bias arise from a focus on assets that have survived or prospered over a particular period, and both can lead to overestimates of index returns and risk premia.

In section 3.5, we focus on the special problems that can arise when measuring inflation rates, as well as long-term returns on bonds, bills and currencies. We conclude in section 3.6 with a summary of the chapter.

3.1 Good indexes and bad

There are five guiding principles that underpin our measures of long-term performance. They are to avoid bias in index construction, to focus on total returns, to ensure the widest possible coverage within each market, to apply appropriate methods of weighting and averaging, and to maximize the extent to which comparisons can be made across national boundaries.

First, equity indexes should avoid bias. Good indexes follow an investment strategy that could be followed in real life. Apart from dealing costs, an investor should in principle have been able to replicate index performance. Indexes, especially when they are constructed retrospectively, must therefore be free of any look-ahead bias. They must be constructed solely from information that would have been available at the time of investment. Serious bias can arise if index constituents are tilted toward companies that subsequently survived or

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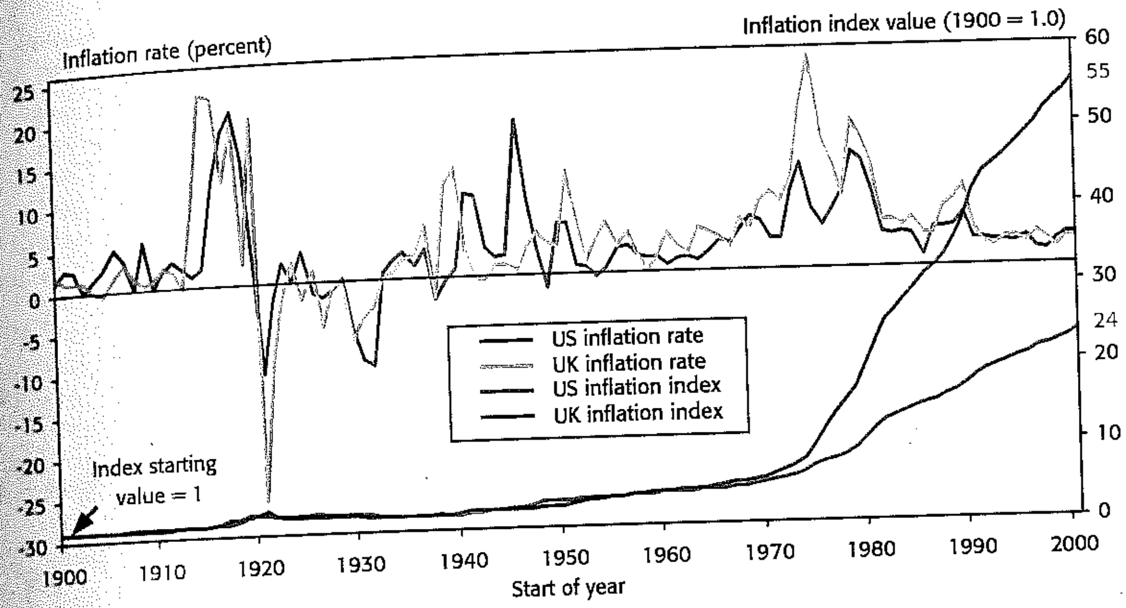
Figure 5-2: US and UK annual inflation rates and cumulative inflation, 1900–2000



much higher than in the United States, peaking at 25 percent in 1975. The cumulative impact of these higher rates can be seen by comparing the two inflation indexes, which are plotted against the right-hand axis in Figure 5-2. The red line for the United Kingdom and the blue line for the United States are virtually coincident until the mid-1960s. From that point onward, the UK index rises to a value of fifty-five by end-2000, compared with twenty-four for the United States. From 1900–2000, UK consumer prices thus rose 55-fold, a factor of 2.3 times more than in the United States, with the difference almost entirely attributable to the last thirty-five years. Over the full 101-year period, the average annualized UK inflation rate was 4.1 percent per year, versus 3.2 percent for the United States.

5.2 Inflation around the world

While inflation was fairly similar in the United States and the United Kingdom, a number of other countries had quite different experiences. Table 5-1 provides international inflation rate comparisons across all sixteen countries covered in this book, showing the mean inflation rates from 1900–2000. Clearly, there were large differences between countries. At the same time, the standard deviations for each country show that there was also considerable variation in inflation rates over time. Taking the full 101-year period as a whole, there have been four high inflation rate countries, France, Germany, Italy, and Japan; two runner-ups, Belgium and Spain; and one low inflation country, Switzerland. The other countries fall in between, with inflation rates in the region of 3–4 percent per year. Note that the true 101-year means and standard deviations for inflation in Germany are much higher than shown in Table 5-1 since the statistics in the table omit the hyper-inflationary years of 1922–23.

and some of the studies span periods as brief as five years. These differences in research periods, methodologies, and definitions of "smallness" mean that the premia shown in Figure 9-5 are not directly comparable. In particular, it would be unwise to make inferences about the magnitudes or significance of any apparent size premium differences between countries.

In spite of this caveat, Figure 9-5 paints a very clear picture, namely, that the size premium was not restricted to the United States but was present in almost every country studied by the researchers. The sole exception was Korea, where a negative premium was reported, although this study used just five years of data. Furthermore, in most countries, researchers also looked at risk differences. They concluded, like Banz (1981), that the size premium could not be explained away by risk.

The pervasiveness and magnitude of the size effect, and the striking outperformance of smaller companies in most countries around the world, meant that the size effect rapidly became recognized as the premier stock market anomaly.

9.4 The reversal of the size premium

The "discovery" of the size effect in the United States by Banz (1981) and Reinganum (1981), and the publication and dissemination of their research, led to considerable interest in small-caps among investors in the United States. This spurred the launch of significant new small-cap investment vehicles led by Dimensional Fund Advisors, who raised several billion dollars within a couple of years of their 1981 launch. This honeymoon period lasted for approximately two years, until the end of 1983, and during this period, US small-caps continued to outperform. But subsequently, and over much of the period since, US small-caps have underperformed.

The UK experience was remarkably similar. When the HGSC was launched in 1987, its back-history showed that smaller companies had outperformed the UK market by 5.2 percent per year. This dramatic outperformance attracted substantial media attention, and there were over two hundred follow-up articles in the UK press. By the end of 1988, at least thirty open- and closed-end funds had been launched to exploit the perceived outperformance of small-caps, and numerous investment institutions developed a strategy of investing in smaller companies as a distinct asset class. Again, the honeymoon lasted just two years. In the decade that followed, smaller companies were to underperform by a large margin.

This reversal in the fortunes of US and UK small-cap stocks led us to write an article in 1999 entitled "Murphy's Law and Market Anomalies." Murphy's Law is often summarized as "bread always falls with the buttered side down." Figure 9-6 shows the performance record of US and UK small-caps at the time of our article, and shows why this appeared like a classic case of Murphy's Law. The left-hand side of Figure 9-6 shows the historical small- and micro-cap premia for the United States and the United Kingdom from the start date of the original research studies until the end of the post publication honeymoon period (i.e., 1926–

Our subsequent research has shown that the small-cap reversal extended beyond the United Kingdom and United States, and was a worldwide phenomenon. The line of investigation we followed here was to revisit all of the research studies that have been conducted into the size effect in different countries, and to estimate the size premium over the years since the research was published. These studies were discussed earlier in section 9.3 and their findings were summarized in Figure 9-5. We found that they showed evidence of a significant size premium in every country examined, with the sole exception of Korea, where the research covered just a five-year period. Most of these research studies were published in the 1980s.

To update these studies, we estimated the size premium in each country over the period since each study was published. For consistency, we again measured the size premium as the difference between the average monthly returns on the smallest and the largest stocks. For the United States, we use the CRSP NYSE Decile 10 and Decile 1 returns as our respective measures of small and large stock returns, as this most closely approximates Banz's (1981) earlier research, and gives results close to his over his earlier period. Similarly, for the United Kingdom we adopt the same definition as was used in Figure 9-5, namely, the difference between HGSC returns and overall UK equity returns.

For all other countries, we use the size-based indexes published by either Independence International Associates (IIA) or by FTSE International. IIA publish large- and small-cap indexes for a number of countries starting in 1975. They define small as the bottom 30 percent by capitalization of their universe, and large as the top 70 percent. FTSE publish a similar set of large and medium-small-cap indexes for a larger population of countries, but only from 1987, with some countries starting even later. FTSE define medium/small-cap as the bottom 25 percent by capitalization, and large-cap as the balancing 75 percent. For countries where we had a choice between both IIA and FTSE Indexes, we have used the IIA series since they provide a longer time series and generally have somewhat wider coverage.

The results of our research are shown in Figure 9-7. Countries are listed in alphabetical order, and for each country, the size premium reported by the original research studies and plotted earlier in Figure 9-5 in shown in green. Alongside this, the yellow bar shows the size premium calculated over the period since the original research was published, that is, over the period starting at the beginning of the year immediately following publication and ending at New Year 2001. No size-based indexes were available for Korea or Taiwan, so we omitted these countries. We have, however, included the four countries covered in this book, but which did not feature in Figure 9-5 due to the absence of any research study on the size premium. For these countries, we have omitted the "initial research" bars in Figure 9-7, while the "subsequent period" bars show the size premium over the period from 1990–2000.

It is clear from Figure 9-7 that there was a global reversal of the size effect in virtually every country, with the size premium not just disappearing but going into reverse. Researchers around the world universally fell victim to Murphy's Law, with the very effect they were documenting—and inventing explanations for—promptly reversing itself shortly after their studies were published. The only country experiencing a size premium, as opposed to a size discount, in the period subsequent to the original research was Switzerland. However, the Swiss size premium was statistically insignificant, and its magnitude was just 0.05 percent.

Table 12-2: Worldwide equity risk premia relative to long bond returns, 1900–2000

	Annual equity risk premium relative to long-term bonds						Ten year risk premium		
Country	Geometric mean	Arithmetic mean	Standard error	Standard deviation	Minimum premium	Maximum premium	Geometric mean	Arithmetic mean	Standard deviation
Australia	6.3	8.0	1.9	18.9	-30.6	66.3	6.3	6.4	4.6
Belgium	2.9	4.8	2.1	20.7	-35.1	76.6	3.0	3.2	5.1
Canada	4.5	6.0	1.8	17.8	-36.8	54.7	4.6	4.7	5.4
Denmark	2.0	3.3	1.7	16.9	-35.9	74.9	1.8	1.9	4.1
France	4.9	7.0	2.1	21.6	-32.7	83.7	4.9	5.1	6.8
Germany*	6.7	9.9	2.9	28.4	-38.6	117.6	8.2	8.5	9.1
Ireland	3.2	4.6	1.7	17.4	-37.0	73.3	3.0	3.2	4.8
Italy	5.0	8.4	3.0	30.0	-39.6	152.2	5.0	5.4	9.2
Japan	6.2	10.3	3.3	33.2	-43.3	193.0	6.7	7.2	11.5
The Netherlands	4.7	6.7	2.1	21.4	-43.9	107.6	4.3	4.5	6.5
South Africa	5.4	7.1	2.0	19.7	-29.2	70.9	6.2	6.3	5.0
Spain	2.3	4.2	2.0	20.3	-34.0	69.1	2.2	2.3	5.5
Sweden	5.2	7.4	2.2	22.1	-38.3	87.8	4.8	5.0	7.7
Switzerland†	2.7	4.2	1.9	17.9	-34.4	52.2	2.0	2.1	5.1
United Kingdom	4.4	5.6	1.7	16.7	-38.0	80.8	4.8	4.9	4.5
United States	5.0	7.0	2.0	20.0	-40.8	57.7	4.9	5.0	5.2
World	4.6	5.6	1.4	14.5	-31.2	37.4	4.6	4.7	4.8

* All statistics for Germany exclude 1922–23. † Premia for Switzerland are from 1911.

In this table, the first six columns give summary statistics for the annual premia, while the last three columns relate to rolling ten-year premia. The first column shows the geometric means that were plotted as bars in Figure 12-6. The fourth column shows the standard deviations. The 20.0 percent figure for the United States is close to the 19.6 percent standard deviation for the premia relative to bills shown earlier in Table 12-1. For some countries, however, the distribution of premia relative to bonds is narrower than relative to bills. For the United Kingdom, for example, the standard deviation is 16.7 percent, compared with 19.9 percent relative to bills. This is because, in the United Kingdom, there was a fairly high correlation between annual equity returns and long bond returns (0.56), while the correlation between equities and bills was lower (0.29). This propensity for good bond years to coincide with good equity years, and vice versa, has tended to lower the annual difference between equity and bond returns in the United Kingdom. This was particularly marked in the best and worst years on record for UK equities, namely, 1975 and 1974 respectively.

12.5 Summary

In this chapter, we have used 101 years of stock market history for sixteen different countries and for the world index to take a fresh look at the equity risk premium. In the past, the historical evidence for the US market, and to a lesser extent for the United Kingdom, has heavily influenced views about the magnitude of the risk premium. For the United States, the most widely cited source is Ibbotson Associates (2000), who estimate a geometric risk premium of

The chapter addresses four questions: Which historical equity risk premium should one use as the starting point? Why has it typically been so high? What is a good forward-looking predictor for the future? How can one use variables such as the dividend yield to improve forecasts of the risk premium?

We stress the central role in finance of the equity premium. The historical premium is often summarized in the form of an annualized rate of return. This is a geometric mean. It provides information on past performance. For the future, what is required is the arithmetic mean of the distribution of equity premia, which is larger than the geometric mean. For markets that have been particularly volatile, the arithmetic mean of past equity premia may exceed the geometric mean premium by several percentage points. We adjust the arithmetic mean for (i) the differences between the variability of the stock market over the last 101 years, and the variability that we might anticipate today, and (ii) the impact of unanticipated cash flows and of declines in the required risk premium. The result is a forward-looking, *geometric* mean risk premium for the United States, United Kingdom and world of around 2½ to 4 percent and an *arithmetic* mean risk premium for US, UK, and world equities that falls within a range from a little below 4 to a little above 5 percent.

These equity risk premia are lower than those cited in surveys of finance academics. They are also lower than frequently quoted historical averages, such as those from Ibbotson Associates, which cover a somewhat briefer interval. We show that the historical risk premium, even if it embraces countries that have been less successful than the United States, is supported by two factors. Over the second half of the last century, equity cash flows almost certainly exceeded expectations, and the required rate of return doubtless fell as investment risk declined and the scope for diversification increased. Stock markets rose, in both the United States and other countries, for reasons that are unlikely to be repeated. Even after the setbacks of 2001, the prospective risk premium is markedly lower than the historical risk premium.

CAPITAL EQUIPMENT ANALYSIS: THE REQUIRED RATE OF PROFIT

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The interest in capital equipment analysis that has been evident in the business literature of the past five years is the product of numerous social, economic, and business developments of the postwar period. No conclusive listing of these developments can be attempted here. However, four should be mentioned which are of particular importance in this search for a more systematic method for discovering, evaluating, and selecting investment opportunities. These are: (1) the high level of capital outlays (in absolute terms); (2) the growth in the size of business firms; (3) the delegation of responsibility for initiating recommendations from top management to the profit center, which has been part of the general movement toward decentralization; and (4) the growing use of "scientific management" in the operations of the business firm.

These developments have motivated the current attempt to develop objective criteria whereby the executive committee in a decentralized firm can arrive at a capital budget. Since each of its profit centers submits capital proposals, the executive committee must screen these and establish an allocation and a level of capital outlays that is consistent with top management's criteria for rationing the firm's funds. Capital budgeting affords the promise that this screening process can be made amenable to some established criteria that are understandable to all the component parts of the firm. Consequently, capital budgeting appeals to top management, for, in the first place, each plant manager can see his proposal in the light of all competing proposals for the funds of the enterprise. This may not completely eliminate irritation among the various parts of the firm, but a rational capital budgeting program can go a long way toward maintaining initiative on the part of a plant manager, even though the executive committee may veto one or all of his proposals. In the second place, the use of a capital budgeting program serves to satisfy top management that each accepted proposal meets adequate predetermined standards and that the budget as a whole is part of a sound, long-run plan for the firm.

What specifically does a capital budgeting program entail? The focal points of capital budgeting are: (1) ascertaining the profit abilities of the array of capital outlay alternatives, and (2) determining the least profitability required to make an investment, i.e., a cut-off point. Capital budgeting also involves administrative procedures and organization designed to discover investment opportunities, process information, and carry out the budget; however, these latter aspects of the subject have been discussed in detail by means of case studies that have appeared in publications of the American Management Association and the

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National Industrial Conference Board and in periodicals such as the *N.A.C.A. Bulletin*.[1] Hence, we will not concern ourselves with them here.

There are at least four methods for establishing an order-preference array of the capital expenditure suggestions. They are: (1) the still popular "payoff period"; (2) the average investment formula; (3) the present value formula with the rate of interest given; and (4) the present value formula used to find the rate of profit. It is not our intention in this paper to discuss these various methods specifically, since critical analyses of these alternatives are to be found in papers by Dean, by Lorie and Savage, and by Gordon in a recent issue of the *Journal of Business*,[2] which is devoted exclusively to the subject of capital budgeting.

However, it is of interest to note that in each of these methods the future revenue streams generated by the proposed outlays must be amenable to measurement if the method is to be operational. However, improvements in quality, more pleasant working conditions, strategic advantages of integration, and other types of benefits from a capital outlay are still recognized only in qualitative terms, and there is a considerable hiatus in the literature of capital budgeting with respect to the solution of this problem. Hence, in the absence of satisfactory methods for quantifying these types of benefits, the evaluation of alternative proposals is still characterized by intuitive judgments on the part of management, and a general quantitative solution to the capital budgeting problem is not now feasible. It appears to us that this problem affords one of the most promising opportunities for the application of the methods of management science. In fact, we anticipate that techniques for the quantification of the more important factors now treated qualitatively will soon be found.

Given the rate of profit on each capital outlay proposal, the size of the budget and its allocation are automatically determined with the establishment of the rate of profit required for the inclusion of a proposal in the budget. In the balance of this paper, a method for determining this quantity is proposed and its use in capital budgeting is analyzed.

II

We state that the objective of a firm is the maximization of the value of the stockholders' equity. While there may be legitimate differences of opinion as to whether this is the sole motivation of management, we certainly feel that there can be no quarrel with the statement that it is a dominant variable in manage-

[1] American Management Association, *Tested Approaches to Capital Equipment Replacement*, Special Report No. 1, 1954; American Management Association, *Capital Equipment Replacement*; *AMA Special Conference*, May 3–4, 1954 (New York, 1954, American Management Association, 105 pp.); J. H. Watson, III, National Industrial Conference Board, *Controlling Capital Expenditures*, Studies in Business Policy, No. 62, April, 1953; C. I. Fellers, "Problems of Capital Expenditure Budgeting", *N.A.C.A. Bulletin*, 26 (May, 1955), 918–24; E. N. Martin, "Equipment Replacement Policy and Application", *N.A.C.A. Bulletin*, 35 (February, 1954), 715–30.

[2] *Journal of Business*, Vol. XXVIII, No. 3 (October, 1955).

ment's decisions. It has been shown by Lutz and Lutz in their *Theory of the Investment of the Firm*[3] and by others[4] that this objective is realized in capital budgeting when the budget is set so as to equate the marginal return on investment with the rate of return at which the corporation's stock is selling in the market. The logic and operation of this criterion will be discussed later. Now, we only wish to note the role assigned in capital budgeting to the rate of profit that is required by the market.

At the present time, the dividend yield (the current dividend divided by the price) and the earnings yield (the current income per share divided by the price) are used to measure the rate of profit at which a share is selling. However, both these yields fail to recognize that a share's payments can be expected to grow, and the earnings yield fails to recognize that the corporation's earnings per share are not the payments made to the stockholder.

The practical significance of these failures is evidenced by the qualifications with which these two rate-of-profit measures are used by investment analysts. In the comparative analysis of common stocks for the purpose of arriving at buy or sell recommendations, the conclusions indicated by the dividend and/or the earnings yield are invariably qualified by the presence or absence of the prospect of growth. If it is necessary to qualify a share's yield as a measure of the rate of profit one might expect to earn by buying the share, then it must follow that current yield, whether income or dividend, is inadequate for the purposes of capital budgeting, which is also concerned with the future. In short, it appears to us that the prospective growth in a share's revenue stream should be reflected in a measure of the rate of profit at which the share is selling. Otherwise, its usefulness as the required rate of profit in capital budgeting is questionable.

In his *Theory of Investment Value*[5], a classic on the subject, J. B. Williams tackled this problem of growth. However, the models he developed were arbitrary and complicated so that the problem of growth remained among the phenomena dealt with qualitatively. It is our belief that the following proposal for a definition of the rate of profit that takes cognizance of prospective growth has merit.

The accepted definition of the rate of profit on an asset is the rate of discount that equates the asset's expected future payments with its price. Let P_0 = a share's price at $t = 0$, let D_t = the dividend expected at time t, and let k = the rate of profit. Then, the rate of profit on a share of stock is the value of k that satisfies

$$P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+k)^t}. \tag{1}$$

[3] Friedrich and Vera Lutz, *The Theory of Investment of the Firm* (Princeton, N. J., 1951, Princeton University Press, 253 pp.), 41–43.

[4] Joel Dean, *Capital Budgeting: Top Management Policy on Plant, Equipment, and Product Development* (New York, 1951, Columbia University Press, 174 pp.); Roland P. Soule, "Trends in the Cost of Capital", *Harvard Business Review*, 31 (March, April, 1953), 33–47.

[5] J. B. Williams, *The Theory of Investment Value*, (Cambridge, Massachusetts, 1938, Harvard University Press), 87–96.

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It is mathematically convenient to assume that the dividend is paid and discounted continuously at the annual rates D_t and k, in which case

$$P_0 = \int_0^\infty D_t e^{-kt}\, dt. \tag{2}$$

Since P_0 is known, estimating the rate of profit at which a share of stock is selling requires the determination of D_t, $t = 1, 2, \cdots, \infty$.

At the outset it should be made clear that our objective is not to find the rate of profit that *will actually be earned* by buying a share of stock. This requires knowledge of the dividends that will be paid in the future, the price at which the share will be sold, and when it will be sold. Unfortunately, such information is not available to us. The rate of profit of interest here is a relation between the present known price and the *expected future dividends*. The latter will vary among individuals with the information they have on a host of variables and with their personality. Therefore, by expected future dividends we mean an estimate that (1) is derivable from known data in an objective manner, (2) is derived by methods that appear reasonable, i.e., not in conflict with common sense knowledge of corporation financial behavior, and (3) can be used to arrive at a manageable measure of the rate of profit implicit in the expectation.

We arrive at D_t by means of two assumptions. One, a corporation is expected to retain a fraction b of its income after taxes; and two, a corporation is expected to earn a return of r on the book value of its common equity. Let Y_t equal a corporation's income per share of common after taxes at time t. Then the expected dividend at time t is

$$D_t = (1 - b)Y_t \tag{3}$$

The income per share at time t is the income at $(t - 1)$ plus r percent of the income at $(t - 1)$ retained, or

$$Y_t = Y_{t-1} + rbY_{t-1} \tag{4}$$

Equation (4) is simply a compound interest expression so that, if Y_t grows continuously at the rate $g = br$,

$$Y_t = Y_0 e^{gt}. \tag{5}$$

From Equations (3) and (5)

$$D_t = D_0 e^{gt}. \tag{6}$$

Substituting this expression for D_t in Equation (2) and integrating, yields

$$\begin{aligned} P_0 &= \int_0^\infty D_0 e^{gt}\, e^{-kt}\, dt \\ &= D_0 \int_0^\infty e^{-t(k-g)}\, dt \qquad (7) \\ &= \frac{D_0}{k - g}. \end{aligned}$$

The condition for a solution is $k > g$, a condition that is easily satisfied, for otherwise, P_0 would be infinite or negative.

Solving Equation (7) for k we find that

$$k = \frac{D_0}{P_0} + g. \tag{8}$$

Translated, this means that the rate of profit at which a share of common stock is selling is equal to the current dividend, divided by the current price (the dividend yield), plus the rate at which the dividend is expected to grow. Since there are other possible empirical definitions of the market rate of profit on a share of stock, we will refer to k as the growth rate of profit.

III

Let us now review and evaluate the rationale of the model we have just established. Estimating the rate of profit on a share of stock involves estimating the future dividend stream that it provides, and the fundamental difference between this model and the dividend yield is the assumption of growth. The latter, as can be seen, assumes that the dividend will remain constant. Since growth is generally recognized as a factor in the value of a share and since it is used to explain differences in dividend yield among shares, its explicit recognition appears desirable. Future dividends are uncertain, but the problem cannot be avoided by ignoring it. To assume a constant rate of growth and estimate it to be equal to the current rate appears to be a better alternative.

Under this model the dividend will grow at the rate br, which is the product of the fraction of income retained and the rate of return earned on net worth. It is mathematically true that the dividend will grow at this rate if the corporation retains b and earns r. While we can be most certain that the dividend will not grow uniformly and continuously at some rate, unless we believe that an alternative method for estimating the future dividend stream is superior, the restriction of the model to the assumption that it will grow uniformly at some rate is no handicap. Furthermore, the future is discounted; hence, an error in the estimated dividend for a year in the distant future results in a considerably smaller error in k than an error in estimating the dividend in a near year.

It should be noted that this measure of the rate of profit is suspect, when *both* income and dividend are zero, and it may also be questioned when either falls to very low (or negative) values. In such cases, the model yields a lower rate of profit than one might believe that the market requires on a corporation in such difficulties. It is evident that the dividend and the income yields are even more suspect under these conditions and, hence, are subject to the same limitations.

There are other approaches to the estimation of future dividends than the extrapolation of the current dividend on the basis of the growth rate implicit in b and r. In particular, one can arrive at g directly by taking some average of the past rate of growth in a corporation's dividend. Whether or not this or some other measure of the expected future dividends is superior to the one presented earlier will depend on their relative usefulness in such purposes as the analysis of variation in prices among shares and the preferences of those who want an objective measure of a share's rate of profit.

So far, we have compared the growth rate of profit with the income and dividend yields on theoretical grounds. Let us now consider how they differ in practice, using the same measurement rules for the variables in each case. The numerical difference between the growth rate of profit and the dividend yield is simply the growth rate. However, the income yield, which is the measure of the rate of profit commonly recommended for capital budgeting, differs from the growth rate of profit in a more complex manner, and to establish this difference we first note that

$$b = \frac{Y - D}{Y} \text{ and } r = \frac{Y}{B} \tag{9}$$

where B = the net worth or book value per share. The growth rate of profit, therefore, may be written as

$$k = \frac{D}{P} + br = \frac{D}{P} + \frac{Y - D}{B}. \tag{10}$$

Next, the income yield can be decomposed as follows:

$$y = \frac{Y}{P} = \frac{D}{P} + \frac{Y - D}{P}. \tag{11}$$

We see then that y and k will be equal when book and market values are equal. It can be argued that the income yield overstates a share's payment stream by assuming that each payment is equal to the income per share and understates the payment stream by assuming that it will not grow. Hence, in this special case where book and market values are equal, the two errors exactly compensate each other.

Commonly market and book values differ, and y will be above k when market is below book, and it will be below k when market is above book. Hence, a share of IBM, for example, that is priced far above book had had an earnings yield of two to three percent in 1955. We know that the market requires a higher rate of profit on a common stock, even on IBM, and its growth rate of profit, k, is more in accord with the value suggested by common sense. Conversely, when U. S. Steel was selling at one-half of book value in 1950, the high income yield grossly overstated the rate of profit that the market was, in fact, requiring on the stock.

Furthermore, the growth rate of profit will fluctuate in a narrower range than the earnings yield. For instance, during the last few years, income, dividends, and book value have gone up more or less together, but market price has gone up at a considerably higher rate. Consequently, the growth rate of profit, dependent in part on book value, has fallen less than the earnings yield. Conversely, in a declining market k would rise less rapidly than y.

There is a widespread feeling that many accounting figures, particularly book value per share, are insensitive to the realities of the world, and some may feel

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that the comparative stability of k is merely a consequence of the limitations of accounting data. This is not true! The behavior of k is not a consequence of the supposed lack of realism in accounting data. Rather, book value appears in the model because it, and not market value, is used to measure the rate of return the corporation earns on investment, which, we have seen, is the rate of return that enters into the determination of the rate at which the dividend will grow. The comparative stability of k follows from the simple fact that, when a revenue stream is expected to grow, a change in the required rate of profit will give rise to a more than proportional change in the asset's price. Conversely, a change in the price reflects a less than proportional change in the rate of profit.

IV

Given the rate of profit expected on each item in the schedule of available investment opportunities and given the rate of profit at which the corporation's stock is selling, what should the capital budget be? As stated earlier, the accepted theory is that the budget should be set so as to equate the marginal return on investment with the rate of profit at which the stock is selling. The reasoning is, if the market requires, let us say, a 10 percent return on investment in the corporation's stock, and if the corporation can earn 15 percent on additional investment, obtaining the funds and making the investment will increase the earnings per share. As the earnings and the dividend per share increase or as the market becomes persuaded that they will increase, the price of the stock will rise. The objective, it will be recalled, is the maximization of the value of the stockholder's equity.

The conclusion drawn implicitly assumes that the corporation can sell additional shares at or above the prevailing market, or if a new issue depresses the market, the fall will be slight, and the price will soon rise above the previous level. However, some other consideration may argue against a new stock issue; for example, the management may be concerned with dilution of control, or the costs of floating a new issue may be very high, or a new issue may be expected to depress the price severely and indefinitely for reasons not recognized in the theory. Hence, it does not automatically follow that a new issue should be floated when a firm's demand for funds exceeds, according to the above criterion, those that are internally available.

In determining whether the required rate of profit is above or below r', the marginal return on investment, one can use y, the earnings yield, or k, the growth rate of profit as the required rate of profit. If y and k differ and if the reasoning in support of k presented earlier is valid, using y to estimate the direction in which a new issue will change the price of the stock may result in a wrong conclusion.

In arriving at the optimum size of a stock issue, the objective is to equate r' and y or k, depending on which is used. Internal data may be used to estimate the marginal efficiency of capital schedule. If the required rate of profit is considered a constant, its definition, $y = Y/P$ or $k = D/P + br$, provides its value. However, the required rate of profit may vary with the size of the stock issue or with the variables that may change as a consequence of the issue. In this event,

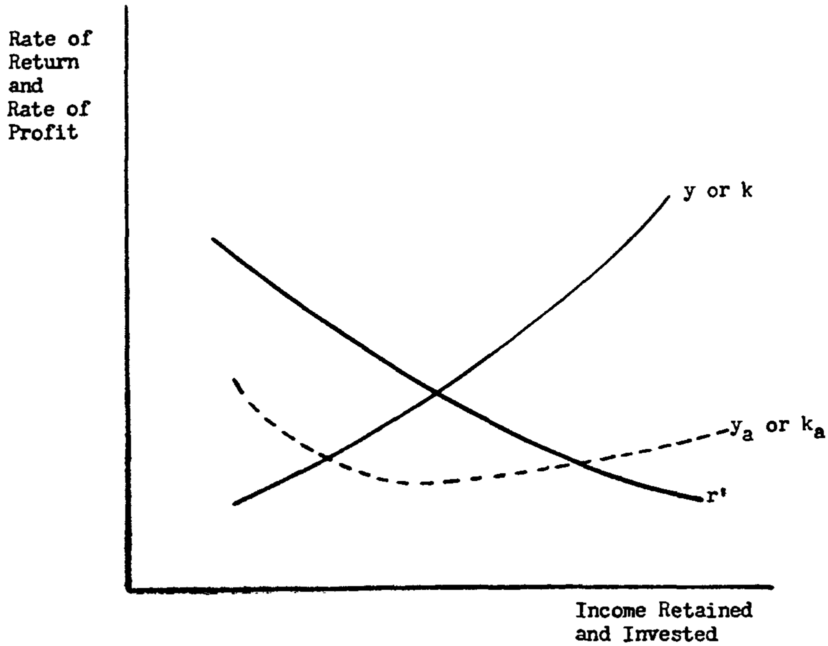

FIG. 1

finding the optimum size of a stock issue requires a model that predicts the variation in the required rate of profit with the relevant variables.

Borrowing is an alternative source of funds for investment. However, an analysis of this alternative requires the measurement of both (1) the variation in risk with debt, and (2) the difference between the rate of profit and the rate of interest needed to cover a given increase in risk. This has not been done as yet, which may explain the widespread practice of arbitrarily establishing a "satisfactory" financial structure and only borrowing to the extent allowed by it.

It has been stated by Dean[6] and Terborgh[7] that the long-term ceiling on a firm's capital outlays is the amount of its internally available funds. However, the share of its income a corporation retains is not beyond the control of its management; and, among the things we want from a capital budgeting model is guidance on whether the share of a corporation's income that is retained for investment should be raised or lowered.

Proceeding along traditional lines, the problem may be posed as follows. A firm estimates its earnings and depreciation allowances for the coming year and deducts the planned dividend to arrive at a preliminary figure for the capital budget. The marginal rate of return on investment in excess of this amount may be above or below the required rate of profit. We infer from theory that the two rates should be equated by (1) raising the budget and reducing the dividend

[6] Dean, *op. cit.*, 53–55.

[7] George Willard Terborgh, *Dynamic Equipment Policy* (New York, 1949, McGraw-Hill, 290 pp.), 228–29.

when the marginal return on investment is above the required rate of return, and (2) raising the dividend and reducing the budget when the reverse holds. The conditions under which this process yields an equilibrium are illustrated in Figure 1. The marginal return on investment, r', should fall as the budget is increased, and the required rate of profit, y or k, should increase or it should fall at a lower rate than r'. The latter case is illustrated by the line y_a or k_a.

Changing the dividend so as to equate r' and say y should maximize the price of the stock. For instance, if r' is above y, the company can earn a higher return on investment than stockholders require, and a dollar used this way is worth more to the stockholders than the dollar distributed in dividends. In other words, the price should go up by more than the income retained.

There are, of course, a number of problems connected with the use of this model for arriving at the optimum dividend rate. First, there is the question whether y or k should be used to measure the required rate of profit. Second, there is no question that the required rate of profit varies with the dividend rate. Hence, the current rate of profit given by the definition does not tell what profit rate will be required with a different dividend rate. This requires a model which predicts the variation in y or k with the dividend rate and other variables. Third, there is a very nasty problem of the short and the long run. It is widely believed, though the evidence has limitations, that the price of a share of stock varies with the dividend rate, in which case a corporation should distribute all of its income. However, it is quite possible that a change in the dividend gives rise to the expectation that earnings and future dividends are changing in the same direction. Further, in the short run, the market is not likely to be informed on a firm's marginal efficiency of capital schedule. For these and other reasons, it is likely that the dividend rate should not be made to vary with short-run changes in the marginal efficiency of capital, and more sophisticated methods than those now in use are needed to establish the variation in price or required rate of profit with the dividend rate.

V

The major points developed in this paper may be summarized as follows. We presented a definition of the rate of profit required by the market on a share of common stock, and we noted some of its advantages. It is theoretically superior to the income and dividend yields because it recognizes that the revenue stream provided by a share can be expected to grow. Furthermore, its empirical characteristics are also superior to those of the income and dividend yields since its value is generally in closer agreement with common sense notions concerning the prevailing rate of profit on a share of stock and since its value fluctuates in a narrower range over time. We next examined some of the problems involved in using this definition of the rate of profit and the earnings yield in capital budgeting models. Finally, we saw that, before capital budgeting theory can be made a reliable guide to action, we must improve our techniques for estimating the future revenue on a capital outlay proposal, and we must learn a good deal more about how the rate of profit the market requires on a share of stock varies with the dividend, the growth rate, and other variables that may influence it.

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Equity Risk Premiums (ERP): Determinants, Estimation and Implications – The 2015 Edition

Updated: March 2015

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Electronic copy available at: http://ssrn.com/abstract=2581517

Equity Risk Premiums (ERP): Determinants, Estimation and Implications – The 2015 Edition

Equity risk premiums are a central component of every risk and return model in finance and are a key input in estimating costs of equity and capital in both corporate finance and valuation. Given their importance, it is surprising how haphazard the estimation of equity risk premiums remains in practice. We begin this paper by looking at the economic determinants of equity risk premiums, including investor risk aversion, information uncertainty and perceptions of macroeconomic risk. In the standard approach to estimating the equity risk premium, historical returns are used, with the difference in annual returns on stocks versus bonds over a long time period comprising the expected risk premium. We note the limitations of this approach, even in markets like the United States, which have long periods of historical data available, and its complete failure in emerging markets, where the historical data tends to be limited and volatile. We look at two other approaches to estimating equity risk premiums – the survey approach, where investors and managers are asked to assess the risk premium and the implied approach, where a forward-looking estimate of the premium is estimated using either current equity prices or risk premiums in non-equity markets. In the next section, we look at the relationship between the equity risk premium and risk premiums in the bond market (default spreads) and in real estate (cap rates) and how that relationship can be mined to generated expected equity risk premiums. We close the paper by examining why different approaches yield different values for the equity risk premium, and how to choose the "right" number to use in analysis.

(This is the eighth update of this piece. The first update was in the midst of the financial crisis in 2008 and there have been annual updates at the start of each year from 2009 through 2014.)

Electronic copy available at: http://ssrn.com/abstract=2581517

The notion that risk matters, and that riskier investments should have higher expected returns than safer investments, to be considered good investments, is intuitive and central to risk and return models in finance. Thus, the expected return on any investment can be written as the sum of the riskfree rate and a risk premium to compensate for the risk. The disagreement, in both theoretical and practical terms, remains on how to measure the risk in an investment, and how to convert the risk measure into an expected return that compensates for risk. A central number in this debate is the premium that investors demand for investing in the 'average risk' equity investment (or for investing in equities as a class), i.e., the equity risk premium.

In this paper, we begin by examining competing risk and return models in finance and the role played by equity risk premiums in each of them. We argue that equity risk premiums are central components in every one of these models and consider what the determinants of these premiums might be. We follow up by looking at three approaches for estimating the equity risk premium in practice. The first is to survey investors or managers with the intent of finding out what they require as a premium for investing in equity as a class, relative to the riskfree rate. The second is to look at the premiums earned historically by investing in stocks, as opposed to riskfree investments. The third is to back out an equity risk premium from market prices today. We consider the pluses and minuses of each approach and how to choose between the very different numbers that may emerge from these approaches.

Equity Risk Premiums: Importance and Determinants

Since the equity risk premium is a key component of every valuation, we should begin by looking at not only why it matters in the first place but also the factors that influence its level at any point in time and why that level changes over time. In this section, we look at the role played by equity risk premiums in corporate financial analysis, valuation and portfolio management, and then consider the determinants of equity risk premiums.

Why does the equity risk premium matter?

The equity risk premium reflects fundamental judgments we make about how much risk we see in an economy/market and what price we attach to that risk. In the process, it affects the expected return on every risky investment and the value that we estimate for that investment. Consequently, it makes a difference in both how we allocate wealth across different asset classes and which specific assets or securities we invest in within each asset class.

A Price for Risk

To illustrate why the equity risk premium is the price attached to risk, consider an alternate (though unrealistic) world where investors are risk neutral. In this world, the value of an asset would be the present value of expected cash flows, discounted back at a risk free rate. The expected cash flows would capture the cash flows under all possible scenarios (good and bad) and there would be no risk adjustment needed. In the real world, investors are risk averse and will pay a lower price for risky cash flows than for riskless cash flows, with the same expected value. How much lower? That is where equity risk premiums come into play. In effect, the equity risk premium is the premium that investors demand for the average risk investment, and by extension, the discount that they apply to expected cash flows with average risk. When equity risk premiums rise, investors are charging a higher price for risk and will therefore pay lower prices for the same set of risky expected cash flows.

Expected Returns and Discount Rates

Building on the theme that the equity risk premium is the price for taking risk, it is a key component into the expected return that we demand for a risky investment. This expected return, is a determinant of both the cost of equity and the cost of capital, essential inputs into corporate financial analysis and valuation.

While there are several competing risk and return models in finance, they all share some common assumptions about risk. First, they all define risk in terms of variance in actual returns around an expected return; thus, an investment is riskless when actual returns are always equal to the expected return. Second, they argue that risk has to be measured from the perspective of the marginal investor in an asset, and that this marginal investor is well diversified. Therefore, the argument goes, it is only the risk that an investment adds on to a diversified portfolio that should be measured and compensated. In fact, it is this view of risk that leads us to break the risk in any investment into two components. There is a firm-specific component that measures risk that relates only to that investment or to a few investments like it, and a market component that contains risk that affects a large subset or all investments. It is the latter risk that is not diversifiable and should be rewarded.

All risk and return models agree on this fairly crucial distinction, but they part ways when it comes to how to measure this market risk. In the capital asset pricing model (CAPM), the market risk is measured with a beta, which when multiplied by the equity risk premium yields the total risk premium for a risky asset. In the competing models, such as the arbitrage pricing and multi-factor models, betas are estimated against individual market risk factors, and each factor has it own price (risk premium). Table 1 summarizes four models, and the role that equity risk premiums play in each one:

Table 1: Equity Risk Premiums in Risk and Return Models

	Model	*Equity Risk Premium*
The CAPM	Expected Return = Riskfree Rate + $Beta_{Asset}$ (Equity Risk Premium)	Risk Premium for investing in the market portfolio, which includes all risky assets, relative to the riskless rate.
Arbitrage pricing model (APM)	$\text{Expected Return} = \text{Riskfree Rate} + \sum_{j=1}^{j=k} \beta_j (\text{Risk Premium}_j)$	Risk Premiums for individual (unspecified) market risk factors.
Multi-Factor Model	$\text{Expected Return} = \text{Riskfree Rate} + \sum_{j=1}^{j=k} \beta_j (\text{Risk Premium}_j)$	Risk Premiums for individual (specified) market risk factors
Proxy Models	Expected Return = a + b (Proxy 1) + c (Proxy 2) (where the proxies are firm characteristics such as market capitalization, price to book ratios or return momentum)	No explicit risk premium computation, but coefficients on proxies reflect risk preferences.

All of the models other than proxy models require three inputs. The first is the riskfree rate, simple to estimate in currencies where a default free entity exists, but more complicated in markets where there are no default free entities. The second is the beta (in the CAPM) or betas (in the APM or multi-factor models) of the investment being analyzed, and the third is the appropriate risk premium for the portfolio of all risky assets (in the CAPM) and the factor risk premiums for the market risk factors in the APM and multi-factor models. While I examine the issues of riskfree rate and beta estimation in companion pieces, I will concentrate on the measurement of the risk premium in this paper.

Note that the equity risk premium in all of these models is a market-wide number, in the sense that it is not company specific or asset specific but affects expected returns on all risky investments. Using a larger equity risk premium will increase the expected returns for all risky investments, and by extension, reduce their value. Consequently, the choice of an equity risk premium may have much larger consequences for value than firm-specific inputs such as cash flows, growth and even firm-specific risk measures (such as betas).

Investment and Policy Implications

It may be tempting for those not in the midst of valuation or corporate finance analysis to pay little heed to the debate about equity risk premium, but it would be a mistake to do so, since its effects are far reaching.

- The amounts set aside by both corporations and governments to meet future pension fund and health care obligations are determined by their expectations of returns from investing in equity markets, i.e., their views on the equity risk premium. Assuming that the equity risk premium is 6% will lead to far less being set aside each year to cover future obligations than assuming a premium of 4%. If the actual premium delivered by equity markets is only 2%, the fund's assets will be insufficient to meet its liabilities, leading to fund shortfalls which have to be met by raising taxes (for governments) or reducing profits (for corporations) In some cases, the pension benefits can be put at risk, if plan administrators use unrealistically high equity risk premiums, and set aside too little each year.
- Business investments in new assets and capacity is determined by whether the businesses think they can generate higher returns on those investments than the cost that they attach to the capital in that investment. If equity risk premiums increase, the cost of equity and capital will have to increase with them, leading to less overall investment in the economy and lower economic growth.
- Regulated monopolies, such as utility companies, are often restricted in terms of the prices that they charge for their products and services. The regulatory commissions that determine "reasonable" prices base them on the assumption that these companies have to earn a fair rate of return for their equity investors. To come up with this fair rate of return, they need estimates of equity risk premiums; using higher equity risk premiums will translate into higher prices for the customers in these companies.[1]
- Judgments about how much you should save for your retirement or health care and where you should invest your savings are clearly affected by how much return you think you can make on your investments. Being over optimistic about equity risk premiums will lead you to save too little to meet future needs and to over investment in risky asset classes.

Thus, the debate about equity risk premiums has implications for almost every aspect of our lives.

Market Timing and Risk Premiums

Any one who invests has a view on equity risk premiums, though few investors are explicit about their views. In particular, if you believe that markets are efficient, you

[1] The Society of Utility and Regulatory Financial Analysts (SURFA) has annual meetings of analysts involved primarily in this debate. Not surprisingly, they spend a good chunk of their time discussing equity risk premiums, with analysts working for the utility firms arguing for higher equity risk premiums and analysts working for the state or regulatory authorities wanting to use lower risk premiums.

are arguing that the equity risk premiums built into market prices today are correct. If you believe that stock markets are over valued or in a bubble, you are asserting that the equity risk premiums built into prices today are too low, relative to what they should be (based on the risk in equities and investor risk aversion). Conversely, investors who believe that stocks are collectively underpriced or cheap are also making a case that the equity risk premium in the market today is much higher than what you should be making (again based on the risk in equities and investor risk aversion). Thus, every debate about the overall equity market can be translated into a debate about equity risk premiums.

Put differently, asset allocation decisions that investors make are explicitly or implicitly affected by investor views on risk premiums and how they vary across asset classes and geographically. Thus, if you believe that equity risk premiums are low, relative to the risk premiums in corporate bond markets (which take the form or default spreads on bonds), you will allocated more of your overall portfolio to bonds. Your allocation of equities across geographical markets are driven by your perceptions of equity risk premiums in those markets, with more of your portfolio going into markets where the equity risk premium is higher than it should be (given the risk of those markets). Finally, if you determine that the risk premiums in financial assets (stocks and bonds) are too low, relative to what you can earn in real estate or other real assets, you will redirect more of your portfolio into the latter.

By making risk premiums the focus of asset allocation decisions, you give focus to those decisions. While it is very difficult to compare PE ratios for stocks to interest rates on bonds and housing price indicators, you can compare equity risk premiums to default spreads to real estate capitalization rates to make judgments about where you get the best trade off on risk and return. In fact, we will make these comparisons later in this paper.

What are the determinants of equity risk premiums?

Before we consider different approaches for estimating equity risk premiums, we should examine the factors that determine equity risk premiums. After all, equity risk premiums should reflect not only the risk that investors see in equity investments but also the price they attach to that risk.

Risk Aversion and Consumption Preferences

The first and most critical factor, obviously, is the risk aversion of investors in the markets. As investors become more risk averse, equity risk premiums will climb, and as risk aversion declines, equity risk premiums will fall. While risk aversion will vary across investors, it is the collective risk aversion of investors that determines equity risk premium, and changes in that collective risk aversion will manifest themselves as changes in the equity risk premium. While there are numerous variables that influence risk aversion, we will focus on the variables most likely to change over time.

a. Investor Age: There is substantial evidence that individuals become more risk averse as they get older. The logical follow up to this proposition is that markets with older investors, in the aggregate, should have higher risk premiums than markets with younger investors, for any given level of risk. Bakshi and Chen (1994), for instance, examined risk premiums in the United States and noted an increase in risk premiums as investors aged.[2] Liu and Spiegel computed the ratio of the middle-age cohort (40-49 years) to the old-age cohort (60-69) and found that PE ratios are closely and positively related to the MO ratio for the US equity market from 1954 to 2010; since the equity risk premium is inversely related to the PE, this would suggest that investor age does play a role in determining equity risk premiums.[3]
b. Preference for current consumption: We would expect the equity risk premium to increase as investor preferences for current over future consumption increase. Put another way, equity risk premiums should be lower, other things remaining equal, in markets where individuals are net savers than in markets where individuals are net consumers. Consequently, equity risk premiums should increase as savings rates decrease in an economy. Rieger, Wang and Hens (2012) compare equity risk premiums and time discount factors across 27 countries and find that premiums are higher in countries where investors are more short term.[4]

Relating risk aversion to expected equity risk premiums is not straightforward. While the direction of the relationship is simple to establish – higher risk aversion should translate into higher equity risk premiums- getting beyond that requires us to be more precise in our judgments about investor utility functions, specifying how investor utility relates to wealth (and variance in that wealth). As we will see later in this paper, there has been a significant angst among financial economics that most conventional utility models do not do a good job of explaining observed equity risk premiums.

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[2] Bakshi, G. S., and Z. Chen, 1994, *Baby Boom, Population Aging, and Capital Markets*, The Journal of Business, LXVII, 165-202.

[3] Liu, Z. and M.M. Siegel, 2011, *Boomer Retirement: Headwinds for US Equity Markets?* FRBSF Economic Letters, v26.

[4] Rieger, M.O., M. Wang and T. Hens, 2012, International Evidence on the Equity Risk Premium Puzzle and Time Discounting, SSRN Working Paper, http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2120442

Economic Risk

The risk in equities as a class comes from more general concerns about the health and predictability of the overall economy. Put in more intuitive terms, the equity risk premium should be lower in an economy with predictable inflation, interest rates and economic growth than in one where these variables are volatile. Lettau, Ludwigson and Wachter (2008) link the changing equity risk premiums in the United States to shifting volatility in the real economy.[5] In particular, they attribute that that the lower equity risk premiums of the 1990s (and higher equity values) to reduced volatility in real economic variables including employment, consumption and GDP growth. One of the graphs that they use to illustrate the correlation looks at the relationship between the volatility in GDP growth and the dividend/ price ratio (which is the loose estimate that they use for equity risk premiums), and it is reproduced in figure 1.

Figure 1: Volatility in GDP growth and Equity Risk Premiums (US)

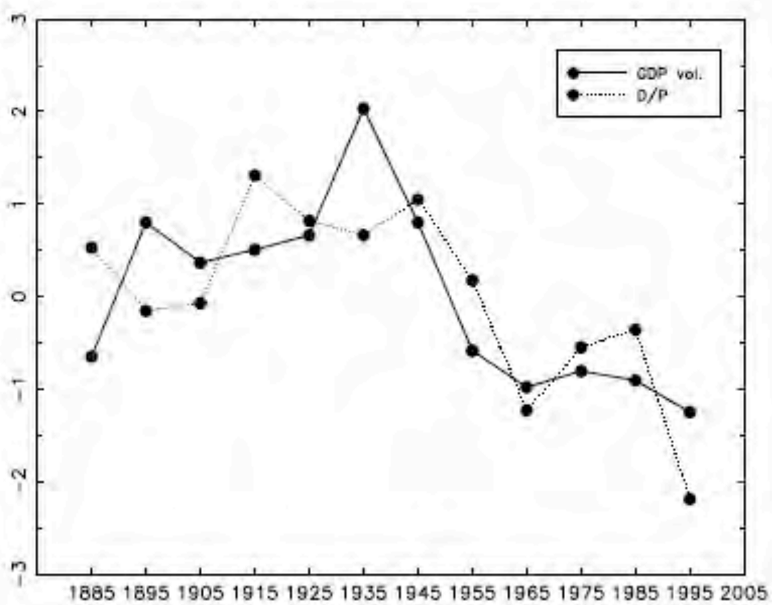

Figure 3
GDP volatility and the D/P ratio—Prewar evidence
This figure plots the standard deviations of GDP growth and the mean D/P ratio by decade starting in 1880 until 2000. Both series are demeaned and divided by their standard deviation. The GDP data are from Ray Fair's website (http://fairmodel.econ.yale.edu/RAYFAIR/PDF/2002DTBL.HTM) based on Balke and Gordon (1989). The dividend yield data is from Robert Shiller's website (http://aida.econ.yale.edu/~shiller/data/ie_data.htm).

Note how closely the dividend yield has tracked the volatility in the real economy over this very long time period.

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5 Lettau, M., S.C. Ludvigson and J.A. Wachter, 2008. *The Declining Equity Risk Premium: What role does macroeconomic risk play?* Review of Financial Studies, v21, 1653-1687.

Gollier (2001) noted that the linear absolute risk tolerance often assumed in standard models breaks down when there is income inequality and the resulting concave absolute risk tolerance should lead to higher equity risk premiums.[6] Hatchondo (2008) attempted to quantify the impact on income inequality on equity risk premiums. In his model, which is narrowly structured, the equity risk premium is higher in an economy with unequal income than in an egalitarian setting, but only by a modest amount (less than 0.50%).[7]

A related strand of research examines the relationship between equity risk premium and inflation, with mixed results. Studies that look at the relationship between the level of inflation and equity risk premiums find little or no correlation. In contrast, Brandt and Wang (2003) argue that news about inflation dominates news about real economic growth and consumption in determining risk aversion and risk premiums.[8] They present evidence that equity risk premiums tend to increase if inflation is higher than anticipated and decrease when it is lower than expected. Another strand of research on the Fisher equation, which decomposes the riskfree rate into expected inflation and a real interest rate, argues that when inflation is stochastic, there should be a third component in the risk free rate: an inflation risk premium, reflecting uncertainty about future inflation.[9] Reconciling the findings, it seems reasonable to conclude that it is not so much the level of inflation that determines equity risk premiums but uncertainty about that level, and that some of the inflation uncertainty premium may be captured in the risk free rate, rather than in the equity risk premiums.

Since the 2008 crisis, with its aftermath oflow government bond rates and a simmering economic crisis, equity risk premiums in the United States have behaved differently than they have historically. Connolly and Dubofsky (2015) find that equity risk premiums have increased (decreased) as US treasury bond rates decrease (increase), and have moved inversely with inflation (with higher inflation leading to lower equity risk premiums), both behaviors at odds with the relationship in the pre-2008 time period, suggesting a structural break in 2008.[10]

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[6] Gollier, C., 2001. *Wealth Inequality and Asset Pricing*, Review of Economic Studies, v68, 181–203.
[7] Hatchondo, J.C., 2008, *A Quantitative Study of the Role of Income Inequality on Asset Prices*, Economic Quarterly, v94, 73–96.
[8] Brandt, M.W. and K.Q. Wang. 2003. *Time-varying risk aversion and unexpected inflation*, Journal of Monetary Economics, v50, pp. 1457-1498.
[9] Benninga, S., and A. Protopapadakis, 1983, *Real and Nominal Interest Rates under Uncertainty: The Fisher Problem and the Term Structure*, Journal of Political Economy, vol. 91, pp. 856–67.
[10] Connolly, R. and D. Dubofsky, 2015, *Risk Perceptions, Inflation and Financial Asset Returns: A Tale of Two Connections*, Working Paper, SSRN #2527213.

Information

When you invest in equities, the risk in the underlying economy is manifested in volatility in the earnings and cash flows reported by individual firms in that economy. Information about these changes is transmitted to markets in multiple ways, and it is clear that there have been significant changes in both the quantity and quality of information available to investors over the last two decades. During the market boom in the late 1990s, there were some who argued that the lower equity risk premiums that we observed in that period were reflective of the fact that investors had access to more information about their investments, leading to higher confidence and lower risk premiums in 2000. After the accounting scandals that followed the market collapse, there were others who attributed the increase in the equity risk premium to deterioration in the quality of information as well as information overload. In effect, they were arguing that easy access to large amounts of information of varying reliability was making investors less certain about the future.

As these contrary arguments suggest, the relationship between information and equity risk premiums is complex. More precise information should lead to lower equity risk premiums, other things remaining equal. However, precision here has to be defined in terms of what the information tells us about future earnings and cash flows. Consequently, it is possible that providing more information about last period's earnings may create more uncertainty about future earnings, especially since investors often disagree about how best to interpret these numbers. Yee (2006) defines earnings quality in terms of volatility of future earnings and argues that equity risk premiums should increase (decrease) as earnings quality decreases (increases).[11]

Empirically, is there a relationship between earnings quality and observed equity risk premiums? The evidence is mostly anecdotal, but there are several studies that point to the deteriorating quality of earnings in the United States, with the blame distributed widely. First, the growth of technology and service firms has exposed inconsistencies in accounting definitions of earnings and capital expenditures – the treatment of R&D as an operating expense is a prime example. Second, audit firms have been accused of conflicts of interest leading to the abandonment of their oversight responsibility. Finally, the earnings game, where analysts forecast what firms will earn and firms then try to beat these forecasts has led to the stretching (and breaking) of accounting rules and standards. If earnings have become less informative in the aggregate, it stands to reason that equity

[11] Yee, K. K.,, 2006, *Earnings Quality and the Equity Risk Premium: A Benchmark Model*, Contemporary Accounting Research, 23: 833–877.

investors will demand large equity risk premiums to compensate for the added uncertainty.

Information differences may be one reason why investors demand larger risk premiums in some emerging markets than in others. After all, markets vary widely in terms of transparency and information disclosure requirements. Markets like Russia, where firms provide little (and often flawed) information about operations and corporate governance, should have higher risk premiums than markets like India, where information on firms is not only more reliable but also much more easily accessible to investors. Lau, Ng and Zhang (2011) look at time series variation in risk premiums in 41 countries and conclude that countries with more information disclosure, measured using a variety of proxies, have less volatile risk premiums and that the importance of information is heightened during crises (illustrated using the 1997 Asian financial crisis and the 2008 Global banking crisis).[12]

Liquidity and Fund Flows

In addition to the risk from the underlying real economy and imprecise information from firms, equity investors also have to consider the additional risk created by illiquidity. If investors have to accept large discounts on estimated value or pay high transactions costs to liquidate equity positions, they will be pay less for equities today (and thus demand a large risk premium).

The notion that market for publicly traded stocks is wide and deep has led to the argument that the net effect of illiquidity on aggregate equity risk premiums should be small. However, there are two reasons to be skeptical about this argument. The first is that not all stocks are widely traded and illiquidity can vary widely across stocks; the cost of trading a widely held, large market cap stock is very small but the cost of trading an over-the-counter stock will be much higher. The second is that the cost of illiquidity in the aggregate can vary over time, and even small variations can have significant effects on equity risk premiums. In particular, the cost of illiquidity seems to increase when economies slow down and during periods of crisis, thus exaggerating the effects of both phenomena on the equity risk premium.

While much of the empirical work on liquidity has been done on cross sectional variation across stocks (and the implications for expected returns), there have been attempts to extend the research to look at overall market risk premiums. Gibson and Mougeot (2004) look at U.S. stock returns from 1973 to 1997 and conclude that liquidity

[12] Lau. S.T., L. Ng and B. Zhang, 2011, *Information Environment and Equity Risk Premium Volatility around the World*, Management Science, Forthcoming.

accounts for a significant component of the overall equity risk premium, and that its effect varies over time.[13] Baekart, Harvey and Lundblad (2006) present evidence that the differences in equity returns (and risk premiums) across emerging markets can be partially explained by differences in liquidity across the markets.[14]

Another way of framing the liquidity issue is in terms of funds flows, where the equity risk premium is determined by funds flows into and out of equities. Thus, if more funds are flowing into an equity market, either from other asset classes or other geographies, other things remaining equal, the equity risk premium should decrease, whereas funds flowing out of an equity market will lead to higher equity risk premiums.

Catastrophic Risk

When investing in equities, there is always the potential for catastrophic risk, i.e. events that occur infrequently but can cause dramatic drops in wealth. Examples in equity markets would include the great depression from 1929-30 in the United States and the collapse of Japanese equities in the last 1980s. In cases like these, many investors exposed to the market declines saw the values of their investments drop so much that it was unlikely that they would be made whole again in their lifetimes.[15] While the possibility of catastrophic events occurring may be low, they cannot be ruled out and the equity risk premium has to reflect that risk.

Rietz (1988) uses the possibility of catastrophic events to justify higher equity risk premiums and Barro (2006) extends this argument. In the latter's paper, the catastrophic risk is modeled as both a drop in economic output (an economic depression) and partial default by the government on its borrowing.[16] Gabaix (2009) extends the Barro-Rietz model to allow for time varying losses in disasters.[17] Barro, Nakamura, Steinsson and Ursua (2009) use panel data on 24 countries over more than 100 years to examine the empirical effects of disasters.[18] They find that the average length of a disaster is six years

[13] Gibson R., Mougeot N., 2004, *The Pricing of Systematic Liquidity Risk: Empirical Evidence from the US Stock Market*. Journal of Banking and Finance, v28: 157–78.

[14] Bekaert G., Harvey C. R., Lundblad C., 2006, *Liquidity and Expected Returns: Lessons from Emerging Markets*, The Review of Financial Studies.

[15] An investor in the US equity markets who invested just prior to the crash of 1929 would not have seen index levels return to pre-crash levels until the 1940s. An investor in the Nikkei in 1987, when the index was at 40000, would still be facing a deficit of 50% (even after counting dividends) in 2008,

[16] Rietz, T. A., 1988, *The equity premium~: A solution*, Journal of Monetary Economics, v22, 117-131; Barro R J., 2006, *Rare Disasters and Asset Markets in the Twentieth Century*, Quarterly Journal of Economics, August, 823-866.

[17]Gabaix, Xavier, 2012, *Variable Rare Disasters: An Exactly Solved Framework for Ten Puzzles in Macro-Finance*, The Quarterly Journal of Economics, v127, 645-700.

[18] Barro, R., E. Nakamura, J. Steinsson and J. Ursua, 2009, *Crises and Recoveries in an Empirical Model of Consumption Disasters*, Working Paper, http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1594554.

and that half of the short run impact is reversed in the long term. Investigating the asset pricing implications, they conclude that the consequences for equity risk premiums will depend upon investor utility functions, with some utility functions (power utility, for instance) yielding low premiums and others generating much higher equity risk premiums. Barro and Ursua (2008) look back to 1870 and identify 87 crises through 2007, with an average impact on stock prices of about 22%, and estimate that investors would need to generate an equity risk premium of 7% to compensate for risk taken.[19] Wachter (2012) builds a consumption model, where consumption follows a normal distribution with low volatility most of the time, with a time-varying probability of disasters that explains high equity risk premiums.[20]

There have been attempts to measure the likelihood of catastrophic risk and incorporate them into models that predict equity risk premiums. In a series of papers with different co-authors, Bollerslev uses the variance risk premium, i.e., the difference between the implied variance in stock market options and realized variance, as a proxy for expectations of catastrophic risk, and documents a positive correlation with equity risk premiums.[21] Kelly (2012) looks at extreme stock market movements as a measure of expected future jump (catastrophic) risk and finds a positive link between jump risk and equity risk premiums.[22] Guo, Liu, Wang, Zhou and Zuo (2014) refine this analysis by decomposing jumps into bad (negative) and good (positive) ones and find that it is the risk of downside jumps that determines equity risk premiums..[23] Maheu, McCurdy and Zhao (2013) used a time-varying jump-arrival process and a two-component GARCH model on US stock market data from 1926 to 2011, and estimated that each additional jump per year increased the equity risk premium by 0.1062% and that there were, on average, 34 jumps a year, leading to a jump equity risk premium of 3.61%.[24]

The banking and financial crisis of 2008, where financial and real estate markets plunged in the last quarter of the year, has provided added ammunition to this school. As

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[19] Barro, R. and J. Ursua, 2008, Macroeconomic Crises since 1870, Working Paper, http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1124864.

[20] Wachter, J.A., 2013, *Can time-varying risk of rare disasters explain aggregate stock market volatility?* Journal of Finance, v68, 987-1035.

[21] Bollerslev, T. M., T. H. Law, and G. Tauchen, 2008, *Risk, Jumps, and Diversification*, Journal of Econometrics, 144, 234-256; Bollerslev, T. M., G. Tauchen, and H. Zhou, 2009, *Expected Stock Returns and Variance Risk Premia*, Review of Financial Studies, 101-3, 552-573; Bollerselv, T.M., and V. Todorov, 2011, *Tails, Fears, and Risk Premia*, Journal of Finance, 66-6, 2165-2211.

[22] Kelly, B., 2012, *Tail Risk and Asset Prices*, Working Paper, University of Chicago.

[23] Guo, H., Z. Liu, K. Wang, H. Zhou and H. Zuo, 2014, *Good Jumps, Bad Jumps and Conditional Equity Risk Premium*, Working Paper, SSRN #2516074.

[24] Maheu, J.M., T.H. McCurdy and X. Wang, 2013, *Do Jumps Contribute to the Dynamics of the Equity Premium*, Journal of Financial Economics, v110, 457-477.

we will see later in the paper, risk premiums in all markets (equity, bond and real estate) climbed sharply during the weeks of the market crisis. In fact, the series of macro crises in the last four years that have affected markets all over the world has led some to hypothesize that the globalization may have increased the frequency and probability of disasters and by extension, equity risk premiums, in all markets.

Government Policy

The prevailing wisdom, at least until 2008, was that while government policy affected equity risk premiums in emerging markets, it was not a major factor in determining equity risk premiums in developed markets. The banking crisis of 2008 and the government responses to it have changed some minds, as both the US government and European governments have made policy changes that at times have calmed markets and at other times roiled them, potentially affecting equity risk premiums.

Pastor and Veronesi (2012) argue that uncertainty about government policy can translate into higher equity risk premiums.[25] The model they develop has several testable implications. First, government policy changes will be more likely just after economic downturns, thus adding policy uncertainty to general economic uncertainty and pushing equity risk premiums upwards. Second, you should expect to see stock prices fall, on average, across all policy changes, with the magnitude of the negative returns increasing for policy changes create more uncertainty. Third, policy changes will increase stock market volatility and the correlation across stocks.

Lam and Zhang (2014) try to capture the potential policy shocks from either an unstable government (government stability) or an incompetent bureaucracy (bureaucracy quality) in 49 countries from 1995 to 2006, using two measures of policy uncertainty drawn from the international country risk guide (ICG). They do find that equity risk premiums are higher in countries with more policy risk from either factor, with more bureaucratic risk increasing the premium by approximately 8%.[26]

The behavioral/ irrational component

Investors do not always behave rationally, and there are some who argue that equity risk premiums are determined, at least partially, by quirks in human behavior. While there are several strands to this analysis, we will focus on three:

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[25] Pástor, L. and P. Veronesi, 2012. *Uncertainty about Government policy and Stock Prices*. Journal of Finance 67: 1219-1264.

[26] Lam, S.S. and W. Zhang, 2014, *Does Policy Uncertainty matter for International Equity Markets?* Working Paper, SSRN #2297133.

a. The Money Illusion: As equity prices declined significantly and inflation rates increased in the late 1970s, Modigliani and Cohn (1979) argued that low equity values of that period were the consequence of investors being inconsistent about their dealings with inflation. They argued that investors were guilty of using historical growth rates in earnings, which reflected past inflation, to forecast future earnings, but current interest rates, which reflected expectations of future inflation, to estimate discount rates.[27] When inflation increases, this will lead to a mismatch, with high discount rates and low cash flows resulting in asset valuations that are too low (and risk premiums that are too high). In the Modigliani-Cohn model, equity risk premiums will rise in periods when inflation is higher than expected and drop in periods when inflation in lower than expected. Campbell and Voulteenaho (2004) update the Modigliani-Cohn results by relating changes in the dividend to price ratio to changes in the inflation rate over time and find strong support for the hypothesis.[28]

b. Narrow Framing: In conventional portfolio theory, we assume that investors assess the risk of an investment in the context of the risk it adds to their overall portfolio, and demand a premium for this risk. Behavioral economists argue that investors offered new gambles often evaluate those gambles in isolation, separately from other risks that they face in their portfolio, leading them to over estimate the risk of the gamble. In the context of the equity risk premium, Benartzi and Thaler (1995) use this "narrow framing" argument to argue that investors over estimate the risk in equity, and Barberis, Huang and Santos (2001) build on this theme.[29]

The Equity Risk Premium Puzzle

While many researchers have focused on individual determinants of equity risk premiums, there is a related question that has drawn almost as much attention. Are the equity risk premiums that we have observed in practice compatible with the theory? Mehra and Prescott (1985) fired the opening shot in this debate by arguing that the observed historical risk premiums (which they estimated at about 6% at the time of their analysis) were too high, and that investors would need implausibly high risk-aversion

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[27] Modigliani, Franco and Cohn, Richard. 1979, *Inflation, Rational Valuation, and the Market*, Financial Analysts Journal, v37(3), pp. 24-44.

[28] Campbell, J.Y. and T. Vuolteenaho, 2004, *Inflation Illusion and Stock Prices,* American Economic Review, v94, 19-23.

[29] Benartzi, S. and R. Thaler, 1995, *Myopic Loss Aversion and the Equity Premium Puzzle*, Quarterly Journal of Economics; Barberis, N., M. Huang, and T. Santos, 2001, *Prospect Theory and Asset Prices*, Quarterly Journal of Economics, v 116(1), 1-53.

coefficients to demand these premiums.[30] In the years since, there have been many attempts to provide explanations for this puzzle:

1. Statistical artifact: The historical risk premium obtained by looking at U.S. data is biased upwards because of a survivor bias (induced by picking one of the most successful equity markets of the twentieth century). The true premium, it is argued, is much lower. This view is backed up by a study of large equity markets over the twentieth century, which concluded that the historical risk premium is closer to 4% than the 6% cited by Mehra and Prescott.[31] However, even the lower risk premium would still be too high, if we assumed reasonable risk aversion coefficients.
2. Disaster Insurance: A variation on the statistical artifact theme, albeit with a theoretical twist, is that the observed volatility in an equity market does not fully capture the potential volatility, which could include rare but disastrous events that reduce consumption and wealth substantially. Reitz, referenced earlier, argues that investments that have dividends that are proportional to consumption (as stocks do) should earn much higher returns than riskless investments to compensate for the possibility of a disastrous drop in consumption. Prescott and Mehra (1988) counter than the required drops in consumption would have to be of such a large magnitude to explain observed premiums that this solution is not viable. [32] Berkman, Jacobsen and Lee (2011) use data from 447 international political crises between 1918 and 2006 to create a crisis index and note that increases in the index increase equity risk premiums, with disproportionately large impacts on the industries most exposed to the crisis.[33]
3. Taxes: One possible explanation for the high equity returns in the period after the Second World War is the declining marginal tax rate during that period. McGrattan and Prescott (2001), for instance, provide a hypothetical illustration where a drop in the tax rate on dividends from 50% to 0% over 40 years would cause equity prices to rise about 1.8% more than the growth rate in GDP; adding the dividend yield to this expected price appreciation generates returns similar to

[30] Mehra, Rajnish, and Edward C.Prescott, 1985, *The Equity Premium: A Puzzle*, Journal of Monetary Economics, v15, 145–61. Using a constant relative risk aversion utility function and plausible risk aversion coefficients, they demonstrate the equity risk premiums should be much lower (less than 1%).

[31] Dimson, E., P. March and M. Staunton, 2002, *Triumph of the Optimists*, Princeton University Press.

[32] Mehra, R. and E.C. Prescott, 1988, *The Equity Risk Premium: A Solution?* Journal of Monetary Economics, v22, 133-136.

[33] Berkman, H., B. Jacobsen and J. Lee, 2011, *Time-varying Disaster Risk and Stock Returns*, Journal of Financial Economics, v101, 313-332

the observed equity risk premium.[34] In reality, though, the drop in marginal tax rates was much smaller and cannot explain the surge in equity risk premiums.

4. Alternative Preference Structures: There are some who argue that the equity risk premium puzzle stems from its dependence upon conventional expected utility theory to derive premiums. In particular, the constant relative risk aversion (CRRA) function used by Mehra and Prescott in their paper implies that if an investor is risk averse to variation in consumption across different states of nature at a point in time, he or she will also be equally risk averse to consumption variation across time. Epstein and Zin consider a class of utility functions that separate risk aversion (to consumption variation at a point in time) from risk aversion to consumption variation across time. They argue that individuals are much more risk averse when it comes to the latter and claim that this phenomenon explain the larger equity risk premiums.[35] Put in more intuitive terms, individuals will choose a lower and more stable level of wealth and consumption that they can sustain over the long term over a higher level of wealth and consumption that varies widely from period to period. Constantinides (1990) adds to this argument by noting that individuals become used to maintaining past consumption levels and that even small changes in consumption can cause big changes in marginal utility. The returns on stocks are correlated with consumption, decreasing in periods when people have fewer goods to consume (recessions, for instance); the additional risk explains the higher observed equity risk premiums.[36]
5. Myopic Loss Aversion: Myopic loss aversion refers to the finding in behavioral finance that the loss aversion already embedded in individuals becomes more pronounced as the frequency of their monitoring increases. Thus, investors who receive constant updates on equity values actually perceive more risk in equities, leading to higher risk premiums. The paper that we cited earlier by Benartzi and Thaler yields estimates of the risk premium very close to historical levels using a one-year time horizon for investors with plausible loss aversion characteristics (of about 2, which is backed up by the experimental research).

In conclusion, it is not quite clear what to make of the equity risk premium puzzle. It is true that historical risk premiums are higher than could be justified using conventional

[34] McGrattan, E.R., and E.C. Prescott. 2001, *Taxes, Regulations, and Asset Prices*, Working Paper, http://papers.ssrn.com/sol3/papers.cfm?abstract_id=292522.

[35] Epstein, L.G., and S.E. Zin. 1991. *Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: An Empirical Analysis*, Journal of Political Economy, v99, 263–286.

[36] Constantinides, G.M. 1990. *Habit Formation: A Resolution of the Equity Premium Puzzle*, Journal of Political Economy, v98, no. 3 (June):519–543.

utility models for wealth. However, that may tell us more about the dangers of using historical data and the failures of classic utility models than they do about equity risk premiums. In fact, the last decade of poor stock returns in the US and declining equity risk premiums may have made the equity risk premium puzzle less of a puzzle, since explaining a historical premium of 4% (the premium in 2011) is far easier than explaining a historical premium of 6% (the premium in 1999).

Estimation Approaches

There are three broad approaches used to estimate equity risk premiums. One is to survey subsets of investors and managers to get a sense of their expectations about equity returns in the future. The second is to assess the returns earned in the past on equities relative to riskless investments and use this historical premium as the expectation. The third is to attempt to estimate a forward-looking premium based on the market rates or prices on traded assets today; we will categorize these as implied premiums.

Survey Premiums

If the equity risk premium is what investors demand for investing in risky assets today, the most logical way to estimate it is to ask these investors what they require as expected returns. Since investors in equity markets number in the millions, the challenge is often finding a subset of investors that best reflects the aggregate market. In practice, se see surveys of investors, managers and even academics, with the intent of estimating an equity risk premium.

Investors

When surveying investors, we can take one of two tacks. The first is to focus on individual investors and get a sense of what they expect returns on equity markets to be in the future. The second is to direct the question of what equities will deliver as a premium at portfolio managers and investment professionals, with the rationale that their expectations should matter more in the aggregate, since they have the most money to invest.

a. Individual Investors: The oldest continuous index of investor sentiment about equities was developed by Robert Shiller in the aftermath of the crash of 1987 and has been updated since.[37] UBS/Gallup has also polled individual investors since 1996 about their optimism about future stock prices and reported a measure of investor

[37] The data is available at http://bit.ly/NcgTW7.

sentiment.[38] While neither survey provides a direct measure of the equity risk premium, they both yield broad measure of where investors expect stock prices to go in the near future. The Securities Industry Association (SIA) surveyed investors from 1999 to 2004 on the expected return on stocks and yields numbers that can be used to extract equity risk premiums. In the 2004 survey, for instance, they found that the median expected return across the 1500 U.S. investors they questioned was 12.8%, yielding a risk premium of roughly 8.3% over the treasury bond rate at that time.[39]

b. Institutional Investors/ Investment Professionals: Investors Intelligence, an investment service, tracks more than a hundred newsletters and categorizes them as bullish, bearish or neutral, resulting in a consolidated advisor sentiment index about the future direction of equities. Like the Shiller and UBS surveys, it is a directional survey that does not yield an equity risk premium. Merrill Lynch, in its monthly survey of institutional investors globally, explicitly poses the question about equity risk premiums to these investors. In its February 2007 report, for instance, Merrill reported an average equity risk premium of 3.5% from the survey, but that number jumped to 4.1% by March, after a market downturn.[40] As markets settled down in 2009, the survey premium has also settled back to 3.76% in January 2010. Through much of 2010, the survey premium stayed in a tight range (3.85% - 3.90%) but the premium climbed to 4.08% in the January 2012 update. In February 2014, the survey yielded a risk premium of 4.6%, though it may not be directly comparable to the earlier numbers because of changes in the survey.[41]

While survey premiums have become more accessible, very few practitioners seem to be inclined to use the numbers from these surveys in computations and there are several reasons for this reluctance:

1. Survey risk premiums are responsive to recent stock prices movements, with survey numbers generally increasing after bullish periods and decreasing after market decline. Thus, the peaks in the SIA survey premium of individual investors occurred in the bull market of 1999, and the more moderate premiums of 2003 and 2004 occurred after the market collapse in 2000 and 2001.
2. Survey premiums are sensitive not only to whom the question is directed at but how the question is asked. For instance, individual investors seem to have higher

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38 The data is available at http://www.ubs.com/us/en/wealth/misc/investor-watch.html

39 See http://www.sifma.org/research/surveys.aspx. The 2004 survey seems to be the last survey done by SIA. The survey yielded expected stock returns of 10% in 2003, 13% in 2002, 19% in 2001, 33% in 2000 and 30% in 1999.

40 See http://www.ml.com/index.asp?id=7695_8137_47928.

41 Global Fund Manager Survey, Bank of America Merrill Lynch, February 2014.

(and more volatile) expected returns on equity than institutional investors and the survey numbers vary depending upon the framing of the question.[42]

3. In keeping with other surveys that show differences across sub-groups, the premium seems to vary depending on who gets surveyed. Kaustia, Lehtoranta and Puttonen (2011) surveyed 1,465 Finnish investment advisors and note that not only are male advisors more likely to provide an estimate but that their estimated premiums are roughly 2% lower than those obtained from female advisors, after controlling for experience, education and other factors.[43]
4. Studies that have looked at the efficacy of survey premiums indicate that if they have any predictive power, it is in the wrong direction. Fisher and Statman (2000) document the negative relationship between investor sentiment (individual and institutional) and stock returns.[44] In other words, investors becoming more optimistic (and demanding a larger premium) is more likely to be a precursor to poor (rather than good) market returns.

As technology aids the process, the number and sophistication of surveys of both individual and institutional investors will also increase. However, it is also likely that these survey premiums will be more reflections of the recent past rather than good forecasts of the future.

Managers

As noted in the first section, equity risk premiums are a key input not only in investing but also in corporate finance. The hurdle rates used by companies – costs of equity and capital – are affected by the equity risk premiums that they use and have significant consequences for investment, financing and dividend decisions. Graham and Harvey have been conducting annual surveys of Chief Financial Officers (CFOs) or companies for roughly the last decade with the intent of estimating what these CFOs think is a reasonable equity risk premium (for the next 10 years over the ten-year bond rate). In their March 2014 survey, they report an average equity risk premium of 3.73% across survey respondents, down slightly from the average premium of 4.27% a year earlier. The median premium in the March 2014 survey was 3.3%.[45]

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[42] Asking the question "What do you think stocks will do next year?" generates different numbers than asking "What should the risk premium be for investing in stocks?"

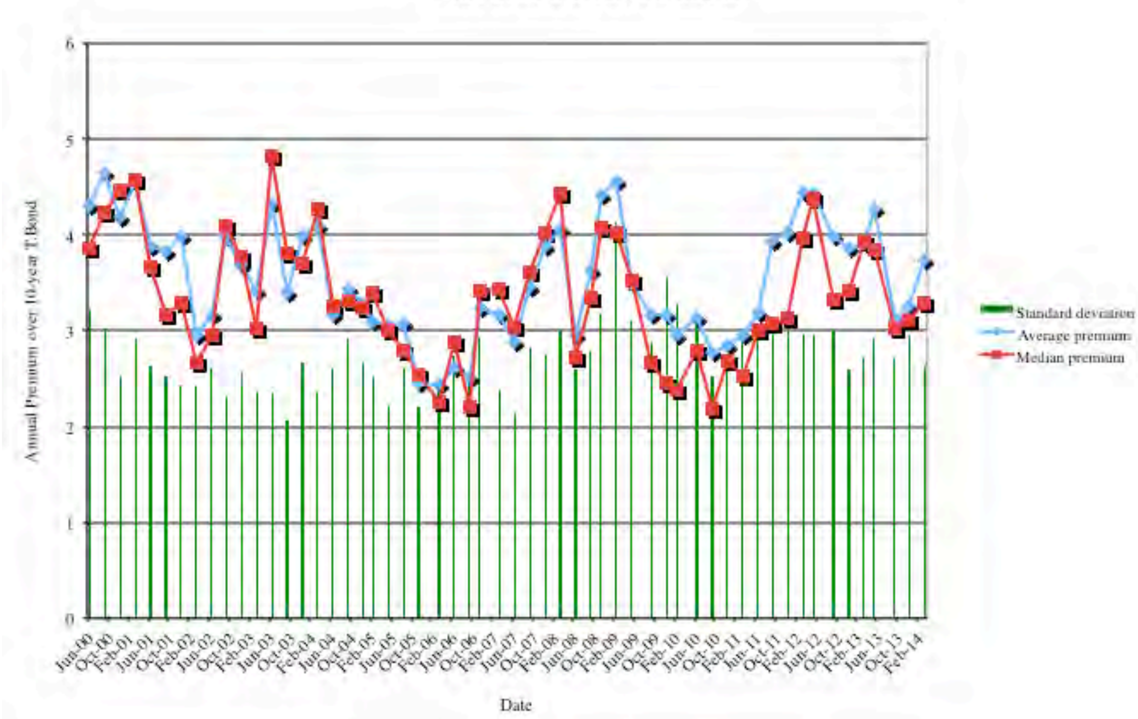
[43] Kaustia, M., A. Lehtoranta and V. Puttonen, 2011, *Sophistication and Gender Effects in Financial Advisers Expectations*, Working Paper, Aalto University.

[44] Fisher, K.L., and M. Statman, 2000, *Investor Sentiment and Stock Returns*, Financial Analysts Journal, v56, 16-23.

[45] Graham, J.R. and C.R. Harvey, 2014, *The Equity Risk Premium in 2014,* Working paper, http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2422008 . See also Graham, J.R. and C.R. Harvey,

To get a sense of how these assessed equity risk premiums have behaved over time, we have graphed the average and median values of the premium and the cross sectional standard deviation in the estimates in each CFO survey, from 2001 to 2014, in Figure 2.

Figure 2: CFO Survey Premiums



Note the survey premium peak was in February 2009, right after the crisis, at 4.56% and had its lowest recording (2.5%) in September 2006. The average across all 14 years of surveys (more than 10,000 responses) was 3.54%, but the standard deviation in the survey responses did increase after the 2008 crisis.

Academics

Most academics are neither big players in equity markets, nor do they make many major corporate finance decisions. Notwithstanding this lack of real world impact, what they think about equity risk premiums may matter for two reasons. The first is that many of the portfolio managers and CFOs that were surveyed in the last two sub-sections received their first exposure to the equity risk premium debate in the classroom and may have been influenced by what was presented as the right risk premium in that setting. The

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2009, *The Equity Risk Premium amid a Global Financial Crisis*, Working paper, http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1405459.

second is that practitioners often offer academic work (textbooks and papers) as backing for the numbers that they use.

Welch (2000) surveyed 226 financial economists on the magnitude of the equity risk premium and reported interesting results. On average, economists forecast an average annual risk premium (arithmetic) of about 7% for a ten-year time horizon and 6-7% for one to five-year time horizons. As with the other survey estimates, there is a wide range on the estimates, with the premiums ranging from 2% at the pessimistic end to 13% at the optimistic end. Interestingly, the survey also indicates that economists believe that their estimates are higher than the consensus belief and try to adjust the premiums down to reflect that view.[46]

Fernandez (2010) examined widely used textbooks in corporate finance and valuation and noted that equity risk premiums varied widely across the books and that the moving average premium has declined from 8.4% in 1990 to 5.7% in 2010.[47] In a more recent survey, Fernandez, Aguirreamalloa and L. Corres (2011) compared both the level and standard deviation of equity risk premium estimates for analysts, companies and academics in the United States:[48]

Group	*Average Equity Risk Premium*	*Standard deviation in Equity Risk Premium estimates*
Academics	5.6%	1.6%
Analysts	5.0%	1.1%
Companies	5.5%	1.6%

The range on equity risk premiums in use is also substantial, with a low of 1.5% and a high of 15%, often citing the same sources. The same authors also report survey responses from the same groups (academics, analysts and companies) in 88 countries in 2014 and note that those in emerging markets use higher risk premiums (not surprisingly) than those in developed markets.[49]

[46] Welch, I., 2000, *Views of Financial Economists on the Equity Premium and on Professional Controversies*, Journal of Business, v73, 501-537.

[47] Fernandez, P., 2010, *The Equity Premium in 150 Textbooks*, Working Paper, http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1473225. He notes that the risk premium actually varies within the book in as many as a third of the textbooks surveyed.

[48] Fernandez, P., J. Aguirreamalloa and L. Corres, 2011, Equity Premium used in 2011 for the USA by Analysts, Companies and Professors: A Survey, Working Paper, http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1805852&rec=1&srcabs=1822182.

[49] Fernandez, P., P. Linares and I.F. Acin, 2014, Market Risk Premium used in 88 countries in 2014, A Survey with 8228 Answers, http://ssrn.com/abstract=2450452.

Historical Premiums

While our task is to estimate equity risk premiums in the future, much of the data we use to make these estimates is in the past. Most investors and managers, when asked to estimate risk premiums, look at historical data. In fact, the most widely used approach to estimating equity risk premiums is the historical premium approach, where the actual returns earned on stocks over a long time period is estimated, and compared to the actual returns earned on a default-free (usually government security). The difference, on an annual basis, between the two returns is computed and represents the historical risk premium. In this section, we will take a closer look at the approach.

Estimation Questions and Consequences

While users of risk and return models may have developed a consensus that historical premium is, in fact, the best estimate of the risk premium looking forward, there are surprisingly large differences in the actual premiums we observe being used in practice, with the numbers ranging from 3% at the lower end to 12% at the upper end. Given that we are almost all looking at the same historical data, these differences may seem surprising. There are, however, three reasons for the divergence in risk premiums: different time periods for estimation, differences in riskfree rates and market indices and differences in the way in which returns are averaged over time.

1. Time Period

Even if we agree that historical risk premiums are the best estimates of future equity risk premiums, we can still disagree about how far back in time we should go to estimate this premium. For decades, Ibbotson Associates was the most widely used estimation service, reporting stock return data and risk free rates going back to 1926,[50] and Duff and Phelps now provides the same service[51]. There are other less widely used databases that go further back in time to 1871 or even to 1792.[52]

While there are many analysts who use all the data going back to the inception date, there are almost as many analysts using data over shorter time periods, such as fifty, twenty or even ten years to come up with historical risk premiums. The rationale

[50] Ibbotson Stocks, Bonds, Bills and Inflation Yearbook (SBBI), 2011 Edition, Morningstar.

[51] Duff and Phelps, 2014 Valuation Handbook, Industry Cost of Capital.

[52] Siegel, in his book, Stocks for the Long Run, estimates the equity risk premium from 1802-1870 to be 2.2% and from 1871 to 1925 to be 2.9%. (Siegel, Jeremy J., Stocks for the Long Run, Second Edition, McGraw Hill, 1998). Goetzmann and Ibbotson estimate the premium from 1792 to 1925 to be 3.76% on an arithmetic average basis and 2.83% on a geometric average basis. Goetzmann. W.N. and R. G. Ibbotson, 2005, History and the Equity Risk Premium, Working Paper, Yale University. Available at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=702341.

presented by those who use shorter periods is that the risk aversion of the average investor is likely to change over time, and that using a shorter and more recent time period provides a more updated estimate. This has to be offset against a cost associated with using shorter time periods, which is the greater noise in the risk premium estimate. In fact, given the annual standard deviation in stock returns[53] between 1928 and 2014 of 19.90% (approximated to 20%), the standard error associated with the risk premium estimate can be estimated in table 2 follows for different estimation periods:[54]

Table 2: Standard Errors in Historical Risk Premiums

Estimation Period	*Standard Error of Risk Premium Estimate*
5 years	20%/ √5 = 8.94%
10 years	20%/ √10 = 6.32%
25 years	20% / √25 = 4.00%
50 years	20% / √50 = 2.83%
80 years	20% / √80 = 2.23%

Even using all of the entire data (about 85 years) yields a substantial standard error of 2.2%. Note that that the standard errors from ten-year and twenty-year estimates are likely to be almost as large or larger than the actual risk premium estimated. This cost of using shorter time periods seems, in our view, to overwhelm any advantages associated with getting a more updated premium.

What are the costs of going back even further in time (to 1871 or before)? First, the data is much less reliable from earlier time periods, when trading was lighter and record keeping more haphazard. Second, and more important, the market itself has changed over time, resulting in risk premiums that may not be appropriate for today. The U.S. equity market in 1871 more closely resembled an emerging market, in terms of volatility and risk, than a mature market. Consequently, using the earlier data may yield premiums that have little relevance for today's markets.

There are two other solutions offered by some researchers. The first is to break the annual data down into shorter return intervals – quarters or even months – with the intent of increasing the data points over any given time period. While this will increase

[53] For the historical data on stock returns, bond returns and bill returns check under "updated data" in http://www.damodaran.com.

[54] The standard deviation in annual stock returns between 1928 and 2014 is 19.90%; the standard deviation in the risk premium (stock return – bond return) is a little higher at 21.59%. These estimates of the standard error are probably understated, because they are based upon the assumption that annual returns are uncorrelated over time. There is substantial empirical evidence that returns are correlated over time, which would make this standard error estimate much larger. The raw data on returns is provided in Appendix 1.

the sample size, the effect on the standard error will be minimal.[55] The second is to use the entire data but to give a higher weight to more recent data, thus getting more updated premiums while preserving the data. While this option seems attractive, weighting more recent data will increase the standard error of the estimate. After all, using only the last ten years of data is an extreme form of time weighting, with the data during that period being weighted at one and the data prior to the period being weighted at zero.

2. Riskfree Security and Market Index

The second estimation question we face relates to the riskfree rate. We can compare the expected return on stocks to either short-term government securities (treasury bills) or long term government securities (treasury bonds) and the risk premium for stocks can be estimated relative to either. Given that the yield curve in the United States has been upward sloping for most of the last eight decades, the risk premium is larger when estimated relative to short term government securities (such as treasury bills) than when estimated against treasury bonds.

Some practitioners and a surprising number of academics (and textbooks) use the treasury bill rate as the riskfree rate, with the alluring logic that there is no price risk in a treasury bill, whereas the price of a treasury bond can be affected by changes in interest rates over time. That argument does make sense, but only if we are interested in a single period equity risk premium (say, for next year). If your time horizon is longer (say 5 or 10 years), it is the treasury bond that provides the more predictable returns.[56] Investing in a 6-month treasury bill may yield a guaranteed return for the next six months, but rolling over this investment for the next five years will create reinvestment risk. In contrast, investing in a ten-year treasury bond, or better still, a ten-year zero coupon bond will generate a guaranteed return for the next ten years.[57]

The riskfree rate chosen in computing the premium has to be consistent with the riskfree rate used to compute expected returns. Thus, if the treasury bill rate is used as the riskfree rate, the premium has to be the premium earned by stocks over that rate. If the treasury bond rate is used as the riskfree rate, the premium has to be estimated relative to that rate. For the most part, in corporate finance and valuation, the riskfree rate will be a

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[55] If returns are uncorrelated over time, the variance in quarterly (monthly) risk premiums will be approximately one-quarter (one twelfth) the variance in annual risk premiums.

[56] For more on risk free rates, see Damodaran, A., 2008, *What is the riskfree rate?* Working Paper, http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1317436.

[57] There is a third choice that is sometimes employed, where the short term government security (treasury bills) is used as the riskfree rate and a "term structure spread" is added to this to get a normalized long term rate.

long-term default-free (government) bond rate and not a short-term rate. Thus, the risk premium used should be the premium earned by stocks over treasury bonds.

The historical risk premium will also be affected by how stock returns are estimated. Using an index with a long history, such as the Dow 30, seems like an obvious solution, but returns on the Dow may not be a good reflection of overall returns on stocks. In theory, at least, we would like to use the broadest index of stocks to compute returns, with two caveats. The first is that the index has to be market-weighted, since the overall returns on equities will be tilted towards larger market cap stocks. The second is that the returns should be free of survivor bias; estimating returns only on stocks that have survived that last 80 years will yield returns that are too high. Stock returns should incorporate those equity investments from earlier years that did not make it through the estimation period, either because the companies in question went bankrupt or were acquired.

Finally, there is some debate about whether the equity risk premiums should be computed using nominal returns or real returns. While the choice clearly makes a difference, if we estimate the return on stocks or the government security return standing alone, it is less of an issue, when computing equity risk premiums, where we look at the difference between the two values.

3. Averaging Approach

The final sticking point when it comes to estimating historical premiums relates to how the average returns on stocks, treasury bonds and bills are computed. The arithmetic average return measures the simple mean of the series of annual returns, whereas the geometric average looks at the compounded return[58]. Many estimation services and academics argue for the arithmetic average as the best estimate of the equity risk premium. In fact, if annual returns are uncorrelated over time, and our objective was to estimate the risk premium for the next year, the arithmetic average is the best and most unbiased estimate of the premium. There are, however, strong arguments that can be made for the use of geometric averages. First, empirical studies seem to indicate that returns on stocks are negatively correlated[59] over time. Consequently, the arithmetic

[58] The compounded return is computed by taking the value of the investment at the start of the period (Value_0) and the value at the end (Value_N), and then computing the following:

$$\text{Geometric Average} = \left(\frac{\text{Value}_N}{\text{Value}_0}\right)^{1/N} - 1$$

[59] In other words, good years are more likely to be followed by poor years, and vice versa. The evidence on negative serial correlation in stock returns over time is extensive, and can be found in Fama and French (1988). While they find that the one-year correlations are low, the five-year serial correlations are strongly

average return is likely to over state the premium. Second, while asset pricing models may be single period models, the use of these models to get expected returns over long periods (such as five or ten years) suggests that the estimation period may be much longer than a year. In this context, the argument for geometric average premiums becomes stronger. Indro and Lee (1997) compare arithmetic and geometric premiums, find them both wanting, and argue for a weighted average, with the weight on the geometric premium increasing with the time horizon.[60]

In closing, the averaging approach used clearly matters. Arithmetic averages will be yield higher risk premiums than geometric averages, but using these arithmetic average premiums to obtain discount rates, which are then compounded over time, seems internally inconsistent. In corporate finance and valuation, at least, the argument for using geometric average premiums as estimates is strong.

Estimates for the United States

The questions of how far back in time to go, what riskfree rate to use and how to average returns (arithmetic or geometric) may seem trivial until you see the effect that the choices you make have on your equity risk premium. Rather than rely on the summary values that are provided by data services, we will use raw return data on stocks, treasury bills and treasury bonds from 1928 to 2014 to make this assessment.[61] In figure 3, we begin with a chart of the annual returns on stock, treasury bills and bonds for each year:

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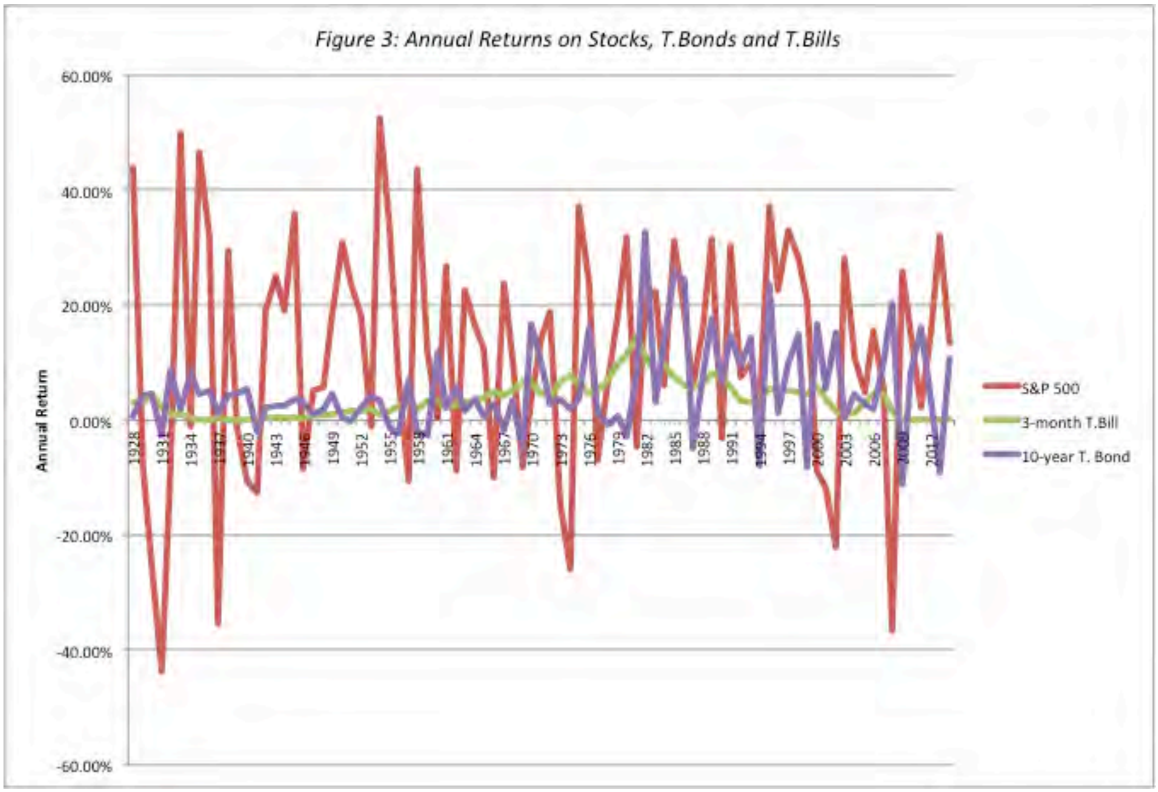
negative for all size classes. Fama, E.F. and K.R. French, 1992, *The Cross-Section of Expected Returns*, Journal of Finance, Vol 47, 427-466.

[60] Indro, D.C. and W. Y. Lee, 1997, *Biases in Arithmetic and Geometric Averages as Estimates of Long-run Expected Returns and Risk Premium*, Financial Management, v26, 81-90.

[61] The raw data for treasury rates is obtained from the Federal Reserve data archive (http://research.stlouisfed.org/fred2/) at the Fed site in St. Louis, with the 3-month treasury bill rate used for treasury bill returns and the 10-year treasury bond rate used to compute the returns on a constant maturity 10-year treasury bond. The stock returns represent the returns on the S&P 500. Appendix 1 provides the returns by year on stocks, bonds and bills, by year, from 1928 through the current year.



It is difficult to make much of this data other than to state the obvious, which is that stock returns are volatile, which is at the core of the demand for an equity risk premium in the first place. In table 3, we present summary statistics for stock, 3-month Treasury bill and ten-year Treasury bond returns from 1928 to 2014:

Table 3: Summary Statistics- U.S. Stocks, T.Bills and T. Bonds- 1928-2014

	Stocks	T. Bills	T. Bonds
Mean	11.53%	3.53%	5.28%
Standard Error	2.13%	0.33%	0.84%
Median	14.22%	3.11%	3.61%
Standard Deviation	19.90%	3.06%	7.83%
Kurtosis	2.98	3.82	4.39
Skewness	-0.41	0.96	0.94
Minimum	-43.84%	0.03%	-11.12%
Maximum	52.56%	14.30%	32.81%
25th percentile	-1.19%	1.01%	2.20%
75th percentile	26.11%	5.32%	8.93%

While U.S. equities have delivered much higher returns than treasuries over this period, they have also been more volatile, as evidenced both by the higher standard deviation in returns and by the extremes in the distribution. Using this table, we can take a first shot at estimating a risk premium by taking the difference between the average returns on stocks and the average return on treasuries, yielding a risk premium of 8.00% for stocks over T.Bills (11.53%-3.53%) and 6.25% for stocks over T.Bonds (11.53%-5.28%). Note, though, that these represent arithmetic average, long-term premiums for stocks over treasuries.

How much will the premium change if we make different choices on historical time periods, riskfree rates and averaging approaches? To answer this question, we estimated the arithmetic and geometric risk premiums for stocks over both treasury bills and bonds over different time periods in table 4, with standard errors reported in brackets below the arithmetic averages:

Table 4: Historical Equity Risk Premiums (ERP) –Estimation Period, Riskfree Rate and Averaging Approach

	Arithmetic Average		*Geometric Average*	
	Stocks - T. Bills	Stocks - T. Bonds	Stocks - T. Bills	Stocks - T. Bonds
1928-2014	8.00%	6.25%	6.11%	4.60%
	(2.17%)	(2.32%)		
1965-2014	6.19%	4.12%	4.84%	3.14%
	(2.42%	(2.74%)		
2005-2014	7.94%	4.06%	6.18%	2.73%
	(6.05%)	(8.65%)		

Note that even with only three slices of history considered, the premiums range from 2.73% to 8.00%, depending upon the choices made. If we take the earlier discussion about the “right choices” to heart, and use a long-term geometric average premium over the long-term rate as the risk premium to use in valuation and corporate finance, the equity risk premium that we would use would be 4.60%. The caveats that we would offer, though, are that this estimate comes with significant standard error and is reflective of time periods (such as 1920s and 1930s) when the U.S. equity market (and investors in it) had very different characteristics.

There have been attempts to extend the historical time period to include years prior to 1926 (the start of the Ibbotson database). Goetzmann and Jorion (1999) estimate the returns on stocks and bonds between 1792 and 1925 and report an arithmetic average premium, for stocks over bonds, of 2.76% and a geometric average premium of 2.83%.[62] The caveats about data reliability and changing market characteristics that we raised in an earlier section apply to these estimates.

[62] Jorion, Philippe and William N. Goetzmann, 1999, *Global Stock Markets in the Twentieth Century*, Journal of Finance, 54(3), 953-980.

There is one more troublesome (or at least counter intuitive) characteristic of historical risk premiums. The geometric average equity risk premium through the end of 2007 was 4.79%, higher than the 3.88% estimated though the end of 2008; in fact, every single equity risk premium number in this table would have been much higher, if we had stopped with 2007 as the last year. Adding the data for 2008, an abysmal year for stocks and a good year for bonds, lowers the historical premium dramatically, even when computed using a long period of history. In effect, the historical risk premium approach would lead investors to conclude, after one of worst stock market crisis in several decades, that stocks were less risky than they were before the crisis and that investors should therefore demand lower premiums. In contrast, adding the data for 2009, a good year for stocks (+25.94%) and a bad year for bonds (-11.12%) would have increased the equity risk premium from 3.88% to 4.29%. As a general rule, historical risk premiums will tend to rise when markets are buoyant and investors are less risk averse and will fall as markets collapse and investor fears rise.

Global Estimates

If it is difficult to estimate a reliable historical premium for the US market, it becomes doubly so when looking at markets with short, volatile and transitional histories. This is clearly true for emerging markets, where equity markets have often been in existence for only short time periods (Eastern Europe, China) or have seen substantial changes over the last few years (Latin America, India). It also true for many West European equity markets. While the economies of Germany, Italy and France can be categorized as mature, their equity markets did not share the same characteristics until recently. They tended to be dominated by a few large companies, many businesses remained private, and trading was thin except on a few stocks.

Notwithstanding these issues, services have tried to estimate historical risk premiums for non-US markets with the data that they have available. To capture some of the danger in this practice, Table 5 summarizes historical arithmetic average equity risk premiums for major non-US markets below for 1976 to 2001, and reports the standard error in each estimate:[63]

Table 5: Risk Premiums for non-US Markets: 1976- 2001

Country	*Weekly average*	*Weekly standard deviation*	*Equity Risk Premium*	*Standard error*
Canada	0.14%	5.73%	1.69%	3.89%

[63] Salomons, R. and H. Grootveld, 2003, *The equity risk premium: Emerging vs Developed Markets,* Emerging Markets Review, v4, 121-144.

France	0.40%	6.59%	4.91%	4.48%
Germany	0.28%	6.01%	3.41%	4.08%
Italy	0.32%	7.64%	3.91%	5.19%
Japan	0.32%	6.69%	3.91%	4.54%
UK	0.36%	5.78%	4.41%	3.93%
India	0.34%	8.11%	4.16%	5.51%
Korea	0.51%	11.24%	6.29%	7.64%
Chile	1.19%	10.23%	15.25%	6.95%
Mexico	0.99%	12.19%	12.55%	8.28%
Brazil	0.73%	15.73%	9.12%	10.69%

Before we attempt to come up with rationale for why the equity risk premiums vary across countries, it is worth noting the magnitude of the standard errors on the estimates, largely because the estimation period includes only 25 years. Based on these standard errors, we cannot even reject the hypothesis that the equity risk premium in each of these countries is zero, let alone attach a value to that premium.

If the standard errors on these estimates make them close to useless, consider how much more noise there is in estimates of historical risk premiums for some emerging market equity markets, which often have a reliable history of ten years or less, and very large standard deviations in annual stock returns. Historical risk premiums for emerging markets may provide for interesting anecdotes, but they clearly should not be used in risk and return models.

The survivor bias

Given how widely the historical risk premium approach is used, it is surprising that the flaws in the approach have not drawn more attention. Consider first the underlying assumption that investors' risk premiums have not changed over time and that the average risk investment (in the market portfolio) has remained stable over the period examined. We would be hard pressed to find anyone who would be willing to sustain this argument with fervor. The obvious fix for this problem, which is to use a more recent time period, runs directly into a second problem, which is the large noise associated with historical risk premium estimates. While these standard errors may be tolerable for very long time periods, they clearly are unacceptably high when shorter periods are used.

Even if there is a sufficiently long time period of history available, and investors' risk aversion has not changed in a systematic way over that period, there is a final problem. Markets such as the United States, which have long periods of equity market history, represent "survivor markets". In other words, assume that one had invested in the largest equity markets in the world in 1926, of which the United States was one.[64] In the period extending from 1926 to 2000, investments in many of the other equity markets would have earned much smaller premiums than the US equity market, and some of them would have resulted in investors earning little or even negative returns over the period. Thus, the survivor bias will result in historical premiums that are larger than expected premiums for markets like the United States, even assuming that investors are rational and factor risk into prices.

How can we mitigate the survivor bias? One solution is to look at historical risk premiums across multiple equity markets across very long time periods. In the most comprehensive attempt of this analysis, Dimson, Marsh and Staunton (2002, 2008) estimated equity returns for 17 markets and obtained both local and a global equity risk premium.[65] In their most recent update in 2015, they provide the risk premiums from 1900 to 2014 for 20 markets, with standard errors on each estimate (reported in table 6):[66]

Table 6: Historical Risk Premiums across Equity Markets – 1900 – 2014 (in %)

	Stocks minus Short term Governments				*Stocks minus Long term Governments*			
Country	*Geometric Mean*	*Arithmetic Mean*	*Standard Error*	*Standard Deviation*	*Geometric Mean*	*Arithmetic Mean*	*Standard Error*	*Standard Deviation*
Australia	6.6%	8.1%	1.6%	17.5%	5.6%	7.5%	1.9%	20.0%
Austria	5.5%	10.4%	3.5%	37.4%	2.5%	21.5%	14.4%	153.5%
Belgium	3.0%	5.4%	2.2%	23.9%	2.3%	4.4%	2.0%	21.1%
Canada	4.2%	5.6%	1.6%	16.9%	3.5%	5.1%	1.7%	18.2%
Denmark	3.1%	5.0%	1.9%	20.5%	2.0%	3.6%	1.7%	17.9%
Finland	5.9%	9.5%	2.8%	29.9%	5.1%	8.7%	2.8%	30.1%
France	6.1%	8.7%	2.3%	24.2%	3.0%	5.3%	2.1%	22.8%
Germany	6.0%	9.9%	3.0%	31.5%	5.0%	8.4%	2.7%	28.6%
Ireland	3.5%	5.8%	2.0%	21.3%	2.6%	4.5%	1.8%	19.6%
Italy	5.7%	9.5%	2.9%	31.6%	3.1%	6.5%	2.7%	29.5%

[64] Jorion, Philippe and William N. Goetzmann, 1999, *Global Stock Markets in the Twentieth Century*, Journal of Finance, 54(3), 953-980. They looked at 39 different equity markets and concluded that the US was the best performing market from 1921 to the end of the century. They estimated a geometric average premium of 3.84% across all of the equity markets that they looked at, rather than just the US and estimated that the survivor bias added 1.5% to the US equity risk premium (with arithmetic averages) and 0.9% with geometric averages.

[65] Dimson, E.,, P Marsh and M Staunton, 2002, *Triumph of the Optimists: 101 Years of Global Investment Returns*, Princeton University Press, NJ; Dimson, E.,, P Marsh and M Staunton, 2008, The Worldwide Equity Risk Premium: a smaller puzzle, Chapter 11 in the Handbook of the Equity Risk Premium, edited by R. Mehra, Elsevier.

[66] *Credit Suisse Global Investment Returns Sourcebook,* 2015, Credit Suisse/ London Business School. Summary data is accessible at the Credit Suisse website.

Japan	6.1%	9.3%	2.6%	27.7%	5.1%	9.1%	3.0%	32.6%
Netherlands	4.4%	6.5%	2.1%	22.5%	3.2%	5.6%	2.1%	22.3%
New Zealand	4.4%	5.9%	1.7%	18.1%	3.9%	5.5%	1.7%	17.9%
Norway	3.1%	5.9%	2.4%	26.1%	2.3%	5.3%	2.6%	27.7%
South Africa	6.3%	8.4%	2.0%	21.7%	5.4%	7.1%	1.8%	19.6%
Spain	3.4%	5.5%	2.0%	21.6%	1.9%	3.9%	1.9%	20.7%
Sweden	3.9%	5.9%	1.9%	20.5%	3.0%	5.3%	2.0%	21.5%
Switzerland	3.7%	5.3%	1.7%	18.7%	2.1%	3.6%	1.6%	17.5%
U.K.	4.3%	6.1%	1.8%	19.7%	3.7%	5.0%	1.6%	17.3%
U.S.	5.6%	7.5%	1.8%	19.6%	4.4%	6.5%	1.9%	20.7%
Europe	3.4%	5.2%	1.8%	19.3%	3.1%	4.4%	1.5%	16.1%
World-ex U.S.	3.6%	5.2%	1.7%	18.6%	2.8%	3.9%	1.4%	14.7%
World	4.3%	5.7%	1.6%	17.0%	3.2%	4.5%	1.5%	15.5%

In making comparisons of the numbers in this table to prior years, note that this database was modified in two ways: the world estimates are now weighted by market capitalization and the issue of survivorship bias has been dealt with frontally by incorporating the return histories of three markets (Austria, China and Russia) where equity investors would have lost their entire investment during the century. Note that the risk premiums, averaged across the markets, are lower than risk premiums in the United States. For instance, the geometric average risk premium for stocks over long-term government bonds, across the non-US markets, is only 2.8%, lower than the 4.4% for the US markets. The results are similar for the arithmetic average premium, with the average premium of 3.9% across non-US markets being lower than the 6.5% for the United States. In effect, the difference in returns captures the survivorship bias, implying that using historical risk premiums based only on US data will results in numbers that are too high for the future. Note that the "noise" problem persists, even with averaging across 20 markets and over 112 years. The standard error in the global equity risk premium estimate is 1.5%, suggesting that the range for the historical premium remains a large one.

Decomposing the historical equity risk premium

As the data to compute historical risk premiums has become richer, those who compute historical risk premiums have also become more creative, breaking down the historical risk premiums into its component parts, partly to understand the drivers of the premiums and partly to get better predictors for the future. Ibbotson and Chen (2013) started this process by breaking down the historical risk premium into four components:[67]

1. The income return is the return earned by stockholders from dividends and stock buybacks.
2. The second is the inflation rate during the estimation time period
3. The third is the growth rate in real earnings (earnings cleansed of inflation) during the estimation period
4. The change in PE ratio over the period, since an increase (decrease) in the PE ratio will raise (lower) the realized return on stocks during an estimation period.

Using the argument that the first three are sustainable and generated by "the productivity of corporations in the economy" and the fourth is not, they sum up the first three components to arrive at what they term a "supply-side" equity risk premium.

Following the same playbook, Dimson, Marsh and Staunton decompose the realized equity risk premium in each market into three components: the level of dividends, the growth in those dividends and the effects on stock price of a changing multiple for dividend (price to dividend ratio). For the United States, they attribute 1.67% of the overall premium of 5.59% (for stocks over treasury bills) to growth in real dividends and 0.57% to expansion in the price to dividend ratio. Of the global premium of 4.32%, 0.57% can be attributed to growth in dividends and 0.53% to increases in the price to dividend ratio.

While there is some value in breaking down a historical risk premium, notice that none of these decompositions remove the basic problems with historical risk premiums, which is that they are backward looking and noisy. Thus, a supply side premium has to come with all of the caveats that a conventional historical premium with the added noise created by the decomposition, i.e, measuring inflation and real earnings.

Historical Premium Plus

If we accept the proposition that historical risk premiums are the best way to estimate future risk premiums and also come to terms with the statistical reality that we need long time periods of history to get reliable estimates, we are trapped when it comes to estimating risk premiums in most emerging markets, where historical data is either non-existent or unreliable. Furthermore, the equity risk premium that we estimate becomes the risk premium that we use for all stocks within a market, no matter what their

[67] Ibbotson, R. and P. Chen, 2003, *Long-Run Stock Returns: Participating in the Real Economy*, Financial Analysts Journal, pp.88-98.

differences are on market capitalization and growth potential; in effect, we assume that the betas we use will capture differences in risk across companies.

In this section, we consider one way out of this box, where we begin with the US historical risk premium (4.60%) or the global premium from the DMS data (3.20%) as the base premium for a mature equity market and then build additional premiums for riskier markets or classes of stock. For the first part of this section, we stay within the US equity market and consider the practice of adjusting risk premiums for company-specific characteristics, with market capitalization being the most common example. In the second part, we extend the analysis to look at emerging markets in Asia, Latin American and Eastern Europe, and take a look at the practice of estimating country risk premiums that augment the US equity risk premium. Since many of these markets have significant exposures to political and economic risk, we consider two fundamental questions in this section. The first relates to whether there should be an additional risk premium when valuing equities in these markets, because of the country risk. As we will see, the answer will depend upon whether we think country risk is diversifiable or non-diversifiable, view markets to be open or segmented and whether we believe in a one-factor or a multi-factor model. The second question relates to estimating equity risk premiums for emerging markets. Depending upon our answer to the first question, we will consider several solutions.

Small cap and other risk premiums

In computing an equity risk premium to apply to all investments in the capital asset pricing model, we are essentially assuming that betas carry the weight of measuring the risk in individual firms or assets, with riskier investments having higher betas than safer investments. Studies of the efficacy of the capital asset pricing model over the last three decades have cast some doubt on whether this is a reasonable assumption, finding that the model understates the expected returns of stocks with specific characteristics; small market cap companies and companies low price to book ratios, in particular, seem to earn much higher returns than predicted by the CAPM. It is to counter this finding that many practitioners add an additional premium to the required returns (and costs of equity) of smaller market cap companies.

The CAPM and Market Capitalization

In one of very first studies to highlight the failure of the traditional capital asset pricing model to explain returns at small market cap companies, Banz (1981) looked returns on stocks from 1936-1977 and concluded that investing in the smallest companies (the bottom 20% of NYSE firms in terms of capitalization) would have generated about 6% more, after adjusting for beta risk, than larger cap companies.[68] In the years since, there has been substantial research on both the origins and durability of the small cap premium, with mixed conclusions. First, there is evidence of a small firm premium in markets outside the United States as well. Studies find small cap premiums of about 7% from 1955 to 1984 in the United Kingdom,[69] 8.8% in France and 3% in Germany,[70] and a premium of 5.1% for Japanese stocks between 1971 and 1988.[71] Dimson, March and Staunton (2015), in their updated assessment of equity risk premiums in global markets, also compute small cap premiums in 23 markets over long time periods (which range from 113 years for some markets to less for others). Of the 23 markets, small cap stocks have not outperformed the rest of the market in only Norway, Finland and the Netherlands; the small cap premium, over the long term, has been higher in developed markets than in emerging markets. Second, while the small cap premium has been persistent in US equity markets, it has also been volatile, with large cap stocks outperforming small cap stocks for extended periods. In figure 4, we look at the difference in returns between small cap (defined as bottom 10% of firms in terms of market capitalization) and all US stocks between 1927 and 2014; note that the premium was pronounced in the 1970s and disappeared for much of the 1980s.[72]

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[68] Banz, R., 1981, *The Relationship between Return and Market Value of Common Stocks*, Journal of Financial Economics, v9.

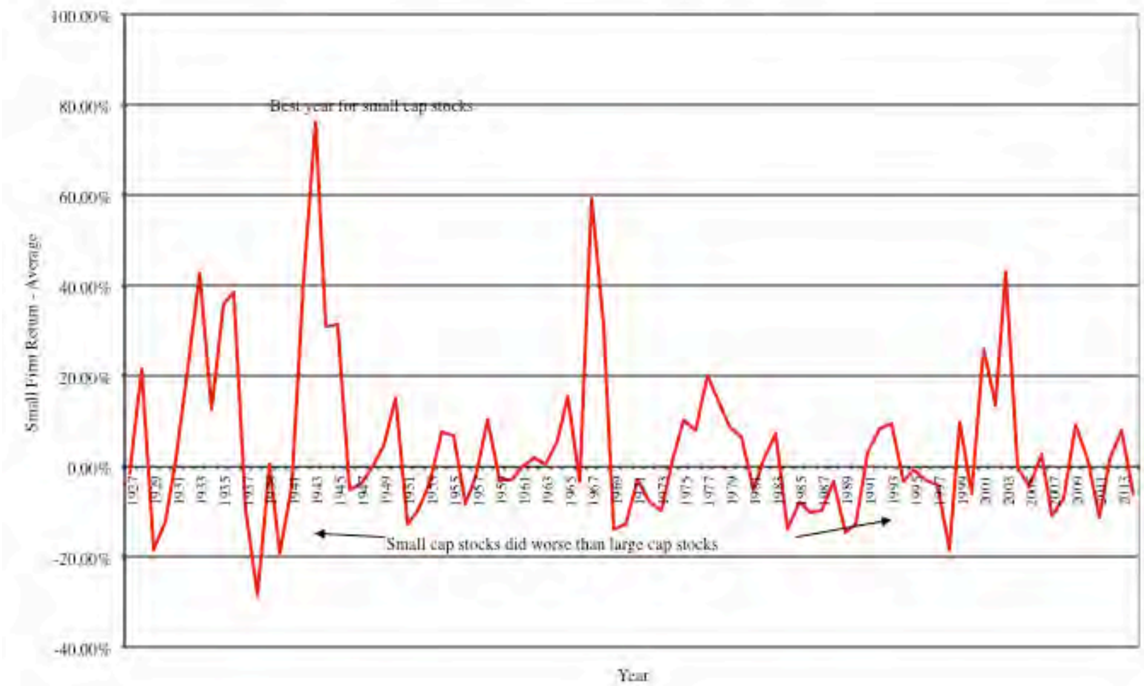
[69] Dimson, E. and P.R. Marsh, 1986, *Event Studies and the Size Effect: The Case of UK Press Recommendations*, Journal of Financial Economics, v17, 113-142.

[70] Bergstrom,G.L., R.D. Frashure and J.R. Chisholm, 1991, *The Gains from international small-company diversification* in Global Portfolios: Quantiative Strategies for Maximum Performance, Edited By R.Z. Aliber and B.R. Bruce, Business One Irwin, Homewood.

[71] Chan, L.K., Y. Hamao, and J. Lakonishok, 1991, *Fundamentals and Stock Returns in Japan*, Journal of Finance. v46. 1739-1789.

[72] The raw data for this table is obtained from Professor Ken French's website at Dartmouth. These premiums are based on value weighted portfolios. If equally weighted portfolios are used, the small cap premium is larger (almost 10.71%).

Figure 4: Small Firm Premium over time- 1927 -2014



The average premium for stocks in the smallest companies, in terms of market capitalization, between 1926 and 2013 was 4.33%, but the standard error in that estimate is 1.96%. Third, much of the premium is generated in one month of the year: January. As Figure 5 shows, eliminating that month from our calculations would essentially dissipate the entire small stock premium. That would suggest that size itself is not the source of risk, since small firms in January remain small firms in the rest of the year, but that the small firm premium, if it exists, comes from some other risk that is more pronounced or prevalent in January than in the rest of the year.

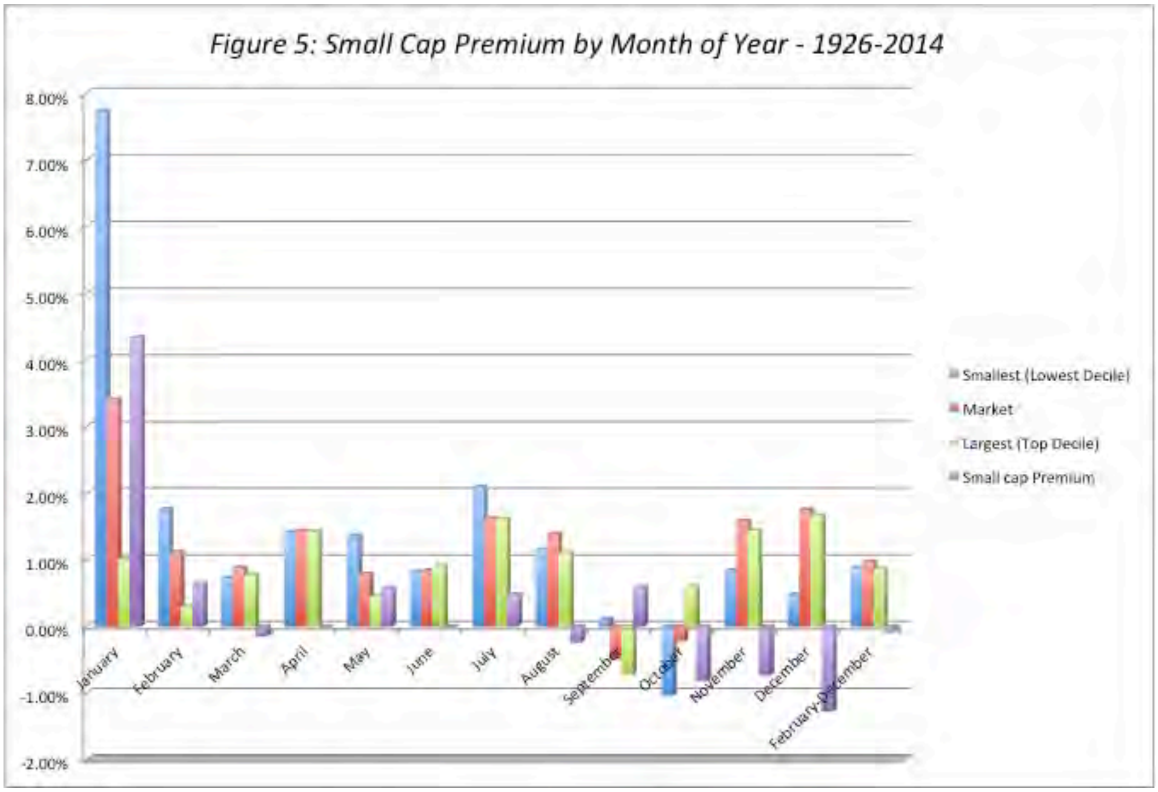


Figure 5: Small Cap Premium by Month of Year - 1926-2014

Source: Raw data from Ken French

Finally, a series of studies have argued that market capitalization, by itself, is not the reason for excess returns but that it is a proxy for other ignored risks such as illiquidity and poor information.

In summary, while the empirical evidence supports the notion that small cap stocks have earned higher returns after adjusting for beta risk than large cap stocks, it is not as conclusive, nor as clean as it was initially thought to be. The argument that there is, in fact, no small cap premium and that we have observed over time is just an artifact of history cannot be rejected out of hand.

The Small Cap Premium

If we accept the notion that there is a small cap premium, there are two ways in which we can respond to the empirical evidence that small market cap stocks seem to earn higher returns than predicted by the traditional capital asset pricing model. One is to view this as a market inefficiency that can be exploited for profit: this, in effect, would require us to load up our portfolios with small market cap stocks that would then proceed to deliver higher than expected returns over long periods. The other is to take the excess returns as evidence that betas are inadequate measures of risk and view the additional returns are compensation for the missed risk. The fact that the small cap premium has endured for as long as it has suggests that the latter is the more reasonable path to take.

If CAPM betas understate the true risk of small cap stocks, what are the solutions? The first is to try and augment the model to reflect the missing risk, but this would require being explicit about this risk. For instance, there are models that include additional factors for illiquidity and imperfect information that claim to do better than the CAPM in predicting future returns. The second and simpler solution that is adopted by many practitioners is to add a premium to the expected return (from the CAPM) of small cap stocks. To arrive at this premium, analysts look at historical data on the returns on small cap stocks and the market, adjust for beta risk, and attribute the excess return to the small cap effect. As we noted earlier, using the data from 1926-2014, we would estimate a small cap premium of 4.33%. Duff and Phelps present a richer set of estimates, where the premiums are computed for stocks in 25 different size classes (with size measured on eight different dimensions including market capitalization, book value and net income). Using the Fama/French data, we present excess returns for firms broken down by ten market value classes in Table 7, with the standard error for each estimate.

Table 7: Excess Returns by Market Value Class: US Stocks from 1927 – 2014

Excess Return = Return on Portfolio – Return on Market

Decile	*Average*	*Standard Error*	*Maximum*	*Minimum*
Smallest	4.33%	1.96%	76.28%	-28.42%
2	1.63%	1.14%	41.25%	-17.96%
3	1.47%	0.77%	41.98%	-13.54%
4	0.64%	0.55%	15.56%	-7.33%
5	0.05%	0.53%	11.63%	-16.05%
6	-0.01%	0.51%	15.21%	-14.01%
7	-0.51%	0.55%	7.48%	-19.50%
8	-1.50%	0.81%	11.20%	-29.42%
9	-2.13%	1.02%	21.96%	-36.09%
Largest	-3.98%	1.56%	31.29%	-65.57%

Note that the market capitalization effect shows up at both extremes – the smallest firms earn higher returns than expected whereas the largest firms earn lower returns than expected. The small firm premium is statistically significant only for the lowest and three highest size deciles.

Perils of the approach

While the small cap premium may seem like a reasonable way of dealing with the failure of the CAPM to capture the risk in smaller companies, there are significant costs to using the approach.

a. Standard Error on estimates: One of the dangers we noted with using historical risk premiums is the high standard error in our estimates. This danger is magnified when we look at sub-sets of stocks, based on market capitalization or any other characteristic, and extrapolate past returns. The standard errors on the small cap premiums that are estimated are likely to be significant, as is evidenced in table 7.

b. Small versus Large Cap: At least in its simplest form, the small cap premium adjustment requires us to divide companies into small market companies and the rest of the market, with stocks falling on one side of the line having much higher required returns (and costs of equity) than stocks falling on the other side.

c. Understanding Risk: Even in its more refined format, where the required returns are calibrated to market cap, using small cap premiums allows analysts to evade basic questions about what it is that makes smaller cap companies riskier, and whether these factors may vary across companies.

d. Small cap companies become large cap companies over time: When valuing companies, we attach high growth rates to revenues, earnings and value over time. Consequently, companies that are small market cap companies now grow to become large market cap companies over time. Consistency demands that we adjust the small cap premium as we go further into a forecast period.

e. Other risk premiums: Using a small cap premium opens the door to other premiums being used to augment expected returns. Thus, we could adjust expected returns upwards for stocks with price momentum and low price to book ratios, reflecting the excess returns that these characteristics seem to deliver, at least on paper. Doing so will deliver values that are closer to market prices, across assets, but undercuts the rationale for intrinsic valuation, i.e., finding market mistakes.

There is another reason why we are wary about adjusting costs of equity for a small cap effect. If, as is the practice now, we add a small cap premium of between 4% to 5% to the cost of equity of small companies, without attributing this premium to any specific risk factor, we are exposed to the risk of double counting risk. For instance, assume that the small cap premium that we have observed over the last few decades is attributable to the lower liquidity (and higher transactions costs) of trading small cap stocks. Adding that premium on to the discount rate will reduce the estimated values of small cap and private businesses. If we attach an illiquidity discount to this value, we are double counting the effect of illiquidity.

The small cap premium is firmly entrenched in practice, with analysts generally adding on 4% to 5% to the conventional cost of equity for small companies, with the definition of small shifting from analyst to analyst. Even if you believe that small cap companies are more exposed to market risk than large cap ones, this is an extremely sloppy and lazy way of dealing with that risk, since risk ultimately has to come from something fundamental (and size is not a fundamental factor). Thus, if you believe that small cap stocks are more prone to failure or distress, it behooves you to measure that risk directly and incorporate it into the cost of equity. If it is illiquidity that is at the heart of the small cap premium, then you should be measuring liquidity risk and incorporating it into the cost of equity and you certainly should not be double counting the risk by first incorporating a small cap premium into the discount rate and then applying an illiquidity discount to value.

The question of whether there is a small cap premium ultimately is not a theoretical one but a practical one. While those who incorporate a small cap premium justify the practice with the historical data, we will present a more forward-looking approach, where we use market pricing of small capitalization stocks to see if the market builds in a small cap premium, later in this paper.

Country Risk Premiums

As both companies and investors get used to the reality of a global economy, they have also been forced to confront the consequences of globalization for equity risk premiums and hurdle rates. Should an investor putting his money in Indian stocks demand a higher risk premium for investing in equities that one investing in German stocks? Should a US consumer product company investing in Brazil demand the same hurdle rates for its Brazilian investments as it does for its US investments? In effect, should we demand one global equity risk premium that we use for investments all over the world or should we use higher equity risk premiums in some markets than in others?

The arguments for no country risk premium

Is there more risk in investing in a Malaysian or Brazilian stock than there is in investing in the United States? The answer, to most, seems to be obviously affirmative, with the solution being that we should use higher equity risk premiums when investing in riskier emerging markets. There are, however, three distinct and different arguments offered against this practice.

1. Country risk is diversifiable

In the risk and return models that have developed from conventional portfolio theory, and in particular, the capital asset pricing model, the only risk that is relevant for purposes of estimating a cost of equity is the market risk or risk that cannot be diversified away. The key question in relation to country risk then becomes whether the additional risk in an emerging market is diversifiable or non-diversifiable risk. If, in fact, the additional risk of investing in Malaysia or Brazil can be diversified away, then there should be no additional risk premium charged. If it cannot, then it makes sense to think about estimating a country risk premium.

But diversified away by whom? Equity in a publicly traded Brazilian, or Malaysian, firm can be held by hundreds or even thousands of investors, some of whom may hold only domestic stocks in their portfolio, whereas others may have more global exposure. For purposes of analyzing country risk, we look at the marginal investor – the investor most likely to be trading on the equity. If that marginal investor is globally diversified, there is at least the potential for global diversification. If the marginal investor does not have a global portfolio, the likelihood of diversifying away country risk declines substantially. Stulz (1999) made a similar point using different terminology.[73] He differentiated between segmented markets, where risk premiums can be different in each market, because investors cannot or will not invest outside their domestic markets, and open markets, where investors can invest across markets. In a segmented market, the marginal investor will be diversified only across investments in that market, whereas in an open market, the marginal investor has the opportunity (even if he or she does not take it) to invest across markets. It is unquestionable that investors today in most markets have more opportunities to diversify globally than they did three decades ago, with international mutual funds and exchange traded funds, and that many more of them take advantage of these opportunities. It is also true still that a significant home bias exists in most investors' portfolios, with most investors over investing in their home markets.

Even if the marginal investor is globally diversified, there is a second test that has to be met for country risk to be diversifiable. All or much of country risk should be country specific. In other words, there should be low correlation across markets. Only then will the risk be diversifiable in a globally diversified portfolio. If, on the other hand, the returns across countries have significant positive correlation, country risk has a market risk component, is not diversifiable and can command a premium. Whether

[73] Stulz, R.M., *Globalization, Corporate finance, and the Cost of Capital,* Journal of Applied Corporate Finance, v12. 8-25.

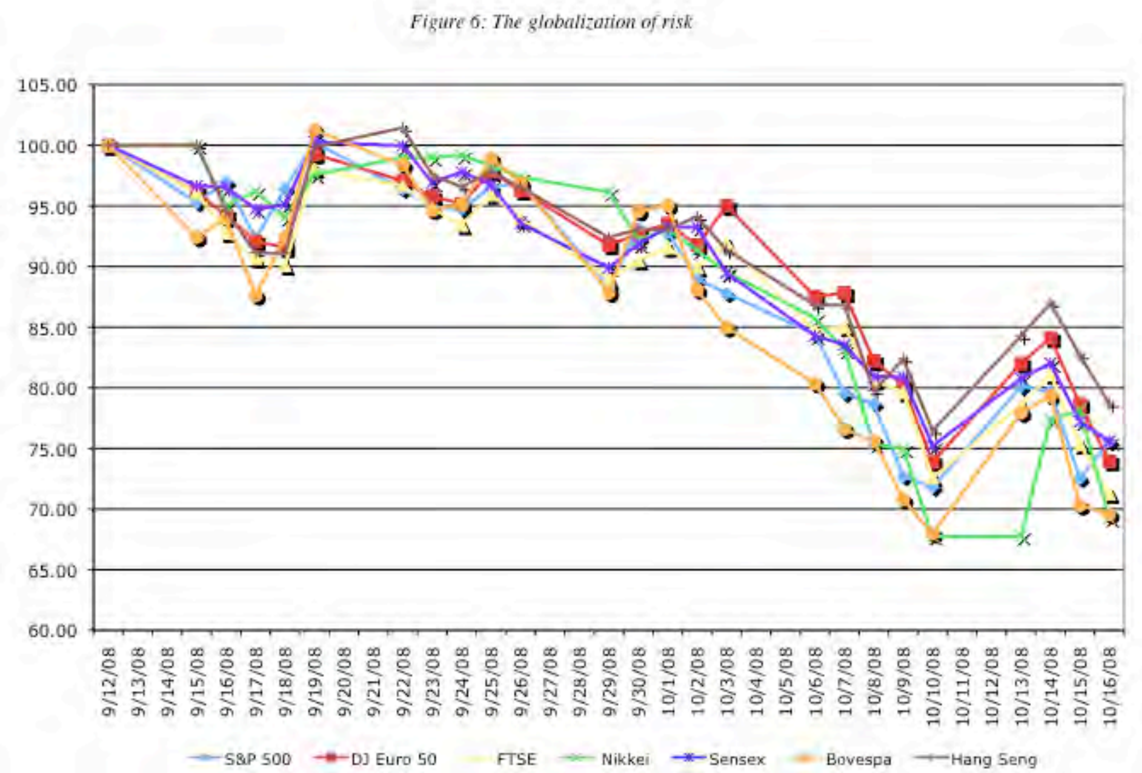
returns across countries are positively correlated is an empirical question. Studies from the 1970s and 1980s suggested that the correlation was low, and this was an impetus for global diversification.[74] Partly because of the success of that sales pitch and partly because economies around the world have become increasingly intertwined over the last decade, more recent studies indicate that the correlation across markets has risen. The correlation across equity markets has been studied extensively over the last two decades and while there are differences, the overall conclusions are as follows:

1. The correlation across markets has increased over time, as both investors and firms have globalized. Yang, Tapon and Sun (2006) report correlations across eight, mostly developed markets between 1988 and 2002 and note that the correlation in the 1998-2002 time period was higher than the correlation between 1988 and 1992 in every single market; to illustrate, the correlation between the Hong Kong and US markets increased from 0.48 to 0.65 and the correlation between the UK and the US markets increased from 0.63 to 0.82.[75] In the global returns sourcebook, from Credit Suisse, referenced earlier for historical risk premiums for different markets, the authors estimate the correlation between developed and emerging markets between 1980 and 2013, and note that it has increased from 0.57 in 1980 to 0.88 in 2013.
2. The correlation across equity markets increases during periods of extreme stress or high volatility.[76] This is borne out by the speed with which troubles in one market, say Russia, can spread to a market with little or no obvious relationship to it, say Brazil. The contagion effect, where troubles in one market spread into others is one reason to be skeptical with arguments that companies that are in multiple emerging markets are protected because of their diversification benefits. In fact, the market crisis in the last quarter of 2008 illustrated how closely bound markets have become, as can be seen in figure 6:

[74] Levy, H. and M. Sarnat, 1970, *International Diversification of Investment Portfolios*, American Economic Review 60(4), 668-75.

[75] Yang, Li , Tapon, Francis and Sun, Yiguo, 2006, *International correlations across stock markets and industries: trends and patterns 1988-2002*, Applied Financial Economics, v16: 16, 1171-1183

[76] Ball, C. and W. Torous, 2000, *Stochastic correlation across international stock markets,* Journal of Empirical Finance. v7, 373-388.



Between September 12, 2008 and October 16, 2008, markets across the globe moved up and down together, with emerging markets showing slightly more volatility.

3. The downside correlation increases more than upside correlation: In a twist on the last point, Longin and Solnik (2001) report that it is not high volatility per se that increases correlation, but downside volatility. Put differently, the correlation between global equity markets is higher in bear markets than in bull markets.[77]
4. Globalization increases exposure to global political uncertainty, while reducing exposure to domestic political uncertainty: In the most direct test of whether we should be attaching different equity risk premiums to different countries due to systematic risk exposure, Brogaard, Dai, Ngo and Zhang (2014) looked at 36 countries from 1991-2010 and measured the exposure of companies in these countries to global political uncertainty and domestic political uncertainty.[78] They find that the costs of capital of companies in integrated markets are more highly

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[77] Longin, F. and B. Solnik, 2001, *Extreme Correlation of International Equity Markets*, Journal of Finance, v56 , pg 649-675.
[78] Brogaard, J., L. Dai, P.T.H. Ngo, B. Zhuang, 2014, *The World Price of Political Uncertainty*, SSRN #2488820.

influenced by global uncertainty (increasing as uncertainty increases) and those in segmented markets are more highly influenced by domestic uncertainty.[79]

2. A Global Capital Asset Pricing Model

The other argument against adjusting for country risk comes from theorists and practitioners who believe that the traditional capital asset pricing model can be adapted fairly easily to a global market. In their view, all assets, no matter where they are traded, should face the same global equity risk premium, with differences in risk captured by differences in betas. In effect, they are arguing that if Malaysian stocks are riskier than US stocks, they should have higher betas and expected returns.

While the argument is reasonable, it flounders in practice, partly because betas do not seem capable of carry the weight of measuring country risk.

1. If betas are estimated against local indices, as is usually the case, the average beta within each market (Brazil, Malaysia, US or Germany) has to be one. Thus, it would be mathematically impossible for betas to capture country risk.
2. If betas are estimated against a global equity index, such as the Morgan Stanley Capital Index (MSCI), there is a possibility that betas could capture country risk but there is little evidence that they do in practice. Since the global equity indices are market weighted, it is the companies that are in developed markets that have higher betas, whereas the companies in small, very risky emerging markets report low betas. Table 8 reports the average beta estimated for the ten largest market cap companies in Brazil, India, the United States and Japan against the MSCI.

Table 8: Betas against MSCI – Large Market Cap Companies

Country	*Average Beta (against local index)*	*Average Beta (against MSCI)*
India	0.97	0.83
Brazil	0.98	0.81
United States	0.96	1.05
Japan	0.94	1.03

[a] The betas were estimated using two years of weekly returns from January 2006 to December 2007 against the most widely used local index (Sensex in India, Bovespa in Brazil, S&P 500 in the US and the Nikkei in Japan) and the MSCI using two years of weekly returns.

The emerging market companies consistently have lower betas, when estimated against global equity indices, than developed market companies. Using these betas with a global equity risk premium will lead to lower costs of equity for emerging market companies than developed market companies. While there are creative fixes

[79] The implied costs of capital for companies in the 36 countries were computed and related to global political uncertainty, measured using the US economic policy uncertainty index, and to domestic political uncertainty, measured using domestic national elections.

that practitioners have used to get around this problem, they seem to be based on little more than the desire to end up with higher expected returns for emerging market companies.[80]

3. *Country risk is better reflected in the cash flows*

The essence of this argument is that country risk and its consequences are better reflected in the cash flows than in the discount rate. Proponents of this point of view argue that bringing in the likelihood of negative events (political chaos, nationalization and economic meltdowns) into the expected cash flows effectively risk adjusts the cashflows, thus eliminating the need for adjusting the discount rate.

This argument is alluring but it is wrong. The expected cash flows, computed by taking into account the possibility of poor outcomes, is not risk adjusted. In fact, this is exactly how we should be calculating expected cash flows in any discounted cash flow analysis. Risk adjustment requires us to adjust the expected cash flow further for its risk, i.e. compute certainty equivalent cash flows in capital budgeting terms. To illustrate why, consider a simple example where a company is considering making the same type of investment in two countries. For simplicity, let us assume that the investment is expected to deliver $ 90, with certainty, in country 1 (a mature market); it is expected to generate $ 100 with 90% probability in country 2 (an emerging market) but there is a 10% chance that disaster will strike (and the cash flow will be $0). The expected cash flow is $90 on both investments, but only a risk neutral investor would be indifferent between the two. A risk averse investor would prefer the investment in the mature market over the emerging market investment, and would demand a premium for investing in the emerging market.

In effect, a full risk adjustment to the cash flows will require us to go through the same process that we have to use to adjust discount rates for risk. We will have to estimate a country risk premium, and use that risk premium to compute certainty equivalent cash flows.[81]

The arguments for a country risk premium

There are elements in each of the arguments in the previous section that are persuasive but none of them is persuasive enough.

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[80] There are some practitioners who multiply the local market betas for individual companies by a beta for that market against the US. Thus, if the beta for an Indian chemical company is 0.9 and the beta for the Indian market against the US is 1.5, the global beta for the Indian company will be 1.35 (0.9*1.5). The beta for the Indian market is obtained by regressing returns, in US dollars, for the Indian market against returns on a US index (say, the S&P 500).

[81] In the simple example above, this is how it would work. Assume that we compute a country risk premium of 3% for the emerging market to reflect the risk of disaster. The certainty equivalent cash flow on the investment in that country would be $90/1.03 = $87.38.

- Investors have become more globally diversified over the last three decades and portions of country risk can therefore be diversified away in their portfolios. However, the significant home bias that remains in investor portfolios exposes investors disproportionately to home country risk, and the increase in correlation across markets has made a portion of country risk into non-diversifiable or market risk.
- As stocks are traded in multiple markets and in many currencies, it is becoming more feasible to estimate meaningful global betas, but it also is still true that these betas cannot carry the burden of capturing country risk in addition to all other macro risk exposures.
- Finally, there are certain types of country risk that are better embedded in the cash flows than in the risk premium or discount rates. In particular, risks that are discrete and isolated to individual countries should be incorporated into probabilities and expected cash flows; good examples would be risks associated with nationalization or related to acts of God (hurricanes, earthquakes etc.).

After you have diversified away the portion of country risk that you can, estimated a meaningful global beta and incorporated discrete risks into the expected cash flows, you will still be faced with residual country risk that has only one place to go: the equity risk premium.

There is evidence to support the proposition that you should incorporate additional country risk into equity risk premium estimates in riskier markets:

1. Historical equity risk premiums: Donadelli and Prosperi (2011) look at historical risk premiums in 32 different countries (13 developed and 19 emerging markets) and conclude that emerging market companies had both higher average returns and more volatility in these returns between 1988 and 2010 (see table 9).

Table 9: Historical Equity Risk Premiums (Monthly) by Region

Region	*Monthly ERP*	*Standard deviation*
Developed Markets	0.62%	4.91%
Asia	0.97%	7.56%
Latin America	2.07%	8.18%
Eastern Europe	2.40%	15.66%
Africa	1.41%	6.03%

While we remain cautious about using historical risk premiums over short time periods (and 22 years is short in terms of stock market history), the evidence is consistent with the argument that country risk should be incorporated into a larger equity risk premium.[82]

2. Survey premiums: Earlier in the paper, we referenced a paper by Fernandez et al (2014) that surveyed academics, analysts and companies in 82 countries on equity risk premiums. The reported average premiums vary widely across markets and are higher for riskier emerging markets, as can be seen in table 10.

Table 10: Survey Estimates of Equity Risk Premium: By Region

Region	Number	Average	Median
Africa	11	10.14%	9.85%
Developed Markets	20	5.44%	5.29%
Eastern Europe	15	8.29%	8.25%
Emerging Asia	12	8.33%	8.08%
EU Troubled	7	8.36%	8.31%
Latin America	15	9.45%	9.39%
Middle East	8	7.14%	6.79%
Grand Total	**88**	**7.98%**	**7.82%**

Again, while this does not conclusively prove that country risk commands a premium, it does indicate that those who do valuations in emerging market countries seem to act like it does. Ultimately, the question of whether country risk matters and should affect the equity risk premium is an empirical one, not a theoretical one, and for the moment, at least, the evidence seems to suggest that you should incorporate country risk into your discount rates. This could change as we continue to move towards a global economy, with globally diversified investors and a global equity market, but we are not there yet.

Estimating a Country Risk Premium

If country risk is not diversifiable, either because the marginal investor is not globally diversified or because the risk is correlated across markets, we are then left with the task of measuring country risk and considering the consequences for equity risk premiums. In this section, we will consider three approaches that can be used to estimate country risk premiums, all of which build off the historical risk premiums estimated in the last section. To approach this estimation question, let us start with the basic proposition that the risk premium in any equity market can be written as:

[82] Donadelli, M. and L. Prosperi, 2011, *The Equity Risk Premium: Empirical Evidence from Emerging Markets*, Working Paper, http://ssrn.com/abstract=1893378.

Equity Risk Premium = Base Premium for Mature Equity Market + Country Risk Premium

The country premium could reflect the extra risk in a specific market. This boils down our estimation to estimating two numbers – an equity risk premium for a mature equity market and the additional risk premium, if any, for country risk. To estimate a mature market equity risk premium, we can look at one of two numbers. The first is the historical risk premium that we estimated for the United States, which yielded 4.60% as the geometric average premium for stocks over treasury bonds from 1928 to 2014. If we do this, we are arguing that the US equity market is a mature market, and that there is sufficient historical data in the United States to make a reasonable estimate of the risk premium. The other is the average historical risk premium across 20 equity markets, approximately 3.3%, that was estimated by Dimson et al (see earlier reference), as a counter to the survivor bias that they saw in using the US risk premium. Consistency would then require us to use this as the equity risk premium, in every other equity market that we deem mature; the equity risk premium in January 2015 would be 4.60% in Germany, France and the UK, for instance. For markets that are not mature, however, we need to measure country risk and convert the measure into a country risk premium, which will augment the mature market premium.

Measuring Country Risk

There are at least three measures of country risk that we can use. The first is the sovereign rating attached to a country by ratings agencies. The second is to subscribe to services that come up with broader measures of country risk that explicitly factor in the economic, political and legal risks in individual countries. The third is go with a market-based measure such as the volatility in the country's currency or markets.

i. Sovereign Ratings

One of the simplest and most accessible measures of country risk is the rating assigned to a country's debt by a ratings agency (S&P, Moody's and Fitch, among others, all provide country ratings). These ratings measure default risk (rather than equity risk) but they are affected by many of the factors that drive equity risk – the stability of a country's currency, its budget and trade balances and political uncertainty, among other variables[83].

[83] The process by which country ratings are obtained in explained on the S&P web site at http://www.ratings.standardpoor.com/criteria/index.htm.

To get a measure of country ratings, consider six countries – Germany, Brazil, China, India, Russia and Greece. In January 2015, the Moody's ratings for the countries are summarized in table 11:

Table 11: Sovereign Ratings in January 2015 – Moody's

Country	*Foreign Currency Rating*	*Local Currency Rating*
Brazil	Baa2	Baa2
China	Aa3	Aa3
Germany	Aaa	Aaa
Greece	Caa1	Caa1
India	Baa3	Baa3
Russia	Baa2	Baa2

What do these ratings tell us? First, the local currency and foreign currency ratings are identical for all of the countries on the list. There are a few countries (not on this list) where the two ratings diverge, and when they do, the local currency ratings tend to be higher (or at worst equal to) the foreign currency ratings for most countries, because a country should be in a better position to pay off debt in the local currency than in a foreign currency. Second, at least based on Moody's assessments in 2015, Germany is the safest company in this group, followed by China, Russia, Brazil, India and Greece, in that order. Third, ratings do change over time. In fact, Brazil's rating has risen from B1 in 2001 to its current rating of Baa2, reflecting both strong economic growth and a more robust political system. Appendix 2 contains the current ratings – local currency and foreign currency – for the countries that are tracked by Moody's in January 2015.[84]

While ratings provide a convenient measure of country risk, there are costs associated with using them as the only measure. First, ratings agencies often lag markets when it comes to responding to changes in the underlying default risk. The ratings for India, according to Moody's, were unchanged from 2004 to 2007, though the Indian economy grew at double-digit rates over that period. Similarly, Greece's ratings did not plummet until the middle of 2011, though their financial problems were visible well before that time. Second, the ratings agency focus on default risk may obscure other risks that could still affect equity markets. For instance, rising commodity (and especially oil) prices pushed up the ratings for commodity supplying countries (like Russia), even

[84] In a disquieting reaction to the turmoil of the market crisis in the last quarter of 2008, Moody's promoted the notion that Aaa countries were not all created equal and slotted these countries into three groups – resistant Aaa (the stongest), resilient Aaa (weaker but will probably survive intact) and vulnerable Aaa (likely to face additional default risk.

though there was little improvement in the rest of the economy. Finally, not all countries have ratings; much of sub-Saharan Africa, for instance, is unrated.

ii. Country Risk Scores

Rather than focus on just default risk, as rating agencies do, some services have developed numerical country risk scores that take a more comprehensive view of risk. These risk scores are often estimated from the bottom-up by looking at economic fundamentals in each country. This, of course, requires significantly more information and, as a consequence, most of these scores are available only to commercial subscribers.

The Political Risk Services (PRS) group, for instance, considers political, financial and economic risk indicators to come up with a composite measure of risk (ICRG) for each country that ranks from 0 to 100, with 0 being highest risk and 100 being the lowest risk.[85] Appendix 3 classifies countries based on composite country risk measures from the PRS Group in January 2014.[86] Harvey (2005) examined the efficacy of these scores and found that they were correlated with costs of capital, but only for emerging market companies.

The Economist, the business newsmagazine, also operates a country risk assessment unit that measures risk from 0 to 100, with 0 being the least risk and 100 being the most risk. In September 2008, Table 12 the following countries were ranked as least and most risky by their measure:

[85] The PRS group considers three types of risk – political risk, which accounts for 50% of the index, financial risk, which accounts for 25%, and economic risk, which accounts for the balance. While this table is dated, updated numbers are available for a hefty price. We have used the latest information in the public domain. Some university libraries have access to the updated data. While we have not updated the numbers, out of concerns about publishing proprietary data, you can get the latest PRS numbers by paying $99 on their website (http://www.prsgroup.com).

[86] Harvey, C.R., ***Country Risk Components, the Cost of Capital, and Returns in Emerging Markets***, Working paper, Duke University. Available at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=620710.

Table 12: Country Risk Scores – The Economist

Economist.com rankings

Country risk

Selected countries and territories, September 2008 (except where noted)

Least risky Rank		Score*	Most risky Rank		Score
1	Switzerland †	12	**120**	Zimbabwe	86
2	Finland **	14	**119**	Iraq	80
	Norway **	14	**118**	Sudan	76
	Sweden ††	14	**117**	Myanmar	75
5	Canada **	17	**116**	Nicaragua	69
	Denmark †	17	**115**	Jamaica	68
	Netherlands §	17	**114**	Kenya	66
8	Germany ††	18	**113**	Cuba	64
9	Austria **	19	**112**	Cambodia	62
	France ††	19	**111**	Côte d'Ivoire	61
11	Belgium ††	20		Ecuador	61
12	Singapore	21		Pakistan	61
13	Japan **	23		Venezuela	61
14	Ireland #	24		Vietnam	61
	Britain	24	**106**	Syria	60
	United States †	24			

*Out of 100, with higher numbers indicating more risk. Scores are based on indicators from three categories: currency risk, sovereign debt risk and banking risk.

† May 2008; ** July 2008; †† June 2008; § August 2008; # February 2008

In fact, comparing the PRS and Economist measures of country risk provides some insight into the problems with using their risk measures. The first is that the measures may be internally consistent but are not easily comparable across different services. The Economist, for instance, assigns its lowest scores to the safest countries whereas PRS assigns the highest scores to these countries. The second is that, by their very nature, a significant component of these measures have to be black boxes to prevent others from replicating them at no cost. Third, the measures are not linear and the services do not claim that they are; a country with a risk score of 60 in the Economist measure is not twice as risky as a country with a risk score of 30.

iii. Market-based Measures

To those analysts who feel that ratings agencies are either slow to respond to changes in country risk or take too narrow a view of risk, there is always the alternative of using market based measures.

- Bond default spread: We can compute a default spread for a country if it has bonds that are denominated in currencies such as the US dollar, Euro or Yen, where there is a riskfree rate to compare it to. In January 2015, for instance, a 10-year US dollar denominated bond issued by the Brazilian government had a yield to maturity of 3.87%, giving it a default spread of 1.70% over the 10-year US treasury bond rate (2.17%), as of the same time.
- Credit Default Swap Spreads: In the last few years, credit default swaps (CDS) markets have developed, allowing us to obtain updated market measures of default risk in different entities. In particular, there are CDS spreads for countries (governments) that yield measures of default risk that are more updated and precise, at least in some cases, than bond default spreads.[87] Table 13 summarizes the CDS spreads for all countries where a CDS spread was available, in January 2015:

Table 13: Credit Default Swap Spreads (in basis points)– January 2015

Country	Moody's rating	CDS Spread	CDS Spread adj for US	Country	Moody's rating	CDS Spread	CDS Spread adj for US	Country	Moody's rating	CDS Spread	Cl Spr adj U
Abu Dhabi	Aa2	1.43%	1.12%	Hungary	Ba1	2.64%	2.33%	Poland	A2	1.46%	1.1
Argentina	Caa1	83.48%	83.17%	Iceland	Baa3	2.27%	1.96%	Portugal	Ba1	3.09%	2.7
Australia	Aaa	0.97%	0.66%	India	Baa3	2.64%	2.33%	Qatar	Aa2	1.57%	1.2
Austria	Aaa	0.81%	0.50%	Indonesia	Baa3	2.82%	2.51%	Romania	Baa3	2.23%	1.9
Bahrain	Baa2	3.18%	2.87%	Ireland	Baa1	1.26%	0.95%	Russia	Baa2	5.63%	5.3
Belgium	Aa3	1.20%	0.89%	Israel	A1	0.42%	0.11%	Saudi Arabia	Aa3	1.39%	1.0
Brazil	Baa2	3.17%	2.86%	Italy	Baa2	2.34%	2.03%	Slovakia	A2	1.32%	1.0
Bulgaria	Baa2	2.99%	2.68%	Japan	A1	1.55%	1.24%	Slovenia	Ba1	2.14%	1.8
Chile	Aa3	1.77%	1.46%	Kazakhstan	Baa2	4.16%	3.85%	South Africa	Baa2	2.96%	2.6
China	Aa3	1.78%	1.47%	Korea	Aa3	1.17%	0.86%	Spain	Baa2	1.79%	1.4
Colombia	Baa2	2.57%	2.26%	Latvia	Baa1	1.92%	1.61%	Sweden	Aaa	0.65%	0.3
Costa Rica	Ba1	3.58%	3.27%	Lebanon	B2	4.69%	4.38%	Switzerland	Aaa	0.72%	0.4
Croatia	Ba1	3.65%	3.34%	Lithuania	Baa1	1.88%	1.57%	Thailand	Baa1	1.91%	1.6
Cyprus	B3	6.35%	6.04%	Malaysia	A3	2.15%	1.84%	Tunisia	Ba3	3.38%	3.0
Czech Republic	A1	1.25%	0.94%	Mexico	A3	2.05%	1.74%	Turkey	Baa3	2.77%	2.4
Denmark	Aaa	0.79%	0.48%	Morocco	Ba1	2.55%	2.24%	Uganda	B1	0.31%	0.0
Egypt	Caa1	3.56%	3.25%	Netherlands	Aaa	0.78%	0.47%	Ukraine	Caa3	15.74%	15.
Estonia	A1	1.20%	0.89%	New Zealand	Aaa	1.01%	0.70%	UAE	Aa2	1.54%	1.2
Finland	Aaa	0.81%	0.50%	Norway	Aaa	0.61%	0.30%	United Kingdom	Aa1	0.77%	0.4
France	Aa1	1.22%	0.91%	Pakistan	Caa1	10.41%	10.10%	United States	Aaa	0.31%	0.0
Germany	Aaa	0.74%	0.43%	Panama	Baa2	2.09%	1.78%	Venezuela	Caa1	18.06%	17.
Greece	Caa1	10.76%	10.45%	Peru	A3	2.23%	1.92%	Vietnam	B1	3.15%	2.8

[87] The spreads are usually stated in US dollar or Euro terms.

Hong Kong	Aa1	1.12%	0.81%	Philippines	Baa2	1.98%	1.67%

Source: Bloomberg
Spreads are for 10-year US $ CDS.

In January 2015, for instance, the CDS market yielded a spread of 3.17% for the Brazilian Government, higher than the 1.70% that we obtained from the 10-year dollar denominated Brazilian bond. However, the CDS market does have some counterparty risk exposure and other risk exposures that are incorporated into the spreads. In fact, there is no country with a zero CDS spread, indicating either that there is no entity with default risk or that the CDS spread is not a pure default spread. To counter that problem, we netted the US CDS spread of 0.31% from each country's CDS to get a modified measure of country default risk.[88] Using this approach for Brazil, for instance, yields a netted CDS spread of 2.86% for the country.

- Market volatility: In portfolio theory, the standard deviation in returns is generally used as the proxy for risk. Extending that measure to emerging markets, there are some analysts who argue that the best measure of country risk is the volatility in local stock prices. Stock prices in emerging markets will be more volatile that stock prices in developed markets, and the volatility measure should be a good indicator of country risk. While the argument makes intuitive sense, the practical problem with using market volatility as a measure of risk is that it is as much a function of the underlying risk as it is a function of liquidity. Markets that are risky and illiquid often have low volatility, since you need trading to move stock prices. Consequently, using volatility measures will understate the risk of emerging markets that are illiquid and overstate the risk of liquid markets.

Market-based numbers have the benefit of constant updating and reflect the points of view of investors at any point in time. However, they also are also afflicted with all of the problems that people associate with markets – volatility, mood shifts and at times, irrationality. They tend to move far more than the other two measures – sovereign ratings and country risk scores – sometimes for good reasons and sometimes for no reason at all.

b. Estimating Country Risk Premium (for Equities)

How do we link a country risk measure to a country risk premium? In this section, we will look at three approaches. The first uses default spreads, based upon

[88] If we assume that there is default risk in the US, we would subtract the default spread associated with this risk from the 0.67% first, before netting the value against other CDS spreads. Thus, if the default spread for the US is 0.15%, we would subtract out only 0.52% (0.67% - 0.15%) from each country's CDS spread to get to a corrected default spread for that country.

country bonds or ratings, whereas the latter two use equity market volatility as an input in estimating country risk premiums.

1. Default Spreads

The simplest and most widely used proxy for the country risk premium is the default spread that investors charge for buying bonds issued by the country. This default spread can be estimated in one of three ways.

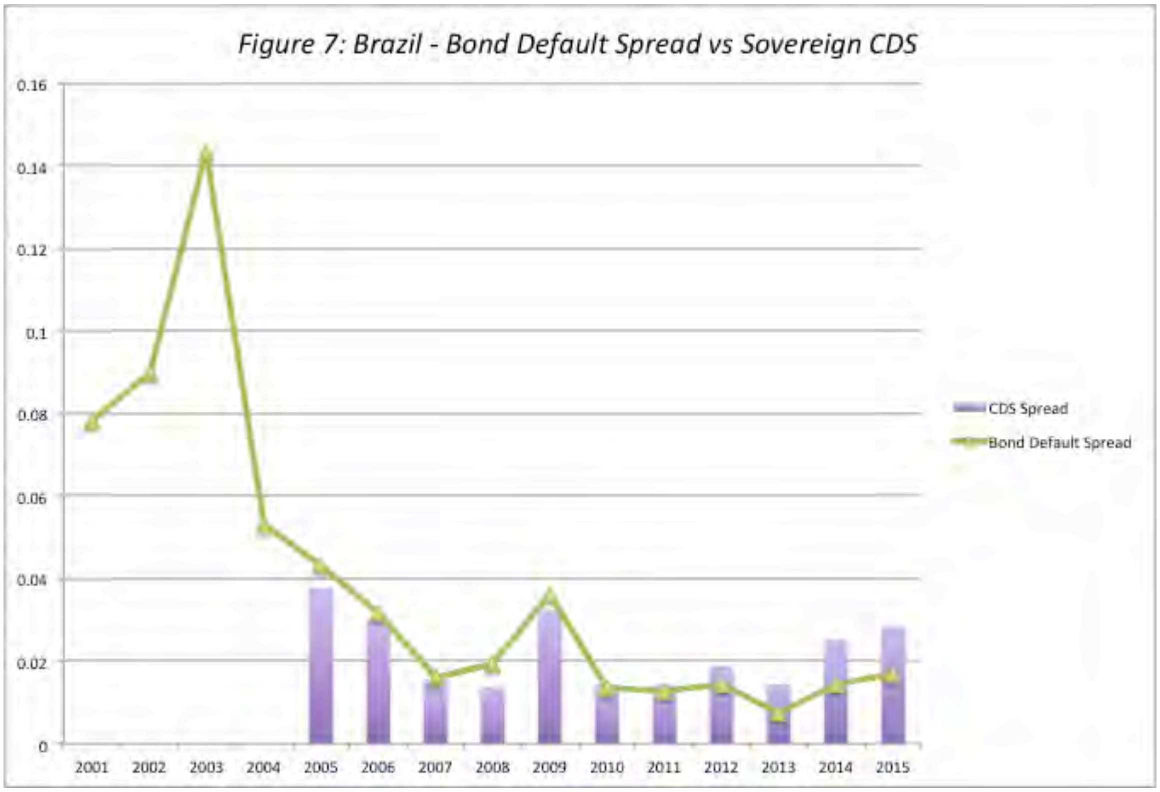
a. Current Default Spread on Sovereign Bond or CDS market: As we noted in the last section, the default spread comes from either looking at the yields on bonds issued by the country in a currency where there is a default free bond yield to which it can be compared or spreads in the CDS market.[89] With the 10-year US dollar denominated Brazilian bond that we cited as an example in the last section, the default spread would have amounted to 1.70% in January 2015: the difference between the interest rate on the Brazilian bond and a treasury bond of the same maturity. The netted CDS market spread on the same day for the default spread was 2.86%. Bekaert, Harvey, Lundblad and Siegel (2014) break down the sovereign bond default spread into four components, including global economic conditions, country-specific economic factors, sovereign bond liquidity and policial risk, and find that it is the political risk component that best explain money flows into and out of the country equity markets.[90]

b. Average (Normalized) spread on bond: While we can make the argument that the default spread in the dollar denominated is a reasonable measure of the default risk in Brazil, it is also a volatile measure. In figure 7, we have graphed the yields on the dollar denominated ten-year Brazilian Bond and the U.S. ten-year treasury bond and highlighted the default spread (as the difference between the two yields) from January 2000 to January 2015. In the same figure, we also show the 10-year CDS spreads from 2005 to 2015,[91] the spreads have also changed over time but move with the bond default spreads.

[89] You cannot compare interest rates across bonds in different currencies. The interest rate on a peso bond cannot be compared to the interest rate on a dollar denominated bond.

[90] Bekaert, G., C.R. Harvey, C.T. Lundblad and S. Siegel, 2014, *Political Risk Spreads*, Journal of International Business Studies, v45, 471-493.

[91] Data for the sovereign CDS market is available only from the last part of 2004.



Note that the bond default spread widened dramatically during 2002, mostly as a result of uncertainty in neighboring Argentina and concerns about the Brazilian presidential elections.[92] After the elections, the spreads decreased just as quickly and continued on a downward trend through the middle of last year. Since 2004, they have stabilized, with a downward trend; they spiked during the market crisis in the last quarter of 2008 but have settled back into pre-crisis levels. Given this volatility, a reasonable argument can be made that we should consider the average spread over a period of time rather than the default spread at the moment. If we accept this argument, the normalized default spread, using the average spreads over the last 5 years of data would be 1.65% (bond default spread) or 1.99% (CDS spread). Using this approach makes sense only if the economic fundamentals of the country have not changed significantly (for the better or worse) during the period but will yield misleading values, if there have been structural shifts in the economy. In 2008, for instance, it would have made sense to use averages over time for a country like Nigeria, where oil price movements created volatility in spreads over time, but not for countries like China and India, which saw their economies expand and mature dramatically over the period or Venezuela, where government capriciousness

[92] The polls throughout 2002 suggested that Lula Da Silva who was perceived by the market to be a leftist would beat the establishment candidate. Concerns about how he would govern roiled markets and any poll that showed him gaining would be followed by an increase in the default spread.

made operating private businesses a hazardous activity (with a concurrent tripling in default spreads). In fact, the last year has seen a spike in the Brazilian default spread, partly the result of another election and partly because of worries about political corruption and worse in large Brazilian companies.

c. Imputed or Synthetic Spread: The two approaches outlined above for estimating the default spread can be used only if the country being analyzed has bonds denominated in US dollars, Euros or another currency that has a default free rate that is easily accessible. Most emerging market countries, though, do not have government bonds denominated in another currency and some do not have a sovereign rating. For the first group (that have sovereign rating but no foreign currency government bonds), there are two solutions. If we assume that countries with the similar default risk should have the same sovereign rating, we can use the typical default spread for other countries that have the same rating as the country we are analyzing and dollar denominated or Euro denominated bonds outstanding. Thus, Bulgaria, with a Baa2 rating, would be assigned the same default spread as Brazil, which also has Baa2 rating, and dollar denominated bonds and CDS prices from which we can extract a default spread. For the second group, we are on even more tenuous grounds. Assuming that there is a country risk score from the Economist or PRS for the country, we could look for other countries that are rated and have similar scores and assign the default spreads that these countries face. For instance, we could assume that Cuba and Cameroon, which fall within the same score grouping from PRS, have similar country risk; this would lead us to attach Cuba's rating of Caa1 to Cameroon (which is not rated) and to use the same default spread (based on this rating) for both countries.

In table 14, we have estimated the typical default spreads for bonds in different sovereign ratings classes in January 2015. One problem that we had in obtaining the numbers for this table is that relatively few emerging markets have dollar or Euro denominated bonds outstanding. Consequently, there were some ratings classes where there was only one country with data and several ratings classes where there were none. To mitigate this problem, we used spreads from the CDS market, referenced in the earlier section. We were able to get default spreads for 65 countries, categorized by rating class, and we averaged the spreads across multiple countries in the same ratings class.[93] An alternative approach to estimating default spread is to assume that sovereign ratings are

[93] There were thirteen Baa2 rated countries, with ten-year CDS spreads, in January 2015. The average spread across the these countries is 2.68%. We noticed wide variations across countries in the same ratings class, and no discernible trend when compared to the January 2014 averages. Consequently, we decided to use the same default spreads that we used last year.

comparable to corporate ratings, i.e., a Ba1 rated country bond and a Ba1 rated corporate bond have equal default risk. In this case, we can use the default spreads on corporate bonds for different ratings classes. Table 14 summarizes the typical default spreads by sovereign rating class in January 2015, and compares it to the default spreads for similar corporate ratings.

Table 14: Default Spreads by Ratings Class – Sovereign vs. Corporate in January 2015

Moody's rating	*Sovereign Bonds/CDS*	Corporate Bonds
Aaa/AAA	0.00%	0.42%
Aa1/AA+	0.40%	0.60%
Aa2/AA	0.50%	0.78%
Aa3/AA-	0.60%	0.87%
A1/A+	0.70%	0.96%
A2/A	0.85%	0.97%
A3/A-	1.20%	1.10%
Baa1/BBB+	1.60%	1.36%
Baa2/BBB	1.90%	1.67%
Baa3/BBB-	2.20%	2.22%
Ba1/BB+	2.50%	2.61%
Ba2/BB	3.00%	2.97%
Ba3/BB-	3.60%	3.33%
B1/B+	4.50%	3.74%
B2/B	5.50%	4.10%
B3/B-	6.50%	4.45%
Caa1/ CCC+	7.50%	4.86%
Caa2/CCC	9.00%	7.50%
Caa3/ CCC-	10.00%	10.00%

Note that the corporate bond spreads, at least in January 2015, were slightly larger than the sovereign spreads for the higher ratings classes, converge for the intermediate ratings and widen again at the lowest ratings. Using this approach to estimate default spreads for Brazil, with its rating of Baa2 would result in a spread of 1.90% (1.67%), if we use sovereign spreads (corporate spreads). These spreads are down from post-crisis levels at the end of 2008 but are still larger than the actual spreads on Brazilian sovereign bonds in early 2014.

Figure 8 depicts the alternative approaches to estimating default spreads for four countries, Brazil, China, India and Poland, in early 2015:

Figure 8: Approaches for estimating Sovereign Default Spreads

Estimating a default spread for a country or sovereign entity

Market Based estimates

Sovereign Bond spread
1. Find a bond issued by the country, denominated in US$ or Euros.
2. Compute the default spread by comparing to US treasury bond (if US $) or German Euro bond (if Euros).

CDS Market
1. Find a 10-year CDS for the country (if one exists)
2. Net out US CDS
2. This is your default spread.

Country	*Sovereign Bond Yield*	*Currency*	*Risk free rate*	*Default Spread*	*CDS Spread (net of US)*
Brazil	3.87%	US $	2.17%	1.70%	2.86%
China	NA	NA	NA	NA	1.47%
India	NA	NA	NA	NA	2.33%
Poland	1.70%	Euro	0.50%	1.20%	1.15%

Rating/Risk score based estimates
Step 1: Find a sovereign rating (local currency) for the country (on Moody's or S&P)
Step 2: Look up the default spread for that rating in the lookup table below:

Moody's rating	*Sovereign Bonds/CDS*
Aaa/AAA	0.00%
Aa1/AA+	0.40%
Aa2/AA	0.50%
Aa3/AA-	0.60%
A1/A+	0.70%
A2/A	0.85%
A3/A-	1.20%
Baa1/BBB+	1.60%
Baa2/BBB	1.90%
Baa3/BBB-	2.20%
Ba1/BB+	2.50%
Ba2/BB	3.00%
Ba3/BB-	3.60%
B1/B+	4.50%
B2/B	5.50%
B3/B-	6.50%
Caa1/ CCC+	7.50%
Caa2/CCC	9.00%
Caa3/ CCC-	10.00%

Country	*Moody's Rating*	*Default Spread*
Brazil	Baa2	1.90%
China	Aa3	0.60%
India	Baa3	2.20%
Poland	A2	0.85%

With some countries, without US-dollar (or Euro) denominated sovereign bonds or CDS spreads, you don't have a choice since the only estimate of the default spread comes from the sovereign rating. With other countries, such as Brazil, you have multiple estimates of the default spreads: 1.70% from the dollar denominated bond, 3.17% from the CDS spread, 2.86% from the netted CDS spread and 1.90% from the sovereign rating look up table (table 14). You could choose one of these approaches and stay consistent over time or average across them.

Analysts who use default spreads as measures of country risk typically add them on to both the cost of equity and debt of every company traded in that country. Thus, the cost of equity for an Indian company, estimated in U.S. dollars, will be 2.2% higher than the cost of equity of an otherwise similar U.S. company, using the January 2015 measure of the default spread, based upon the rating. In some cases, analysts add the default spread to the U.S. risk premium and multiply it by the beta. This increases the cost of equity for high beta companies and lowers them for low beta firms.[94]

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[94] In a companion paper, I argue for a separate measure of company exposure to country risk called lambda that is scaled around one (just like beta) that is multiplied by the country risk premium to estimate the cost

While many analysts use default spreads as proxies for country risk, the evidence for its use is still thin. Abuaf (2011) examines ADRs from ten emerging markets and relates the returns on these ADRs to returns on the S&P 500 (which yields a conventional beta) and to the CDS spreads for the countries of incorporation. He finds that ADR returns as well as multiples (such as PE ratios) are correlated with movement in the CDS spreads over time and argues for the addition of the CDS spread (or some multiple of it) to the costs of equity and capital to incorporate country risk.[95]

2. Relative Equity Market Standard Deviations

There are some analysts who believe that the equity risk premiums of markets should reflect the differences in equity risk, as measured by the volatilities of these markets. A conventional measure of equity risk is the standard deviation in stock prices; higher standard deviations are generally associated with more risk. If you scale the standard deviation of one market against another, you obtain a measure of relative risk. For instance, the relative standard deviation for country X (against the US) would be computed as follows:

$$\text{Relative Standard Deviation}_{\text{Country X}} = \frac{\text{Standard Deviation}_{\text{Country X}}}{\text{Standard Deviation}_{\text{US}}}$$

If we assume a linear relationship between equity risk premiums and equity market standard deviations, and we assume that the risk premium for the US can be computed (using historical data, for instance) the equity risk premium for country X follows:

$$\text{Equity risk premium}_{\text{Country X}} = \text{Risk Premum}_{\text{US}} * \text{Relative Standard Deviation}_{\text{Country X}}$$

Assume, for the moment, that you are using an equity risk premium for the United States of 5.75%. The annualized standard deviation in the S&P 500 in two years preceding January 2014, using weekly returns, was 10.85%, whereas the standard deviation in the Bovespa (the Brazilian equity index) over the same period was 22.25%.[96] Using these values, the estimate of a total risk premium for Brazil would be as follows.

$$\text{Equity Risk Premium}_{Brazil} = 5.75\% * \frac{22.25\%}{10.85\%} = 11.77\%$$

The country risk premium for Brazil can be isolated as follows:

$$\text{Country Risk Premium}_{Brazil} = 11.77\% - 5.75\% = 6.02\%$$

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of equity. See Damodaran, A., 2007, Measuring Company Risk Exposure to Country Risk, Working Paper, http://papers.ssrn.com/sol3/papers.cfm?abstract_id=889388.

[95] Abuaf, N., 2011, *Valuing Emerging Market Equities – The Empirical Evidence*, Journal of Applied Finance, v21, 123-138.

[96] If the dependence on historical volatility is troubling, the options market can be used to get implied volatilities for both the US market (14.16%) and for the Bovespa (24.03%).

Table 15 lists country volatility numbers for some of the Latin American markets and the resulting total and country risk premiums for these markets, based on the assumption that the equity risk premium for the United States is 5.75%. Appendix 4 contains a more complete list of emerging markets, with equity risk premiums and country risk premiums estimated for each.

Table 15: Equity Market Volatilities and Risk Premiums (Weekly returns: Feb 13-Feb 15): Latin American Countries

Country	*Standard deviation in Equities (weekly)*	*Relative Volatility (to US)*	*Total Equity Risk Premium*	*Country risk premium*
Argentina	35.50%	3.27	18.78%	13.03%
Brazil	22.25%	2.05	11.77%	6.02%
Chile	13.91%	1.28	7.36%	1.61%
Colombia	16.00%	1.47	8.46%	2.71%
Costa Rica	8.78%	0.81	4.64%	-1.11%
Mexico	14.81%	1.36	7.83%	2.08%
Panama	6.18%	0.57	3.27%	-2.48%
Peru	16.15%	1.49	8.54%	2.79%
US	**10.87%**	**1.00**	5.75%	0.00%
Venezuela	40.03%	3.68	21.18%	15.43%

While this approach has intuitive appeal, there are problems with using standard deviations computed in markets with widely different market structures and liquidity. Since equity market volatility is affected by liquidity, with more liquid markets often showing higher volatility, this approach will understate premiums for illiquid markets and overstate the premiums for liquid markets. For instance, the standard deviations for Panama and Costa Rica are lower than the standard deviation in the S&P 500, leading to equity risk premiums for those countries that are lower than the US. The second problem is related to currencies since the standard deviations are usually measured in local currency terms; the standard deviation in the U.S. market is a dollar standard deviation, whereas the standard deviation in the Brazilian market is based on nominal Brazilian Real returns. This is a relatively simple problem to fix, though, since the standard deviations can be measured in the same currency – you could estimate the standard deviation in dollar returns for the Brazilian market.

3. Default Spreads + Relative Standard Deviations

In the first approach to computing equity risk premiums, we assumed that the default spreads (actual or implied) for the country were good measures of the additional risk we face when investing in equity in that country. In the second approach, we argued that the information in equity market volatility can be used to compute the country risk premium. In the third approach, we will meld the first two, and try to use the information in both the country default spread and the equity market volatility.

The country default spreads provide an important first step in measuring country equity risk, but still only measure the premium for default risk. Intuitively, we would expect the country equity risk premium to be larger than the country default risk spread. To address the issue of how much higher, we look at the volatility of the equity market in a country relative to the volatility of the bond market used to estimate the spread. This yields the following estimate for the country equity risk premium.

$$\text{Country Risk Premium}=\text{Country Default Spread}*\left(\frac{\sigma_{\text{Equity}}}{\sigma_{\text{Country Bond}}}\right)$$

To illustrate, consider again the case of Brazil. As noted earlier, the default spread for Brazil in January 2015, based upon its sovereign rating, was 1.90%. We computed annualized standard deviations, using two years of weekly returns, in both the equity market and the government bond, in early March 2015. The annualized standard deviation in the Brazilian dollar denominated ten-year bond was 11.97%, well below the standard deviation in the Brazilian equity index of 22.25%. The resulting country equity risk premium for Brazil is as follows:

$$\text{Brazil Country Risk Premium} = 1.90\% * \frac{22.25\%}{11.97\%} = 3.53\%$$

Unlike the equity standard deviation approach, this premium is in addition to a mature market equity risk premium. Thus, assuming a 5.75% mature market premium, we would compute a total equity risk premium for Brazil of 8.22%:

Brazil's Total Equity Risk Premium = 5.75% + 3.53% = 9.28%

Note that this country risk premium will increase if the country rating drops or if the relative volatility of the equity market increases.

Why should equity risk premiums have any relationship to country bond spreads? A simple explanation is that an investor who can make 1.90% risk premium on a dollar-denominated Brazilian government bond would not settle for a risk premium of 1.90% (in dollar terms) on Brazilian equity. Playing devil's advocate, however, a critic could argue that the interest rate on a country bond, from which default spreads are extracted, is not really an expected return since it is based upon the promised cash flows (coupon and principal) on the bond rather than the expected cash flows. In fact, if we wanted to estimate a risk premium for bonds, we would need to estimate the expected return based upon expected cash flows, allowing for the default risk. This would result in a lower default spread and equity risk premium. Both this approach and the last one use the standard deviation in equity of a market to make a judgment about country risk premium, but they measure it relative to different bases. This approach uses the country bond as a base, whereas the previous one uses the standard deviation in the U.S. market. This approach assumes that investors are more likely to choose between Brazilian bonds and Brazilian equity, whereas the previous approach assumes that the choice is across equity markets.

There are two potential measurement problems with using this approach. The first is that the relative standard deviation of equity is a volatile number, both across countries (ranging from 4.04 for India to 0.48 for the Phillipines) and across time (Brazil's relative volatility numbers have ranged from close to one to well above 2). The second is that computing the relative volatility requires us to estimate volatility in the government bond, which, in turn, presupposes that long-term government bonds not only exist but are also traded.[97] In countries where this data item is not available, we have three choices. One is to fall back on one of the other two approaches. The second is to use a different market measure of default risk, say the CDS spread, and compute the standard deviation in the spread; this number can be standardized by dividing the level of the spread. The third is to compute a cross sectional average of the ratio of stock market to bond market volatility across countries, where both items are available, and use that average. In 2015, for instance, there were 26 emerging markets, where both the equity market volatility and the government bond volatility numbers were available, at least for 100 trading weeks; the numbers are summarized in Appendix 5. The median ratio, across these markets, of equity market volatility to bond price volatility was approximately 1.88.[98] We also computed a second measure of relative volatility: equity volatility divided by the coefficient of variation in the CDS spread.

	$\sigma_{Equity} / \sigma_{Bond}$	$\sigma_{Equity} / \sigma_{CDS}$
Number of countries with data	26	46

[97] One indication that the government bond is not heavily traded is an abnormally low standard deviation on the bond yield.

[98] The ratio seems to be lowest in the markets with the highest default spreads and higher in markets with lower default spreads. The median ratio this year is higher than it has been historically. On my website, I continue to use a multiple of 1.50, reflecting the historical value for this ratio.

Average	1.86	2.11
Median	1.88	0.97
Maximum	4.04	23.49
Minimum	0.48	0.51

Looking at the descriptive statistics, the need to adjust default spreads seems to be smaller, at least in the cross section, if you use the CDS spread as your measure of the default spread for a country; the median ratio is close to one.

Choosing between the approaches

The three approaches to estimating country risk premiums will usually give you different estimates, with the bond default spread and relative equity standard deviation approaches generally yielding lower country risk premiums than the melded approach that uses both the country bond default spread and the equity and bond standard deviations. Table 16 summarizes the estimates of country equity and total risk premium using the three approaches for Brazil in March 2014:

Table 16: Country and Total Equity Risk Premium: Brazil in January 2013

Approach	*Mature Market Equity Premium*	*Brazil Country Risk Premium*	*Total Equity Risk Premium*
Country Bond Default Spread	5.75%	1.90%	7.65%
Relative Equity Market Standard Deviations	5.75%	6.02%	11.77%
Melded Approach (Bond default spread X Relative Standard $Deviation_{Bond}$)	5.75%	1.90%*1.86 = 3.53%	9.28%
Melded Approach (CDS X Relative Standard $Deviation_{CDS}$)	5.75%	3.37% *1.87= 6.30%	12.05%

The CDS and relative equity market approaches yield similar equity risk premiums, but that is more the exception than the rule. In particular, the melded CDS approach offers more promise going forward, as more countries have CDS traded on them. With all three approaches, just as companies mature and become less risky over time, countries can mature and become less risky as well.

One way to adjust country risk premiums over time is to begin with the premium that emerges from the melded approach and to adjust this premium down towards either the country bond default spread or the country premium estimated from equity standard deviations. Thus, the equity risk premium will converge to the country bond default spread as we look at longer term expected returns. As an illustration, the country risk premium for Brazil would be 3.53% for the next year but decline over time to 1.90% (country default spread) or perhaps even lower, depending upon your assessment of how Brazil's economy will evolve over time.

Implied Equity Premiums

The problem with any historical premium approach, even with substantial modifications, is that it is backward looking. Given that our objective is to estimate an updated, forward-looking premium, it seems foolhardy to put your faith in mean reversion and past data. In this section, we will consider three approaches for estimating equity risk premiums that are more forward looking.

1. DCF Model Based Premiums

When investors price assets, they are implicitly telling you what they require as an expected return on that asset. Thus, if an asset has expected cash flows of $15 a year in perpetuity, and an investor pays $75 for that asset, he is announcing to the world that his required rate of return on that asset is 20% (15/75). In this section, we expand on this intuition and argue that the current market prices for equity, in conjunction with expected cash flows, should yield an estimate on the equity risk premium.

A Stable Growth DDM Premium

It is easiest to illustrated implied equity premiums with a dividend discount model (DDM). In the DDM, the value of equity is the present value of expected dividends from the investment. In the special case where dividends are assumed to grow at a constant rate forever, we get the classic stable growth (Gordon) model:

$$\text{Value of equity} = \frac{\text{Expected Dividends Next Period}}{(\text{Required Return on Equity - Expected Growth Rate})}$$

This is essentially the present value of dividends growing at a constant rate. Three of the four inputs in this model can be obtained or estimated - the current level of the market (value), the expected dividends next period and the expected growth rate in earnings and dividends in the long term. The only "unknown" is then the required return on equity; when we solve for it, we get an implied expected return on stocks. Subtracting out the riskfree rate will yield an implied equity risk premium.

To illustrate, assume that the current level of the S&P 500 Index is 900, the expected dividend yield on the index is 2% and the expected growth rate in earnings and dividends in the long term is 7%. Solving for the required return on equity yields the following:

$$900 = (.02*900) /(r - .07)$$

Solving for r,

$$r = (18+63)/900 = 9\%$$

If the current riskfree rate is 6%, this will yield a premium of 3%.

In fact, if we accept the stable growth dividend discount model as the base model for valuing equities and assume that the expected growth rate in dividends should equate to the riskfree rate in the long term, the dividend yield on equities becomes a measure of the equity risk premium:

$$\text{Value of equity} = \frac{\text{Expected Dividends Next Period}}{(\text{Required Return on Equity - Expected Growth Rate})}$$

$$\begin{aligned} \text{Dividends/ Value of Equity} &= \text{Required Return on Equity – Expected Growth rate} \\ \text{Dividend Yield} &= \text{Required Return on Equity – Riskfree rate} \\ &= \text{Equity Risk Premium} \end{aligned}$$

Rozeff (1984) made this argument[99] and empirical support has been claimed for dividend yields as predictors of future returns in many studies since.[100] Note that this simple equation will break down if (a) companies do not pay out what they can afford to in dividends, i.e., they hold back cash or (b) if earnings are expected to grow at extraordinary rates for the short term.

There is another variant of this model that can be used, where we focus on earnings instead of dividends. To make this transition, though, we have to state the expected growth rate as a function of the payout ratio and return on equity (ROE) :[101]

$$\begin{aligned} \text{Growth rate} &= (1 – \text{Dividends/ Earnings}) (\text{Return on equity}) \\ &= (1 – \text{Payout ratio}) (\text{ROE}) \end{aligned}$$

Substituting back into the stable growth model,

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[99] Rozeff, M. S. 1984. *Dividend yields are equity risk premiums*, Journal of Portfolio Management, v11, 68-75.

[100] Fama, E. F., and K. R. French. 1988. *Dividend yields and expected stock returns*. Journal of Financial Economics, v22, 3-25.

[101] This equation for sustainable growth is discussed more fully in Damodaran, A., 2002, Investment Valuation, John Wiley and Sons.

$$\text{Value of equity} = \frac{\text{Expected Earnings Next Period (Payout ratio)}}{\text{(Required Return on Equity - (1-Payout ratio) (ROE))}}$$

If we assume that the return on equity (ROE) is equal to the required return on equity (cost of equity), i.e., that the firm does not earn excess returns, this equation simplifies as follows:

$$\text{Value of equity} = \frac{\text{Expected Earnings Next Period}}{\text{Required Return on Equity}}$$

In this case, the required return on equity can be written as:

$$\text{Required return on equity} = \frac{\text{Expected Earnings Next Period}}{\text{Value of Equity}}$$

In effect, the inverse of the PE ratio (also referenced as the earnings yield) becomes the required return on equity, <u>if firms are in stable growth and earning no excess returns</u>. Subtracting out the riskfree rate should yield an implied premium:

Implied premium (EP approach) = Earnings Yield on index – Riskfree rate

In January 2015, the first of these approaches would have delivered a very low equity risk premium for the US market.

Dividend Yield = 1.87%

The second approach of netting the earnings yield against the risk free rate would have generated a more plausible number[102]:

Earnings Yield = 5.57%:

Implied premium = Earnings yield – 10-year US Treasury Bond rate

= 5.57% - 2.17% = 3.40%

Both approaches, though, draw on the dividend discount model and make strong assumptions about firms being in stable growth and/or long-term excess returns.

A Generalized Model: Implied Equity Risk Premium

To expand the model to fit more general specifications, we would make the following changes: Instead of looking at the actual dividends paid as the only cash flow to equity, we would consider <u>potential dividends instead of actual dividends</u>. In my earlier work (2002, 2006), the free cash flow to equity (FCFE), i.e, the cash flow left over after taxes, reinvestment needs and debt repayments, was offered as a measure of potential dividends.[103] Over the last decade, for instance, firms have paid out only about half their FCFE as dividends. If this poses too much of an estimation challenge, there is a

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[102] The earnings yield in January 2015 is estimated by dividing the aggregated earnings for the index by the index level.

[103] Damodaran, A., 2002, *Investment Valuation*, John Wiley and Sons; Damodaran, A., 2006, *Damodaran on Valuation*, John Wiley and Sons.

simpler alternative. Firms that hold back cash build up large cash balances that they use over time to fund stock buybacks. Adding stock buybacks to aggregate dividends paid should give us a better measure of total cash flows to equity. The model can also be expanded to allow for a high growth phase, where earnings and dividends can grow at rates that are very different (usually higher, but not always) than stable growth values. With these changes, the value of equity can be written as follows:

$$\text{Value of Equity} = \sum_{t=1}^{t=N} \frac{E(FCFE_t)}{(1+k_e)^t} + \frac{E(FCFE_{N+1})}{(k_e - g_N)\,(1+k_e)^N}$$

In this equation, there are N years of high growth, $E(FCFE_t)$ is the expected free cash flow to equity (potential dividend) in year t, k_e is the rate of return expected by equity investors and g_N is the stable growth rate (after year N). We can solve for the rate of return equity investors need, given the expected potential dividends and prices today. Subtracting out the riskfree rate should generate a more realistic equity risk premium.

In a variant of this approach, the implied equity risk premium can be computed from excess return or residual earnings models. In these models, the value of equity today can be written as the sum of capital invested in assets in place and the present value of future excess returns:[104]

$$\text{Value of Equity} = \text{Book Equity today} + \sum_{t=1}^{t=\infty} \frac{\text{Net Income}_t - k_e(\text{Book Equity}_{t-1})}{(1+k_e)^t}$$

If we can make estimates of the book equity and net income in future periods, we can then solve for the cost of equity and use that number to back into an implied equity risk premium. Claus and Thomas (2001) use this approach, in conjunction with analyst forecasts of earnings growth, to estimate implied equity risk premiums of about 3% for the market in 2000.[105] Easton (2007) provides a summary of possible limitations of models that attempt to extract costs of equity from accounting data including the unreliability of book value numbers and the use of optimistic estimates of growth from analysts.[106]

Implied Equity Risk Premium: S&P 500

Given its long history and wide following, the S&P 500 is a logical index to use to try out the implied equity risk premium measure. In this section, we will begin by

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[104] For more on excess return models, see Damodaran, A, 2006, *Valuation Approaches and Metrics: A Survey of the Theory and Evidence,* Working Paper, www.damodaran.com.

[105] Claus, J. and J. Thomas, 2001,'*Equity premia as low as three percent? Evidence from analysts' earnings forecasts for domestic and international stock markets*, Journal of Finance 56(5), 1629–1666.

[106] Easton, P., 2007, *Estimating the cost of equity using market prices and accounting data*, Foundations and Trends in Accounting, v2, 241-364.

estimating implied equity risk premiums at the start of the years 2008-2015, and follow up by looking at the volatility in that estimate over time.

Implied Equity Risk Premiums: Annual Estimates from 2008 to 2015

On December 31, 2007, the S&P 500 Index closed at 1468.36, and the dividend yield on the index was roughly 1.89%. In addition, the consensus estimate of growth in earnings for companies in the index was approximately 5% for the next 5 years.[107] Since this is not a growth rate that can be sustained forever, we employ a two-stage valuation model, where we allow growth to continue at 5% for 5 years, and then lower the growth rate to 4.02% (the riskfree rate) after that.[108] Table 17 summarizes the expected dividends for the next 5 years of high growth, and for the first year of stable growth thereafter:

Table 17: Estimated Dividends on the S&P 500 Index – January 1, 2008

Year	*Dividends on Index*
1	29.12
2	30.57
3	32.10
4	33.71
5	35.39
6	36.81

[a]Dividends in the first year = 1.89% of 1468.36 (1.05)

If we assume that these are reasonable estimates of the expected dividends and that the index is correctly priced, the value can be written as follows:

$$1468.36 = \frac{29.12}{(1+r)} + \frac{30.57}{(1+r)^2} + \frac{32.10}{(1+r)^3} + \frac{33.71}{(1+r)^4} + \frac{35.39}{(1+r)^5} + \frac{36.81}{(r-.0402)(1+r)^5}$$

Note that the last term in the equation is the terminal value of the index, based upon the stable growth rate of 4.02%, discounted back to the present. Solving for required return in this equation yields us a value of 6.04%. Subtracting out the ten-year treasury bond rate (the riskfree rate) yields an implied equity premium of 2.02%.

The focus on dividends may be understating the premium, since the companies in the index have bought back substantial amounts of their own stock over the last few years. Table 18 summarizes dividends and stock buybacks on the index, going back to 2001.

Table 18: Dividends and Stock Buybacks: 2001- 2007

Year	*Dividend*	*Stock Buyback*	*Total Yield*

[107] We used the average of the analyst estimates for individual firms (bottom-up). Alternatively, we could have used the top-down estimate for the S&P 500 earnings.

[108] The treasury bond rate is the sum of expected inflation and the expected real rate. If we assume that real growth is equal to the real interest rate, the long term stable growth rate should be equal to the treasury bond rate.

	Yield	Yield	
2001	1.37%	1.25%	2.62%
2002	1.81%	1.58%	3.39%
2003	1.61%	1.23%	2.84%
2004	1.57%	1.78%	3.35%
2005	1.79%	3.11%	4.90%
2006	1.77%	3.39%	5.16%
2007[a]	1.89%	4.00%	5.89%
Average total yield between 2001-2007 =			4.02%

[a]Trailing 12-month data, from September 2006 through September 2007. In January 2008, this was the information that would have been available. The actual cash yield for all of 2007 was 6.49%.

In 2007, for instance, firms collectively returned more than twice as much in the form of buybacks than they paid out in dividends. Since buybacks are volatile over time, and 2007 may represent a high-water mark for the phenomenon, we recomputed the expected cash flows, in table 19, for the next 6 years using the average total yield (dividends + buybacks) of 4.11%, instead of the actual dividends, and the growth rates estimated earlier (5% for the next 5 years, 4.02% thereafter):

Table 19: Cashflows on S&P 500 Index

Year	*Dividends+ Buybacks on Index*
1	63.37
2	66.54
3	69.86
4	73.36
5	77.02

Using these cash flows to compute the expected return on stocks, we derive the following:

$$1468.36 = \frac{63.37}{(1+r)} + \frac{66.54}{(1+r)^2} + \frac{69.86}{(1+r)^3} + \frac{73.36}{(1+r)^4} + \frac{77.02}{(1+r)^5} + \frac{77.02(1.0402)}{(r-.0402)(1+r)^5}$$

Solving for the required return and the implied premium with the higher cash flows:

Required Return on Equity = 8.39%

Implied Equity Risk Premium = Required Return on Equity - Riskfree Rate

= 8.48% - 4.02% = 4.46%

This value (4.46%) would have been our estimate of the equity risk premium on January 1, 2008.

During 2008, the S&P 500 lost just over a third of its value and ended the year at 903.25 and the treasury bond rate plummeted to close at 2.21% on December 31, 2008. Firms also pulled back on stock buybacks and financial service firms in particular cut dividends during the year. The inputs to the equity risk premium computation reflect these changes:

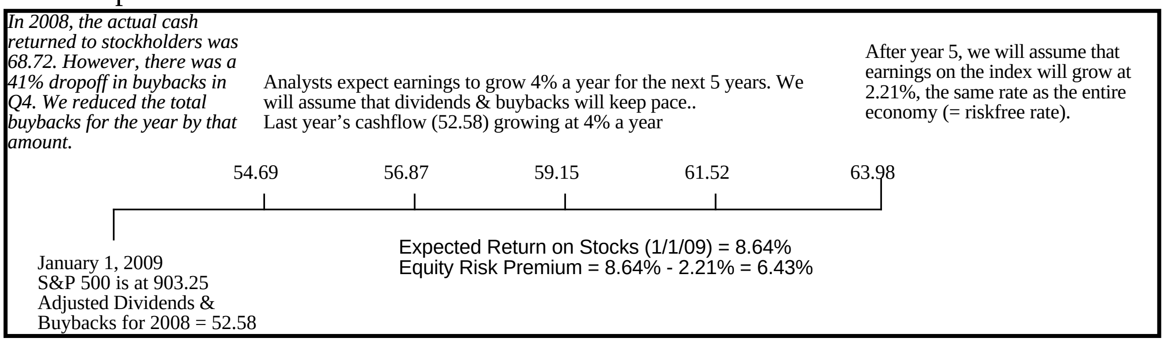
Level of the index = 903.25 (Down from 1468.36)

Treasury bond rate = 2.21% (Down from 4.02%)

Updated dividends and buybacks on the index = 52.58 (Down about 15%)

Expected growth rate = 4% for next 5 years (analyst estimates) and 2.21% thereafter (set equal to riskfree rate).

The computation is summarized below:



The resulting equation is below:

$$903.25 = \frac{54.69}{(1+r)} + \frac{56.87}{(1+r)^2} + \frac{59.15}{(1+r)^3} + \frac{61.52}{(1+r)^4} + \frac{63.98}{(1+r)^5} + \frac{63.98(1.0221)}{(r-.0221)(1+r)^5}$$

Solving for the required return and the implied premium with the higher cash flows:

Required Return on Equity = 8.64%

Implied Equity Risk Premium = Required Return on Equity - Riskfree Rate

= 8.64% - 2.21% = 6.43%

The implied premium rose more than 2%, from 4.37% to 6.43%, over the course of the year, indicating that investors perceived more risk in equities at the end of the year, than they did at the start and were demanding a higher premium to compensate.

By January 2010, the fears of a banking crisis had subsided and the S&P 500 had recovered to 1115.10. However, a combination of dividend cuts and a decline in stock buybacks had combined to put the cash flows on the index down to 40.38 in 2009. That was partially offset by increasing optimism about an economic recovery and expected earnings growth for the next 5 years had bounced back to 7.2%.[109] The resulting equity risk premium is 4.36%:

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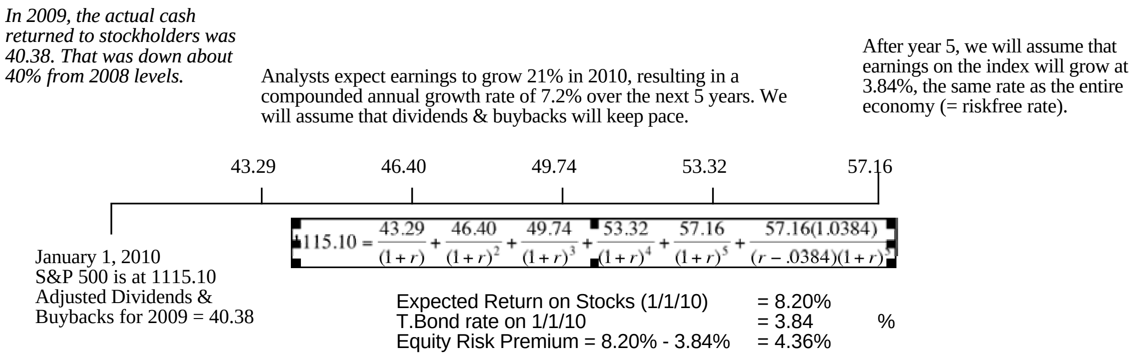
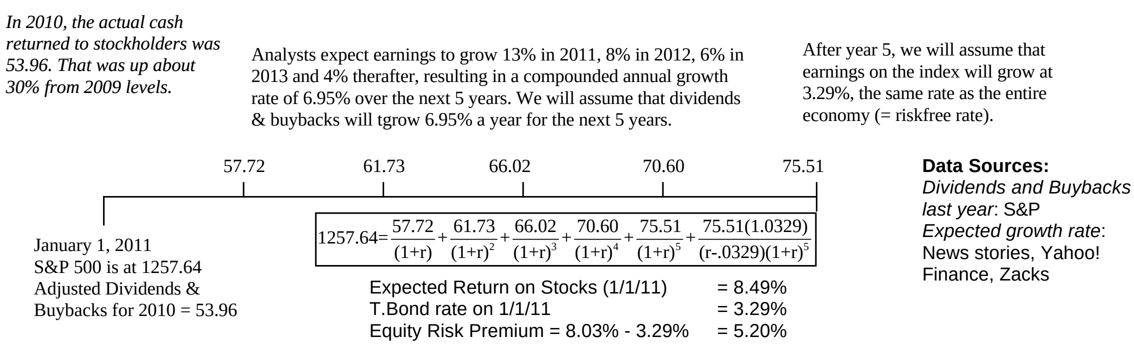
[109] The expected earnings growth for just 2010 was 21%, primarily driven by earnings bouncing back to pre-crisis levels, followed by a more normal 4% earnings growth in the following years. The compounded average growth rate is $((1.21)(1.04)^4)^{1/5}-1 = .072$ or 7.2%.



In effect, equity risk premiums have reverted back to what they were before the 2008 crisis.

Updating the numbers to January 2011, the S&P 500 had climbed to 1257.64, but cash flows on the index, in the form of dividends and buybacks, made an even more impressive comeback, increasing to 53.96 from the depressed 2009 levels. The implied equity risk premium computation is summarized below:



The implied equity risk premium climbed to 5.20%, with the higher cash flows more than offsetting the rise in equity prices.

The S&P 500 ended 2011 at 1257.60, almost unchanged from the level at the start of the year. The other inputs into the implied equity risk premium equation changed significantly over the year:

a. The ten-year treasury bond rate dropped during the course of the year from 3.29% to 1.87%, as the European debt crisis caused a "flight to safety". The US did lose its AAA rating with Standard and Poor's during the course of the year, but we will continue to assume that the T.Bond rate is risk free.
b. Companies that had cut back dividends and scaled back stock buybacks in 2009, after the crisis, and only tentatively returned to the fray in 2010, returned to buying back stocks at almost pre-crisis levels. The total dividends and buybacks

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for the trailing 12 months leading into January 2012 climbed to 72.23, a significant increase over the previous year.[110]

c. Analysts continued to be optimistic about earnings growth, in the face of signs of a pickup in the US economy, forecasting growth rate of 9.6% for 2012 (year 1), 11.9% in 2013, 8.2% in 2014, 4% in 2015 and 2.5% in 2016, leading to a compounded annual growth rate of 7.18% a year:.

Incorporating these inputs into the implied equity risk premium computation, we get an expected return on stocks of 9.29% and an implied equity risk premium of 7.32%:

In the trailing 12 months, the cash returned to stockholders was 72.23.

Analysts expect earnings to grow 9.6% in 2012, 11.9% in 2013, 8.2% in 2014, 4.5% in 2015 and 2% therafter, resulting in a compounded annual growth rate of 7.18% over the next 5 years. We will assume that dividends & buybacks will grow 7.18% a year for the next 5 years.

After year 5, we will assume that earnings on the index will grow at 1.87%, the same rate as the entire economy (= riskfree rate).

77.41 | 82.97 | 88.93 | 95.31 | 102.16

Data Sources:
Dividends and Buybacks last year: S&P
Expected growth rate: News stories, Yahoo! Finance, Bloomberg

January 1, 2012
S&P 500 is at 1257.60
Dividends & Buybacks for 2011 = 72.23

$$\boxed{1257.60 = \frac{77.41}{(1+r)} + \frac{82.97}{(1+r)^2} + \frac{88.93}{(1+r)^3} + \frac{95.31}{(1+r)^4} + \frac{102.16}{(1+r)^5} + \frac{102.16(1.0187)}{(r-.0187)(1+r)^5}}$$

Expected Return on Stocks (1/1/12) = 9.19%
T.Bond rate on 1/1/12 = 1.87%
Equity Risk Premium = 7.91% - 1.87% = 7.32%

Since the index level did not change over the course of the year, the jump in the equity risk premium from 5.20% on January 1, 2011 to 7.32% on January 1, 2012, was precipitated by two factors. The first was the drop in the ten-year treasury bond rate to a historic low of 1.87% and the second was the surge in the cash returned to stockholders, primarily in buybacks. With the experiences of the last decade fresh in our minds, we considered the possibility that the cash returned during the trailing 12 months may reflect cash that had built up during the prior two years, when firms were in their defensive posture. If that were the case, it is likely that buybacks will decline to a more normalized value in future years. To estimate this value, we looked at the total cash yield on the S&P 500 from 2002 to 2011 and computed an average value of 4.69% over the decade in table 20.

Table 20: Dividends and Buybacks on S&P 500 Index: 2002-2011

Year	Dividend Yield	Buybacks/Index	Yield
2002	1.81%	1.58%	3.39%
2003	1.61%	1.23%	2.84%
2004	1.57%	1.78%	3.35%
2005	1.79%	3.11%	4.90%
2006	1.77%	3.39%	5.16%

[110] These represented dividends and stock buybacks from October 1, 2010 to September 30, 2011, based upon the update from S&P on December 22, 2011. The data for the last quarter is not made available until late March of the following year.

2007	1.92%	4.58%	6.49%
2008	3.15%	4.33%	7.47%
2009	1.97%	1.39%	3.36%
2010	1.80%	2.61%	4.42%
2011	2.00%	3.53%	5.54%
Average: Last 10 years =			4.69%

Assuming that the cash returned would revert to this yield provides us with a lower estimate of the cash flow (4.69% of 1257.60= 59.01) and an equity risk premium of 6.01%:

In the trailing 12 months, the cash returned to stockholders was 72.23. Using the average cash yield of 4.69% for 2002-2011 the cash returned would have been 59.01.

Analysts expect earnings to grow 9.6% in 2012, 11.9% in 2013, 8.2% in 2014, 4.5% in 2015 and 2.5% therafter, resulting in a compounded annual growth rate of 7.18% over the next 5 years. We will assume that dividends & buybacks will grow 7.18% a year for the next 5 years.

After year 5, we will assume that earnings on the index will grow at 1.87%, the same rate as the entire economy (= riskfree rate).

63.24 67.78 72.65 77.87 83.46

January 1, 2012
S&P 500 is at 1257.60
Normalized Dividends & Buybacks for 2011 = 59.01

$$1257.60 = \frac{63.24}{(1+r)} + \frac{67.78}{(1+r)^2} + \frac{72.65}{(1+r)^3} + \frac{77.87}{(1+r)^4} + \frac{83.46}{(1+r)^5} + \frac{83.46(1.0287)}{(r-.0187)(1+r)^5}$$

Expected Return on Stocks (1/1/12) = 7.88%
T.Bond rate on 1/1/12 = 1.87%
Equity Risk Premium = 7.91% - 1.87% = 7.32%

Data Sources:
Dividends and Buybacks last year: S&P
Expected growth rate: News stories, Yahoo! Finance, Bloomberg

So, did the equity risk premium for the S&P 500 jump from 5.20% to 7.32%, as suggested by the raw cash yield, or from 5.20% to 6.01%, based upon the normalized yield? We would be more inclined to go with the latter, especially since the index remained unchanged over the year. Note, though, that if the cash returned by firms does not drop back in the next few quarters, we will revisit the assumption of normalization and the resulting lower equity risk premium.

By January 1, 2013, the S&P 500 climbed to 1426.19 and the treasury bond rate had dropped to 1.76%. The dividends and buybacks were almost identical to the prior year and the smoothed out cash returned (using the average yield over the prior 10 years) climbed to 69.46. Incorporating the lower growth expectations leading into 2013, the implied equity risk premium dropped to 5.78% on January 1, 2013:

In 2012, the actual cash returned to stockholders was 72.25. Using the average total yield for the last decade yields 69.46

Analysts expect earnings to grow 7.67% in 2013, 7.28% in 2014, scaling down to 1.76% in 2017, resulting in a compounded annual growth rate of 5.27% over the next 5 years. We will assume that dividends & buybacks will tgrow 5.27% a year for the next 5 years.

After year 5, we will assume that earnings on the index will grow at 1.76%, the same rate as the entire economy (= riskfree rate).

73.12 76.97 81.03 85.30 89.80

January 1, 2013
S&P 500 is at 1426.19
Adjusted Dividends & Buybacks for base year = 69.46

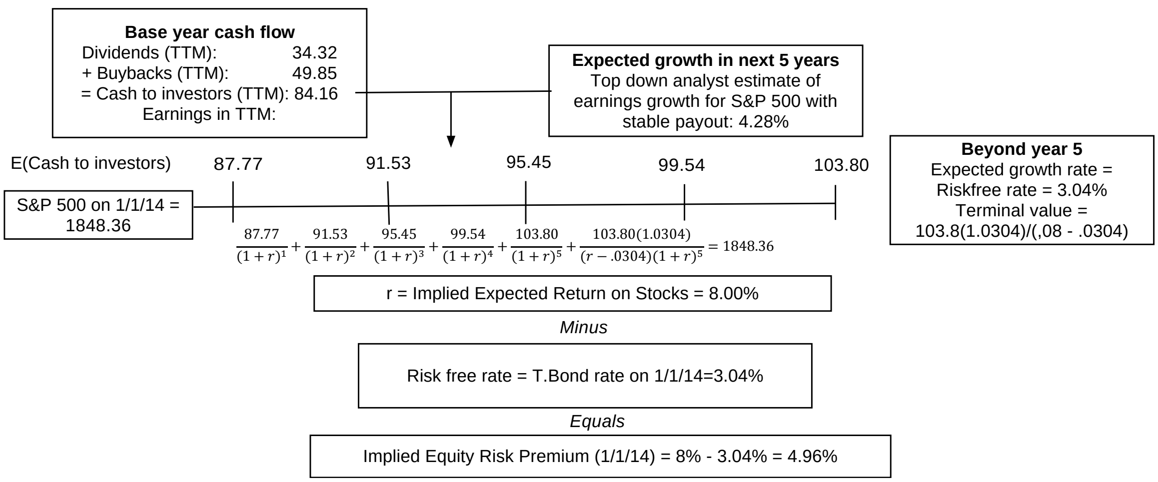
$$1426.19 = \frac{73.12}{(1+r)} + \frac{76.97}{(1+r)^2} + \frac{81.03}{(1+r)^3} + \frac{85.30}{(1+r)^4} + \frac{89.80}{(1+r)^5} + \frac{89.80(1.0176)}{(r-.0176)(1+r)^5}$$

Expected Return on Stocks (1/1/13) = 7.54%
T.Bond rate on 1/1/13 = 1.76%
Equity Risk Premium = 7.54% - 1.76% = 5.78%

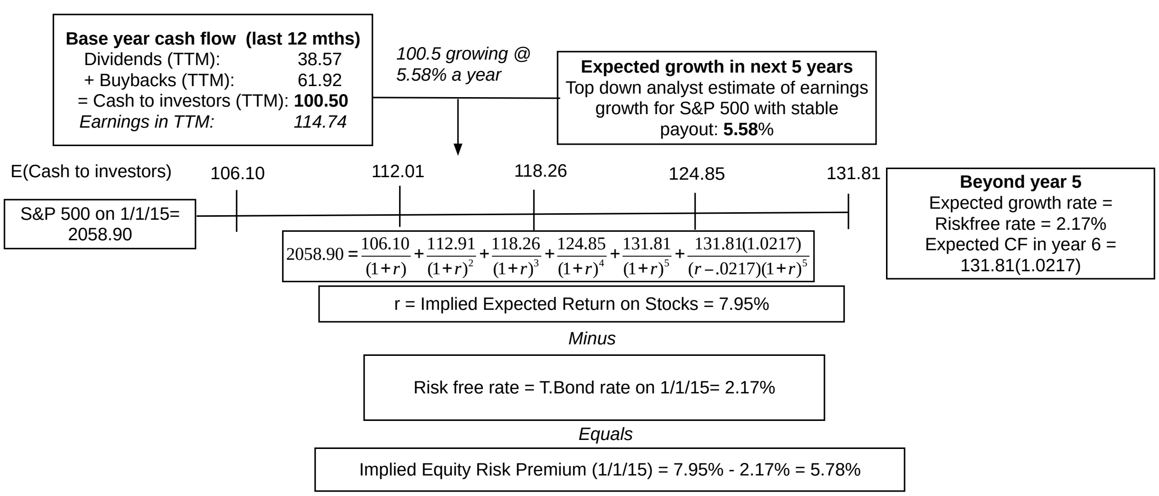
Data Sources:
Dividends and Buybacks last year: S&P
Expected growth rate: S&P, Media reports, Factset, Thomson-Reuters

Note that the chasm between the trailing 12-month cash flow premium and the smoother cash yield premium that had opened up at the start of 2012 had narrowed. The trailing 12-month cash flow premium was 6%, just 0.22% higher than the 5.78% premium obtained with the smoothed out cash flow.

After a good year for stocks, the S&P 500 was at 1848.36 on January 1, 2014, up 29.6% over the prior year, and cash flows also jumped to 84.16 over the trailing 12 months (ending September 30, 2013), up 16.48% over the prior year. Incorporating an increase in the US ten-year treasury bond rate to 3.04%, the implied equity risk premium at the start of 2014 was 4.96%.



During 2014, stocks continued to rise, albeit at a less frenetic pace, and the US ten-year treasury bond rate dropped back again to 2.17%. Since buybacks and dividends grew at higher rate than prices, the net effect was an increase in the implied equity risk premium to 5.78% at the start of 2015:



A Term Structure for Equity Risk Premiums

When we estimate an implied equity risk premium, from the current level of the index and expected future cash flows, we are estimating a compounded average equity risk premium over the long term. Thus, the 5.78% estimate of the equity risk premium at the start of 2015 is the geometric average of the annualized equity risk premiums in future years and is analogous to the yield to maturity on a long term bond.

But is it possible that equity risk premiums have a term structure, just as interest rates do? Absolutely. In a creative attempt to measure the slope of the term structure of equity risk premiums, Binsberger, Brandt and Koijen (2012) use dividend strips, i.e., short term assets that pay dividends for finite time periods (and have no face value), to extract equity risk premiums for the short term as opposed to the long term. Using dividend strips on the S&P 500 to extract expected returns from 1996 to 2009, they find that equity risk premiums are higher for shorter term claims than for longer term claims, by approximately 2.75%.[111] Their findings are contested by Boguth, Carlson, Fisher and Simutin (2011), who note that small market pricing frictions are amplified when valuing synthetic dividend strips and that using more robust return measures results in no significant differences between short term and longer term equity risk premiums.[112]

While this debate will undoubtedly continue, the relevance to valuation and corporate finance practice is questionable. Even if you could compute period-specific equity risk premiums, the effect on value of using these premiums (instead of the compounded average premium) would be small in most valuations. To illustrate, your valuation of an asset, using an equity risk premium of 7% for the first 3 years and 5.5% thereafter[113], at the start of 2015, would be very similar to the value you would have obtained using 5.78% as your equity risk premium for all time periods. The only scenario where using year-specific premiums would make a material difference would be in the valuation of an asset or investment with primarily short-term cash flows, where using a higher short term premium will yield a lower (and perhaps more realistic) value for the asset.

[111] Binsbergen, J. H. van, Michael W. Brandt, and Ralph S. J. Koijen, 2012, *On the timing and pricing of dividends*, American Economic Review, v102, 1596-1618.

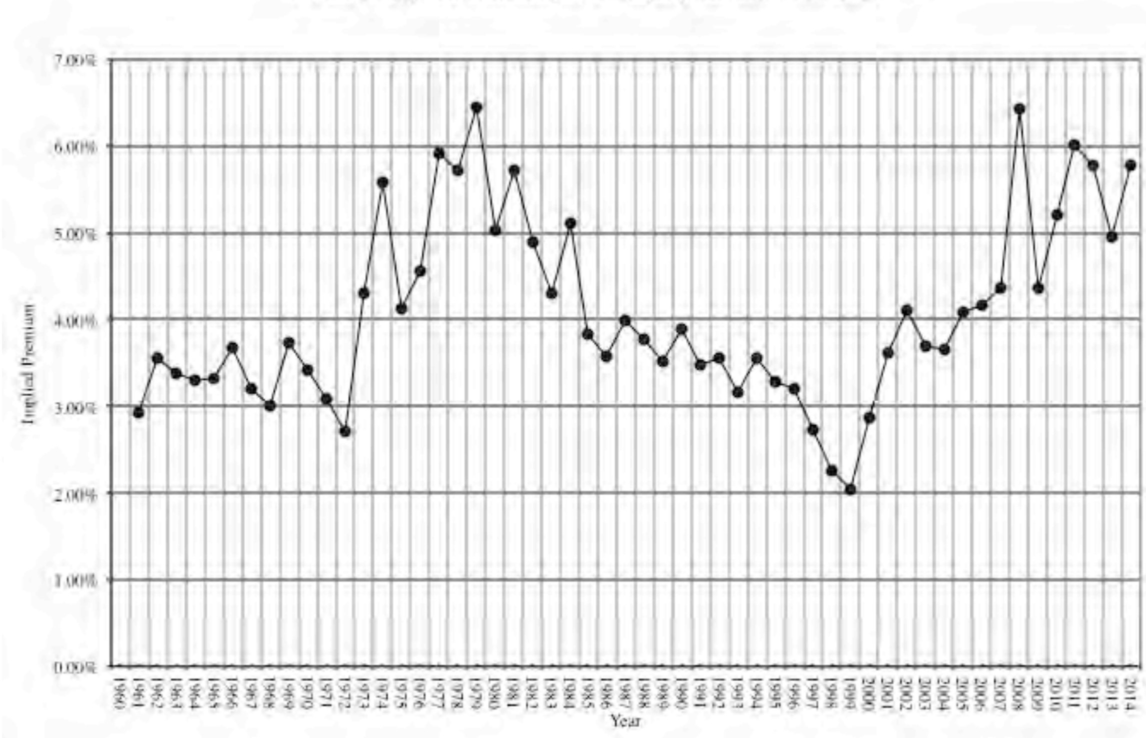
[112] Boguth, O., M. Carlson, A. Fisher and M. Simutin, 2011, *Dividend Strips and the Term Structure of Equity Risk Premia: A Case Study of Limits to Arbitrage*, Working Paper, http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1931105. In a response, Binsbergen, Brandt and Koijen argue that their results hold even if traded dividend strips (rather than synthetic strips) are used.

[113] The compounded average premium over time, using a 7% equity risk premium for the first 3 years and 5.88% thereafter, is roughly 6.01%.

Time Series Behavior for S&P 500 Implied Premium

As the inputs to the implied equity risk premium, it is quite clear that the value for the premium will change not just from day to day but from one minute to the next. In particular, movements in the index will affect the equity risk premium, with higher (lower) index values, other things remaining equal, translating into lower (higher) implied equity risk premiums. In Figure 9, we chart the implied premiums in the S&P 500 from 1960 to 2014 (year ends):

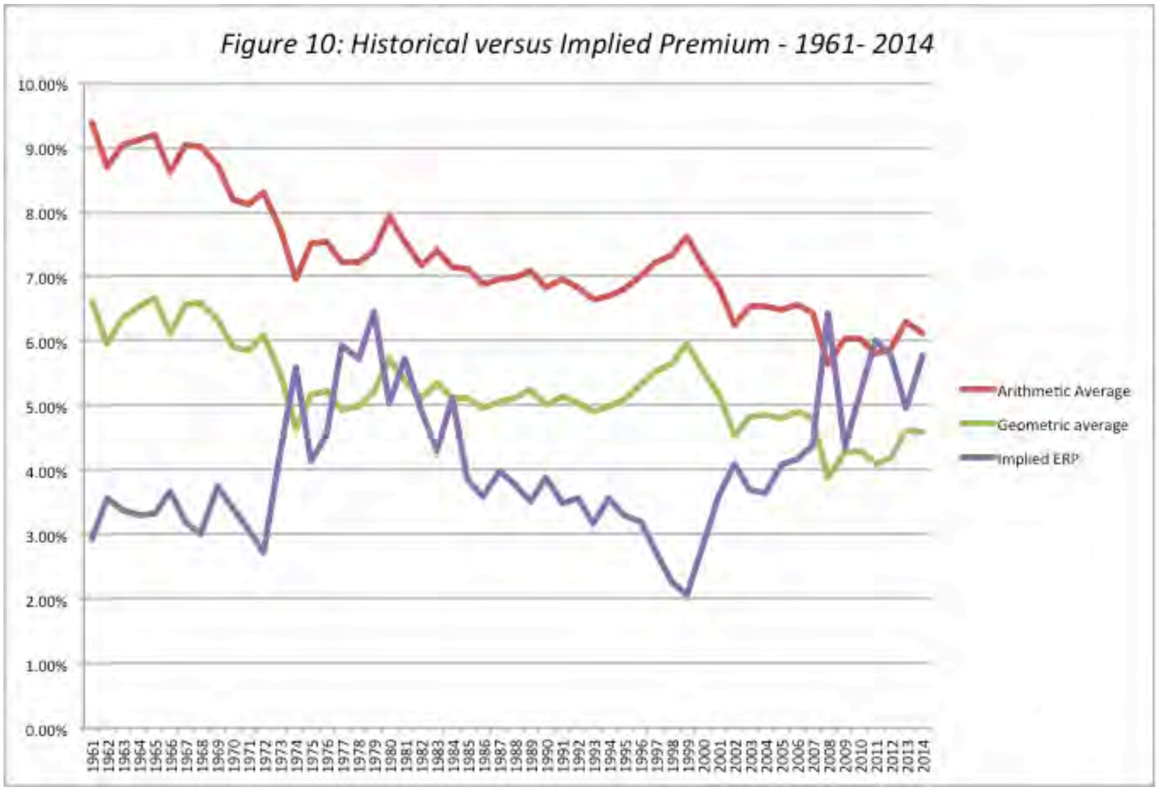
Figure 9: Implied Premium for US Equity Market: 1960-2014



In terms of mechanics, we used potential dividends (including buybacks) as cash flows, and a two-stage discounted cash flow model; the estimates for each year are in appendix 6.[114] Looking at these numbers, we would draw the following conclusions:

- The implied equity premium has generally been lower than the historical risk premium for the US equity market for most of the last few decades. To provide a contrast, we compare the implied equity risk premiums each year to the historical risk premiums for stocks over treasury bonds, using both geometric and arithmetic averages, each year from 1961 to 2014 in figure 10:

[114] We used analyst estimates of growth in earnings for the 5-year growth rate after 1980. Between 1960 and 1980, we used the historical growth rate (from the previous 5 years) as the projected growth, since analyst estimates were difficult to obtain. Prior to the late 1980s, the dividends and potential dividends were very similar, because stock buybacks were uncommon. In the last 20 years, the numbers have diverged.



The arithmetic average premium, which is used by many practitioners, has been significantly higher than the implied premium over almost the entire fifty-year period (with 2009 and 2011 being the only exceptions). The geometric premium does provide a more interesting mix of results, with implied premiums exceeding historical premiums in the mid-1970s and again since 2008.

- The implied equity premium did increase during the seventies, as inflation increased. This does have interesting implications for risk premium estimation. Instead of assuming that the risk premium is a constant, and unaffected by the level of inflation and interest rates, which is what we do with historical risk premiums, would it be more realistic to increase the risk premium if expected inflation and interest rates go up? We will come back and address this question in the next section.
- While historical risk premiums have generally drifted down for the last few decades, there is a strong tendency towards mean reversion in implied equity premiums. Thus, the premium, which peaked at 6.5% in 1978, moved down towards the average in the 1980s. By the same token, the premium of 2% that we observed at the end of the dot-com boom in the 1990s quickly reverted back to the average, during the market correction from 2000-2003.[115] Given this tendency, it is possible that we can end up

[115] Arnott, Robert D., and Ronald Ryan, 2001, *The Death of the Risk Premium: Consequences of the*

with a far better estimate of the implied equity premium by looking at not just the current premium, but also at historical trend lines. We can use the average implied equity premium over a longer period, say ten to fifteen years. Note that we do not need as many years of data to make this estimate as we do with historical premiums, because the standard errors tend to be smaller.

Finally, the crisis of 2008 was unprecedented in terms of its impact on equity risk premiums. Implied equity risk premiums rose more during 2008 than in any one of the prior 50 years, with much of the change happening in a fifteen week time period towards the end of the year. While much of that increase dissipated in 2009, as equity risk premiums returned to pre-crisis levels, equity risk premiums have remained more volatile since 2008. In the next section, we will take a closer look at this time period.

Implied Equity Risk Premiums during a Market Crisis and Beyond

When we use historical risk premiums, we are, in effect, assuming that equity risk premiums do not change much over short periods and revert back over time to historical averages. This assumption was viewed as reasonable for mature equity markets like the United States, but was put under a severe test during the market crisis that unfolded with the fall of Lehman Brothers on September 15, and the subsequent collapse of equity markets, first in the US, and then globally.

Since implied equity risk premiums reflect the current level of the index, the 75 trading days between September 15, 2008, and December 31, 2008, offer us an unprecedented opportunity to observe how much the price charged for risk can change over short periods. In figure 11, we depict the S&P 500 on one axis and the implied equity risk premium on the other. To estimate the latter, we used the level of the index and the treasury bond rate at the end of each day and used the total dollar dividends and buybacks over the trailing 12 months to compute the cash flows for the most recent year.[116] We also updated the expected growth in earnings for the next 5 years, but that number changed only slowly over the period. For example, the total dollar dividends and buybacks on the index for the trailing 12 months of 52.58 resulted in a dividend yield of 4.20% on September 12 (when the index closed at 1252) but jumped to 4.97% on October 6, when the index closed at 1057.[117]

1990s, Journal of Portfolio Management, v27, 61-74. They make the same point about reduction in implied equity risk premiums that we do. According to their calculations, though, the implied equity risk premium in the late 1990s was negative.

[116] This number, unlike the index and treasury bond rate, is not updated on a daily basis. We did try to modify the number as companies in the index announced dividend suspensions or buyback modifications.

[117] It is possible, and maybe even likely, that the banking crisis and resulting economic slowdown was leading some companies to reassess policies on buybacks. Alcoa, for instance, announced that it was terminating stock buybacks. However, other companies stepped up buybacks in response to lower stock

Figure 11: Implied Equity Risk Premium - 9/12- 12/31/08

In a period of a month, the implied equity risk premium rose from 4.20% on September 12 to 6.39% at the close of trading of October 10 as the S&P moved from 1250 down to 903. Even more disconcertingly, there were wide swings in the equity risk premium within a day; in the last trading hour just on October 10, the implied equity risk premium ranged from a high of 6.6% to a low of 6.1%. Over the rest of the year, the equity risk premium gyrated, hitting a high of 8% in late November, before settling into the year-end level of 6.43%.

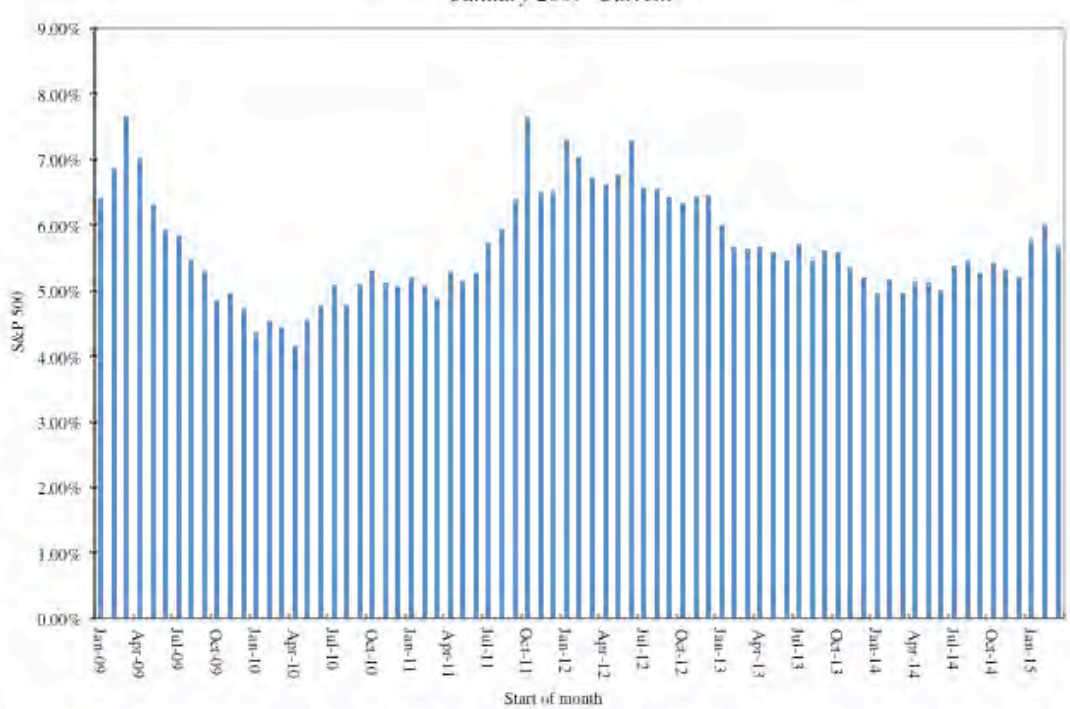
The volatility captured in figure 12 was not restricted to just the US equity markets. Global equity markets gyrated with and sometimes more than the US, default spreads widened considerably in corporate bond markets, commercial paper and LIBOR rates soared while the 3-month treasury bill rate dropped close to zero and the implied volatility in option markets rose to levels never seen before. Gold surged but other commodities, such as oil and grains, dropped. Not only did we discover how intertwined equity markets are around the globe but also how markets for all risky assets are tied together. We will explicitly consider these linkages as we go through the rest of the paper.

prices. If the total cash return was dropping, as the market was, the implied equity risk premiums should be lower than the numbers that we have computed.

There are two ways in which we can view this volatility. One the one side, proponents of using historical averages (either of actual or implied premiums) will use the day-to-day volatility in market risk premiums to argue for the stability of historical averages. They are implicitly assuming that when the crisis passes, markets will return to the status quo. On the other hand, there will be many who point to the unprecedented jump in implied premiums over a few weeks and note the danger of sticking with a "fixed" premium. They will argue that there are sometimes structural shifts in markets, i.e. big events that change market risk premiums for long periods, and that we should be therefore be modifying the risk premiums that we use in valuation as the market changes around us. In January 2009, in the context of equity risk premiums, the first group would have argued we should ignore history (both in terms of historical returns and implied equity risk premiums) and move to equity risk premiums of 6%+ for mature markets (and higher for emerging markets whereas the second would have made a case for sticking with a historical average, which would have been much lower than 6.43%.

The months since the crisis ended in 2008 have seen ups and downs in the implied premium, with clear evidence that the volatility in the equity risk premium has increased over the last few years. In figure 12, we report on the monthly equity risk premiums for the S&P 500 from January 2009 through March 2015:

Figure 12: Implied ERP by month: S&P 500
January 2009- Current



Note that the equity risk premium dropped from its post-crisis highs in 2010 but climbed back in 2011 to 6% or higher, before dropping back to 5% in 2013, before rising again in the last year.

On a personal note, I believe that the very act of valuing companies requires taking a stand on the appropriate equity risk premium to use. For many years prior to September 2008, I used 4% as my mature market equity risk premium when valuing companies, and assumed that mean reversion to this number (the average implied premium over time) would occur quickly and deviations from the number would be small. Though mean reversion is a powerful force, I think that the banking and financial crisis of 2008 has created a new reality, i.e., that equity risk premiums can change quickly and by large amounts even in mature equity markets. Consequently, I have forsaken my practice of staying with a fixed equity risk premium for mature markets, and I now vary it year-to-year, and even on an intra-year basis, if conditions warrant. After the crisis, in the first half of 2009, I used equity risk premiums of 6% for mature markets in my valuations. As risk premiums came down in 2009, I moved back to using a 4.5% equity risk premium for mature markets in 2010. With the increase in implied premiums at the start of 2011, my valuations for the year were based upon an equity risk premium of 5% for mature markets and I increased that number to 6% for 2012. In 2015, I will be using a lower equity risk premium (5.75%), reflecting the implied premium at the start o the year but will remain vigilant by computing the premium on a monthly basis. While some may view this shifting equity risk premium as a sign of weakness, I would frame it differently. When valuing individual companies, I want my valuations to reflect my assessments of the company and not my assessments of the overall equity market. Using equity risk premiums that are very different from the implied premium will introduce a market view into individual company valuations.

Determinants of Implied Premiums

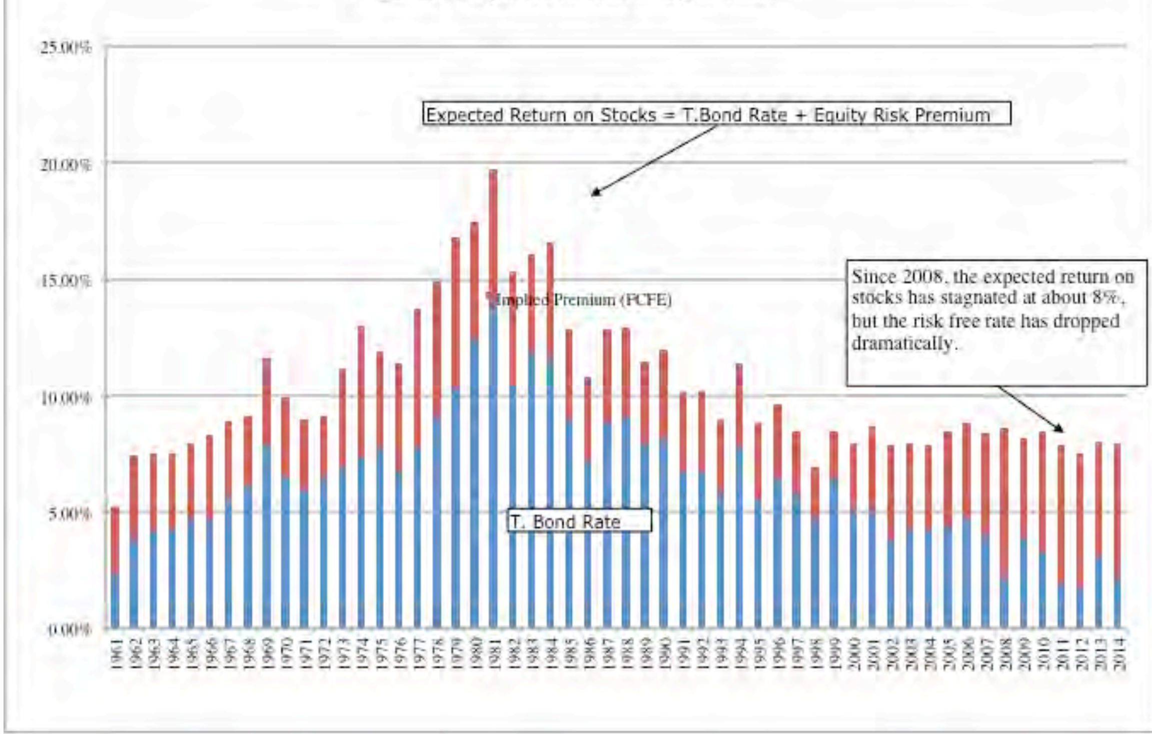
One of the advantages of estimating implied equity risk premiums, by period, is that we can track year to year changes in that number and relate those changes to shifts in interest rates, the macro environment or even to company characteristics. By doing so, not only can we get a better understanding of what causes equity risk premiums to change over time, but we are also able to come up with better estimates of future premiums.

Implied ERP and Interest rates

In much of valuation and corporate finance practice, we assume that the equity risk premium that we compute and use is unrelated to the level of interest rates. In particular, the use of historical risk premiums, where the premium is based upon an average premium earned over shifting risk free rates, implicitly assumes that the level of the premium is unchanged as the risk free rate changes. Thus, we use the same equity risk premium of 4.2% (the historical average for 1928-2012) on a risk free rate of 1.76% in 2012, as we would have, if the risk free rate had been 10%.

But is this a reasonable assumption? How much of the variation in the premium over time can be explained by changes in interest rates? Put differently, do equity risk premiums increase as the risk free rate increases or are they unaffected? To answer this question, we looked at the relationship between the implied equity risk premium and the treasury bond rate (risk free rate). As can be seen in figure 13, the implied equity risk premiums were highest in the 1970s, when interest rates and inflation were also high. However, there is contradictory evidence between 2008 and 2014, when high equity risk premiums accompanied low risk free rates.

Figure 13: Implied ERP and Risk free Rates



To examine the relationship between equity risk premiums and risk free rates, we ran a regression of the implied equity risk premium against both the level of long-term rates (the treasury bond rate) and the slope of the yield curve (captured as the difference between the 10-year treasury bond rate and the 3-month T.Bill rate), with the t statistics reported in brackets below each coefficient:

$$\text{Implied ERP} = \underset{(8.45)}{3.62\%} + \underset{(1.05)}{0.0570}\,(\text{T.Bond Rate}) + \underset{(0.37)}{0.0731}\,(\text{T.Bond} - \text{T.Bill}) \quad R^2 = 2.54\%$$

There is a mildly positive relationship between the T.Bond rate and implied equity risk premiums: every 1% increase in the treasury bond rate increases the equity risk premium by 0.06%. The slope of the yield curve seems to have little impact on the implied equity risk premium. Removing the latter variable and running the regression again:

Implied ERP = 3.74% + 0.0531 (T.Bond Rate) R^2=1.88%
(10.27) (1.00)

This regression does provide very weak support for the view that equity risk premiums should not be constant but should be linked to the level of interest rates. In fact, the regression can be used to estimate an equity risk premium, conditional on current interest rates. On March 14, 2015, for instance, when the 10-year treasury bond rate was 2.75%, the implied equity risk premium would have been computed as follows:

Implied ERP = 3.74% + 0.0531 (2.25%) = 3.86%

This would have been below the observed implied equity risk premium of about 5.78% and the average implied equity risk premium of 4.1% between 1960 and 2014. Put differently, given the low level of risk free rates in 2015 and the historical relationship between equity risk premiums and risk free rates, we would have expected the equity risk premium to be a much lower number (3.86%) than the actual number (5.78%).

Implied ERP and Macroeconomic variables

While we considered the interaction between equity risk premiums and interest rates in the last section, the analysis can be expanded to include other macroeconomic variables including economic growth, inflation rates and exchange rates. Doing so may give us a way of estimating an "intrinsic' equity risk premium, based upon macro economic variables, that is less susceptible to market moods and perceptions.

To explore the relationship, we estimated the correlation, between the implied equity risk premiums that we estimated for the S&P 500 and three macroeconomic variables – real GDP growth for the US, inflation rates (CPI) and exchange rates (trade weighted dollar), using data from 1973 to 2014, in table 21 (t statistics in brackets):

Table 21: Correlation Matrix: ERP and Macroeconomic variables: 1973-2015

	ERP	*Weighted Dollar*	*Real GDP*	*CPI*
ERP	1.0000			
Weighted dollar	-0.3492 (2.33)**	1.0000		
Real GDP	0.3883 (2.63)**	-0.1608 (01.02)	1.0000	
CPI	0.1452	-0.1550	0.0123	1.0000

	(0.92)	(0.98)	(0.08)	

** Statistically significant

The implied equity risk premium is positively correlated with GDP growth, decreasing as GDP growth increases and negatively correlated with the US dollar, with a stronger dollar going with lower implied equity risk premiums. The ERP is also mildly affected by inflation, with higher inflation going hand-in-hand with higher equity risk premiums.[118]

Following up on this analysis, we regressed equity risk premiums against the inflation rate, the weighted dollar and GDP growth, using data from 1974 to 2014:

Implied ERP = 4.21% - 0.1419 Real GDP growth + 0.1204 CPI + 0.0149 Weighted $ R^2= 30.68%
(12.13) (1.90) (2.36) (0.67)

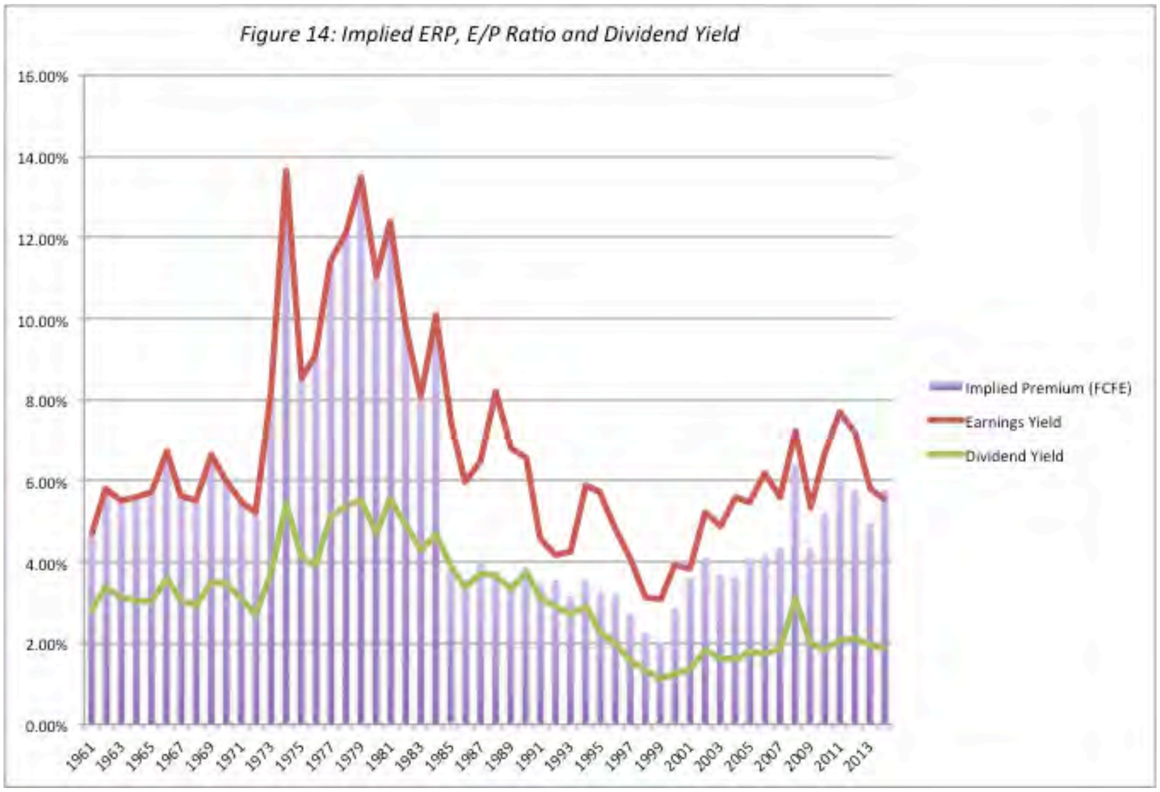
Based on this regression, every 1% increase in the inflation rate increases the equity risk premium by approximately 0.1204%, whereas every 1% increase in the growth rate in real GDP decreases the implied equity risk premium by 0.1419%.

From a risk perspective, it is not the level of GDP growth that matters, but uncertainty about that level; you can have low and stable economic growth and high and unstable economic growth. Since 2008, the economies of both developed and emerging markets have become more unstable over time and upended long held beliefs about developed economies. It will be interesting to see if equity risk premiums become more sensitive to real economic growth in this environment.

Implied ERP, Earnings Yields and Dividend Yields

Earlier in the paper, we noted that the dividend yield and the earnings yield (net of the risk free rate) can be used as proxies for the equity risk premium, if we make assumptions about future growth (stable growth, with the dividend yield) or expected excess returns (zero, with the earnings yield). In figure 14, we compare the implied equity risk premiums that we computed to the earnings and dividend yields for the S&P 500 from 1961 to 2014:

[118] The correlation was also computed for lagged and leading versions of these variables, with two material differences: the equity risk premium is negatively correlated with leading inflation rates and positively correlated with a leading weighted dollar.



Note that the dividend yield is a very close proxy for the implied equity risk premium until the late 1980s, when the two measures decoupled, a phenomenon that is best explained by the rise of stock buybacks as an alternative way of returning cash to stockholders.

The earnings yield, with the riskfree rate netted out, has generally not been a good proxy for the implied equity risk premium and would have yielded negative values for the equity risk premium (since you have to subtract out the risk free rate from it) through much of the 1990s. However, it does move with the implied equity risk premium. The difference between the earnings to price measure and the implied ERP can be attributed to a combination of higher earnings growth and excess returns that investors expect companies to deliver in the future. Analysts and academic researchers who use the earnings to price ratio as a proxy for forward-looking equity risk premiums may therefore end up with significant measurement error in their analyses.

Implied ERP and Technical Indicators

Earlier in the paper, we noted that any market timing forecast can be recast as a view on the future direction of the equity risk premium. Thus, a view that the market is under (over) priced and likely to go higher (lower is consistent with a belief that equity risk premiums will decline (increase) in the future. Many market timers do rely on technical indicators, such as moving averages and momentum measures, to make their judgment about market direction. To evaluate whether these approaches have a basis, you would need to look at how these measures are correlated with changes in equity risk premiums.

In a test of the efficacy of technical indicators, Neely, Rapach, Tu and Zhou (2011) compare the predictive power of macroeconomic/fundamental indications (including the interest rate, inflation, GDP growth and earnings/dividend yield numbers) with those of technical indicators (moving average, momentum and trading volume) and conclude that the latter better explain movements in stock returns.[119] They conclude that a composite prediction, that incorporates both macroeconomic and technical indicators, is superior to using just one set or the other of these variables. Note, however, that their study focused primarily on the predictability of stock returns over the next year and not on longer term equity risk premiums.

Extensions of Implied Equity Risk Premium

The process of backing out risk premiums from current prices and expected cashflows is a flexible one. It can be expanded into emerging markets to provide estimates of risk premiums that can replace the country risk premiums we developed in the last section. Within an equity market, it can be used to compute implied equity risk premiums for individual sectors or even classes of companies.

Other Equity Markets

The advantage of the implied premium approach is that it is market-driven and current, and does not require any historical data. Thus, it can be used to estimate implied equity premiums in any market, no matter how short its history, It is, however, bounded by whether the model used for the valuation is the right one and the availability and reliability of the inputs to that model. Earlier in this paper, we estimated country risk premiums for Brazil, using default spreads and equity market volatile. To provide a contrast, we estimated the implied equity risk premium for the Brazilian equity market in September 2009, from the following inputs.

- The index (Bovespa) was trading at 61,172 on September 30, 2009, and the dividend yield on the index over the previous 12 months was approximately 2.2%. While stock buybacks represented negligible cash flows, we did compute the FCFE for companies in the index, and the aggregate FCFE yield across the companies was 4.95%.

[119] Neely, C.J., D.E. Rapach, J. Tu and G. Zhou, 2011, *Forecasting the Equity Risk Premium: The Role of Technical Indicators*, Working Paper, http://ssrn.com/abstract=1787554.

- Earnings in companies in the index are expected to grow 6% (in US dollar terms) over the next 5 years, and 3.45% (set equal to the treasury bond rate) thereafter.
- The riskfree rate is the US 10-year treasury bond rate of 3.45%.

The time line of cash flows is shown below:

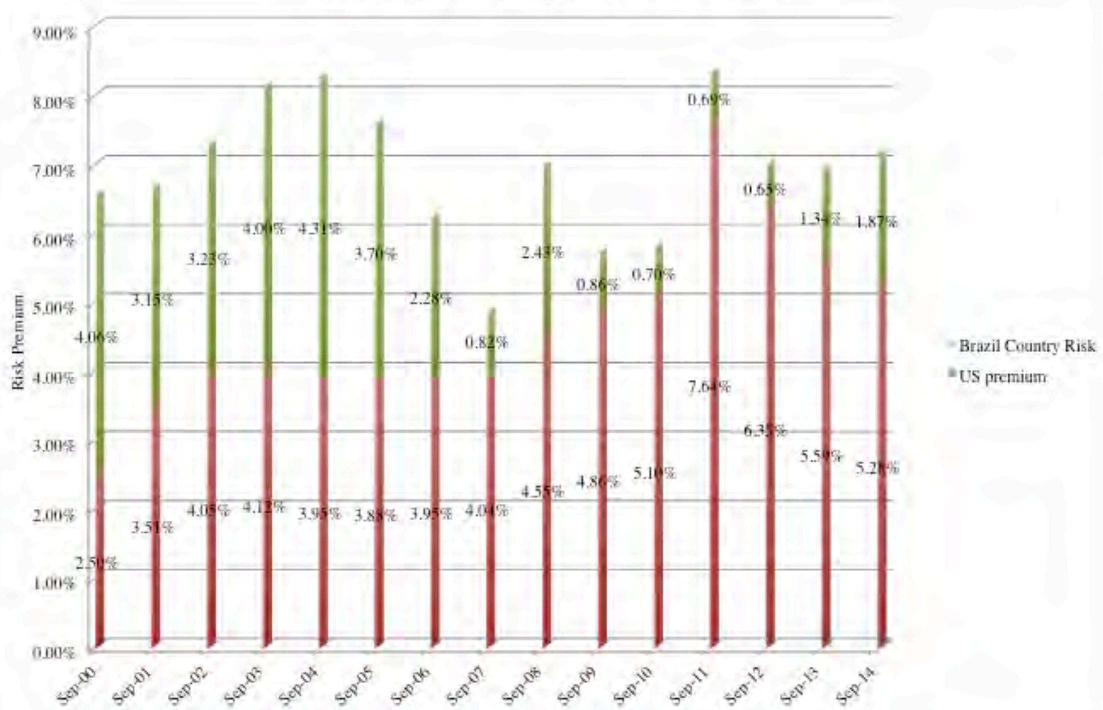
$$61,272 = \frac{3210}{(1+r)} + \frac{3,402}{(1+r)^2} + \frac{3,606}{(1+r)^3} + \frac{3,821}{(1+r)^4} + \frac{4,052}{(1+r)^5} + \frac{4,052(1.0345)}{(r-.0345)(1+r)^5}$$

These inputs yield a required return on equity of 9.17%, which when compared to the treasury bond rate of 3.45% on that day results in an implied equity premium of 5.72%. For simplicity, we have used nominal dollar expected growth rates[120] and treasury bond rates, but this analysis could have been done entirely in the local currency.

One of the advantages of using implied equity risk premiums is that that they are more sensitive to changing market conditions. The implied equity risk premium for Brazil in September 2007, when the Bovespa was trading at 73512, was 4.63%, lower than the premium in September 2009, which in turn was much lower than the premium prevailing in September 2014. In figure 15, we trace the changes in the implied equity risk premium in Brazil from September 2000 to September 2014 and compare them to the implied premium in US equities:

[120] The input that is most difficult to estimate for emerging markets is a long-term expected growth rate. For Brazilian stocks, I used the average consensus estimate of growth in earnings for the largest Brazilian companies which have ADRs listed on them. This estimate may be biased, as a consequence.

Figure 15: Implied Equity Risk Premium - Brazil



Implied equity risk premiums in Brazil declined steadily from 2003 to 2007, with the September 2007 numbers representing a historic low. They surged in September 2008, as the crisis unfolded, fell back in 2009 and 2010 but increased again in 2011. In fact, the Brazil portion of the implied equity risk premium fell to its lowest level in ten years in September 2010, a phenomenon that remained largely unchanged in 2011 and 2012. Political turmoil and corruptions scandals have combined to push the premium back up again in the last year or two.

Computing and comparing implied equity risk premiums across multiple equity markets allows us to pinpoint markets that stand out, either as over priced (because their implied premiums are too low, relative to other markets) or under priced (because their premiums at too high, relative to other markets). In September 2007, for instance, the implied equity risk premiums in India and China were roughly equal to or even lower than the implied premium for the United States, computed at the same time. Even an optimist on future growth these countries would be hard pressed to argue that equity markets in these markets and the United States were of equivalent risk, which would lead us to conclude that these stocks were overvalued relative to US companies.

One final note is worth making. Over the last decade, the implied equity risk premiums in the largest emerging markets – India, China and Brazil- have all declined substantially, relative to developed markets. In table 22, we summarize implied equity risk premiums for developed and emerging markets from 2001 and 2013, making simplistic assumptions about growth and stable growth valuation models:[121]

Table 22: Developed versus Emerging Market Equity Risk Premiums

Start of year	*PBV Developed*	*PBV Emerging*	*ROE (Dev)*	*ROE (Emerg)*	*US T.Bond*	*Cost of Equity (Developed)*	*Cost of Equity (Emerging)*	*Differential ERP*
2004	2.00	1.19	10.81%	11.65%	4.25%	7.28%	10.63%	3.35%
2005	2.09	1.27	11.12%	11.93%	4.22%	7.26%	10.50%	3.24%
2006	2.03	1.44	11.32%	12.18%	4.39%	7.55%	10.11%	2.56%
2007	1.67	1.67	10.87%	12.88%	4.70%	8.19%	10.00%	1.81%
2008	0.87	0.83	9.42%	11.12%	4.02%	10.30%	12.37%	2.07%
2009	1.20	1.34	8.48%	11.02%	2.21%	7.35%	9.04%	1.69%
2010	1.39	1.43	9.14%	11.22%	3.84%	7.51%	9.30%	1.79%
2011	1.12	1.08	9.21%	10.04%	3.29%	8.52%	9.61%	1.09%
2012	1.17	1.18	9.10%	9.33%	1.88%	7.98%	8.35%	0.37%
2013	1.56	1.63	8.67%	10.48%	1.76%	6.02%	7.50%	1.48%
2014	1.95	1.50	9.27%	9.64%	3.04%	6.00%	7.77%	1.77%
2015	1.88	1.56	9.69%	9.75%	2.17%	5.94%	7.39%	1.45%

The trend line from 2004 to 2012 is clear as the equity risk premiums, notwithstanding a minor widening in 2008, have converged in developed and emerging markets, suggesting that globalization has put "emerging market risk" into developed markets, while creating "developed markets stability factors" (more predictable government policies, stronger legal and corporate governance systems, lower inflation and stronger currencies) in emerging markets. In the last two years, we did see a correction in emerging markets that pushed the premium back up, albeit to a level that was still lower than it was prior to 2010.

Sector premiums

Using current prices and expected future cash flows to back out implied risk premiums is not restricted to market indices. We can employ the approach to estimate the implied equity risk premium for a specific sector at a point in time. In September 2008,

[121] We start with the US treasury bond rate as the proxy for global nominal growth (in US dollar terms), and assume that the expected growth rate in developed markets is 0.5% lower than that number and the expected growth rate in emerging markets is 1% higher than that number. The equation used to compute the ERP is a simplistic one, based on the assumptions that the countries are in stable growth and that the return on equity in each country is a predictor of future return on equity:

PBV = (ROE – g)/ (Cost of equity –g)

Cost of equity = (ROE –g + PBV(g))/ PBV

for instance, there was a widely held perception that investors were attaching much higher equity risk premiums to commercial bank stocks, in the aftermath of the failures of Fannie Mae, Freddie Mac, Bear Stearns and Lehman. To test this proposition, we took a look at the S&P Commercial Bank index, which was trading at 318.26 on September 12, 2008, with an expected dividend yield of 5.83% for the next 12 months. Assuming that these dividends will grow at 4% a year for the next 5 years and 3.60% (the treasury bond rate) thereafter, well below the nominal growth rate in the overall economy, we arrived at the following equation:

$$318.26 = \frac{19.30}{(1+r)} + \frac{20.07}{(1+r)^2} + \frac{20.87}{(1+r)^3} + \frac{21.71}{(1+r)^4} + \frac{22.57}{(1+r)^5} + \frac{22.57(1.036)}{(r-.036)(1+r)^5}$$

Solving for the expected return yields a value of 9.74%, which when netted out against the riskfree rate at the time (3.60%) yields an implied premium for the sector:

Implied ERP for Banking in September 2008 = 9.74% - 3.60% = 6.14%

How would we use this number? One approach would be to compare it to the average implied premium in this sector over time, with the underlying assumption that the value will revert back to the historical average for the sector. The implied equity risk premium for commercial banking stocks was close to 4% between 2005 and 2007, which would lead to the conclusion that banking stocks were undervalued in September 2008. The other is to assume that the implied equity premium for a sector is reflective of perceptions of future risk in that sector; in September 2008, there can be no denying that financial service companies faced unique risks and the market was reflecting these risks in prices. As a postscript, the implied equity risk premium for financial service firms was 5.80% in January 2012, just below the market-implied premium at the time (6.01%), suggesting that some of the post-crisis fear about banking stocks had receded.

A note of caution has to be added to about sector-implied premiums. Since these risk premiums consolidate both sector risk and market risk, it would be inappropriate to multiply these premiums by conventional betas, which are measures of sector risk. Thus, multiplying the implied equity risk premium for the technology sector (which will yield a high value) by a market beta for a technology company (which will also be high for the same reason) will result in double counting risk.[122]

Firm Characteristics

Earlier in this paper, we talked about the small firm premium and how it has been estimated using historical data, resulting in backward looking estimates with substantial

[122] You could estimate betas for technology companies against a technology index (rather than the market index) and use these betas with the implied equity risk premium for technology companies.

standard error. We could use implied premiums to arrive at more forward looking estimates, using the following steps:

Step 1: Compute the implied equity risk premium for the overall market, using a broad index such as the S&P 500. Earlier in this paper, we estimated this, as of January 2015, to be 5.78%.

Step 2: Compute the implied equity risk premium for an index containing primarily or only small cap firms, such as the S&P 600 Small Cap Index. On January 1, 2015, the index was trading at 695.08, with an aggregated FCFE yield of about 3.76% (yielding a FCFE for the most recent year of 26.14), and an expected growth rate in earnings of 10.25% for the next 5 years. Using these values, in conjunction with the prevailing riskfree rate of 2.17%, yields the following equation:

$$695.08 = \frac{28.81}{(1+r)} + \frac{31.77}{(1+r)^2} + \frac{35.02}{(1+r)^3} + \frac{38.61}{(1+r)^4} + \frac{42.57}{(1+r)^5} + \frac{42.57\ (1.0217)}{(r-.0217)(1+r)^5}$$

Solving for the expected return, we get:

Expected return on small cap stocks = 7.61%

Implied equity risk premium for small cap stocks = 7.61% -2.17% = 5.44%

Step 3: The forward-looking estimate of the small cap premium should be the difference between the implied premium for small cap stocks (in step 2) and the implied premium for the market (in step 1).

Small cap premium = 5.44% - 5.78% = -0.34%

With the numbers in January 2015, small caps are priced to generate an expected return that is lower than the rest of the market, thus putting into question the wisdom of using the 4-5% small cap premium in computing costs of equity.

This approach to estimating premiums can be extended to other variables. For instance, one of the issues that has challenged analysts in valuation is how to incorporate the illiquidity of an asset into its estimated value. While the conventional approach is to attach an illiquidity discount, an alternative is to adjust the discount rate upwards for illiquid assets. If we compute the implied equity risk premiums for stocks categorized by illiquidity, we may be able to come up with an appropriate adjustment. For instance, you could estimate the implied equity risk premium for the stocks that rank in the lowest decile in terms of illiquidity, defined as turnover ratio.[123] Comparing this value to the implied premium for the S&P 500 of 5.78% should yield an implied illiquidity risk premium. Adding this premium to the cost of equity for relatively illiquid investments will then discount the value of these investments for illiquidity.

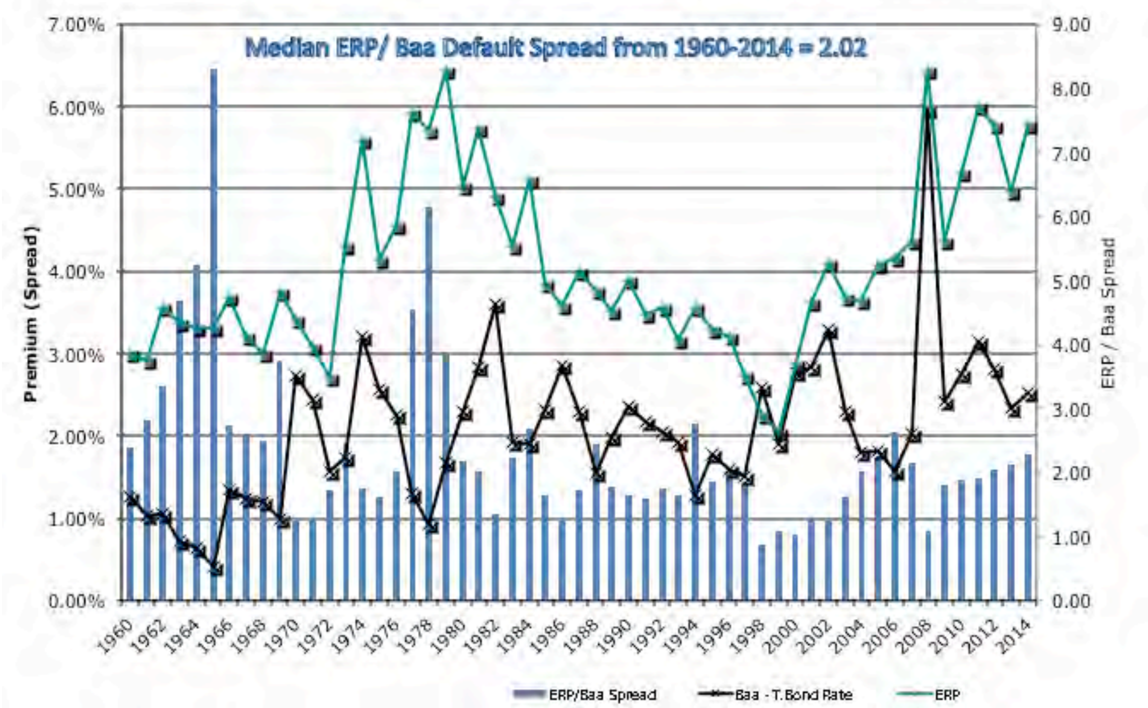
[123] The turnover ratio is obtained by dividing $ trading volume in a stock by its market capitalization at that time.

2. Default Spread Based Equity Risk Premiums

While we think of corporate bonds, stocks and real estate as different asset classes, it can be argued that they are all risky assets and that they should therefore be priced consistently. Put another way, there should be a relationship across the risk premiums in these asset classes that reflect their fundamental risk differences. In the corporate bond market, the default spread, i.e, the spread between the interest rate on corporate bonds and the treasury bond rate, is used as the risk premium. In the equity market, as we have seen through this paper, historical and implied equity premiums have tussled for supremacy as the measure of the equity risk premium. In the real estate market, no mention is made of an explicit risk premium, but real estate valuations draw heavily on the "capitalization rate", which is the discount rate applied to a real estate property's earnings to arrive at an estimate of value. The use of higher (lower) capitalization rates is the equivalent of demanding a higher (lower) risk premium.

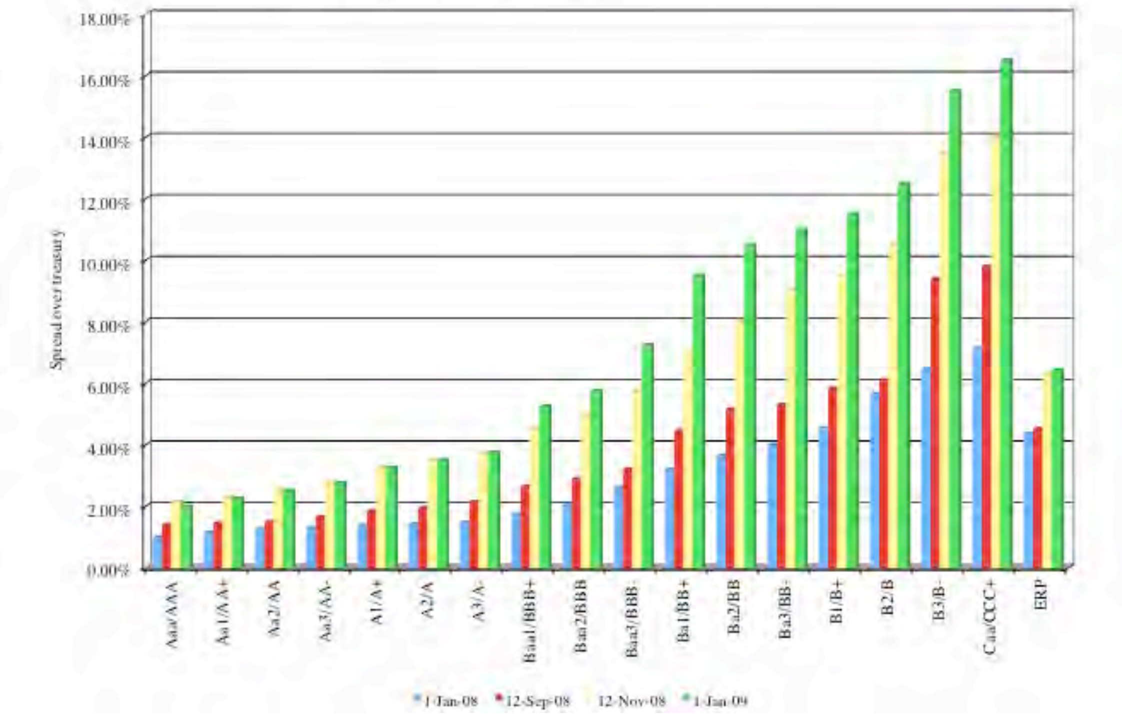
Of these three premiums, the default spread is the less complex and the most widely accessible data item. If equity risk premiums could be stated in terms of the default spread on corporate bonds, the estimation of equity risk premiums would become immeasurably simpler. For instance, assume that the default spread on Baa rated corporate bonds, relative to the ten-year treasury bond, is 2.2% and that equity risk premiums are routinely twice as high as Baa bonds, the equity risk premium would be 4.4%. Is such a rule of thumb even feasible? To answer this question, we looked at implied equity risk premiums and Baa-rated corporate bond default spreads from 1960 to 2014 in Figure 16.

Figure 16: Equity Risk Premiums and Bond Default Spreads



Note that both default spreads and equity risk premiums jumped in 2008, with the former increasing more on a proportionate basis. The ratio of 1.08 (ERP/ Baa Default Spread) at the end of 2008 was close to the lowest value in the entire series, suggesting that either equity risk premiums were too low or default spreads were too high. At the end of 2013, both the equity risk premium and the default spread increased, and the ratio moved back to 2.12, a little higher than the median value of 2.02 for the entire time period. The connection between equity risk premiums and default spreads was most obvious during 2008, where changes in one often were accompanied by changes in the other. Figure 17 graphs out changes in default spreads and ERP over the tumultuous year:

Figure 17: Default Spreads on Ratings Classes



How could we use the historical relationship between equity risk premiums and default spreads to estimate a forward-looking equity risk premium? On January 1, 2015, the default spread on a Baa rated bond was 2.52%. Applying the median ratio of 2.02, estimated from 1960-2014 numbers, to the Baa default spread of 2.52% results in the following estimate of the ERP:

Default Spread on Baa bonds (over treasury) on 1/1/2015 = 2.52%

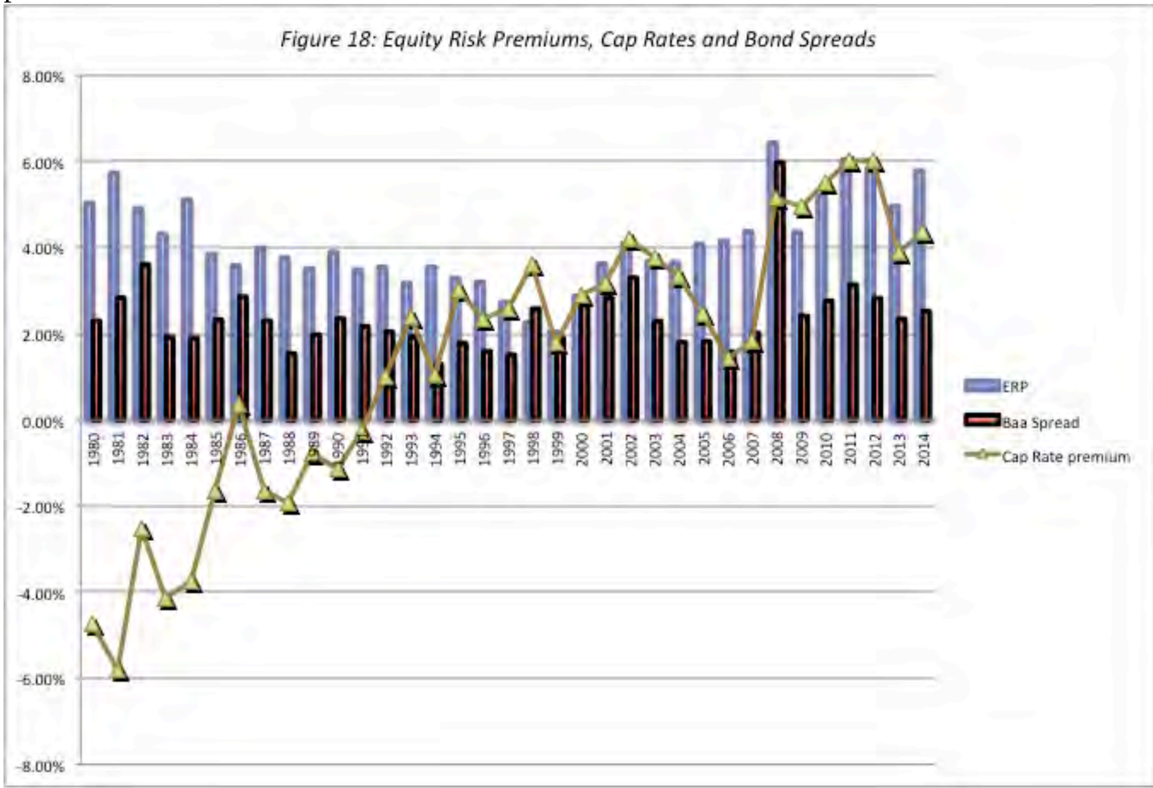
Imputed Equity Risk Premium = Default Spread * Median ratio or ERP/Spread

= 2.52%* 2.02 = 5.10%

This is a little lower than the implied equity risk premium of 5.78% that we computed in January 2015. Note that there is significant variation in the ratio (of ERP to default spreads) over time, with the ratio dropping below one at the peak of the dot.com boom (when equity risk premiums dropped to 2%) and rising to as high as 2.63 at the end of 2006; the standard error in the estimate is 0.20. Whenever the ratio has deviated significantly from the average, though, there is reversion back to that median over time.

The capitalization rate in real estate, as noted earlier, is a widely used number in the valuation of real estate properties. For instance, a capitalization rate of 10%, in conjunction with an office building that generates income of $ 10 million, would result in a property value of $ 100 million ($10/.10). The difference between the capitalization ratio and the treasury bond rate can be considered a real estate market risk premium, In

Figure 18, we used the capitalization rate in real estate ventures and compared the risk premiums imputed for real estate with both bond default spreads and implied equity risk premiums between 1980 and 2014.



The story in this graph is the convergence of the real estate and financial asset risk premiums. In the early 1980s, the real estate market seems to be operating in a different risk/return universe than financial assets, with the cap rates being less than the treasury bond rate. For instance, the cap rate in 1980 was 8.1%, well below the treasury bond rate of 12.8%, resulting in a negative risk premium for real estate. The risk premiums across the three markets - real estate, equity and bonds - starting moving closer to each other in the late 1980s and the trend accelerated in the 1990s. We would attribute at least some of this increased co-movement to the securitization of real estate in this period. In 2008, the three markets moved almost in lock step, as risk premiums in the markets rose and prices fell. The housing bubble of 2004-2008 is manifested in the drop in the real estate equity risk premium during those years, bottoming out at less than 2% at the 2006. The correction in housing prices since has pushed the premium back up. Both equity and bond premiums have adjusted quickly to pre-crisis levels in 2009 and 2010, and real estate premiums are following, albeit at a slower pace.

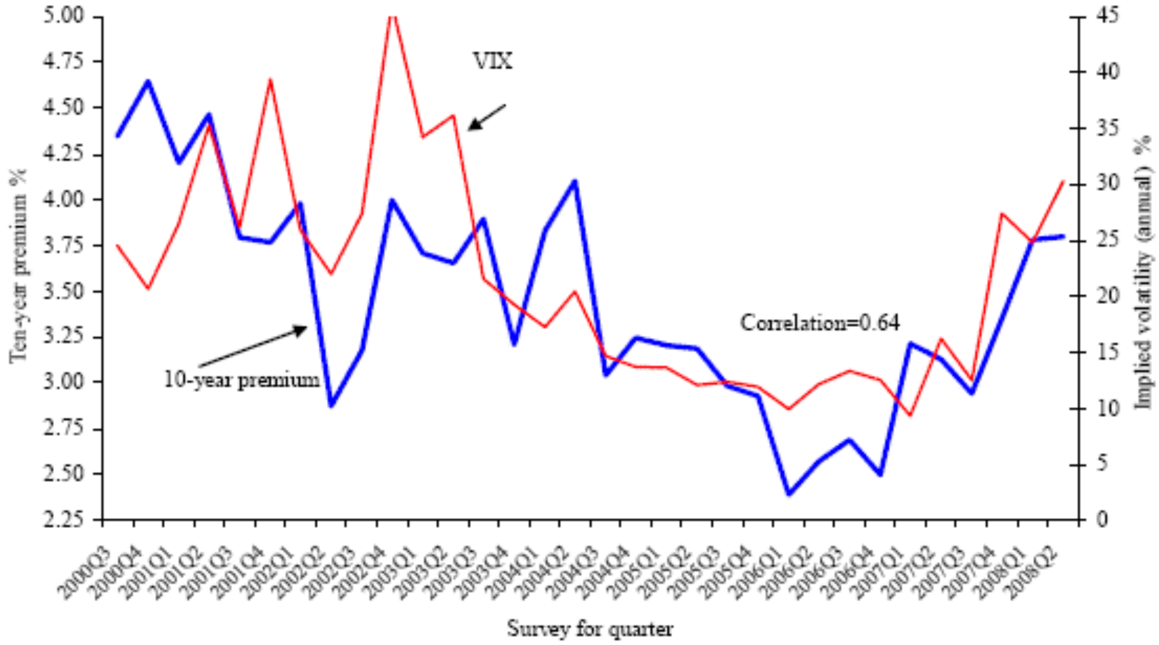
While the noise in the ratios (of ERP to default spreads and cap rates) is too high for us to develop a reliable rule of thumb, there is enough of a relationship here that we would suggest using this approach as a secondary one to test to see whether the equity risk premiums that we are using in practice make sense, given how risky assets are being priced in other markets. Thus, using an equity risk premium of 2%, when the Baa default spread is approximately at the same level strikes us as imprudent, given history. For macro strategists, there is a more activist way of using these premiums. When risk premiums in markets diverge, there is information in the relative pricing. Thus, the drop in equity risk premiums in the late 1990s, as default spreads stayed stable, would have signaled that the equity markets were overvalued (relative to bonds), just as the drop in default spreads between 2004 and 2007, while equity risk premiums were stagnant, would have suggested the opposite.

3. Option Pricing Model based Equity Risk Premium

There is one final approach to estimating equity risk premiums that draws on information in the option market. In particular, option prices can be used to back out implied volatility in the equity market. To the extent that the equity risk premium is our way of pricing in the risk of future stock price volatility, there should be a relationship between the two.

The simplest measure of volatility from the options market is the volatility index (VIX), which is a measure of 30—day volatility constructed using the implied volatilities in traded S&P 500 index options. The CFO survey premium from Graham and Harvey that we referenced earlier in the paper found a high degree of correlation between the premiums demanded by CFOs and the VIX value (see figure 19 below):

Figure 19: Volatility Index (VIX) and Survey Risk Premiums

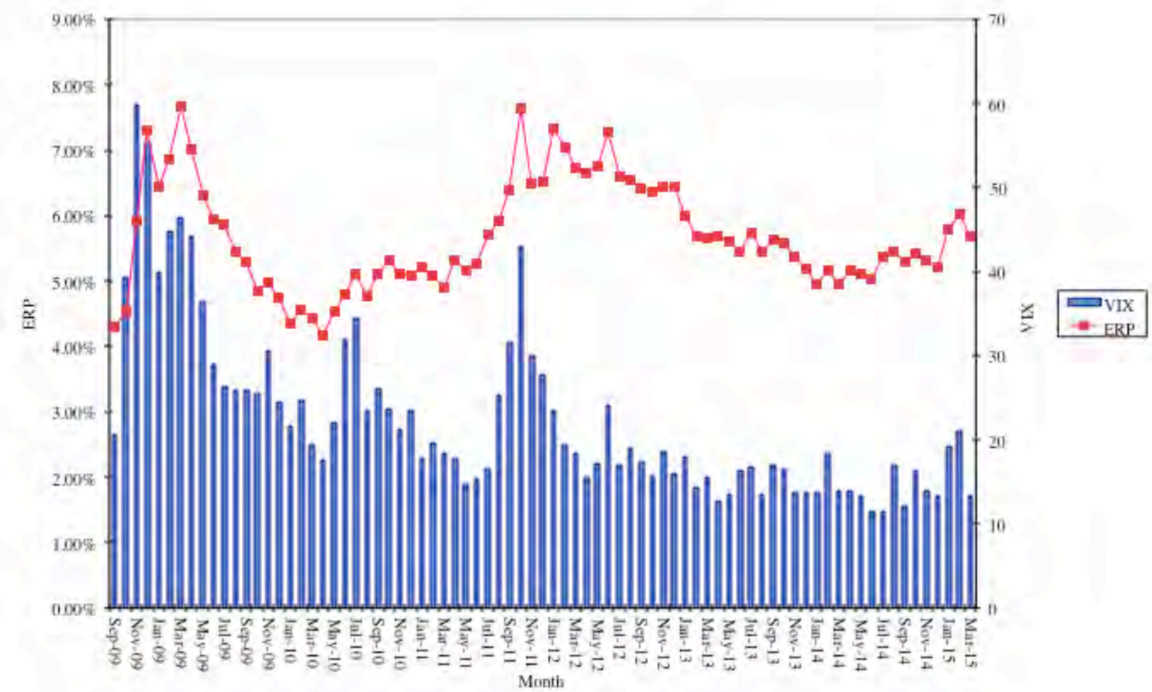


Santa-Clara and Yan (2006) use options on the S&P 500 to estimate the ex-ante risk assessed by investors from 1996 and 2002 and back out an implied equity risk premium on that basis.[124] To estimate the ex-ante risk, they allow for both continuous and discontinuous (or jump) risk in stocks, and use the option prices to estimate the probabilities of both types of risk. They then assume that investors share a specific utility function (power utility) and back out a risk premium that would compensate for this risk. Based on their estimates, investors should have demanded an equity risk premium of 11.8% for their perceived risk and that the perceived risk was about 70% higher than the realized risk over this period.

The link between equity market volatility and the equity risk premium also became clearer during the market meltdown in the last quarter of 2008. Earlier in the paper, we noted the dramatic shifts in the equity risk premiums, especially in the last year, as the financial crisis has unfolded. In Figure 20, we look at the implied equity risk premium each month from September 2008 to March 2014 and the volatility index (VIX) for the S&P 500:

[124] Santa-Clara, P. and S. Yan, 2006, *Crashes, Volatility, and the Equity Premium: Lessons from S&P 500 Options*, Review of Economics and Statistics, v92, pg 435-451.

Figure 20: ERP versus VIX



Note that the surge in equity risk premiums between September 2008 and December 2008 coincided with a jump in the volatility index and that both numbers have declined in the years since the crisis. The drop in the VIX between September 2011 and March 2012 was not accompanied by a decrease in the implied equity risk premium, but equity risk premiums drifted down in the year after. While the VIX stayed low for much of 2014, equity risk premiums climbed through the course of the year.

In a paper referenced earlier, Bollerslev, Tauchen and Zhou (2009) take a different tack and argue that it is not the implied volatility per se, but the variance risk, i.e., the difference between the implied variance (in option prices) and the actual variance, that drives expected equity returns.[125] Thus, if the realized variance in a period is far higher (lower) than the implied variance, you should expect to see higher (lower) equity risk premiums demanded for subsequent periods. While they find evidence to back this proposition, they also note the relationship is strongest for short term returns (next quarter) and are weaker for longer-term returns. Bekaert and Hoerova (2013) decomposed the squared VIX into two components, a conditional variance of the stock

125 Bollerslev, T. G. Tauchen and H. Zhou, 2009, *Expected Stock Returns and Variance Risk Premia*, Review of Financial Studies, v22, 4463-4492.

market and an equity variance premium, and conclude that while the latter is a significant predictor of stock returns but the former is not.[126]

Choosing an Equity Risk Premium

We have looked at three different approaches to estimating risk premiums, the survey approach, where the answer seems to depend on who you ask and what you ask them, the historical premium approach, with wildly different results depending on how you slice and dice historical data and the implied premium approach, where the final number is a function of the model you use and the assumptions you make about the future. Ultimately, thought, we have to choose a number to use in analysis and that number has consequences. In this section, we consider why the approaches give you different numbers and a pathway to use to devise which number is best for you.

Why do the approaches yield different values?

The different ways of estimating equity risk premium provide cover for analysts by providing justification for almost any number they choose to use in practice. No matter what the premium used by an analyst, whether it be 3% or 12%, there is back-up evidence offered that the premium is appropriate. While this may suffice as a legal defense, it does not pass muster on common sense grounds since not all risk premiums are equally justifiable. To provide a measure of how the numbers vary, the values that we have attached to the US equity risk premium, using different approaches, in January 2013 are summarized in table 23.

Table 23: Equity Risk Premium (ERP) for the United States – January 2013

Approach Used	*ERP*	*Additional information*
Survey: CFOs	3.73%	Campbell and Harvey survey of CFOs (2014); Average estimate. Median was 3.4%.
Survey: Global Fund Managers	4.60%	Merrill Lynch (January 2014) survey of global managers
Historical - US	4.60%	Geometric average - Stocks over T.Bonds: 1928-2014
Historical – Multiple Equity Markets	2.80%	Average premium across 20 markets from 1900-2014: Dimson, Marsh and Staunton (2015)
Current Implied premium	5.78%	From S&P 500 – January 1, 2015
Average Implied premium	4.13%	Average of implied equity risk premium: 1960-2014

[126] Bekaert, G. and M. Hoerova, 2013, *The VIX, Variance Premium and Stock Market Volatility*, SSRN Working Paper, http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2342200.

Implied premium adjusted for T.Bond rate and term structure	3.86%	Using regression of implied premium on T.Bond rate
Default spread based premium	5.10%	Baa Default Spread * Median value of (ERP/ Default Spread)

The equity risk premiums, using the different approaches, yield a range, with the lowest value being 2.80% and the highest being 5.78%. Note that the range would have been larger if we used other measures of historical risk premiums: different time periods, arithmetic instead of geometric averages.

There are several reasons why the approaches yield different answers much of time and why they converge sometimes.

1. When stock prices enter an extended phase of upward (downward) movement, the historical risk premium will climb (drop) to reflect past returns. Implied premiums will tend to move in the opposite direction, since higher (lower) stock prices generally translate into lower (higher) premiums. In 1999, for instance, after the technology induced stock price boom of the 1990s, the implied premium was 2% but the historical risk premium was almost 6%.
2. Survey premiums reflect historical data more than expectations. When stocks are going up, investors tend to become more optimistic about future returns and survey premiums reflect this optimism. In fact, the evidence that human beings overweight recent history (when making judgments) and overreact to information can lead to survey premiums overshooting historical premiums in both good and bad times. In good times, survey premiums are even higher than historical premiums, which, in turn, are higher than implied premiums; in bad times, the reverse occurs.
3. When the fundamentals of a market change, either because the economy becomes more volatile or investors get more risk averse, historical risk premiums will not change but implied premiums will. Shocks to the market are likely to cause the two numbers to deviate. After the terrorist attack on the World Trade Center in September 2001, for instance, implied equity risk premiums jumped almost 0.50% but historical premiums were unchanged (at least until the next update).

In summary, we should not be surprised to see large differences in equity risk premiums as we move from one approach to another, and even within an approach, as we change estimation parameters.

Which approach is the "best" approach?

If the approaches yield different numbers for the equity risk premium, and we have to choose one of these numbers, how do we decide which one is the "best" estimate? The answer to this question will depend upon several factors:

a. Predictive Power: In corporate finance and valuation, what we ultimately care about is the equity risk premium for the future. Consequently, the approach that has the best predictive power, i.e. yields forecasts of the risk premium that are closer to realized premiums, should be given more weight. So, which of the approaches does best on this count?

   Campbell and Shiller (1988) suggested that the dividend yield, a simplistic measure of the implied equity risk premium, had significant predictive power for future returns.[127] However, Goyal and Welch (2007) examined many of the measures suggested as predictors of the equity risk premium in the literature, including the dividend yield and the earnings to price ratio, and find them all wanting.[128] Using data from 1926 to 2005, they conclude that while the measures do reasonably well in sample, they perform poorly out of sample, suggesting that the relationships in the literature are either spurious or unstable. Campbell and Thompson (2008) disagree, noting that putting simple restrictions on the predictive regressions improve out of sample performance for many predictive variables.[129]

   To answer this question, we looked at the implied equity risk premiums from 1960 to 2014 and considered four predictors of this premium – the historical risk premium through the end of the prior year, the implied equity risk premium at the end of the prior year, the average implied equity risk premium over the previous five years and the premium implied by the Baa default spread. Since the survey data does not go back very far, we could not test the efficacy of the survey premium. Our results are summarized in table 24:

[127] Campbell, J. Y. and R. J. Shiller. 1988, *The Dividend-Price Ratio And Expectations Of Future Dividends And Discount Factors*, Review of Financial Studies, v1(3), 195-228.
[128] Goyal, A. and I. Welch, 2007, *A Comprehensive Look at the Empirical Performance of Equity Premium Prediction*, Review of Financial Studies, v21, 1455-1508.
[129] Campbell, J.Y., and S.B. Thompson, 2008, *Predictive Excess Stock Returns Out of Sample: Can Anything Beat the Historical Average?* Review of Financial Studies, v21, 150-9-1531.

Table 24: Predictive Power of different estimates- 1960 - 2014

Predictor	*Correlation with implied premium next year*	*Correlation with actual return- next 5 years*	*Correlation with actual return – next 10 years*[130]
Current implied premium	0.736	0.352	0.500
Average implied premium: Last 5 years	0.684	0.238	0.449
Historical Premium	-0.460	-0.365	-0.466
Default Spread based premium	0.047	0.148	0.165

Over this period, the implied equity risk premium at the end of the prior period was the best predictor of the implied equity risk premium in the next period, whereas historical risk premiums did worst. If we extend our analysis to make forecasts of the actual return premium earned by stocks over bonds for the next five or ten years, the current implied equity risk premium still yields the best forecast for the future, though default spread based premiums improve as predictors. Historical risk premiums perform even worse as forecasts of actual risk premiums over the next 5 or 10 years. If predictive power were the only test, historical premiums clearly fail the test.

b. Beliefs about markets: Implicit in the use of each approach are assumptions about market efficiency or lack thereof. If you believe that markets are efficient in the aggregate, or at least that you cannot forecast the direction of overall market movements, the current implied equity premium is the most logical choice, since it is estimated from the current level of the index. If you believe that markets, in the aggregate, can be significantly overvalued or undervalued, the historical risk premium or the average implied equity risk premium over long periods becomes a better choice. If you have absolutely no faith in markets, survey premiums will be the choice.

c. Purpose of the analysis: Notwithstanding your beliefs about market efficiency, the task for which you are using equity risk premiums may determine the right risk premium to use. In acquisition valuations and equity research, for instance, you are

[130] I computed the compounded average return on stocks in the following five (ten) years and netted out the compounded return earned on T.Bonds over the following five (ten) years. This was a switch from the simple arithmetic average of returns over the next 10 years that I was using until last year's survey.

asked to assess the value of an individual company and not take a view on the level of the overall market. This will require you to use the current implied equity risk premium, since using any other number will bring your market views into the valuation. To see why, assume that the current implied premium is 4% and you decide to use a historical premium of 6% in your company valuation. Odds are that you will find the company to be over valued, but a big reason for your conclusion is that you started off with the assumption that the market itself is over valued by about 25-30%.[131] To make yourself market neutral, you will have to stick with the current implied premium. In corporate finance, where the equity risk premium is used to come up with a cost of capital, which in turn determines the long-term investments of the company, it may be more prudent to build in a long-term average (historical or implied) premium.

In conclusion, there is no one approach to estimating equity risk premiums that will work for all analyses. If predictive power is critical or if market neutrality is a pre-requisite, the current implied equity risk premium is the best choice. For those more skeptical about markets, the choices are broader, with the average implied equity risk premium over a long time period having the strongest predictive power. Historical risk premiums are very poor predictors of both short-term movements in implied premiums or long-term returns on stocks.

As a final note, there are papers that report consensus premiums, often estimated by averaging across approaches. I remain skeptical about these estimates, since the approaches vary not only in terms of accuracy and predictive power but also in their philosophy. Averaging a historical risk premium with an implied premium may give an analyst a false sense of security but it really makes no sense since they represent different views of the world and push in different directions.

Five myths about equity risk premiums

There are widely held misconceptions about equity risk premiums that we would like to dispel in this section.

1. Services "know" the risk premium: When Ibbotson and Sinquefield put together the first database of historical returns on stocks, bonds and bills in the 1970s, the data that they used was unique and not easily replicable, even for professional money managers. The niche they created, based on proprietary data, has led some to believe that Ibbotson Associates, and data services like them, have the capacity to read the

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[131] If the current implied premium is 4%, using a 6% premium on the market will reduce the value of the index by about 25-30%.

historical data better than the rest of us, and therefore come up with better estimates. Now that the access to data has been democratized, and we face a much more even playing field, there is no reason to believe that any service has an advantage over any other, when it comes to historical premiums. Analysts should no longer be allowed to hide behind the defense that the equity risk premiums they use come from a reputable service and are thus beyond questioning.

2. There is no right risk premium: The flip side of the "services know it best" argument is that the data is so noisy that no one knows what the right risk premium is, and that any risk premium within a wide range is therefore defensible. As we have noted in this paper, it is indeed possible to arrive at outlandishly high or low premiums, but only if you use estimation approaches that do not hold up to scrutiny. The arithmetic average premium from 2005 to 2014 for stocks over treasury bonds is an equity risk premium estimate, but it is not a good one.
3. The equity risk premium does not change much over time: Equity risk premiums reflect both economic fundamentals and investor risk aversion and they do change over time, sometimes over very short intervals, as evidenced by what happened in the last quarter of 2008. Shocks to the system – a collapse of a large company or sovereign entity or a terrorist attack – can cause premiums to shoot up overnight. A failure to recognize this reality will lead to analyses that lag reality.
4. Using the same premium is more important than using the right premium: Within many investment banks, corporations and consulting firms, the view seems to be that getting all analysts to use the same number as the risk premium is more important than testing to see whether that number makes sense. Thus, if all equity research analysts use 5% as the equity risk premium, the argument is that they are all being consistent. There are two problems with this argument. The first is that using a premium that is too high or low will lead to systematic errors in valuation. For instance, using a 5% risk premium across the board, when the implied premium is 4%, will lead you to find that most stocks are overvalued. . The second is that the impact of using too high a premium can vary across stocks, with growth stocks being affected more negatively than mature companies. A portfolio manager who followed the recommendations of these analysts would then be over invested in mature companies and under invested in growth companies.
5. If you adjust the cash flows for risk, there is no need for a risk premium: While statement is technically correct, adjusting cash flows for risk has to go beyond reflecting the likelihood of negative scenarios in the expected cash flow. The risk adjustment to expected cash flows to make them certainty equivalent cash flows

requires us to answer exactly the same questions that we deal with when adjusting discount rates for risk.

Summary

The risk premium is a fundamental and critical component in portfolio management, corporate finance and valuation. Given its importance, it is surprising that more attention has not been paid in practical terms to estimation issues. In this paper, we began by looking at the determinants of equity risk premiums including macro economic volatility, investor risk aversion and behavioral components. We then looked at the three basic approaches used to estimate equity risk premiums – the survey approach, where investors or managers are asked to provide estimates of the equity risk premium for the future, the historical return approach, where the premium is based upon how well equities have done in the past and the implied approach, where we use future cash flows or observed bond default spreads to estimate the current equity risk premium.

The premiums we estimate can vary widely across approaches, and we considered two questions towards the end of the paper. The first is why the numbers vary across approaches and the second is how to choose the "right" number to use in analysis. For the latter question, we argued that the choice of a premium will depend upon the forecast period, whether your believe markets are efficient and whether you are required to be market neutral in your analysis.

Appendix 1: Historical Returns on Stocks, Bonds and Bills – United States

Year	*Stocks*	*T.Bills*	*T.Bonds*	*Stocks - T. Bills*	*Stocks - T.Bonds*	*Arithmetic Average: Stocks versus T. Bonds*	*Geometric average: Stocks vs T.Bonds*
1928	43.81%	3.08%	0.84%	40.73%	42.98%	42.98%	42.98%
1929	-8.30%	3.16%	4.20%	-11.46%	-12.50%	15.24%	12.33%
1930	-25.12%	4.55%	4.54%	-29.67%	-29.66%	0.27%	-3.60%
1931	-43.84%	2.31%	-2.56%	-46.15%	-41.28%	-10.12%	-15.42%
1932	-8.64%	1.07%	8.79%	-9.71%	-17.43%	-11.58%	-15.81%
1933	49.98%	0.96%	1.86%	49.02%	48.13%	-1.63%	-7.36%
1934	-1.19%	0.32%	7.96%	-1.51%	-9.15%	-2.70%	-7.61%
1935	46.74%	0.18%	4.47%	46.57%	42.27%	2.92%	-2.49%
1936	31.94%	0.17%	5.02%	31.77%	26.93%	5.59%	0.40%
1937	-35.34%	0.30%	1.38%	-35.64%	-36.72%	1.36%	-4.22%
1938	29.28%	0.08%	4.21%	29.21%	25.07%	3.51%	-1.87%
1939	-1.10%	0.04%	4.41%	-1.14%	-5.51%	2.76%	-2.17%
1940	-10.67%	0.03%	5.40%	-10.70%	-16.08%	1.31%	-3.30%
1941	-12.77%	0.08%	-2.02%	-12.85%	-10.75%	0.45%	-3.88%
1942	19.17%	0.34%	2.29%	18.84%	16.88%	1.54%	-2.61%
1943	25.06%	0.38%	2.49%	24.68%	22.57%	2.86%	-1.18%
1944	19.03%	0.38%	2.58%	18.65%	16.45%	3.66%	-0.21%
1945	35.82%	0.38%	3.80%	35.44%	32.02%	5.23%	1.35%
1946	-8.43%	0.38%	3.13%	-8.81%	-11.56%	4.35%	0.63%
1947	5.20%	0.57%	0.92%	4.63%	4.28%	4.35%	0.81%
1948	5.70%	1.02%	1.95%	4.68%	3.75%	4.32%	0.95%
1949	18.30%	1.10%	4.66%	17.20%	13.64%	4.74%	1.49%
1950	30.81%	1.17%	0.43%	29.63%	30.38%	5.86%	2.63%
1951	23.68%	1.48%	-0.30%	22.20%	23.97%	6.61%	3.46%
1952	18.15%	1.67%	2.27%	16.48%	15.88%	6.98%	3.94%
1953	-1.21%	1.89%	4.14%	-3.10%	-5.35%	6.51%	3.57%
1954	52.56%	0.96%	3.29%	51.60%	49.27%	8.09%	4.98%
1955	32.60%	1.66%	-1.34%	30.94%	33.93%	9.01%	5.93%
1956	7.44%	2.56%	-2.26%	4.88%	9.70%	9.04%	6.07%
1957	-10.46%	3.23%	6.80%	-13.69%	-17.25%	8.16%	5.23%
1958	43.72%	1.78%	-2.10%	41.94%	45.82%	9.38%	6.39%
1959	12.06%	3.26%	-2.65%	8.80%	14.70%	9.54%	6.66%
1960	0.34%	3.05%	11.64%	-2.71%	-11.30%	8.91%	6.11%
1961	26.64%	2.27%	2.06%	24.37%	24.58%	9.37%	6.62%
1962	-8.81%	2.78%	5.69%	-11.59%	-14.51%	8.69%	5.97%
1963	22.61%	3.11%	1.68%	19.50%	20.93%	9.03%	6.36%
1964	16.42%	3.51%	3.73%	12.91%	12.69%	9.13%	6.53%
1965	12.40%	3.90%	0.72%	8.50%	11.68%	9.20%	6.66%

Year	*Stocks*	*T.Bills*	*T.Bonds*	*Stocks - T. Bills*	*Stocks - T.Bonds*	*Arithmetic Average: Stocks versus T. Bonds*	*Geometric average: Stocks vs T.Bonds*
1966	-9.97%	4.84%	2.91%	-14.81%	-12.88%	8.63%	6.11%
1967	23.80%	4.33%	-1.58%	19.47%	25.38%	9.05%	6.57%
1968	10.81%	5.26%	3.27%	5.55%	7.54%	9.01%	6.60%
1969	-8.24%	6.56%	-5.01%	-14.80%	-3.23%	8.72%	6.33%
1970	3.56%	6.69%	16.75%	-3.12%	-13.19%	8.21%	5.90%
1971	14.22%	4.54%	9.79%	9.68%	4.43%	8.12%	5.87%
1972	18.76%	3.95%	2.82%	14.80%	15.94%	8.30%	6.08%
1973	-14.31%	6.73%	3.66%	-21.03%	-17.97%	7.73%	5.50%
1974	-25.90%	7.78%	1.99%	-33.68%	-27.89%	6.97%	4.64%
1975	37.00%	5.99%	3.61%	31.01%	33.39%	7.52%	5.17%
1976	23.83%	4.97%	15.98%	18.86%	7.85%	7.53%	5.22%
1977	-6.98%	5.13%	1.29%	-12.11%	-8.27%	7.21%	4.93%
1978	6.51%	6.93%	-0.78%	-0.42%	7.29%	7.21%	4.97%
1979	18.52%	9.94%	0.67%	8.58%	17.85%	7.42%	5.21%
1980	31.74%	11.22%	-2.99%	20.52%	34.72%	7.93%	5.73%
1981	-4.70%	14.30%	8.20%	-19.00%	-12.90%	7.55%	5.37%
1982	20.42%	11.01%	32.81%	9.41%	-12.40%	7.18%	5.10%
1983	22.34%	8.45%	3.20%	13.89%	19.14%	7.40%	5.34%
1984	6.15%	9.61%	13.73%	-3.47%	-7.59%	7.13%	5.12%
1985	31.24%	7.49%	25.71%	23.75%	5.52%	7.11%	5.13%
1986	18.49%	6.04%	24.28%	12.46%	-5.79%	6.89%	4.97%
1987	5.81%	5.72%	-4.96%	0.09%	10.77%	6.95%	5.07%
1988	16.54%	6.45%	8.22%	10.09%	8.31%	6.98%	5.12%
1989	31.48%	8.11%	17.69%	23.37%	13.78%	7.08%	5.24%
1990	-3.06%	7.55%	6.24%	-10.61%	-9.30%	6.82%	5.00%
1991	30.23%	5.61%	15.00%	24.62%	15.23%	6.96%	5.14%
1992	7.49%	3.41%	9.36%	4.09%	-1.87%	6.82%	5.03%
1993	9.97%	2.98%	14.21%	6.98%	-4.24%	6.65%	4.90%
1994	1.33%	3.99%	-8.04%	-2.66%	9.36%	6.69%	4.97%
1995	37.20%	5.52%	23.48%	31.68%	13.71%	6.80%	5.08%
1996	23.82%	5.02%	1.43%	18.79%	22.39%	7.02%	5.32%
1997	31.86%	5.05%	9.94%	26.81%	21.92%	7.24%	5.53%
1998	28.34%	4.73%	14.92%	23.61%	13.42%	7.32%	5.63%
1999	20.89%	4.51%	-8.25%	16.38%	29.14%	7.63%	5.96%
2000	-9.03%	5.76%	16.66%	-14.79%	-25.69%	7.17%	5.51%
2001	-11.85%	3.67%	5.57%	-15.52%	-17.42%	6.84%	5.17%
2002	-21.97%	1.66%	15.12%	-23.62%	-37.08%	6.25%	4.53%
2003	28.36%	1.03%	0.38%	27.33%	27.98%	6.54%	4.82%
2004	10.74%	1.23%	4.49%	9.52%	6.25%	6.53%	4.84%

Year	*Stocks*	*T.Bills*	*T.Bonds*	*Stocks - T. Bills*	*Stocks - T.Bonds*	*Arithmetic Average: Stocks versus T. Bonds*	*Geometric average: Stocks vs T.Bonds*
2005	4.83%	3.01%	2.87%	1.82%	1.97%	6.47%	4.80%
2006	15.61%	4.68%	1.96%	10.94%	13.65%	6.57%	4.91%
2007	5.48%	4.64%	10.21%	0.84%	-4.73%	6.42%	4.79%
2008	-36.55%	1.59%	20.10%	-38.14%	-56.65%	5.65%	3.88%
2009	25.94%	0.14%	-11.12%	25.80%	37.05%	6.03%	4.29%
2010	14.82%	0.13%	8.46%	14.69%	6.36%	6.03%	4.31%
2011	2.10%	0.03%	16.04%	2.07%	-13.94%	5.79%	4.10%
2012	15.89%	0.05%	2.97%	15.84%	12.92%	5.88%	4.20%
2013	32.15%	0.07%	-9.10%	32.08%	41.25%	6.29%	4.62%
2014	13.48%	0.05%	10.75%	13.43%	2.73%	6.11%	4.60%

Appendix 2: Sovereign Ratings by Country- January 2015

Sovereign	*Foreign Currency Rating*	*Local Currency Rating*	*Sovereign*	*Foreign Currency Rating*	*Local Currency Rating*
Abu Dhabi	Aa2	Aa2	Czech Republic	A1	A1
Albania	B1	B1	Democratic Republic of the Congo	B3	B3
Angola	Ba2	Ba2	Denmark	Aaa	Aaa
Argentina	Caa1	Caa1	Dominican Republic	B1	B1
Armenia	Ba2	Ba2	Ecuador	B3	-
Australia	Aaa	Aaa	Egypt	Caa1	Caa1
Austria	Aaa	Aaa	El Salvador	Ba3	-
Azerbaijan	Baa3	Baa3	Estonia	A1	A1
Bahamas	Baa2	Baa2	Ethiopia	B1	B1
Bahrain	Baa2	Baa2	Fiji	B1	B1
Bangladesh	Ba3	Ba3	Finland	Aaa	Aaa
Barbados	B3	B3	France	Aa1	Aa1
Belarus	B3	B3	Gabon	Ba3	Ba3
Belgium	Aa3	Aa3	Georgia	Ba3	Ba3
Belize	Caa2	Caa2	Germany	Aaa	Aaa
Bermuda	A1	A1	Ghana	B2	B2
Bolivia	Ba3	Ba3	Greece	Caa1	Caa1
Bosnia and Herzegovina	B3	B3	Guatemala	Ba1	Ba1
Botswana	A2	A2	Honduras	B3	B3
Brazil	Baa2	Baa2	Hong Kong	Aa1	Aa1
Bulgaria	Baa2	Baa2	Hungary	Ba1	Ba1
Cambodia	B2	B2	Iceland	Baa3	Baa3
Canada	Aaa	Aaa	India	Baa3	Baa3
Cayman Islands	Aa3	-	Indonesia	Baa3	Baa3
Chile	Aa3	Aa3	Ireland	Baa1	Baa1
China	Aa3	Aa3	Isle of Man	Aa1	Aa1
Colombia	Baa2	Baa2	Israel	A1	A1
Costa Rica	Ba1	Ba1	Italy	Baa2	Baa2
Côte d'Ivoire	B1	B1	Jamaica	Caa3	Caa3
Croatia	Ba1	Ba1	Japan	A1	A1
Cuba	Caa2	-	Jordan	B1	B1
Cyprus	B3	B3	Kazakhstan	Baa2	Baa2

Appendix 2: Sovereign Ratings by Country- January 2015 (Continued)

Sovereign	*Foreign Currency Rating*	*Local Currency Rating*	*Sovereign*	*Foreign Currency Rating*	*Local Currency Rating*
Kenya	B1	B1	Qatar	Aa2	Aa2
Korea	Aa3	Aa3	Republic of the Congo	Ba3	Ba3
Kuwait	Aa2	Aa2	Romania	Baa3	Baa3
Latvia	Baa1	Baa1	Russia	Baa2	Baa2
Lebanon	B2	B2	Saudi Arabia	Aa3	Aa3
Lithuania	Baa1	Baa1	Senegal	B1	B1
Luxembourg	Aaa	Aaa	Serbia	B1	B1
Macao	Aa2	Aa2	Sharjah	A3	A3
Malaysia	A3	A3	Singapore	Aaa	Aaa
Malta	A3	A3	Slovakia	A2	A2
Mauritius	Baa1	Baa1	Slovenia	Ba1	Ba1
Mexico	A3	A3	South Africa	Baa2	Baa2
Moldova	B3	B3	Spain	Baa2	Baa2
Mongolia	B2	B2	Sri Lanka	B1	-
Montenegro	Ba3	-	St. Maarten	Baa1	Baa1
Morocco	Ba1	Ba1	St. Vincent & the Grenadines	B3	B3
Mozambique	B1	B1	Suriname	Ba3	Ba3
Namibia	Baa3	Baa3	Sweden	Aaa	Aaa
Netherlands	Aaa	Aaa	Switzerland	Aaa	Aaa
New Zealand	Aaa	Aaa	Taiwan	Aa3	Aa3
Nicaragua	B3	B3	Thailand	Baa1	Baa1
Nigeria	Ba3	Ba3	Trinidad and Tobago	Baa1	Baa1
Norway	Aaa	Aaa	Tunisia	Ba3	Ba3
Oman	A1	A1	Turkey	Baa3	Baa3
Pakistan	Caa1	Caa1	Uganda	B1	B1
Panama	Baa2	-	Ukraine	Caa3	Caa3
Papua New Guinea	B1	B1	United Arab Emirates	Aa2	Aa2
Paraguay	Ba2	Ba2	UK	Aa1	Aa1
Peru	A3	A3	USA	Aaa	Aaa
Philippines	Baa2	Baa2	Uruguay	Baa2	Baa2
Poland	A2	A2	Venezuela	Caa1	Caa1
Portugal	Ba1	Ba1	Vietnam	B1	B1
			Zambia	B1	B1

Appendix 3: Country Risk Scores from the PRS Group – January 2015

Country	*PRS Composite Risk Score*	*Country*	*PRS Composite Risk Score*
Albania	66.3	Egypt	59.0
Algeria	68.3	El Salvador	66.8
Angola	65.8	Estonia	69.5
Argentina	63.8	Ethiopia	59.3
Armenia	63.0	Finland	79.0
Australia	78.5	France	70.8
Austria	79.5	Gabon	71.3
Azerbaijan	75.8	Gambia	62.8
Bahamas	75.8	Germany	84.5
Bahrain	70.5	Ghana	61.3
Bangladesh	64.0	Greece	64.3
Belarus	59.3	Guatemala	66.8
Belgium	76.0	Guinea	47.8
Bolivia	73.8	Guinea-Bissau	62.5
Botswana	79.5	Guyana	61.8
Brazil	67.5	Haiti	61.0
Brunei	87.0	Honduras	64.8
Bulgaria	69.3	Hong Kong	81.0
Burkina Faso	63.0	Hungary	72.3
Cameroon	63.5	Iceland	79.8
Canada	82.0	India	68.8
Chile	75.8	Indonesia	67.3
China, Peoples' Rep.	71.8	Iran	61.3
Colombia	68.5	Iraq	61.8
Congo, Dem. Republic	55.3	Ireland	78.5
Congo, Republic	68.8	Israel	72.3
Costa Rica	73.5	Italy	72.5
Cote d'Ivoire	62.3	Jamaica	68.5
Croatia	68.5	Japan	78.8
Cuba	65.5	Jordan	65.0
Cyprus	69.3	Kazakhstan	70.5
Czech Republic	78.3	Kenya	63.3
Denmark	81.3	Korea, D.P.R.	55.8
Dominican Republic	71.5	Korea, Republic	81.5
Ecuador	67.0	Kuwait	81.5

Appendix 3: Country Risk Scores from the PRS Group – January 2015 (Continued)

Country	*PRS Composite Risk Score*	*Country*	*PRS Composite Risk Score*
Latvia	69.0	Russia	64.3
Lebanon	58.5	Saudi Arabia	78.8
Liberia	50.0	Senegal	62.8
Libya	59.3	Serbia	63.0
Lithuania	76.0	Sierra Leone	61.5
Luxembourg	87.5	Singapore	87.0
Madagascar	63.5	Slovakia	74.3
Malawi	61.0	Slovenia	70.0
Malaysia	78.8	Somalia	37.3
Mali	60.5	South Africa	67.3
Malta	75.8	Spain	70.5
Mexico	68.8	Sri Lanka	62.3
Moldova	63.8	Sudan	50.0
Mongolia	64.3	Suriname	72.0
Morocco	67.3	Sweden	82.0
Mozambique	56.0	Switzerland	89.5
Myanmar	62.8	Syria	41.5
Namibia	75.8	Taiwan	83.0
Netherlands	81.0	Tanzania	62.3
New Zealand	83.0	Thailand	67.0
Nicaragua	64.8	Togo	60.3
Niger	55.8	Trinidad & Tobago	76.8
Nigeria	62.5	Tunisia	63.5
Norway	90.0	Turkey	61.5
Oman	81.0	Uganda	58.0
Pakistan	58.5	Ukraine	54.3
Panama	71.8	United Arab Emirates	82.8
Papua New Guinea	64.8	United Kingdom	78.8
Paraguay	69.5	United States	77.3
Peru	71.5	Uruguay	72.0
Philippines	72.3	Venezuela	54.8
Poland	75.3	Vietnam	70.0
Portugal	73.3	Yemen, Republic	59.5
Qatar	82.3	Zambia	67.0
Romania	71.5	Zimbabwe	54.5

Appendix 4: Equity Market volatility, relative to S&P 500: Total Equity Risk Premiums and Country Risk Premiums (Weekly returns from 2/13-2/15)

Country	*Std deviation in Equities (weekly)*	*Relative Volatility (to US)*	*Total Equity Risk Premium*	*Country risk premium*
Argentina	35.50%	3.27	18.78%	13.03%
Bahrain	7.59%	0.70	4.01%	-1.74%
Bangladesh	16.24%	1.49	8.59%	2.84%
Bosnia	8.99%	0.83	4.76%	-0.99%
Botswana	4.19%	0.39	2.22%	-3.53%
Brazil	22.25%	2.05	11.77%	6.02%
Bulgaria	15.33%	1.41	8.11%	2.36%
Chile	13.91%	1.28	7.36%	1.61%
China	17.82%	1.64	9.43%	3.68%
Colombia	16.00%	1.47	8.46%	2.71%
Costa Rica	8.78%	0.81	4.64%	-1.11%
Croatia	7.42%	0.68	3.93%	-1.82%
Cyprus	36.97%	3.40	19.56%	13.81%
Czech Republic	15.39%	1.42	8.14%	2.39%
Egypt	25.47%	2.34	13.47%	7.72%
Estonia	10.26%	0.94	5.43%	-0.32%
Ghana	9.09%	0.84	4.81%	-0.94%
Greece	40.49%	3.72	21.42%	15.67%
Hungary	17.21%	1.58	9.10%	3.35%
Iceland	10.89%	1.00	5.76%	0.01%
India	14.09%	1.30	7.45%	1.70%
Indonesia	16.49%	1.52	8.72%	2.97%
Ireland	16.07%	1.48	8.50%	2.75%
Israel	8.33%	0.77	4.41%	-1.34%
Italy	20.74%	1.91	10.97%	5.22%
Jamaica	10.04%	0.92	5.31%	-0.44%
Jordan	9.88%	0.91	5.23%	-0.52%
Kazakhastan	28.17%	2.59	14.90%	9.15%
Kenya	10.09%	0.93	5.34%	-0.41%
Korea	11.20%	1.03	5.92%	0.17%
Kuwait	10.47%	0.96	5.54%	-0.21%
Laos	14.18%	1.30	7.50%	1.75%
Latvia	12.11%	1.11	6.41%	0.66%
Lebanon	5.89%	0.54	3.12%	-2.63%
Lithuania	8.54%	0.79	4.52%	-1.23%
Macedonia	13.64%	1.25	7.22%	1.47%

Malaysia	8.61%	0.79	4.55%	-1.20%
Malta	6.91%	0.64	3.66%	-2.09%
Mauritius	5.42%	0.50	2.87%	-2.88%
Mexico	14.81%	1.36	7.83%	2.08%
Mongolia	20.05%	1.84	10.61%	4.86%
Montenegro	13.26%	1.22	7.01%	1.26%
Morocco	8.26%	0.76	4.37%	-1.38%
Namibia	15.33%	1.41	8.11%	2.36%
Nigeria	24.07%	2.21	12.73%	6.98%
Oman	17.68%	1.63	9.35%	3.60%
Pakistan	15.07%	1.39	7.97%	2.22%
Palestine	14.08%	1.30	7.45%	1.70%
Panama	6.18%	0.57	3.27%	-2.48%
Peru	16.15%	1.49	8.54%	2.79%
Philippines	14.69%	1.35	7.77%	2.02%
Poland	15.08%	1.39	7.98%	2.23%
Portugal	21.66%	1.99	11.46%	5.71%
Qatar	20.25%	1.86	10.71%	4.96%
Romania	12.29%	1.13	6.50%	0.75%
Russia	21.02%	1.93	11.12%	5.37%
Saudi Arabia	19.02%	1.75	10.06%	4.31%
Serbia	8.58%	0.79	4.54%	-1.21%
Singapore	9.68%	0.89	5.12%	-0.63%
Slovakia	17.07%	1.57	9.03%	3.28%
Slovenia	15.26%	1.40	8.07%	2.32%
South Africa	13.79%	1.27	7.29%	1.54%
Spain	19.38%	1.78	10.25%	4.50%
Sri Lanka	12.40%	1.14	6.56%	0.81%
Taiwan	10.97%	1.01	5.80%	0.05%
Tanzania	18.22%	1.68	9.64%	3.89%
Thailand	16.87%	1.55	8.92%	3.17%
Tunisia	8.23%	0.76	4.35%	-1.40%
Turkey	25.06%	2.31	13.26%	7.51%
UAE	32.50%	2.99	17.19%	11.44%
Ukraine	27.07%	2.49	14.32%	8.57%
US	**10.87%**	**1.00**	**5.75%**	**0.00%**
Venezuela	40.04%	3.68	21.18%	15.43%
Vietnam	16.75%	1.54	8.86%	3.11%

Appendix 5: Equity Market Volatility versus Bond Market/CDS volatility

Standard deviation in equity index (σ_{Equity}) and government bond price (σ_{Bond}) was computed, using 100 trading weeks, where available. To compute the σ_{CDS}, we first computed the standard deviation of the CDS in basis points over 100 weeks and then divided by the level of the CDS to get a coefficient of variation.

Country	σ_{Equity}	σ_{Bond}	$\sigma_{Equity/}\sigma_{Bond}$	σ_{CDS}	$\sigma_{Equity/}\sigma_{CDS}$
Argentina	35.50%		NA	2.95%	12.05
Bahrain	7.59%		NA	14.65%	0.66
Bangladesh	16.24%		NA	NA	NA
Bosnia	8.99%		NA	NA	NA
Botswana	4.19%		NA	NA	NA
Brazil	22.25%	11.97%	1.86	12.78%	1.87
Bulgaria	15.33%	17.49%	0.88	18.69%	1.01
Chile	13.91%	6.66%	2.09	32.46%	0.75
China	17.82%		NA	28.11%	0.92
Colombia	16.00%	6.67%	2.40	23.79%	0.91
Costa Rica	8.78%		NA	11.91%	0.86
Croatia	7.42%		NA	1.05%	7.07
Cyprus	36.97%		NA	16.74%	2.38
Czech Republic	15.39%	7.26%	2.12	5.19%	3.02
Egypt	25.47%		NA	1.08%	23.49
Estonia	10.26%		NA	54.97%	0.74
Ghana	9.09%		NA	NA	NA
Greece	40.49%	56.23%	0.72	12.17%	3.45
Hungary	17.21%		NA	24.13%	0.95
Iceland	10.89%	4.04%	2.70	16.14%	0.84
India	14.09%	3.49%	4.04	11.35%	1.35
Indonesia	16.49%	9.45%	1.74	18.87%	1.06
Ireland	16.07%	5.00%	3.21	7.19%	2.31
Israel	8.33%	5.90%	1.41	220.40%	2.24
Italy	20.74%	7.40%	2.80	31.74%	0.97
Jamaica	10.04%		NA	NA	NA
Jordan	9.88%		NA	NA	NA
Kazakhastan	28.17%		NA	16.96%	1.83
Kenya	10.09%		NA	NA	NA
Korea	11.20%	6.59%	1.70	49.83%	0.72
Kuwait	10.47%		NA	NA	NA
Laos	14.18%		NA	NA	NA
Latvia	12.11%		NA	20.87%	0.79
Lebanon	5.89%	4.44%	1.33	11.82%	0.62
Lithuania	8.54%		NA	21.35%	0.61
Macedonia	13.64%		NA	NA	NA
Malaysia	8.61%		NA	30.24%	0.59

Malta	6.91%		NA	NA	NA
Mauritius	5.42%		NA		
Mexico	14.81%	9.51%	1.56	21.85%	0.90
Mongolia	20.05%		NA	NA	NA
Montenegro	13.26%		NA	NA	NA
Morocco	8.26%		NA	17.27%	0.65
Namibia	15.33%		NA	NA	NA
Nigeria	24.07%		NA	NA	NA
Oman	17.68%		NA	NA	NA
Pakistan	15.07%		NA	15.93%	1.11
Palestine	14.08%		NA	NA	NA
Panama	6.18%		NA	19.13%	0.51
Peru	16.15%	8.51%	1.90	20.04%	1.01
Philippines	14.69%	30.36%	0.48	33.29%	0.77
Poland	15.08%	11.71%	1.29	30.94%	0.80
Portugal	21.66%	10.18%	2.13	36.42%	0.96
Qatar	20.25%		NA	26.85%	1.02
Romania	12.29%		NA	21.61%	0.78
Russia	21.02%	40.10%	0.52	22.87%	1.15
Saudi Arabia	19.02%		NA	36.45%	0.89
Serbia	8.58%		NA	NA	NA
Singapore	9.68%		NA	NA	NA
Slovakia	17.07%	7.91%	2.16	23.18%	0.97
Slovenia	15.26%	13.06%	1.17	8.18%	1.95
South Africa	13.79%		NA	14.78%	1.08
Spain	19.38%	7.30%	2.65	49.92%	0.89
Sri Lanka	12.40%		NA	NA	NA
Taiwan	10.97%		NA	NA	NA
Tanzania	18.22%		NA	NA	NA
Thailand	16.87%	6.49%	2.60	26.79%	0.90
Tunisia	8.23%		NA	13.41%	0.75
Turkey	25.06%	13.17%	1.90	14.83%	1.84
UAE	32.50%		NA	NA	NA
Ukraine	27.07%		NA	6.66%	4.13
US	10.87%		NA	283.38%	2.87
Venezuela	40.04%	36.25%	1.10	10.62%	3.88
Vietnam	16.75%		NA	11.81%	1.54

Appendix 6: Year-end Implied Equity Risk Premiums: 1961-2013

Year	*S&P 500*	*Earnings*[a]	*Dividends*[a]	*T.Bond Rate*	*Estimated Growth*	*Implied Premium*
1961	71.55	3.37	2.04	2.35%	2.41%	2.92%
1962	63.1	3.67	2.15	3.85%	4.05%	3.56%
1963	75.02	4.13	2.35	4.14%	4.96%	3.38%
1964	84.75	4.76	2.58	4.21%	5.13%	3.31%
1965	92.43	5.30	2.83	4.65%	5.46%	3.32%
1966	80.33	5.41	2.88	4.64%	4.19%	3.68%
1967	96.47	5.46	2.98	5.70%	5.25%	3.20%
1968	103.86	5.72	3.04	6.16%	5.32%	3.00%
1969	92.06	6.10	3.24	7.88%	7.55%	3.74%
1970	92.15	5.51	3.19	6.50%	4.78%	3.41%
1971	102.09	5.57	3.16	5.89%	4.57%	3.09%
1972	118.05	6.17	3.19	6.41%	5.21%	2.72%
1973	97.55	7.96	3.61	6.90%	8.30%	4.30%
1974	68.56	9.35	3.72	7.40%	6.42%	5.59%
1975	90.19	7.71	3.73	7.76%	5.99%	4.13%
1976	107.46	9.75	4.22	6.81%	8.19%	4.55%
1977	95.1	10.87	4.86	7.78%	9.52%	5.92%
1978	96.11	11.64	5.18	9.15%	8.48%	5.72%
1979	107.94	14.55	5.97	10.33%	11.70%	6.45%
1980	135.76	14.99	6.44	12.43%	11.01%	5.03%
1981	122.55	15.18	6.83	13.98%	11.42%	5.73%
1982	140.64	13.82	6.93	10.47%	7.96%	4.90%
1983	164.93	13.29	7.12	11.80%	9.09%	4.31%
1984	167.24	16.84	7.83	11.51%	11.02%	5.11%
1985	211.28	15.68	8.20	8.99%	6.75%	3.84%
1986	242.17	14.43	8.19	7.22%	6.96%	3.58%
1987	247.08	16.04	9.17	8.86%	8.58%	3.99%
1988	277.72	24.12	10.22	9.14%	7.67%	3.77%
1989	353.4	24.32	11.73	7.93%	7.46%	3.51%
1990	330.22	22.65	12.35	8.07%	7.19%	3.89%
1991	417.09	19.30	12.97	6.70%	7.81%	3.48%
1992	435.71	20.87	12.64	6.68%	9.83%	3.55%
1993	466.45	26.90	12.69	5.79%	8.00%	3.17%
1994	459.27	31.75	13.36	7.82%	7.17%	3.55%
1995	615.93	37.70	14.17	5.57%	6.50%	3.29%
1996	740.74	40.63	14.89	6.41%	7.92%	3.20%
1997	970.43	44.09	15.52	5.74%	8.00%	2.73%
1998	1229.23	44.27	16.20	4.65%	7.20%	2.26%

1999	1469.25	51.68	16.71	6.44%	12.50%	2.05%
2000	1320.28	56.13	16.27	5.11%	12.00%	2.87%
2001	1148.09	38.85	15.74	5.05%	10.30%	3.62%
2002	879.82	46.04	16.08	3.81%	8.00%	4.10%
2003	1111.91	54.69	17.88	4.25%	11.00%	3.69%
2004	1211.92	67.68	19.407	4.22%	8.50%	3.65%
2005	1248.29	76.45	22.38	4.39%	8.00%	4.08%
2006	1418.3	87.72	25.05	4.70%	12.50%	4.16%
2007	1468.36	82.54	27.73	4.02%	5.00%	4.37%
2008	903.25	65.39	28.05	2.21%	4.00%	6.43%
2009	1115.10	59.65	22.31	3.84%	7.20%	4.36%
2010	1257.64	83.66	23.12	3.29%	6.95%	5.20%
2011	1257.60	97.05	26.02	1.87%	7.18%	6.01%
2012	1426.19	102.47	30.44	1.76%	5.27%	5.78%
2013	1848.36	107.45	36.28	3.04%	4.28%	4.96%
2014	2058.90	114.74	38.57	2.17%	5.58%	5.78%

[a] The earnings and dividend numbers for the S&P 500 represent the estimates that would have been available at the start of each of the years and thus may not match up to the actual numbers for the year. For instance, in January 2011, the estimated earnings for the S&P 500 index included actual earnings for three quarters of 2011 and the estimated earnings for the last quarter of 2011. The actual earnings for the last quarter would not have been available until March of 2011.

FIFTH EDITION

Cost of Capital

APPLICATIONS AND EXAMPLES

Shannon P. Pratt
Roger J. Grabowski

Foreword by Professor Richard Brealey

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following the discrete projection period. Many analysts assume capital expenditures to equal depreciation when estimating the terminal value, which results in overestimation of expected net cash flow and overvaluation, where real growth in excess of inflation is expected.[8]

USING AN UNATTAINABLE GROWTH RATE IN CALCULATING THE TERMINAL VALUE

The growth rate assumed in calculating the terminal value is a compound growth rate *in perpetuity*, which is a very long time. At a growth rate of 20% compounded annually, the company's revenues would soon exceed the gross domestic product (GDP) of the United States and eventually that of the world. Long-term growth rates exceeding the real growth in GDP plus inflation are generally not sustainable. Most analysts use more conservative growth rates in calculating the terminal value. Generally, the long-term growth rate only applies to the existing enterprise or core business net cash flows, consistent with the net cash flow projections in the discounted cash flow method (see discussion in Chapter 34).

Using Market Multiples without Adjusting for Differences in Growth

Some practitioners use a market multiple, such as the industry average multiple of earnings before interest, income taxes, depreciation, and amortization (EBITDA) to estimate a terminal value.

As we discussed in Chapter 4, the authors believe that use of a market-derived multiple for calculation of the terminal value is not appropriate, as it mixes elements of the market and income approaches and does not represent a true income approach.

In addition to mixing valuation approaches, it is not clear that a current average industry multiple reflects a long-term estimate of growth consistent with the sustainable long-term growth rate in net cash flows of the existing enterprise or core business. If the growth rate embedded in the multiple is inconsistent, utilizing this method will either overvalue or undervalue the business.

As an example, current multiples in an industry reflect the consensus growth estimates of the market, which are built upon analysts' estimates of earnings. Analysts include both the earnings of the company expected from the existing business and the earnings expected from reinvestment of retained net cash flows and reinvestment of those retained net cash flows in investments that are unspecified. Typically, the net cash flow estimates used in the DCF method valuation are based on the core business

[8]For a good discussion of this common error, see Gilbert E. Matthews, "Cap X = Depreciation Is Unrealistic Assumption for Most Terminal Values," *Shannon Pratt's Business Valuation Update* (March 2002): 1–3. See also Gilbert E. Matthews, "Errors and Omissions in DCF Calculations: A Critique of Delaware's Dr. Pepper Appraisal," *Business Valuation Update* (October 2007): 1–5. In this article, the author states: "In a perpetuity model with a 3% growth rate and assuming a 10-year average life for fixed assets, capital expenditures would exceed depreciation by 15.5% using straight-line depreciation and 11.6% using double-declining method."

PUBLIC UTILITIES FORTNIGHTLY ENERGY, MONEY, POWER

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What do utility shareholders want? Answer: to earn a total return, dividends plus capital gains, at least commensurate with the risk incurred.

That is, to earn a return equal to, or in excess of, the cost of capital.

Did shareholders earn this in the past? And what do they require now?

In a recent piece written for *Public Utilities Fortnightly*, Steve Huntoon didn't directly answer those questions. Rather he concluded, much more elegantly, that whatever shareholders want, they get too much of it.[1]

Steve is a lawyer. So what does he know?

The authors of this column spent years on Wall Street, complaining that regulators did not provide investors with adequate returns. So we decided to check out the numbers.

Understand first, the market determines cost of capital. Regulators don't.

Second, to determine expected return, investors and academics have lately begun to rely more on historical data.

They are taking into account the tendency of markets to revert to the mean. We will try to apply that technique to answer the questions.

Let's cut to the chase. In the past century or more, globally, common stocks earned real returns of about five and a half percent to six and a half percent. Per year. Adjusted for inflation.

In the U.S., return on stocks have exceeded return on risk-free Treasury bonds. The equity risk premium was roughly two-point-four to five percentage points.

Recent Federal Reserve Bank monetary policy makes Treasuries a dubious benchmark. So we will use seasoned Baa corporate bonds instead.

Those bonds offered yields of one to two percentage points more than Treasuries in the past. And two to three percentage points more recently.

We estimate that investors, over the long term, expect that corporate bonds will earn two percentage points over Treasuries. And equities will earn five percentage points over Treasuries.

For a rule of thumb, equities will earn about three percentage points over corporate bond yields. Why bother with a rate case? Just use that handy rule of thumb.

Two additional points. Bond yields track inflationary expectations. So our calculation in current dollars indirectly takes inflation into account.

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Also, over the post war period, utility stocks have performed at least as well as industrial stocks. So conclusions derived from the general market probably apply to them as well.

The first question is, what did utility investors earn? And was that good enough?

In the postwar period, investors earned just less than ten percent per year. That's six and a half percent in real terms.

Dividends made up about sixty-three percent of this return. See Figure 1.

Our rough-and-ready formula calculated a required return of ten and a half percent per year. That's six-point-nine percent in real terms. See Figure 2.

Utility stocks then earned in-line with long-term market expectations.

But utility stock prices exceeded their book value in fifty-six of the past seventy years. With sub-par pricing during energy and nuclear crises.

This indicates that utilities earned more than the cost of capital in most years.

Thus, utility investors earned an average market return, while taking a lower than average risk. Return probably exceeded the cost of capital.

The numbers tell us about anticipated growth. We define this as expected total return, minus dividend yield.

Over the postwar period, we calculate that investors expected growth of about four and a half percent per year. See Figure 3.

At the end of June 2016, corporate bonds yielded four and a half percent. Utility stocks yielded three-point-four percent.

This indicates, based on historical precedent, that equity investors want a seven and a half percent annual return. Three-point-four percent from dividends. Four-point-one percent from capital gains.

Is seven and a half percent, the number implied by Steve Huntoon, the nominal cost of equity capital? Imagine using that level of return in a utility rate case.

Sooner or later, regulators may see the gap between allowed returns and cost of capital. They might reduce returns.

Or regulators could impose British-style incentive regulation. It would offer utilities the opportunity to take higher risks, in order to maintain returns.

Either option could endanger dividends. That is the downside.

Income-starved investors are looking for means to meet their long-term obligations. They may accept even lower returns than the cost of equity capital we calculated.

The trick is for utilities to find ways to utilize that pool of capital.

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Investors just want a better return on a safe investment than the one and a half percent they can get on ten-year Treasuries. Both utilities and electricity consumers might benefit from this trying financial situation.

And yes, it looks as if Steve Huntoon was right after all. Even if he is a lawyer.

Endnotes:

1. Steve Huntoon, "Nice Work If You Can Get It," *Public Utilities Fortnightly,* August 2016.

Robert D. Arnott and Peter L. Bernstein, "What Risk Premium Is Normal?" *Financial Analysts Journal*, March/April 2002, is a pioneering paper on the topic. It is comprehensive and comprehensible. For more recent data and analysis, see Martin Leibowitz, Andrew W. Lo, Robert C. Merton, Stephen A. Ross, and Jeremy Siegel, "Q Group Panel Discussion: Looking to the Future," *Financial Analysts Journal*, July/August 2016.

Media:

"British-style incentive regulation would offer utilities the opportunity to take higher risks, in order to maintain returns." – Leonard Hyman

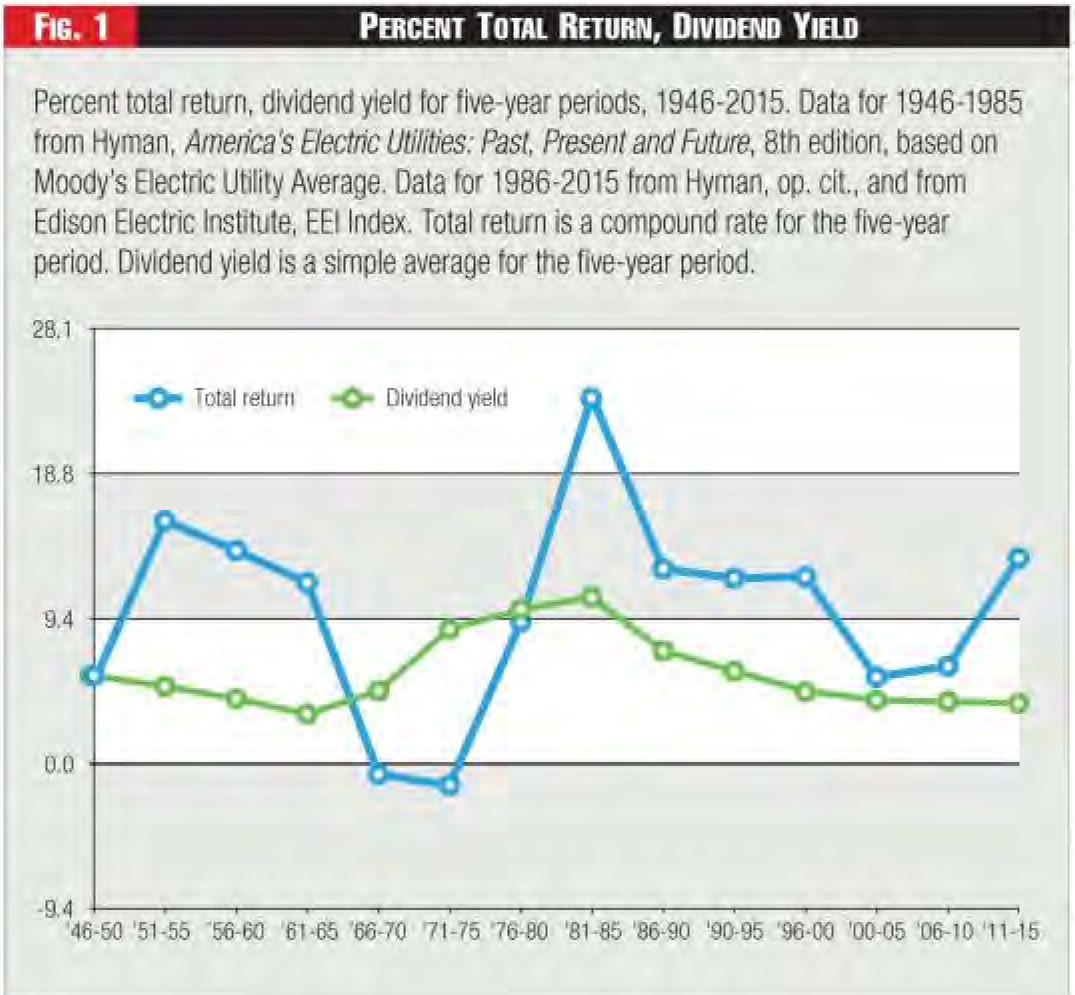

Figure 1 - Percent Total Return, Dividend Yield

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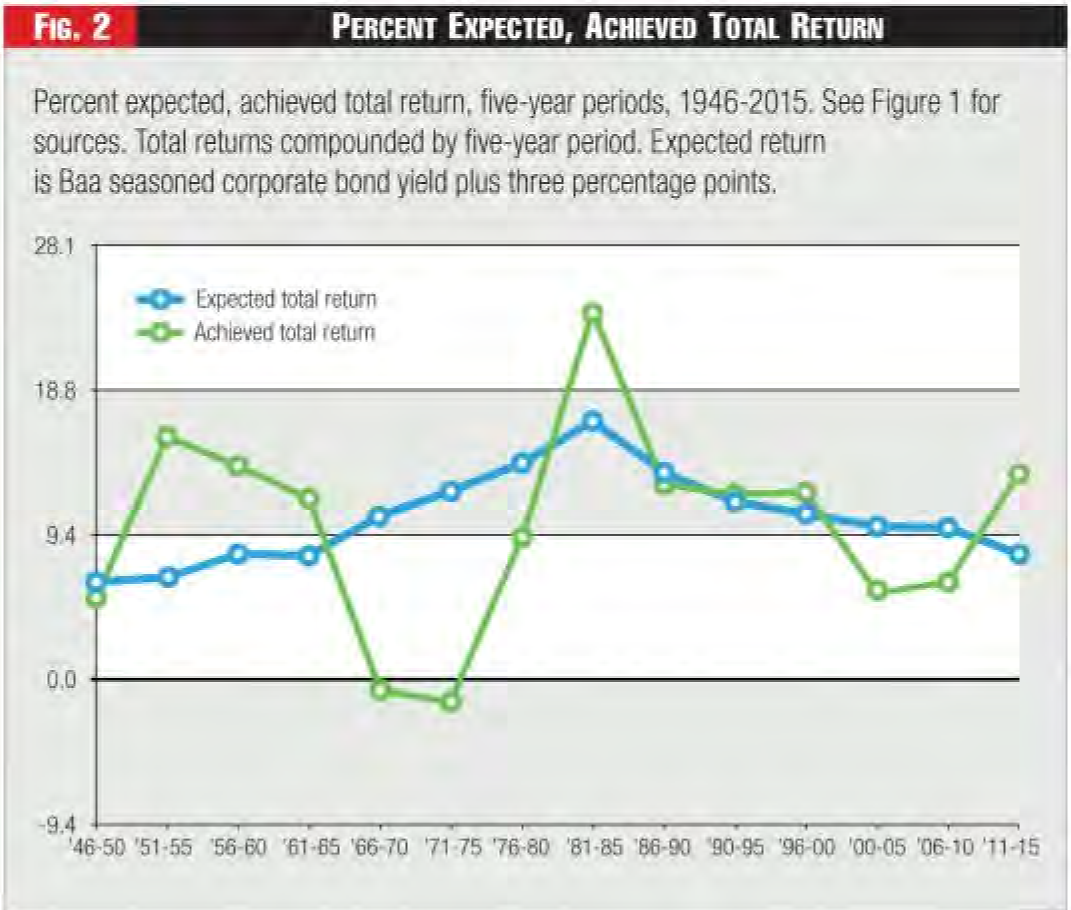

Figure 2 - Percent Expected, Achieved Total Return

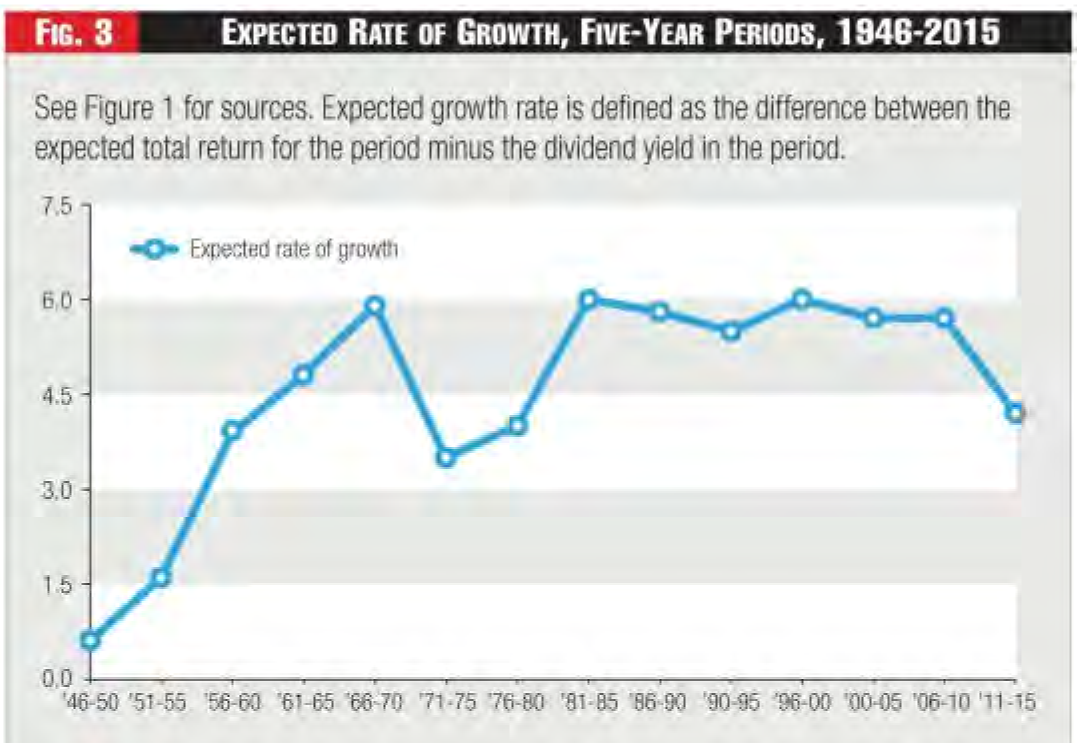

Figure 3 - Expected Rate of Growth, Five-Year Periods, 1946-2015

OPCRESP-POD1d-000339

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PUBLIC UTILITIES FORTNIGHTLY ENERGY, MONEY, POWER
Published on *Fortnightly* (https://www.fortnightly.com)

Let's admit one thing right off the bat. Rate of return is one of the most arcane subjects in utility regulation's ocean of arcania.

But one thing that makes rate of return interesting is the amount of money involved. It's roughly $58 billion each year for electric utilities.[1]

Now you may be thinking, OK, so there's big money involved. But what's in it for me? In the spirit of BLUF, Bottom Line Up Front, let me tackle that question.

There is mounting evidence that investment in utility stocks has outperformed the broader market in the past, and will continue to do so. This is a conundrum. Regulated utilities are less risky than competitive industries, and therefore are supposed to produce a lower total return over time. But instead the opposite is happening.

We'll get into the evidence for this, and then speculate as to how this can be so. But if you want actionable intelligence up front, here it is: invest in regulated utilities.

Vanguard Group gives you low-cost index-fund options for utility investment. The symbol for the mutual fund is VUIAX and for the ETF is VPU. You may now skip the rest of this column if so inclined.

By the way, if your interest is the welfare of utility customers, there is more at stake than just higher than needed equity rates. When allowed equity returns exceed the true cost of equity, utilities have an artificial incentive to expand utility facilities upon which they can earn that extra return, including favoring themselves over others in resource procurement. This is the well-known Averch-Johnson effect first described in 1962.

OK, for those sticking around for the substance here it is. The historical evidence of outperformance comes in three data points:

1. A study released by PJM showing lower-risk regulated generation outperforming higher-risk, market-based generation over a long-term horizon.[2]

2. Broader studies of markets showing lower-beta, lower-risk stocks outperforming higher-beta, higher-risk stocks over a long-term horizon.[3]

3. Utility stocks outperforming the broader market over the last 12 years, the longest period tracked in Google Finance, with the Dow Jones Utility Average at a total return of 161 percent and the Dow Jones Industrial Average at a total return of 133 percent.[4]

These are astounding, counter-intuitive results.

This counter-intuitive past seems destined to continue into the future. Three data points point the way:

1. Jack Bogle, the founder of Vanguard Group and a Wall Street legend, provides rigorous analysis that the long-term total return for the broader market will be around 7 percent going forward.[5] Another Wall Street legend, Professor Burton Malkiel, corroborates that 7 percent in the latest edition of his seminal work, *A Random Walk Down Wall Street*.[6]

2. Institutions like pension funds are validating #1 by piling on risky investments to try and get to a 7.5 percent total return, as reported by the *Wall Street Journal.*[7]

3. Utilities are being granted returns on equity around 10 percent.[8]

Let's reflect on what #3 means relative to #1 and 2.

It means that the less risky utilities are being awarded much higher returns, roughly 40 percent higher, than the broader market is expected to earn. The extra is about $17 billion per year.[9] Not too shabby.

So let's repeat the actionable intelligence. If you're a professional money manager it means you should buy the Vanguard utility index fund (or a comparable fund) and spend the next 10 years in Maui drinking Mai Tai's with those little umbrellas.

The rest of us should make the same investment. But we'll still have to work because we can't drink Mai Tai's in Maui for a living.

Now that we've gotten the practical stuff out of the way, let's think about why this might be so. The efficient market hypothesis says it isn't possible to have an anomaly like lower risk stocks consistently outperforming higher risk stocks. And yet they are.

Why? One thing we know off the bat is that utility stocks are the only stocks where Wall Street analysts actually set earnings, instead of just forecasting earnings. That is because utility regulators use Wall Street analysts' forecasts of earnings and dividend growth to set the "g" factor, and dividend yield plus g becomes the allowed return on equity.

You might observe that there is some circularity to this. If Wall Street analysts set g high, then the allowed return on equity will be high, and then g will be high, etc.

But it's not all circular. There may be some reasons for Wall Street to think g ought to be high. Wall Street forecasts tend to be led by guidance from the companies themselves. Utility companies have decades of experience in maximizing earnings under regulation, and partial deregulation, and they do very well at it.

How exactly? Well, we need to get in the weeds to explore some of the ways, but here goes. Utilities often can take advantage of double leveraging their capital structure. That's pretty esoteric so let's take an example.

Suppose you have an operating utility company with a 50 percent debt, 50 percent equity capital structure, with 5 percent debt cost and 10 percent equity cost. Now, let's suppose a holding company is created that finances the 50 percent operating company equity with 40 percent debt and 60 percent equity. How much does the parent company equity earn on equity? It earns 13.3 percent, not 10 percent, because of the double leverage.[10]

And it also works in reverse. Wall Street forecasts a return of equity of 13.3 percent on the double leveraged parent equity, and that percent is applied to the capital structure of the operating company where the equity cost is only 10 percent. Pretty neat, eh?

Beyond capital structure, the nature of regulation has evolved favorably over time for the regulated. Utilities have been able to enlist regulators in risky endeavors so as to eliminate or mitigate financial losses from failures.

Nuclear and clean coal plants come to mind. New such plants are concentrated in areas of the country where traditional rate regulation for generation has continued. In contrast to areas where generation investment is subject to market conditions and competitive pressures.[11]

Utilities also have exhibited some facility for shifting regulatory paradigms as market conditions change. Ohio and Illinois illustrate this. As part of the deal to allow competition, utilities received stranded cost payments.

Then, rising wholesale prices became a bonus. And now with wholesale prices back down, some of those same utilities are seeking subsidies for their generation. This ability to shift among regulatory paradigms is unique to the utility industry.

Utility rates also tend to be downward sticky. It is easier for a utility to initiate and prosecute rate increases than for consumer advocates to initiate and prosecute rate decreases, with an imbalance in information being one obvious reason why.

And utilities have some ability to influence timing of expenses with, for example, workforce reductions coming a polite period after the resolution of a rate case. And utilities over time have been able to implement automatic pass-through of various types of costs so, for example, some costs can be passed through without comprehensive review of the utility's overall revenues and costs.

All of this is nice work if you can get it.

You may be thinking, is there a risk that regulators look at all this and reduce allowed returns to something closer to what the riskier broader market is expected to earn? So utilities would no longer be an anomalously great investment?

No worries. This is our little secret.

1. According to EEI data, there is $356 billion in electric utility common equity. Assume a 10 percent return on equity plus an income tax allowance of 6.4 percent. The income tax allowance is based on a composite federal or state income tax rate of 39 percent. The 10 percent return is divided by 61 percent (1 minus 39 percent). This gives a pre-tax total return of 16.4 percent, which amounts to $58 billion on the $356 billion in common equity.

2. "... one would expect merchant firms to earn a much higher level of return than the firms that are more tightly regulated. However, the opposite seems to be true as the consistently positive alphas for regulated firms indicates these companies are earning returns higher than what they should be expected to earn given their much lower level of risk." Resource Investment in Competitive Markets, Technical Appendix, May 5, 2016.

3. "In an efficient market, investors earn higher returns only to the extent that they bear higher risk. Despite the intuitive appeal of a positive risk-return relationship, this pattern has been surprisingly hard to find in the data, dating at least to Black (1972). For example, sorting stocks by using measures of market beta or volatility shows just the opposite. Panel A of Figure 1 shows that from 1968 through 2012 in the U.S. equity market, portfolios of low-risk stocks delivered on the promise of lower risk as expected but had surprisingly

higher average returns. A dollar invested in the lowest-risk portfolio grew to $81.66, whereas a dollar invested in the highest-risk portfolio grew to only $9.76." The Low Risk Anomaly: A Decomposition into Micro and Macro Effects, *Financial Analysts Journal,* March/April 2014.

4. These returns are from Google Finance, comparing Dow Jones Utility Average Total Return with Dow Jones Industrial Average Total Return from August 31, 2004, earliest common date, to June 28, 2016.

5. "Thus, the prospective nominal investment return on stocks seems likely to run in the range of 7 percent..." Occam's Razor Redux: Establishing Reasonable Expectations for Financial Market Returns, *Journal of Portfolio Management*. This conclusion is supported by unprecedented lows in the risk-free rate, even negative interest on some sovereign debt. For an excellent summary of the Bogle study see Jason Zweig's column, This Simple Way Is the Best Way to Predict the Market, *Wall Street Journal,* December 24, 2015.

6. "Adding the initial yield and growth rate together, we get a projected total return for the S&P 500 of just under seven percent per year ...." (*A Random Walk*, page 346).

7. "To even come close these days to what is considered a reasonably strong return of 7.5 percent, pension funds and other large endowments are reaching ever further into riskier investments..." *Wall Street Journal,* June 1, 2016.

8. FERC set the base allowed return for New England transmission owners at 10.57 percent in its Opinion Numbers 531, 531-A and 531-B. State commission allowed returns for electric utilities have averaged 9.78 percent according to an analysis of *Public Utilities Fortnightly* data in the PJM Study, earlier referenced.

9. Here's the math: 16.4 percent pretax return on $356 billion equity is $58 billion. If the equity return is 30 percent less, 7 percent versus 10 percent, then the reduction in return is $17 billion.

10. Here's an example of the math. Assume the operating company's equity is $100 million. At a 10 percent allowed return it earns $10 million. Now let's suppose the holding company finances that $100 million with 40 percent debt costing 5 percent and 60 percent equity. The holding company pays $2 million for the debt and thus earns $8 million on the $60 million equity for an actual return on equity of 13.3 percent. The key is the difference between the holding company's consolidated capital structure and the utility operating company's capital structure. Indeed, the leveraging is even more lucrative because the phantom equity also gets a phantom income tax allowance.

11. For more on this see the PJM Study, earlier referenced.

"Rate of return is one of the most arcane subjects in utility regulation's ocean of arcania." – Steve Huntoon

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WHEN "WHAT GOES UP" DOES NOT COME DOWN:
RECENT TRENDS IN UTILITY RETURNS

Charles S. Griffey, P.E., CFA[1]
February 15, 2017

I. Executive Summary

- *Returns on Equity (ROEs) granted to regulated utilities are near an all-time high relative to interest rates.*
- *Yet, the risks faced by regulated utilities are at an all-time low.*
- *Returns achieved by regulated utilities are equal to or greater than the returns of much riskier enterprises.*
- *Utilities could attract necessary capital at much lower awarded ROEs. Excessive ROEs encourage overbuilding and harm utility customers.*
- *Policymakers should reassess the ROEs being granted to utilities, and should be skeptical of requests for additional alternate rate-setting mechanisms without significant ROE reductions.*

II. Overview

Awarded and achieved utility ROEs have been much higher than necessary to induce appropriate investment in recent years. Utility ROEs have failed to track either the utilities' level of regulatory risk or general economic indicators. This trend can drive inefficient investment decisions by utilities and inflates rates for utility customers.

The risks faced by most utilities today are significantly lower than over the last three or four decades.[2] For example, utilities are generally not attempting to place capital-intensive coal and nuclear plants in rates today, as natural-gas-fired generation has emerged as the preferred plant technology. Natural gas plants have a lower up-front capital cost, so they carry significantly less financial risk in a regulatory review than an expensive coal or nuclear plant.[3]

[1] Mr. Griffey is an energy consultant whose clients have included large industrial customers, generators, retail electric providers, electric cooperatives, municipal utilities, and the Staff of the Public Utility Commission of Texas. He is a former utility and energy company executive and is Adjunct Professor of Management at Rice University's Jones Business School.

[2] A view shared by the rating agency Moody's Investor Service (Sector-in-Depth Analysis, March 2015): "Across the US, we continue to see regulators approving mechanisms that allow for more timely recovery of costs, a material credit positive. These mechanisms, which keep utilities' business risk profile low compared to most industrial corporate sectors, include: formulaic rate structures; special purpose trackers or riders; decoupling programs (which delink volumes from revenue); the use of future test years or other pre-approval arrangements. We also see a sustained increase in the frequency of rate case filings."

[3] https://www.eia.gov/forecasts/capitalcost/pdf/updated_capcost.pdf

The risks and uncertainty associated with transitioning to retail competition—such as the potential for stranded utility generation investment—have largely been settled, further reducing utilities' risk. Rate riders, interim "cost recovery factors," and other features that allow a utility to increase its rates without a full rate review have also proliferated over the past two decades, allowing accelerated capital recovery and substantially reducing regulatory lag in the ratemaking process. Over time, these and other factors have materially reduced risk for regulated utilities, making high risk premiums unnecessary to attract capital or induce investment.

Yet, ROEs for regulated utilities are higher than ever relative to US Treasuries. ROEs have not been significantly reduced to recognize the lower risk faced by regulated utilities today, or even general economic trends. Utility ROEs have not fallen at nearly the same rate as interest rates. One cause of this "stickiness" in regulated utility ROEs (compared to interest rates) is the peer-group methodology used by most ROE witnesses and often adopted by regulators. This approach is inherently backward-looking, and when each utility's ROE is based on the ROEs granted to the utility's peers, inflated utility ROEs are self-perpetuating. Further, as Public Utilities Fortnightly observed in its 2016 Annual Rate Case Survey, the trend of sustained, unnecessarily high ROEs for utilities is also a product of utility scare tactics in regulatory proceedings, where risk-averse regulators are led to believe that appropriately reducing ROEs will deter necessary investment—despite robust evidence to the contrary.[4] As a result of these and other factors, utilities are receiving premium ROEs today compared to other industries.

The "risk premium" being granted to utility shareholders is now higher than it has ever been over the last 35 years. Excessive utility ROEs are detrimental to utility customers and the economy as a whole. From a societal standpoint, granting ROEs that are higher than necessary to attract investment creates an inefficient allocation of capital, diverting available funds away from more efficient investments. From the utility customer perspective, if a utility's awarded and/or achieved ROE is higher than necessary to attract capital, customers pay higher rates without receiving any corresponding benefit. Inflated ROEs also encourage utilities to make inefficient investment decisions so that they can earn a return on additional capital, harming both society and customers. As one observer has aptly noted, "When allowed equity returns exceed the true cost of equity, utilities have an artificial incentive to expand utility facilities upon which they can earn that extra return, including favoring themselves over others in resource procurement."[5] This compounds the excess earnings for utilities and further increases rates for customers. In addition, the combination of low debt costs and high utility ROEs in recent years has encouraged a type of arbitrage known as "back-leveraging" or "double-leveraging," where a utility parent or holding company borrows money at a low rate to use as equity at the utility level. This common strategy of translating low cost debt at the parent into equity returns at the utility increases returns for shareholders even beyond the premium levels authorized by regulators.[6]

[4] Cross, P., "2016 Annual Rate Case Survey," Public Utilities Fortnightly (Nov. 2016).

[5] *See* Huntoon, S., "Nice Work If You Can Get It," Public Utilities Fortnightly (Aug. 2016).

[6] Notably, "back-leveraging" also creates significant risk for utility customers by increasing the financial stakes of a default, which could compromise the utility's financial integrity and impede appropriate investment to maintain reliability.

Importantly, an excessive utility ROE has more than a dollar-for-dollar impact on customer rates because rates are grossed up to cover federal income tax liability on utility earnings. Take, for example, a utility with a total rate base (total investment) of $1 billion, and a capital structure of 40% equity, 60% debt, which is common. A one percent increase in this utility's ROE would not just translate to a rate increase of $4 million, but to ***$6.2 million*** because the return would be grossed up to cover corporate federal income tax liability (roughly 35%) on the additional earnings.[7] Investor-owned utilities in Texas have an aggregate rate base of approximately $25 billion.[8] Historically, a typical utility risk premium would be in the range of 450 basis points above Treasuries (in other words, if 30-year treasury bonds yield 3%, the utility ROE would have been 7.5%). However, risk premiums have been on the order of 650 basis points over the last several years, with Treasury bonds at 3% and utility ROEs at 9.5%. In Texas, this 200 basis point differential means, all else being equal, rates could have been reduced by approximately $300 - $350 million[9] annually without adversely impacting investment in utility infrastructure.

As a result of all these factors, utilities have been very profitable investment vehicles in the current economic climate,[10] and investors are eager to provide capital for utility infrastructure. Even if utilities do not achieve their allowed ROE, they have been successful in achieving a return in excess of their cost of capital.[11] Thus, there is no shortage of interest from both traditional utilities and non-traditional players such as pension funds, sovereign wealth funds, and private equity groups to invest in utility projects. This is, generally speaking, because the actual cost of capital required for investment is much lower than the ROEs being granted in the utility sector. A recent analysis concluded that most utility investors are looking for an annual rate of return around 7.5%,[12] while awarded utility ROEs have continued to be around 10%.[13] The result is a risk-adjusted rate of return that is superior to competing investments, and

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[7] $1 billion rate base * 40% equity in capital structure * 1% increase = $4 million. Tax gross-up is $4 million/(1-0.35) = $6.2 million.

[8] *See* Tietjen, D., "Alternative Ratemaking: Is It Time For A Shock To The Rate-Setting System?," presented to Gulf Coast Power Association, November 21, 2016. This figure does not include transmission investments held by municipally owned utilities or electric cooperatives, which are also included in the postage stamp transmission rates in ERCOT. Rate base equals net plant in service of $33 billion from Mr. Tietjen's presentation, less ADFIT of $8 billion, taken from each utility's earnings monitoring reports in the following docket: http://bit.ly/2ibTVke.

[9] $25 billion * 40% equity * 2%/(1-0.35) = $308 million. Non-ERCOT utilities typically have approximately 50% equity in their capital structure, not the 40% used in Transmission and Distribution utilities in ERCOT, so the actual amount would be in excess of $308 million.

[10] Hyman, L. and Tilles, W., "Don't Cry for Utility Shareholders, America," Public Utilities Fortnightly at 65 (Oct. 2016).

[11] The cost of capital is set by the market, not regulators.

[12] Hyman, L. and Tilles, W., "Don't Cry for Utility Shareholders, America," Public Utilities Fortnightly at 65 (Oct. 2016).

[13] *See* Cross, P., "2016 Annual Rate Case Survey," Public Utilities Fortnightly (Nov. 2016); *see also* Huntoon, S., "Nice Work If You Can Get It," Public Utilities Fortnightly (Aug. 2016) at n. 8, citing recent FERC-issues ROEs in the 10% range for New England utilities.

higher than necessary to induce investment. The keen interest of numerous investors in recent utility mergers and acquisitions at premium prices is another sign of this phenomenon.[14]

The evidence showing that awarded utility ROEs far exceed the levels that actual risk factors and general economic trends would support is substantial, and mounting. As one author on this topic has stated, "[r]egulated utilities are less risky than competitive industries, and therefore are supposed to produce a lower total return over time. But instead the opposite is happening."[15]

Mounting evidence indicates that awarded ROEs and actual utility earnings are too high, and that it is time to reevaluate the status quo and reduce utility ROEs to reflect actual risk and economic factors.

III. Current utility ROEs are higher than risk factors and economic trends support.

Rates of return for regulated utilities must achieve two competing goals: (1) they must allow the utility to attract enough capital to make the investments needed to provide reliable, continuous service, and (2) they must protect customers against monopoly pricing by ensuring that rates replicate what a competitive market would produce. A seminal scholar on utility regulation, James Bonbright, famously described the rate-setting process as follows:

> Regulation, it is said, is a substitute for competition. Hence its objective should be to compel a regulated enterprise, despite its possession of complete or partial monopoly, to charge rates approximating those which it would charge if free from regulation but subject to the market forces of competition. In short, regulation should be not only a substitute for competition, but a closely imitative substitute.[16]

If a utility's awarded ROE is too low relative to its risk profile, the utility will not be able to attract capital, which will result in underinvestment. If a utility's awarded ROE is too high, customers will pay more than necessary to incentivize appropriate investment, and the utility will be encouraged to pursue inefficient investments and to "gold plate" infrastructure to inflate its returns. The overall economy is also harmed in these conditions because capital is inefficiently diverted from other potential investments.

With this context, a historical comparison of the returns earned on "risk-free" investments (represented here by thirty-year Treasury yields) and the ROEs granted to regulated utilities strongly suggests that utility ROEs are not appropriately tracking either the risk level of utility investments or general economic trends. As shown in Figure 1, both utility ROEs and Treasuries have fallen since the early 1980s, but the gap has widened because utility ROEs have not declined nearly as quickly as Treasury yields—particularly over the last ten years:

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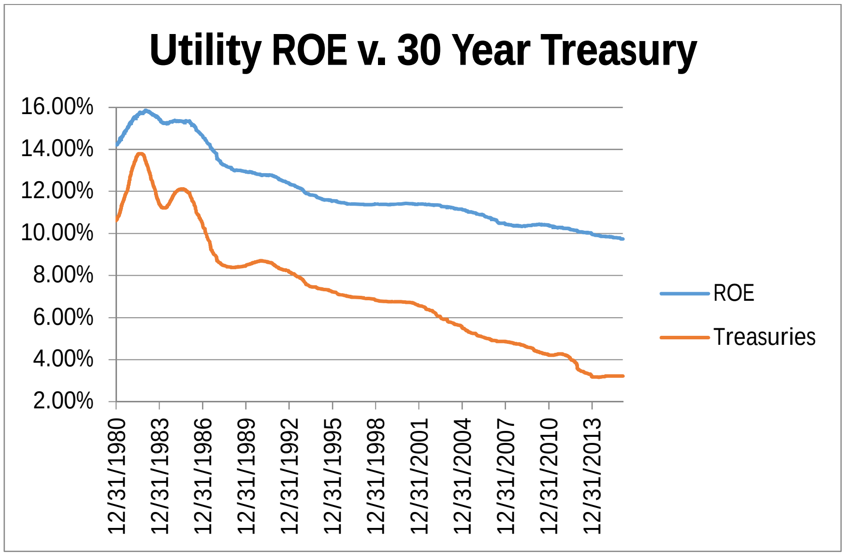
[14] "Recent acquisition activity has been a little troubling, with above-average premiums being paid and, consequently, a more debt-financed profile to the transactions." Standard & Poors Ratings Service, "Industry Top Trends 2016," December 2015 at 22.

[15] Huntoon, S., "Nice Work If You Can Get It," Public Utilities Fortnightly (Aug. 2016).

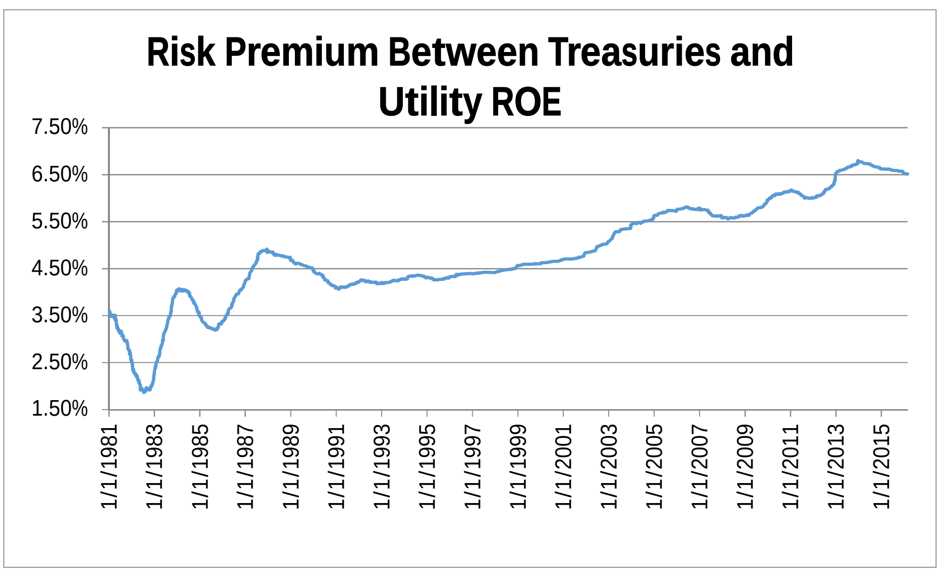
[16] Bonbright, J., Principles of Public Utility Rates at 3 (1966).

Figure 1: Comparison of Utility Allowed ROEs to 30-Year Treasury Yields[17]



This gap between utility ROEs and returns on "risk-free" investments represents a "risk premium." Risk premiums should compensate utility shareholders for the increased risk they bear relative to simply holding a theoretically risk-free asset—the 30-year Treasury bond in this case. As utility risk declines, the difference between utility ROEs and risk-free interest rates should become smaller—but the opposite is happening. The figure below focuses solely on the risk premium:

Figure 2: Comparison of Risk Premiums



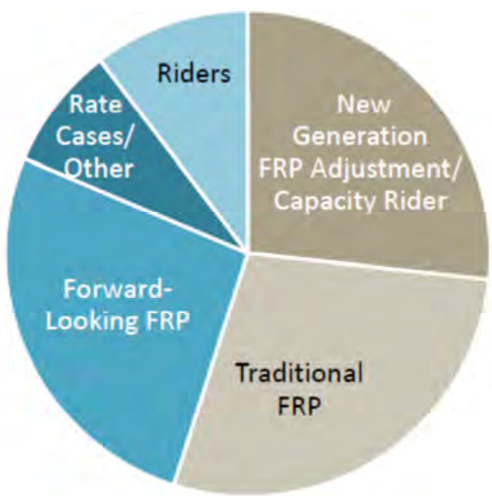
[17] Data is smoothed to be the 12-month moving average for both utility ROEs and Treasuries. Data is from SNL Financial and Bloomberg (see Direct Testimony of Robert Hevert in Docket 45414, Exhibit RBH-8, and Exhibit 1 to March 10, 2015 Moody's Sector-in-Depth Analysis for Electric Power).

As the chart above illustrates, the average risk premium over 1980-2016 was about 4.5%, or 450 basis points. Until the year 2000, risk premiums for utility investments had never exceeded 500 basis points. *Since that time, the gap has steadily increased and stands at approximately 650 basis points today*. If investing in utilities were riskier today than in the past, this result might be appropriate—but the opposite is true, as discussed below. Risk in the utility sector has declined over the last few decades, yet ROEs have not been reduced to reflect this lower risk, or even to track the general decline in expected yields from "risk-free" investments. This strongly suggests that the ROEs being granted to regulated utilities should be reevaluated.

IV. Texas: A Case Study

The utility business in Texas has become significantly less risky over the last two decades. From an investor's viewpoint, "risk" in the utility business includes anything that delays or prevents the investor from earning a return on invested capital. Among other factors, traditional utility risks include the potential that regulators may exclude an investment from rates (*e.g.*, for imprudence in the construction of generating plant), significant delay between the time an investment is made and the time when it is reflected in rates (also called "regulatory lag"),[18] and factors that influence utility revenues such as fluctuations in weather and load growth. Nationally, utilities have been successful in minimizing regulatory lag over the past decade through "alternative" rate mechanisms like future test years, formula rate plans, various riders to collect specific costs, and other forms of piecemeal (or "single-item") ratemaking. The chart below was created by a large multi-jurisdictional utility to show investors how little it relies on traditional rate cases compared to alternate ratemaking mechanisms to recover capital:

Figure 3: Illustrative Recovery of Incremental Utility Capital[19]



[18] Regulatory lag is a complex issue, as it can both hurt and help investors depending on the circumstances. If a utility is over-earning, regulatory lag benefits shareholders by increasing the time it takes to adjust rates downward. When a utility is under-earning, regulatory lag can delay setting rates that reflect the utility's actual revenue requirement.

[19] Entergy Presentation to Investors, February 26, 2016 at 13. http://files.shareholder.com/downloads/ETR/3875534036x0x877819/1D8DC9CC-7551-4A2F-8658-7DDB4147F73A/Handout_-_Investor_Meetings_Feb_26.pdf.

In Texas, there has been a profound trend of declining risk in the utility business over the last 15 years. Regulatory lag has been materially reduced (if not almost completely eliminated) for utilities—inside and outside of ERCOT—through the myriad of riders and cost-recovery factors that are now granted. Utilities can now increase rates without a full rate case to reflect: (1) transmission investment through Transmission Cost of Service (TCOS) and Transmission Cost Recovery Factor (TCRF) updates,[20] (2) distribution investment through Distribution Cost Recovery Factor (DCRF) updates,[21] (3) purchased power contracts through the Purchased Power Cost Recovery Factor (PCRF),[22] (4) changes in fuel costs through the Fuel Factor,[23] and (4) costs of complying with energy efficiency mandates through the energy efficiency cost recovery factor (EECRF).[24] Many of these updates can be filed at the utility's discretion, which means utilities can selectively file only when they believe a rate increase is supported. Some of these mechanisms fail to account for potential reductions in related cost drivers, such as deferred federal income taxes (a reduction to rate base) and load growth. Given that these mechanisms largely eliminate risk and can actually *increase* a utility's earned return, it is indisputable that utilities in Texas face much less regulatory lag or risk than they did in the 1980s or 1990s.

In ERCOT, generation service is now competitive and is no longer provided by rate-regulated utilities. Compared to generation investment, transmission and distribution investment carries a much lower risk of being excluded from rates because: (1) the investments are more granular and gradual, and (2) the utility has significantly less discretion in defining the type of technology and size of the investment. This is particularly true in ERCOT, given that ERCOT independently studies and pre-approves the need for new, large transmission facilities.[25] Outside of ERCOT, utilities still retain some risk and regulatory lag associated with generation investment, but the shorter lead time and lower capital cost for natural gas-fired generation (which has been the leading technology for new utility generation) reduces the impact of regulatory lag and imprudence risk. When combined with the myriad rate riders discussed above, it is hard to dispute that regulatory risk has declined significantly for both ERCOT and non-ERCOT utilities.

Yet, utility ROEs have not declined as ratemaking theory, market factors, and risk analyses would predict. Instead, the risk premiums reflected in utility ROEs have caused regulated utility stocks to closely track the Dow Jones Industrial Average (DJIA), which is comprised of enterprises that are traditionally much riskier than the utility sector. Utilities have historically been "low-beta" stocks, meaning that they are inherently less risky and, accordingly, have traditionally had lower equity returns than the DJIA. But in the recent past, utility stocks

[20] PUC Subst. R. 25.192 and 25.193 (ERCOT) and 25.239 (non-ERCOT).

[21] PUC Subst. R. 25.234 (both ERCOT and non-ERCOT).

[22] PUC Subst. R. 25.238 (non-ERCOT).

[23] PUC Subst. R. 25.235 (non-ERCOT)

[24] PUC Subst. R. 25.181 (both ERCOT and non-ERCOT).

[25] By rule, the PUCT gives "great weight" to ERCOT's need determination. *See* PUC Subst. R. 25.101(b)(3)(ii).

have actually had *higher* returns than the DJIA, strongly indicating that utility ROEs are far above appropriate risk premium levels.[26]

These high risk premiums for utilities allowing equity investor returns equivalent or superior than what is available in the markets generally, but for *a lower level or risk*. This runs completely counter to rationale economics or market theory. As one observer colorfully put it, ". . . if you want actionable [investment] intelligence up front, here it is: invest in regulated utilities."[27]

As discussed below, a large part of the problem appears to be the feedback loop created when ROEs in regulated utility rate cases are set based on the historical ROEs awarded to *other* utilities. This approach makes it difficult to implement a significant change when economic conditions or regulatory changes would merit significant reductions in ROEs. Regulators are understandably hesitant to reduce ROEs relative to what other jurisdictions are awarding for fear of deterring investment, and utilities have been successful in appealing to this conservativism to keep ROEs higher than they should be. However, the data shows that it is imperative to overcome this collective action problem and broadly reevaluate whether regulated ROEs are at appropriate levels.

V. Time to Reassess

The foregoing discussion begs the question: why have utilities continued to receive inflated ROEs in spite of all these compelling factors? The primary drivers behind the "stickiness" of utility ROEs appear to be: (1) the method by which regulated utility ROEs have traditionally been established (the "peer-group" method mentioned previously), and (2) strategic utility appeals to the risk aversion of regulators when it comes to investment and reliability.

Regulators are responsible for making sure customers receive reliable electricity service from their monopoly provider—an issue that is keenly important to the public and policymakers. Because of this, regulators are understandably sensitive to arguments that reducing utility ROEs will decrease investment below an acceptable level, harm a utility's credit profile, or compromise reliability. In recent years, utilities appear to have been particularly successful in persuading regulators that any reduction in ROEs will have unacceptable consequences, despite extensive countervailing data. For example, utilities will often describe an ROE reduction as "credit negative" to deter regulators from pursuing such a reduction. Of course, it is always "credit positive" to grant utilities higher ROEs and "credit negative" to lower ROEs; this says nothing about appropriate return levels. Rebalancing must occur at some point, and reducing ROEs will not harm investment incentives if the reductions appropriately reflect the overall economic climate or the specific risks faced by a utility. Similarly, in its 2016 Annual Rate Case Survey, Public Utilities Fortnightly described a recent case where Michigan regulators set aside extensive record evidence and the Administrative Law Judge's ROE recommendation based on the utility's unsubstantiated claim that investors would view Michigan as a "volatile" regulatory

[26] Some analyses show that utility stocks have outperformed industrial stocks since 2004. *See* Huntoon, S., "Nice Work If You Can Get It," Public Utilities Fortnightly (Aug. 2016).

[27] *See* Huntoon, S., "Nice Work If You Can Get It," Public Utilities Fortnightly (Aug. 2016); Hyman, L. and Tilles, W., "Don't Cry for Utility Shareholders, America," Public Utilities Fortnightly at 65 (Oct. 2016).

environment if its ROE were set at 10%.[28] *It cannot be the case that utility ROEs must only go up and never down, irrespective of industry risk or prevailing economic trends.* Again, this claim of "volatility" was a successful scare tactic that resulted in an excessive awarded ROE.

Structural features of the ratemaking process can also make it difficult to reduce utility earnings to reflect lower risk profiles or overall market trends. As one industry analyst recently noted, "Utility rates also tend to be downward sticky. It is easier for a utility to initiate and prosecute rate increase than for consumer advocates to initiate and prosecute rate decreases, with an imbalance in information being one obvious reason why."[29] Utilities have a natural incentive to file a rate case when they believe a rate increase will be approved, but not when rates would be reduced. Many of the largest regulated utilities in Texas have not had a rate case in many years. For example, Oncor, the state's single largest utility, has not had a rate case in more than five years and still has an awarded ROE of 10.25%.[30] ROEs are still being set in Texas in excess of 9.5%.[31]

Critically, as noted above, the "peer group" method of setting ROEs can create a feedback loop that perpetuates inflated ROEs. The most commonly accepted starting point for setting a utility's ROE is through a peer group analysis, where a survey is conducted of the ROEs for utility companies are claimed to be "peers" of the utility in question. This methodology effectively creates an echo chamber, where past regulatory decisions inform future ROEs and undue conservatism is reinforced—often in the face of contrary market data. As the data discussed above indicates, the ROEs that would be justified by objective market data appears to be in conflict with current awarded ROEs. This indicates that "peer group" ROE methodologies should be revisited to better account for changes in utility risk and other economic factors, rather than relying almost exclusively on the returns that have been awarded in the past.

In fairness, utilities offer a number of arguments to support the current risk premiums in awarded ROEs. For one, utilities argue that the reduction in risk-free ROE yields is an aberration, and utility ROEs should be set based on longer periods or on a lagging/historical basis. While this theory could justify a temporary increase in the observed risk premiums for utility ROEs over one or two years, the trend has far outlasted the limits of this justification. The US has overwhelmingly been a low-interest rate environment since late 2008, and there are a number of structural reasons why these relatively low interest rates may continue.[32,33] Yet, utility

[28] Cross, P., "2016 Annual Rate Case Survey," Public Utilities Fortnightly (Nov. 2016).

[29] Huntoon, S., "Nice Work If You Can Get It," Public Utilities Fortnightly (Aug. 2016).

[30] *Application of Oncor Electric Delivery Company, LLC for Authority to Change Rates*, Docket No. 38929, Final Order at Finding of Fact No. 32 (Aug. 29, 2011).

[31] *See, e.g., Year-end 2015 PUC Earnings Reports for Electric Utilities*, Project No. 45636, Staff Memorandum (Oct. 21, 2016).

[32] Rates for treasury bonds increased immediately following the recent election, but this increase is small (only an increase of about 45-50 basis points) relative to the drop in interest rates over the last decade, which has been hundreds of basis points. These interest rate increases are from historical lows – current treasury yields are at the same level as the beginning of 2016. Some investors are already seeing the Treasuries market as oversold and are recommending bond purchases instead. *See* http://www.wsj.com/articles/government-bond-sell-off-continues-on-trumps-economic-plans-1479114743 *and* http://www.wsj.com/articles/the-trump-trade-is-getting-out-of-hand-buy-some-bonds-1479143922.

ROEs have not been reduced to appropriately track this reduction over the past *eight years*. Utilities also argue that high risk premiums are correlated with low Treasury rates;[34] however, this argument confuses causation with correlation. The historical trend of risk premiums rising as Treasury rates fall is simply a reflection of the "stickiness" of high utility returns relative to interest rates, for the reasons discussed previously, and is not some independent economic principle that regulators should pursue. Utility ROE witnesses will also claim that unique utility business risks or size/scale issues support higher ROEs for particular utilities, but the reality is that there are no persuasive arguments for sustaining high risk premiums when risk in the utility business in Texas has been significantly reduced by legislative and regulatory changes, or when other comparably risky enterprises are earning lower returns in general. Notably, Moody's Investor Service has even concluded that reducing utility ROEs would not harm the credit profile of utilities in general because of the lower business risk and the many credit-positive cost recovery mechanisms that have been adopted.[35] This perspective from an independent bond rating agency reinforces the other substantial data demonstrating that reducing utility ROEs will not harm their ability to attract investment, and is a strong signal that the status quo should be holistically reexamined.

VI. Conclusion

The ROEs awarded to and achieved by regulated utilities are higher than needed to attract appropriate levels of investment. Customers and the economy in general would be well-served by a comprehensive reexamination of utility ROEs in light of relevant risk factors and economic trends. This includes reexamining the application of "peer-group" based ROE analyses, as well critical analysis of utility claims regarding the allegedly adverse impacts of reducing ROEs. Certainly, utility requests for "alternative" or "streamlined" ratemaking should be met with a rigorous analysis of the impacts that existing and proposed mechanisms have in shifting risk from the utility to its customers, and those impacts should translate to lower ROEs. In the world of utility ROEs, "what goes up" should also come down when risk factors and overall economic circumstances overwhelmingly support a lower level of returns.

[33] Structural reasons for low rates include the aging of the US population, persistent excess savings in the rest of the world, and lower productivity growth. *See* http://voxeu.org/article/causes-and-consequences-persistently-low-interest-rates and https://www.allianz.com/v_1453369613000/media/economic_research/publications/working_papers/en/WPRealzinse.pdf.

[34] A utility ROE witness has made this argument in recent rate cases in Texas.

[35] Moody's Investor Service, Sector-in-Depth Analysis, March 2015.

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THE RELATIONSHIP BETWEEN RETURN AND MARKET VALUE OF COMMON STOCKS*

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This study examines the empirical relationship between the return and the total market value of NYSE common stocks. It is found that smaller firms have had higher risk adjusted returns, on average, than larger firms. This 'size effect' has been in existence for at least forty years and is evidence that the capital asset pricing model is misspecified. The size effect is not linear in the market value; the main effect occurs for very small firms while there is little difference in return between average sized and large firms. It is not known whether size *per se* is responsible for the effect or whether size is just a proxy for one or more true unknown factors correlated with size.

1. Introduction

The single-period capital asset pricing model (henceforth CAPM) postulates a simple linear relationship between the expected return and the market risk of a security. While the results of direct tests have been inconclusive, recent evidence suggests the existence of additional factors which are relevant for asset pricing. Litzenberger and Ramaswamy (1979) show a significant positive relationship between dividend yield and return of common stocks for the 1936–1977 period. Basu (1977) finds that price–earnings ratios and risk adjusted returns are related. He chooses to interpret his findings as evidence of market inefficiency but as Ball (1978) points out, market efficiency tests are often joint tests of the efficient market hypothesis and a particular equilibrium relationship. Thus, some of the anomalies that have been attributed to a lack of market efficiency might well be the result of a misspecification of the pricing model.

This study contributes another piece to the emerging puzzle. It examines the relationship between the total market value of the common stock of a firm and its return. The results show that, in the 1936–1975 period, the common stock of small firms had, on average, higher risk-adjusted returns

*This study is based on part of my dissertation and was completed while I was at the University of Chicago. I am grateful to my committee, Myron Scholes (chairman), John Gould, Roger Ibbotson, Jonathan Ingersoll, and especially Eugene Fama and and Merton Miller, for their advice and comments I wish to acknowledge the valuable comments of Bill Schwert on earlier drafts of this paper

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than the common stock of large firms. This result will henceforth be referred to as the 'size effect'. Since the results of the study are not based on a particular theoretical equilibrium model, it is not possible to determine conclusively whether market value *per se* matters or whether it is only a proxy for unknown true additional factors correlated with market value. The last section of this paper will address this question in greater detail.

The various methods currently available for the type of empirical research presented in this study are discussed in section 2. Since there is a considerable amount of confusion about their relative merit, more than one technique is used. Section 3 discusses the data. The empirical results are presented in section 4. A discussion of the relationship between the size effect and other factors, as well as some speculative comments on possible explanations of the results, constitute section 5.

2. Methodologies

The empirical tests are based on a generalized asset pricing model which allows the expected return of a common stock to be a function of risk β and an additional factor ϕ, the market value of the equity.[1] A simple linear relationship of the form

$$E(R_i) = \gamma_0 + \gamma_1 \beta_i + \gamma_2[(\phi_i - \phi_m)/\phi_m], \tag{1}$$

is assumed, where

$E(R_i)$ = expected return on security i,
γ_0 = expected return on a zero-beta portfolio,
γ_1 = expected market risk premium,
ϕ_i = market value of security i,
ϕ_m = average market value, and
γ_2 = constant measuring the contribution of ϕ_i to the expected return of a security.

If there is no relationship between ϕ_i and the expected return, i.e., $\gamma_2 = 0$, (1) reduces to the Black (1972) version of the CAPM.

Since expectations are not observable, the parameters in (1) must be estimated from historical data. Several methods are available for this purpose. They all involve the use of pooled cross-sectional and time series regressions to estimate γ_0, γ_1, and γ_2. They differ primarily in (a) the assumption concerning the residual variance of the stock returns (homoscedastic or heteroscedastic in the cross-sectional), and (b) the treatment of the

[1]In the empirical tests, Φ_i and Φ_m are defined as the market proportion of security i and average market proportion, respectively. The two specifications are, of course, equivalent.

errors-in-variables problem introduced by the use of estimated betas in (1). All methods use a constrained optimization procedure, described in Fama (1976, ch. 9), to generate minimum variance (m.v.) portfolios with mean returns γ_i, $i=0,\ldots,2$. This imposes certain constraints on the portfolio weights, since from (1)

$$E(R_p)\equiv\gamma_i=\gamma_0\sum_j w_j+\gamma_1\sum_j w_j\beta_j + \gamma_2\left[\left(\sum_j w_j\phi_j-\phi_m\sum_j w_j\right)\Big/\phi_m\right], \qquad i=0,\ldots,2, \tag{2}$$

where the w_j are the portfolio proportions of each asset j, $j=1,\ldots,N$. An examination of (2) shows that $\hat{\gamma}_0$ is the mean return of a standard m.v. portfolio ($\sum_j w_j=1$) with zero beta and $\phi_p\equiv\sum_j w_j\phi_j=\phi_m$ [to make the second and third terms of the right-hand side of (2) vanish]. Similarly, $\hat{\gamma}_1$ is the mean return on a zero-investment m.v. portfolio with beta of one and $\phi_p=0$, and $\hat{\gamma}_2$ is the mean return on a m.v. zero-investment, zero-beta portfolio with $\phi_p=\phi_m$. As shown by Fama (1976, ch. 9), this constrained optimization can be performed by running a cross-sectional regression of the form

$$R_{it}=\gamma_{0t}+\gamma_{1t}\beta_{it}+\gamma_{2t}[(\phi_{it}-\phi_{mt})/\phi_{mt}]+\varepsilon_{it}, \qquad i=1,\ldots,N, \tag{3}$$

on a period-by-period basis, using estimated betas $\hat{\beta}_{it}$ and allowing for either homoscedastic or heteroscedastic error terms. Invoking the usual stationarity arguments the final estimates of the gammas are calculated as the averages of the T estimates.

One basic approach involves grouping individual securities into portfolios on the basis of market value and security beta, reestimating the relevant parameters (beta, residual variance) of the portfolios in a subsequent period, and finally performing either an ordinary least squares (OLS) regression [Fama and MacBeth (1973)] which assumes homoscedastic errors, or a generalized least squares (GLS) regression [Black and Scholes (1974)] which allows for heteroscedastic errors, on the portfolios in each time period.[2] Grouping reduces the errors-in-variables problem, but is not very efficient because it does not make use of all information. The errors-in-variables problem should not be a factor as long as the portfolios contain a reasonable number of securities.[3]

Litzenberger and Ramaswamy (1979) have suggested an alternative method which avoids grouping. They allow for heteroscedastic errors in the cross-section and use the estimates of the standard errors of the security

[2]Black and Scholes (1974) do not take account of heteroscedasticity, even though their method was designed to do so.

[3]Black, Jensen and Scholes (1972, p. 116).

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betas as estimates of the measurement errors. As Theil (1971, p. 610) has pointed out, this method leads to unbiased maximum likelihood estimators for the gammas as long as the error in the standard error of beta is small and the standard assumptions of the simple errors-in-variables model are met. Thus, it is very important that the diagonal model is the correct specification of the return-generating process, since the residual variance assumes a critical position in this procedure. The Litzenberger–Ramaswamy method is superior from a theoretical viewpoint; however, preliminary work has shown that it leads to serious problems when applied to the model of this study and is not pursued any further.[4]

Instead of estimating equation (3) with data for all securities, it is also possible to construct arbitrage portfolios containing stocks of very large and very small firms, by combining long positions in small firms with short positions in large firms. A simple time series regression is run to determine the difference in risk-adjusted returns between small and large firms. This approach, long familiar in the efficient markets and option pricing literature, has the advantage that no assumptions about the exact functional relationships between market value and expected return need to be made, and it will therefore be used in this study.

3. Data

The sample includes all common stocks quoted on the NYSE for at least five years between 1926 and 1975. Monthly price and return data and the number of shares outstanding at the end of each month are available in the monthly returns file of the Center for Research in Security Prices (CRSP) of the University of Chicago. Three different market indices are used; this is in response to Roll's (1977) critique of empirical tests of the CAPM. Two of the three are pure common stock indices — the CRSP equally- and value-weighted indices. The third is more comprehensive: a value-weighted combination of the CRSP value-weighted index and return data on corporate and government bonds from Ibbotson and Sinquefield (1977) (henceforth 'market index').[5] The weights of the components of this index are derived from information on the total market value of corporate and government bonds in various issues of the *Survey of Current Business* (updated annually) and from the market value of common stocks in the CRSP monthly index file. The stock indices, made up of riskier assets, have both higher returns

[4]If the diagonal model (or market model) is an incomplete specification of the return generating process, the estimate of the standard error of beta is likely to have an upward bias, since the residual variance estimate is too large The error in the residual variance estimate appears to be related to the second factor. Therefore, the resulting gamma estimates are biased.

[5]No pretense is made that this index is complete, thus, the use of quotation marks It ignores real estate, foreign assets, etc ; it should be considered a first step toward a comprehensive index. See Ibbotson and Fall (1979)

and higher risk than the bond indices and the 'market index'.[6] A time series of commercial paper returns is used as the risk-free rate.[7] While not actually constant through time, its variation is very small when compared to that of the other series, and it is not significantly correlated with any of the three indices used as market proxies.

4. Empirical results

4.1. Results for methods based on grouped data

The portfolio selection procedure used in this study is identical to the one described at length in Black and Scholes (1974). The securities are assigned to one of twenty-five portfolios containing similar numbers of securities, first to one of five on the basis of the market value of the stock, then the securities in each of those five are in turn assigned to one of five portfolios on the basis of their beta. Five years of data are used for the estimation of the security beta; the next five years' data are used for the reestimation of the portfolio betas. Stock price and number of shares outstanding at the end of the five year periods are used for the calculation of the market proportions. The portfolios are updated every year. The cross-sectional regression (3) is then performed in each month and the means of the resulting time series of the gammas could be (and have been in the past) interpreted as the final estimators. However, having used estimated parameters, it is not certain that the series have the theoretical properties, in particular, the hypothesized beta. Black and Scholes (1974, p. 17) suggest that the time series of the gammas be regressed once more on the excess return of the market index. This correction involves running the time series regression (for $\hat{\gamma}_2$)

$$\hat{\gamma}_{2t} - R_{Ft} = \hat{\alpha}_2 + \hat{\beta}_2(R_{mt} - R_{Ft}) + \hat{\varepsilon}_{2t}. \tag{4}$$

It has been shows earlier that the theoretical β_2 is zero. (4) removes the effects of a non-zero $\hat{\beta}_2$ on the return estimate $\hat{\gamma}_2$ and $\hat{\alpha}_2$ is used as the final estimator for $\hat{\gamma}_2 - R_F$. Similar corrections are performed for γ_0 and γ_1. The

[6] Mean monthly returns and standard deviations for the 1926–1975 period are.

	Mean return	Standard deviation
'Market index'	0.0046	0.0178
CRSP value-weighted index	0.0085	0.0588
CRSP equally-weighted index	0.0120	0.0830
Government bond index	0.0027	0.0157
Corporate bond index	0.0032	0.0142

[7] I am grateful to Myron Scholes for making this series available The mean monthly return for the 1926–1975 period is 0.0026 and the standard deviation is 0.0021.

derivations of the $\hat{\beta}_i$, $i=0,\ldots,2$, in (4) from their theoretical values also allow us to check whether the grouping procedure is an effective means to eliminate the errors-in-beta problem.

The results are essentially identical for both OLS and GLS and for all three indices. Thus, only one set of results, those for the 'market index' with GLS, is presented in table 1. For each of the gammas, three numbers are reported: the mean of that time series of returns which is relevant for the test of the hypothesis of interest (i.e., whether or not $\hat{\gamma}_0$ and $\hat{\gamma}_1$ are different from the risk-free rate and the risk premium, respectively), the associated t-statistic, and finally, the estimated beta of the time series of the gamma from (4). Note that the means are corrected for the deviation from the theoretical beta as discussed above.

The table shows a significantly negative estimate for γ_2 for the overall time period. Thus, shares of firms with large market values have had smaller returns, on average, than similar small firms. The CAPM appears to be misspecified. The table also shows that γ_0 is different from the risk-free rate. As both Fama (1976, ch. 9) and Roll (1977) have pointed out, if a test does not use the true market portfolio, the Sharpe–Lintner model might be wrongly rejected. The estimates for γ_0 are of the same magnitude as those reported by Fama and MacBeth (1973) and others. The choice of a market index and the econometric method does not affect the results. Thus, at least within the context of this study, the choice of a proxy for the market portfolio does not seem to affect the results and allowing for heteroscedastic disturbances does not lead to significantly more efficient estimators.

Before looking at the results in more detail, some comments on econometric problems are in order. The results in table 1 are based on the 'market index' which is likely to be superior to pure stock indices from a theoretical viewpoint since it includes more assets [Roll (1977)]. This superiority has its price. The actual betas of the time series of the gammas are reported in table 1 in the columns labeled $\hat{\beta}_i$. Recall that the theoretical values of β_0 and β_1 are zero and one, respectively. The standard zero-beta portfolio with return $\hat{\gamma}_0$ contains high beta stocks in short positions and low beta stocks in long positions, while the opposite is the case for the zero-investment portfolio with return $\hat{\gamma}_1$. The actual betas are all significantly different from the theoretical values. This suggests a regression effect, i.e., the past betas of high beta securities are overestimated and the betas of low beta securities are underestimated.[8] Past beta is not completely uncorrelated with the error of the current beta and the instrumental variable approach to the error-in-variables problem is not entirely successful.[9]

[8]There is no such effect for β_2 because that portfolio has both zero beta and zero investment, i.e., net holdings of both high and low beta securities are, on average, zero

[9]This result is first documented in Brenner (1976) who examines the original Fama–McBeth (1973) time series of $\hat{\gamma}_{0t}$

Table 1

Portfolio estimators for γ_0, γ_1 and γ_2 based on the 'market index' with generalized least squares estimation [a]

$$R_{it} = \hat{\gamma}_{0t} + \hat{\gamma}_{1t}\hat{\beta}_{it} + \hat{\gamma}_{2t}[(\phi_{it} - \phi_{mt})/\phi_{mt}]$$

Period	$\hat{\gamma}_0 - R_F$	$t(\hat{\gamma}_0 - R_F)$	$\hat{\beta}_0$	$\hat{\gamma}_1 - (R_M - R_F)$	$t(\hat{\gamma}_1 - (R_M - R_F))$	$\hat{\beta}_1$	$\hat{\gamma}_2$	$t(\hat{\gamma}_2)$	$\hat{\beta}_2$
1936–1975	0.00450	2.76	0.45	−0.00092	−1.00	0.75	−0.00052	−2.92	0.01
1936–1955	0.00377	1.66	0.43	−0.00060	−0.80	0.80	−0.00043	−2.12	0.01
1956–1975	0.00531	2.22	0.46	−0.00138	−0.82	0.73	−0.00062	−2.09	0.01
1936–1945	0.00121	0.30	0.63	−0.00098	−0.77	0.82	−0.00075	−2.32	−0.01
1946–1955	0.00650	2.89	0.03	−0.00021	−0.26	0.75	−0.00015	−0.65	0.06
1956–1965	0.00494	2.02	0.34	−0.00098	−0.56	0.96	−0.00039	−1.27	−0.01
1966–1975	0.00596	1.43	0.49	−0.00232	−0.80	0.69	−0.00080	−1.55	0.01

[a] $\hat{\gamma}_0 - R_F$ = mean difference between return on zero beta portfolio and risk-free rate. $\hat{\gamma}_1 - (R_M - R_F)$ = mean difference between actual risk premium ($\hat{\gamma}_1$) and risk premium stipulated by Sharpe–Lintner model $(R_M - R_F)$. $\hat{\gamma}_2$ = size premium. $\hat{\beta}_i$ = actual estimated market risk of $\hat{\gamma}_i$ (theoretical values. $\beta_0 = 0$, $\beta_1 = 1$, $\beta_2 = 0$), all β_0, β_1 are significantly different from the theoretical values. $t(\ )$ = t-statistic.

The deviations from the theoretical betas are largest for the 'market index', smaller for the CRSP value-weighted index, and smallest for the CRSP equally-weighted index. This is due to two factors: first, even if the true covariance structure is stationary, betas with respect to a value-weighted index change whenever the weights change, since the weighted average of the betas is constrained to be equal to one. Second, the betas and their standard errors with respect to the 'market index' are much larger than for the stock indices (a typical stock beta is between two and three), which leads to larger deviations -- a kind of 'leverage' effect. Thus, the results in table 1 show that the final correction for the deviation of $\hat{\beta}_0$ and $\hat{\beta}_1$ from their theoretical values is of crucial importance for maket proxies with changing weights.

Estimated portfolio betas and portfolio market proportions are (negatively) correlated. It is therefore possible that the errors in beta induce an error in the coefficient of the market proportion. According to Levi (1973), the probability limit of $\hat{\gamma}_1$ in the standard errors-in-the-variables model is

$$\text{plim}\ \hat{\gamma}_1 = \gamma_1/(1+(\sigma_u^2 \cdot \sigma_2^2)/D) < \gamma_1,$$

with

$$D = (\sigma_1^2 + \sigma_u^2) \cdot \sigma_2^2 - \sigma_{12}^2 > 0,$$

where σ_1^2, σ_2^2 are the variances of the true factors β and ϕ, respectively, σ_u^2 is the variance of the error in beta and σ_{12} is the covariance of β and ϕ. Thus, the bias in $\hat{\gamma}_1$ is unambiguously towards zero for positive γ_1. The probability limit of $\hat{\gamma}_2 - \gamma_2$ is [Levi (1973)]

$$\text{plim}\ (\hat{\gamma}_2 - \gamma_2) = (\sigma_u^2 \cdot \sigma_{12} \cdot \gamma_1)/D.$$

We find that the bias in $\hat{\gamma}_2$ depends on the covariance between β and ϕ and the sign of γ_1. If σ_{12} has the same sign as the covariance between $\hat{\beta}$ and ϕ, i.e., $\sigma_{12} < 0$, and if $\gamma_1 > 0$, then $\text{plim}\,(\hat{\gamma}_2 - \gamma_2) < 0$, i.e., $\text{plim}\,\hat{\gamma}_2 < \gamma_2$. If the grouping procedure is not successful in removing the error in beta, then it is likely that the reported $\hat{\gamma}_2$ overstates the true magnitude of the size effect. If this was a serious problem in this study, the results for the different market indices should reflect the problem. In particular, using the equally-weighted stock index should then lead to the smallest size effect since, as was pointed out earlier, the error in beta problem is apparently less serious for that kind of index. In fact, we find that there is little difference between the estimates.[10]

[10]For the overall time period, $\hat{\gamma}_2$ with the equally-weighted CRSP index is -0.00044, with the value weighted CRSP index -0.00044 as well as opposed to the -0.00052 for the 'market index' reported in table 1 The estimated betas of $\hat{\gamma}_0$ and $\hat{\gamma}_1$ which reflect the degree of the error in beta problems are 0 07 and 0 91, respectively, for the equally-weighted CRSP index and 0 13 and 0.87 for the value-weighted CRSP index.

Thus, it does not appear that the size effect is just a proxy for the unobservable true beta even though the market proportion and the beta of securities are negatively correlated.

The correlation coefficient between the mean market values of the twenty-five portfolios and their betas is significantly negative, which might have introduced a multicollinearity problem. One of its possible consequences is coefficients that are very sensitive to addition or deletion of data. This effect does not appear to occur in this case: the results do not change significantly when five portfolios are dropped from the sample. Revising the grouping procedure — ranking on the basis of beta first, then ranking on the basis of market proportion — also does not lead to substantially different results.

4.2. A closer look at the results

An additional factor relevant for asset pricing — the market value of the equity of a firm — has been found. The results are based on a linear model. Linearity was assumed only for convenience and there is no theoretical reason (since there is no model) why the relationship should be linear. If it is nonlinear, the particular form of the relationship might give us a starting point for the discussion of possible causes of the size effect in the next section. An analysis of the residuals of the twenty-five portfolios is the easiest way to look at the linearity question. For each month t, the estimated residual return

$$\hat{\varepsilon}_{it} = R_{it} - \hat{\gamma}_{0t} - \hat{\gamma}_{1t}\hat{\beta}_{it} - \hat{\gamma}_{2t}[(\phi_{it} - \phi_{mt})/\phi_{mt}], \qquad i = 1, \ldots, 25, \tag{5}$$

is calculated for all portfolios. The mean residuals over the forty-five year sample period are plotted as a function of the mean market proportion in fig. 1. Since the distribution of the market proportions is very skewed, a logarithmic scale is used. The solid line connects the mean residual returns of each size group. The numbers identify the individual portfolios within each group according to beta, '1' being the one with the largest beta, '5' being the one with the smallest beta.

The figure shows clearly that the linear model is misspecified.[11] The residuals are not randomly distributed around zero. The residuals of the portfolios containing the smallest firms are all positive; the remaining ones are close to zero. As a consequence, it is impossible to use $\hat{\gamma}_2$ as a simple size premium in the cross-section. The plot also shows, however, that the misspecification is not responsible for the significance of $\hat{\gamma}_2$ since the linear model underestimates the true size effect present for very small firms. To illustrate this point, the five portfolios containing the smaller firms are

[11]The nonlinearity cannot be eliminated by defining ϕ_i as the log of the market proportion

deleted from the sample and the parameters reestimated. The results, summarized in table 2, show that the $\hat{\gamma}_2$ remain essentially the same. The relationship is still not linear; the new $\hat{\gamma}_2$ still cannot be used as a size premium.

Fig. 1 suggests that the main effect occurs for very small firms. Further support for this conclusion can be obtained from a simple test. We can regress the returns of the twenty-five portfolios in each result on beta alone and examine the residuals. The regression is misspecified and the residuals contain information about the size effect. Fig. 2 shows the plot of those residuals in the same format as fig. 1. The smallest firms have, on average, very large unexplained mean returns. There is no significant difference between the residuals of the remaining portfolios.

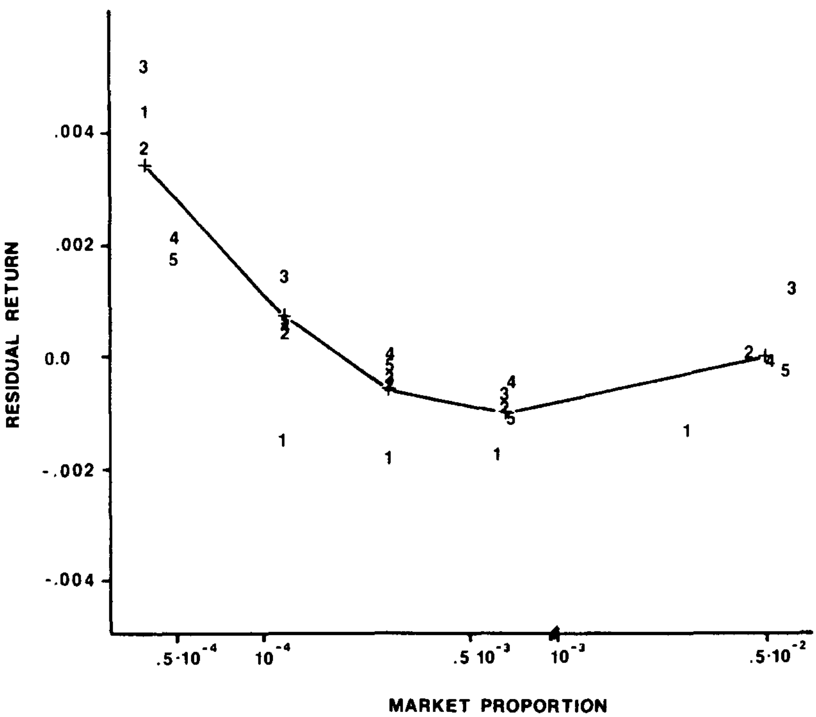

Fig. 1. Mean residual returns of portfolios (1936–1975) with equally-weighted CRSP index as market proxy. The residual is calculated with the three-factor model [eq. (3)]. The numbers 1,...,5 represent the mean residual return for the five portfolios within each size group (1: portfolio with largest beta,. ..,5· portfolio with smallest beta) + represents the mean of the mean residuals of the five portfolios with similar market values.

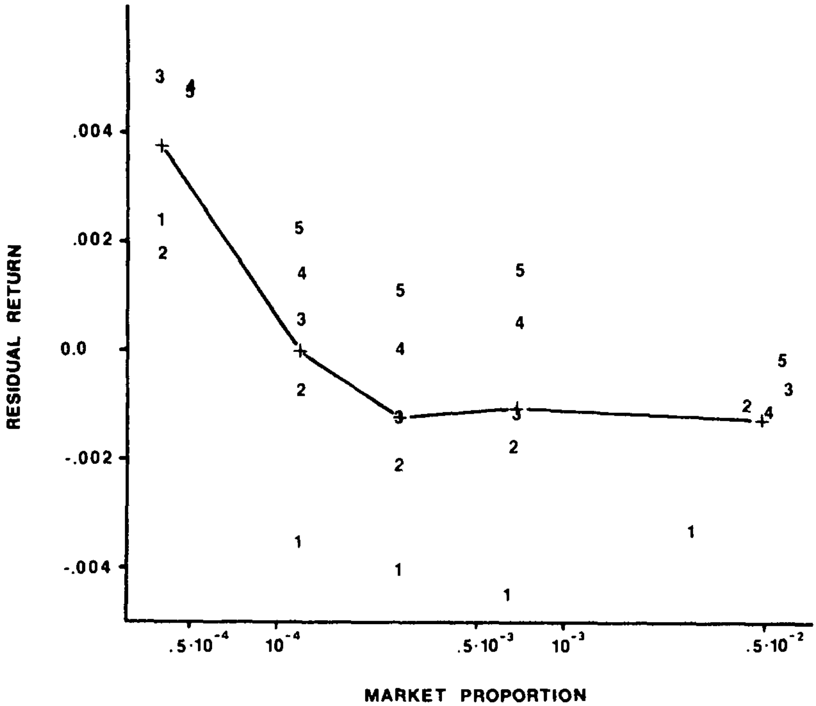

Fig. 2. Mean residual returns of portfolios (1936–1975) with equally-weighted CRSP index as market proxy. The residual is calculated with the two-factor model ($\hat{\varepsilon}_{it} = R_{it} - \hat{\gamma}_{0t} - \hat{\gamma}_{1t}\hat{\beta}_{it}$) The symbols are as defined for fig. 1.

4.3. *'Arbitrage' portfolio returns*

One important empirical question still remains: How important is the size effect from a practical point of view? Fig. 2 suggests that the difference in returns between the smallest firms and the remaining ones is, on average, about 0.4 percent per month. A more dramatic result can be obtained when the securities are chosen solely on the basis of their market value.

As an illustration, consider putting equal dollar amounts into portfolios containing the smallest, largest and median-sized firms at the beginning of a year. These portfolios are to be equally weighted and contain, say, ten, twenty or fifty securities. They are to be held for five years and are rebalanced every month. They are levered or unlevered to have the same beta. We are then interested in the differences in their returns,

$$R_{1t} = R_{st} - R_{lt}, \qquad R_{2t} = R_{st} - R_{at}, \qquad R_{3t} = R_{at} - R_{lt}, \tag{6}$$

Table 2
Portfolio estimators for γ_2 for all 25 portfolios and for 20 portfolios (portfolios containing smallest firms deleted) based on CRSP equally weighted index with generalized least-squares estimation.[a]

	Size premium $\hat{\gamma}_2$ with	
Period	25 portfolios	20 portfolios
1936–1975	−0.00044	−0 00043
	(−2.42)	(−2.54)
1936–1955	−0.00037	−0.00041
	(−1 72)	(−1.88)
1956–1975	−0.00056	−0.00050
	(−1.91)	(−1.91)
1936–1945	−0.00085	−0.00083
	(−2.81)	(−2.48)
1946–1955	0.00003	−0.00003
	(0.12)	(−0 13)
1956–1965	−0.00023)	−0.00017
	(−0.81)	(−0.65)
1966–1975	−0.00091	−0.00085
	(−1.78)	(−1.84)

[a]t-statistic in parentheses

where R_{st}, R_{at} and R_{lt} are the returns on the portfolios containing the smallest, median-sized and largest firms at portfolio formation time (and $R_{1t} = R_{2t} + R_{3t}$). The procedure involves (a) the calculation of the three differences in raw returns in each month and (b) running time series regressions of the differences on the excess returns of the market proxy. The intercept terms of these regressions are then interpreted as the $\bar{R}_i$, $i = 1,\ldots,3$. Thus, the differences can be interpreted as 'arbitrage' returns, since, e.g., R_{1t} is the return obtained from holding the smallest firms long and the largest firms short, representing zero net investment in a zero-beta portfolio.[12] Simple equally weighted portfolios are used rather than more sophisticated minimum variance portfolios to demonstrate that the size effect is not due to some quirk in the covariance matrix.

Table 3 shows that the results of the earlier tests are fully confirmed. $\bar{R}_2$, the difference in returns between very small firms and median-size firms, is typically considerably larger than $\bar{R}_3$, the difference in returns between median-sized and very large firms. The average excess return from holding very small firms long and very large firms short is, on average, 1.52 percent

[12]No *ex post* sample bias is introduced, since monthly rebalancing includes stocks delisted during the five years. Thus, the portfolio size is generally accurate only for the first month of each period

Table 3
Mean monthly returns on 'arbitrage' portfolios.[a]
$R_j - R_k = \hat{\alpha}_i + \hat{\beta}_i (R_m - R_F)$

	$\bar{\alpha}_1$[b]			$\bar{\alpha}_2$[c]			$\bar{\alpha}_3$[d]		
	$n=10$	$n=20$	$n=50$	$n=10$	$n=20$	$n=50$	$n=10$	$n=20$	$n=50$
Overall period									
1931–1975	0.0152	0.0148	0.0101	0.0130	0.0124	0.0089	0.0021	0.0024	0.0012
	(2.99)	(3.53)	(3.07)	(2.90)	(3.56)	(3.64)	(1.06)	(1.41)	(0.85)
Five-year subperiods									
1931–1935	0.0589	0.0597	0.0427	0.0462	0.0462	0.0326	0.0127	0.0134	0.0101
	(2.25)	(2.81)	(2.35)	(1.92)	(2.55)	(2.46)	(1.09)	(1.49)	(1.42)
1936–1940	0.0201	0.0182	0.0089	0.0118	0.0145	0.0064	0.0084	0.0037	0.0025
	(0.82)	(0.97)	(0.67)	(0.55)	(0.90)	(0.65)	(1.20)	(0.62)	(0.49)
1941–1945	0.0430	0.0408	0.0269	0.0381	0.0367	0.0228	0.0049	0.0038	0.0041
	(2.29)	(2.46)	(2.17)	(2.29)	(2.54)	(2.02)	(1.25)	(1.09)	(1.68)
1946–1950	−0.0060	−0.0046	−0.0036	−0.0058	−0.0059	−0.0029	−0.0002	−0.0104	−0.0007
	(−1.17)	(−0.97)	(−0.97)	(−1.03)	(−1.29)	(−0.83)	(−0.07)	(−0.50)	(−0.38)
1951–1955	−0.0067	−0.0011	0.0013	−0.0004	0.0026	0.0010	−0.0062	−0.0037	0.0003
	(−0.89)	(−0.21)	(0.32)	(−0.07)	(0.72)	(0.39)	(−1.29)	(−0.99)	(0.11)
1956–1960	0.0039	0.0008	0.0037	0.0007	−0.0027	0.0011	0.0031	0.0035	0.0026
	(0.67)	(0.15)	(0.89)	(0.14)	(−0.64)	(0.45)	(0.88)	(1.16)	(0.97)
1961–1965	0.0131	0.0060	0.0024	0.0096	0.0046	0.0036	0.0035	0.0014	−0.0012
	(1.38)	(0.67)	(0.31)	(1.11)	(0.72)	(0.77)	(0.59)	(0.24)	(−0.24)
1966–1970	0.0121	0.0117	0.0077	0.0129	0.0110	0.0071	0.0008	0.0007	0.0006
	(1.64)	(2.26)	(1.91)	(1.93)	(2.71)	(2.43)	(0.23)	(0.22)	(0.27)
1971–1975	0.0063	0.0108	0.0098	0.0033	0.0077	0.0083	0.0030	0.0031	0.0015
	(0.60)	(1.23)	(1.45)	(0.39)	(1.18)	(1.79)	(0.64)	(0.72)	(0.43)

[a]Equally-weighted portfolios with n securities, adjusted for differences in market risk with respect to CRSP value-weighted index, t-statistics in parentheses.
[b]Small firms held long, large firms held short.
[c]Small firms held long, median-size firms held short.
[d]Median-size firms held long, large firms held short.

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per month or 19.8 percent on an annualized basis. This strategy, which suggests very large 'profit opportunities', leaves the investor with a poorly diversified portfolio. A portfolio of small firms has typically much larger residual risk with respect to a value-weighted index than a portfolio of very large firms with the same number of securities [Banz (1978, ch. 3)]. Since the fifty largest firms make up more than 25 percent of the total market value of NYSE stocks, it is not surprising that a larger part of the variation of the return of a portfolio of those large firms can be explained by its relation with the value-weighted market index. Table 3 also shows that the strategy would not have been successful in every five year subperiod. Nevertheless, the magnitude of the size effect during the past forty-five years is such that it is of more than just academic interest.

5. Conclusions

The evidence presented in this study suggests that the CAPM is misspecified. On average, small NYSE firms have had significantly larger risk adjusted returns than large NYSE firms over a forty year period. This size effect is not linear in the market proportion (or the log of the market proportion) but is most pronounced for the smallest firms in the sample. The effect is also not very stable through time. An analysis of the ten year subperiods show substantial differences in the magnitude of the coefficient of the size factor (table 1).

There is no theoretical foundation for such an effect. We do not even know whether the factor is size itself or whether size is just a proxy for one or more true but unknown factors correlated with size. It is possible, however, to offer some conjectures and even discuss some factors for which size is suspected to proxy. Recent work by Reinganum (1980) has eliminated one obvious candidate: the price–earnings (P/E) ratio.[13] He finds that the P/E-effect, as reported by Basu (1977), disappears for both NYSE and AMEX stocks when he controls for size but that there is a significant size effect even when he controls for the P/E-ratio, i.e., the P/E-ratio effect is a proxy for the size effect and not vice versa. Stattman (1980), who found a significant negative relationship between the ratio of book value and market value of equity and its return, also reports that this relationship is just a proxy for the size effect. Naturally, a large number of possible factors remain to be tested.[14] But the Reinganum results point out a potential problem with some of the existing negative evidence of the efficient market hypothesis. Basu believed to have identified a market inefficiency but his P/E-effect is

[13]The average correlation coefficient between P/E-ratio and market value is only 0.16 for individual stocks for thirty-eight quarters ending in 1978. But for the portfolios formed on the basis of P/E-ratio, it rises to 0.82. Recall that Basu (1977) used ten portfolios in his study.

[14]E.g., debt–equity ratios, skewness of the return distribution [Kraus and Litzenberger (1976)].

just a proxy for the size effect. Given its longevity, it is not likely that it is due to a market inefficiency but it is rather evidence of a pricing model misspecification. To the extent that tests of market efficiency use data of firms of different sizes and are based on the CAPM, their results might be at least contaminated by the size effect.

One possible explanation involving the size of the firm directly is based on a model by Klein and Bawa (1977). They find that if insufficient information is available about a subset of securities, investors will not hold these securities because of estimation risk, i.e., uncertainty about the true parameters of the return distribution. If investors differ in the amount of information available, they will limit their diversification to different subsets of all securities in the market.[15] It is likely that the amount of information generated is related to the size of the firm. Therefore, many investors would not desire to hold the common stock of very small firms. I have shown elsewhere [Banz (1978, ch. 2)] that securities sought by only a subset of the investors have higher risk-adjusted returns than those considered by all investors. Thus, lack of information about small firms leads to limited diversification and therefore to higher returns for the 'undesirable' stocks of small firms.[16] While this informal model is consistent with the empirical results, it is, nevertheless, just conjecture.

To summarize, the size effect exists but it is not at all clear why it exists. Until we find an answer, it should be interpreted with caution. It might be tempting to use the size effect, e.g., as the basis for a theory of mergers — large firms are able to pay a premium for the stock of small firms since they will be able to discount the same cash flows at a smaller discount rate. Naturally, this might turn out to be complete nonsense if size were to be shown to be just a proxy.

The preceding discussion suggests that the results of this study leave many questions unanswered. Further research should consider the relationship between size and other factors such as the dividend yield effect, and the tests should be expanded to include OTC stocks as well.

[15]Klein and Bawa (1977, p 102)

[16]A similar result can be obtained with the introduction of fixed holding costs which lead to limited diversification as well. See Brennan (1975), Banz (1978, ch. 2) and Mayshar (1979)

References

Ball, Ray, 1978, Anomalies in relationships between securities' yields and yield surrogates, Journal of Financial Economics 6, 103–126

Banz, Rolf W., 1978, Limited diversification and market equilibrium: An empirical analysis, Ph D dissertation (University of Chicago, Chicago, IL)

Basu, S., 1977, Investment performance of common stocks in relation to their price-earnings ratios. A test of market efficiency, Journal of Finance 32, June, 663–682.

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Black, Fischer, 1972, Capital market equilibrium with restricted borrowing, Journal of Business 45, July, 444–454.

Black, Fischer, and Myron Scholes, 1974, The effects of dividend yield and dividend policy on common stock prices and returns, Journal of Financial Economics 1, May, 1–22.

Black, Fischer, Michael C. Jensen and Myron Scholes, 1972, The capital asset pricing model Some empirical tests, in: M.C. Jensen, ed., Studies in the theory of capital markets (Praeger, New York) 79–121.

Brennan, Michael J., 1975, The optimal number of securities in a risky asset portfolio when there are fixed costs of transacting: Theory and some empirical evidence, Journal of Financial and Quantitative Analysis 10, Sept., 483–496.

Brenner, Menachem, 1976, A note on risk, return and equilibrium: Empirical tests, Journal of Political Economy 84, 407–409.

Fama, Eugene F., 1976, Foundations of finance (Basic Books, New York).

Fama, Eugene F. and James D. MacBeth, 1973, Risk return and equilibrium. Some empirical tests, Journal of Political Economy 71, May–June, 607–636.

Ibbotson, Roger G. and Carol L. Fall, 1979, The United States market wealth portfolio, Journal of Portfolio Management 6, 82–92.

Ibbotson, Roger G. and Rex A. Sinquefield, 1977, Stocks, bonds, bills and inflation: The past (1926–1976) and the future (1977–2000) (Financial Analysis Research Foundation)

Klein, Roger W. and Vijay S. Bawa, 1977, The effect of limited information and estimation risk on optimal portfolio diversification, Journal of Financial Economics 5, Aug., 89–111.

Kraus, Alan and Robert H Litzenberger, 1976, Skewness preference and the valuation of risk assets, Journal of Finance 31, 1085–1100

Levi, Maurice D., 1973, Errors in the variables bias in the presence of correctly measured variables, Econometrica 41, Sept., 985–986.

Litzenberger, Robert H and Krishna Ramaswamy, 1979, The effect of personal taxes and dividends on capital asset prices: Theory and empirical evidence, Journal of Financial Economics 7, June, 163–195.

Mayshar, Joram, 1979, Transaction costs in a model of capital market equilibrium, Journal of Political Economy 87, 673–700.

Reinganum, Marc R, 1980, Misspecification of capital asset pricing: Empirical anomalies based on earnings yields and market values, Journal of Financial Economics, this issue

Roll, Richard, 1977, A critique of the asset pricing theory's tests: Part I, Journal of Financial Economics 4, Jan., 120–176.

Stattman, Dennis, 1980, Book values and expected stock returns, Unpublished M.B.A. honors paper (University of Chicago, Chicago, IL).

Theil, Henri, 1971, Principles of econometrics (Wiley, New York).

U.S. Department of Commerce, Office of Business Economics, 1969, 1970, Survey of current business 49, May, 11–12, 50, May, 14.

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Busting the Myth About Size

by Vitali Kalesnik, Ph.D., and Noah Beck

Many market participants (including investors, product providers, and analysts alike) assume that, just as value stocks on average outperform growth, small-cap stocks on average outperform large-caps. Unlike value, however, and contrary to popular opinion, there is little solid evidence that stock size affects performance.

A recent Research Affiliates article by Hsu and Kalesnik (2014) concluded that there are at best three factors from which investors can benefit through passive investing: market, value, and low beta. The size premium was conspicuously missing from that short list. In this article we explore empirical evidence behind the size premium in more detail. The summary below offers a preview of our findings. We let the reader examine the evidence and draw his or her own conclusion. In our opinion the preponderance of evidence does not support the existence of a size premium.

We are not arguing that investors should stop investing in small stocks. A portfolio of small stocks offers a certain level of diversification in an investment program dominated by large-stock strategies. Moreover, major anomalies are stronger in the universe of small stocks (likely because small stocks are more prone to mispricing). Thus, small stocks have the potential to serve as an alpha pool for skilled active managers and rules-based strategies that primarily target factors other than size. Nonetheless, we are skeptical that investors will earn a higher return simply by preferring small stocks over large.

Updating the Evidence

Banz (1981) reported that small-cap stocks outperformed large-cap stocks. For the subsequent decade the phenomenon Banz observed was considered a curious anomaly. The situation changed in 1993, when Eugene Fama and Kenneth French suggested that small stocks may expose investors to some undiversifiable risk that warrants a higher required rate of return. At that moment, the size factor took its place alongside the market and value factors in the original Fama–French three-factor model. Carhart (1997) then made the case for momentum as a fourth return factor. Today the most standard equity pricing model used in academia includes four factors: market, value, size, and momentum.

But consider this: What if a large company were split, on paper only, into two small companies? Suppose there is no change in operations, and imagine that one of the small companies booked all the cash flows on even-numbered days of the month, and the other one accounted for all the cash on odd days. In this scenario, it would be most surprising if the small companies both delivered higher returns than the original large company. Yet the size premium is precisely based on the expectation that small-cap stocks will outperform large-cap stocks!

Summary of Findings on the Size Premium

Arguments in Favor:

1. Over the period July 1926 to July 2014, there was a size premium of 3.4% per annum in the United States.
2. The U.S. size premium is statistically significant (with a p-value of 1.7%), *assuming the returns are normally distributed.*
3. In the 30+ years since the publication of Banz's (1981) article , there has been an average size premium of 1.0% per annum across 18 developed markets including the United States.

Arguments Against:

1. There is an upward bias in size premium estimates due to inaccurate returns on delisted stocks in major databases.
2. Indices and hypothetical portfolios ignore trading costs.
3. The statistical significance of the size premium estimates is likely overstated due to data-mining and reporting bias.
4. Even with the biases that favor small stocks, there is no unquestionably significant evidence in support of the size factor.
   - The estimate of the U.S. size premium is dominated by extreme outliers from the 1930s.
   - The assumption of normality used to obtain statistical significance in the U.S. sample is extremely dubious.
   - There is no statistical significance outside the United States.
5. Even with the biases that favor small stocks, there is no risk-adjusted performance advantage attributable to the size factor.

Source: Research Affiliates.

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For any reasonable economic theory explaining why small-cap stocks are supposed to outperform large-cap stocks, there is an equally plausible theory explaining why the reverse should be true. The source of the specific risk postulated by Fama and French (1993) was unclear 21 years ago, and it is still murky today. Theoretical explanations for the size premium were provided after researchers observed the anomalous regularity in returns—not the other way around. Today investors believe in the size premium on the basis of empirical evidence, not on theoretical arguments. So let's turn to the evidence with updated data.

Following the methodology employed in Fama and French (2012), we grouped stocks in each country by size into two portfolios. The large stock portfolio consists of the top 90% of the market by market capitalization, and the small stock portfolio consists of the bottom 10% of the market. Stocks within the large and small portfolios are weighted by market capitalization. To measure the premium we looked at the arithmetic difference between the small and large stock portfolio returns. We report in **Table 1** the average annualized returns, volatilities, and t-statistics in 18 major developed countries from January 1982 to July 2014. Table 1 also displays data for the United States over the longer period from July 1926 to July 2014.

In the 88-year U.S. sample, the size premium is 3.4% per annum. Assuming a normal distribution of premium estimates (we will discuss later why this assumption may not be warranted), the size premium is statistically significant with a t-stat of 2.38, which corresponds to a p-value of 1.7%. After 1981, when Banz's paper appeared, the premium is positive in the United States and positive on average in the international sample, but it is not statistically significant anywhere. The substantial, statistically significant average return observed in the long-term U.S. dataset is the main reason why size is popularly believed to be one of the most important factors.

Examining the U.S. Data

Existence of the size premium in the United States is practically an article of faith in the practice of asset management as well as the academic literature. The empirical evidence, however, does not stand up very well to closer scrutiny. The data are doubtful for several reasons, including overestimated small-cap returns due to missing data on delisted stocks; the absence of transaction costs in the calculation of index returns; biases resulting from data-mining and the publishing process; and misestimated statistical measures based on the assumption of normality. In addition, there proves to be no return advantage on a risk-adjusted basis.

Table 1. Size Premium: U.S. and International Evidence

Nation	Average Return (Ann.)	Average Volatility (Ann.)	t-stat
Post Publication Period, 1982–2014			
Australia	−1.1%	10.2%	−0.64
Austria	2.0%	13.7%	0.85
Belgium	3.0%	10.7%	1.59
Canada	0.7%	9.2%	0.43
Denmark	−0.2%	13.0%	−0.09
France	2.9%	9.9%	1.67
Germany	−0.5%	10.5%	−0.27
Hong kong	−0.8%	16.5%	−0.26
Ireland	4.9%	18.3%	1.53
Italy	−0.8%	11.0%	−0.39
Japan	3.3%	13.9%	1.36
Netherlands	1.7%	10.8%	0.88
Norway	−0.2%	15.0%	−0.07
Singapore	2.3%	15.6%	0.83
Sweden	0.7%	12.6%	0.34
Switzerland	−2.2%	10.7%	−1.18
United Kingdom	0.8%	9.4%	0.48
United States	1.9%	9.4%	1.15
Equally Weighted Avg. of 18 Countries	1.0%	5.5%	1.05
Full Sample, United States, 1926–2014			
United States	3.4%	13.5%	2.38

Note: Within each country we split stocks into large and small portfolios. Following Fama and French (2012), the portfolio of large stocks comprises 90% of the national market and the small-stock portfolio comprises 10%. Portfolios are capitalization-weighted. The size premium is estimated as the arithmetic average of the differences in return between the small and the large portfolios

Source: Research Affiliates, using CRSP/Compustat and Worldscope/Datastream data.

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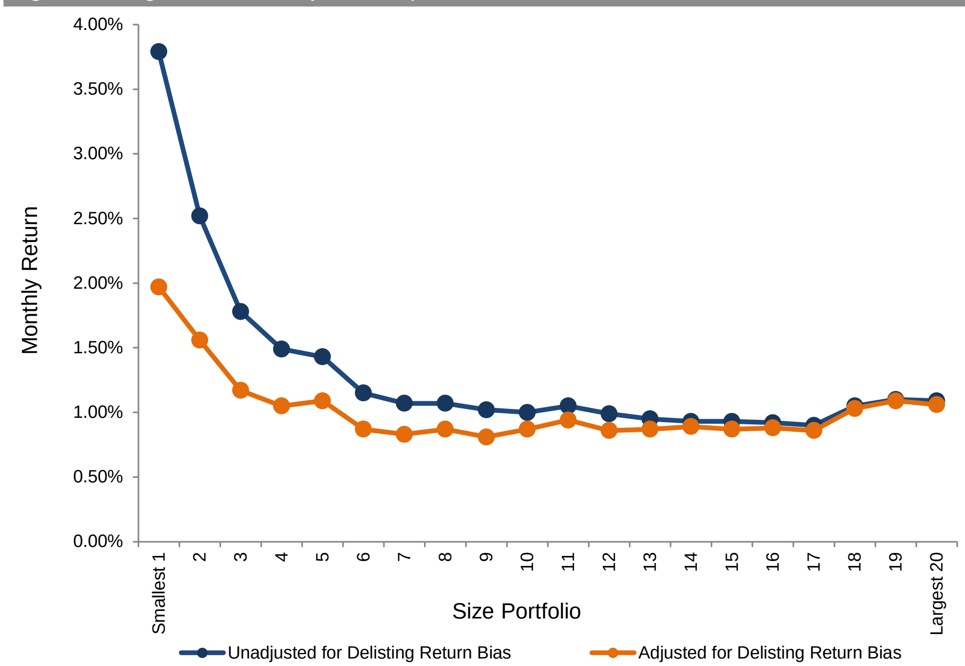
Delisting bias. Shareholders do not necessarily lose the full amount of their investment in a company when it is delisted from a major stock exchange. Often the stock can still be traded in the over-the-counter (OTC) market, and the investor may receive some residual value if the company is liquidated. Nonetheless, returns on stocks after they have been delisted are likely to be very negative. Moreover, all companies are subject to business and financial risks that might result in their stock's falling short of listing requirements, but small stocks by market capitalization are appreciably more likely to be removed from an exchange. Shumway (1997) pointed out that regular performance databases overestimated small-cap stock returns because they did not include returns on delisted stocks. If a database that is used in simulating portfolios omits the strongly negative returns of delisted stocks, the hypothetical results will be better than what actual portfolios can achieve in practice.

To estimate the impact of the delisting bias on the size premium, Shumway and Warther (1999) looked at the smallest and the most distressed stocks for which they could obtain reliable data, namely, stocks listed on the NASDAQ exchange. We represent their findings in **Figure 1**. The chart shows the average monthly returns for 20 groups of stocks sorted by size before and after correcting for the upward bias in the database. Clearly, the smallest stocks are significantly more affected by the delisting bias. After adjusting for the delisting bias, the statistical significance of the size premium completely disappears. It is unreasonable to suppose that the effect Shumway and Warther quantified for NASDAQ stocks is missing from other exchanges.

Transaction costs. Theoretical simulations ignore an important component of investment performance measurement: trading expenses—the actual costs of buying or selling investments. Small stocks by definition have much lower trading capacity and, correspondingly, much higher transaction costs. Soon after the first articles documenting the size effect appeared, researchers asked how much of the premium remains when trading costs are taken into account. Stoll and Whaley (1983) showed that transaction costs accounted for a significant part of the size premium for stocks listed on the New York Stock Exchange and the American Stock Exchange.

Data-mining and reporting bias. There are literally hundreds of known factors in the existing literature, and many papers documenting new factors are published every year. In our opinion the vast majority of these factors are spurious products of data-mining. We are not alone in taking a skeptical position. Lo and MacKinlay (1990), Black (1993), and MacKinlay (1995), among others, have argued that many factors, notably including size, are likely to be a result of data-mining.

Figure 1. Average Stock Returns by Size Group



Source: Research Affiliates, using data from Shumway and Warther (1999).

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And, in finance no less than the physical and biological sciences, striking results—especially new discoveries—tend to win the competition for space in academic journals.

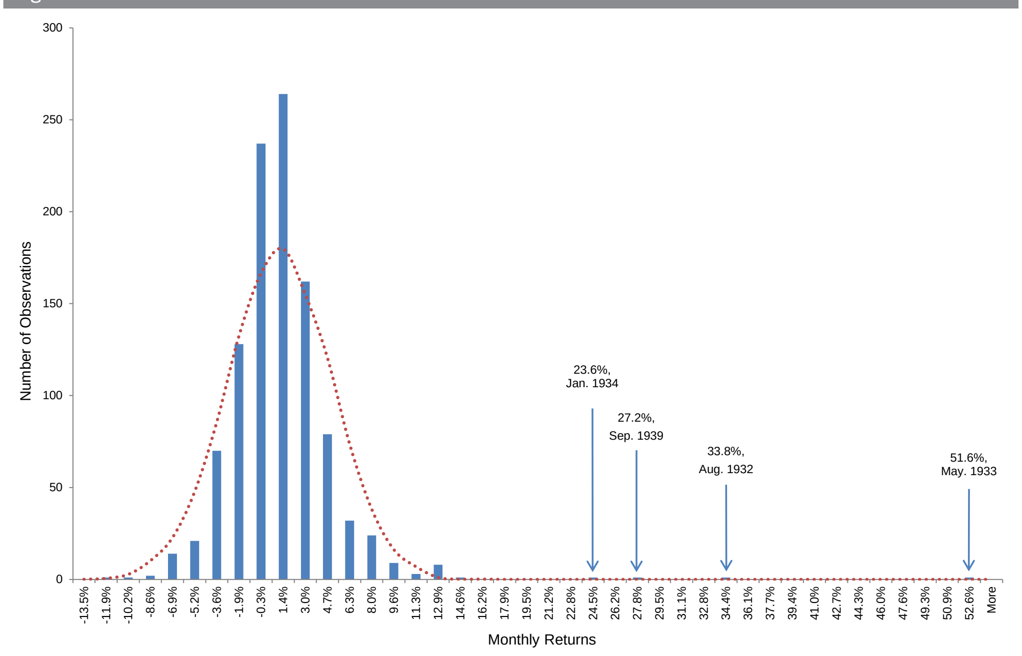
The standard procedure for determining whether a factor is statistically significant is to see if its *t*-stat crosses a certain threshold. Normally the threshold is set at 1.96 for a 5% confidence level. With a *t*-stat of 2.38, the U.S. size premium passes this test for the 1926–2014 sample. But Harvey, Liu, and Zhu (2014) rightly observed that if many researchers are looking for statistical irregularities, then the 1.96 criterion is too low; it allows many inherently random outliers to be misidentified as valid factors. They argue that the threshold for the size factor should have been closer to a *t*-stat of 2.50 in 1993.[1] Size does not pass this test.

Non-normality of returns. Standard statistical testing assumes that the estimate of a variable—in this case, the average of the size premium—quickly converges to a normal distribution.[2] If, however, the underlying data include large outliers, then the assumption of normality is unfounded. The differences between the small and large stock portfolio returns exhibit just such outliers. **Figure 2** is a histogram of the return differences. For comparison, we display on the same chart a normal distribution with the same mean and standard deviation.

We indicate on the chart four extreme outliers of 6 sigma or higher. "Sigma" may be an unfamiliar statistical term, so let us put these outlier returns in perspective. The 23.6% premium registered in January 1934 is a 6-sigma event. If it were drawn from normal distribution, this would be a one-in-67-million-year event, like the one that wiped out the dinosaurs. The 27.2% difference in returns in September 1939 is a 6.9-sigma event; in a normal distribution, it would have about a one-in-five chance of occurring in the 4.5 billion years since the planet earth came into existence. The 33.8% premium in August 1932 is an 8.6-sigma event, and the 51.6% premium in May 1933 is a 13.1-sigma event. If these last two outliers were drawn from a normal distribution, each would have much less than a one-in-a-hundred chance of occurring in the entire 13.8 billion years the universe has existed.

To add to the problem, all four outliers occurred in the 1930s. If they were removed, the estimated size premium in Table 1 would drop from 3.4% to 1.9% and lose statistical significance. (There is a similar outcome in the post-war period: The estimated size premium is about 1.9% premium with a *t*-stat of 1.52.) We do not argue, however, that truncating or otherwise transforming the sample will give us a better estimate. What happened in the 1930s is very valuable information about the economy and the stock market. The average return from the full sample, including the

Figure 2. Distribution of Return Differences



Source: Research Affiliates, using data from Shumway and Warther (1999).

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unadjusted outliers, is the best estimate available as long as the statistical bounds around it are borne in mind. If the size premium is predicated on exceedingly rare events, then we'll have to wait many lifetimes to determine with confidence whether or not it exists.

No risk-adjusted benefit. Academics are interested in the arithmetic average returns in a simulated long/short portfolio, but practitioners are concerned with the actual risk-adjusted returns that they can generate from their investments—and the majority do not engage in short-selling. We display in **Table 2** the average geometrically chained cumulative returns of the long-only portfolios of small and large stocks. These results are produced using the same databases we used earlier in this article, so they contain the same biases that we noted above.

Small stocks outperform large stocks in this sample, but, because small stocks are generally more volatile, the Sharpe ratios reveal that small-cap investing provides a miniscule advantage in the risk-adjusted return. If investors are switching from large stocks to small in the hope of a premium, they should realize that they are increasing the volatility, too. The estimates of average returns are very noisy, and are likely overstated due to the biases we described earlier; the estimates of volatility on the other hand are real. (Estimates of the mean are always less certain than estimates of standard deviation.) We suggest that investors seeking higher returns consider boosting their overall equity allocation rather than chasing the illusory size premium in an attempt to add risk on the cheap within the existing allocation. A large-cap stock portfolio would have higher returns than a mix of small-cap stocks and risk-free assets designed to have the same volatility. In other words, the added risk of small-cap stocks is essentially uncompensated. Note that even in the only data set with a statistically significant size premium (i.e., the U.S. full sample from 1926–2014), the Sharpe ratio is actually lower for small stocks.

Concluding Remarks

We placed our inquiry in a historical context, starting with Banz's (1981) paper, because the widespread belief in a size premium is largely a result of its early discovery. Market capitalization data were readily available to early researchers writing doctoral dissertations and journal articles, and, as we have seen, the performance

Table 2. Average Returns on Long-Only Portfolios

	Small Stocks			Large Stocks			Difference		
Nation	Average Return	Average Volatility	Sharpe Ratio	Average Return	Average Volatility	Sharpe Ratio	Average Return	Average Volatility	Sharpe Ratio
Post Publication Period, 1982–2014									
Australia	10.8%	24.9%	0.26	12.4%	23.4%	0.35	−1.6%	1.5%	−0.08
Austria	13.3%	21.5%	0.42	10.2%	24.4%	0.24	3.1%	−2.9%	0.18
Belgium	15.8%	18.7%	0.62	12.6%	20.3%	0.41	3.2%	−1.6%	0.21
Canada	11.2%	21.4%	0.33	11.1%	18.7%	0.37	0.1%	2.7%	−0.04
Denmark	12.1%	20.1%	0.39	12.6%	19.4%	0.43	−0.4%	0.7%	−0.04
France	15.7%	20.5%	0.56	12.5%	21.0%	0.39	3.2%	−0.5%	0.17
Germany	11.0%	18.4%	0.36	11.0%	21.4%	0.31	0.0%	−3.0%	0.05
Hong kong	10.6%	31.9%	0.20	12.5%	29.2%	0.28	−1.9%	2.7%	−0.08
Ireland	18.3%	23.6%	0.60	12.6%	23.8%	0.35	5.7%	−0.2%	0.24
Italy	8.1%	23.6%	0.16	8.7%	24.9%	0.18	−0.6%	−1.3%	−0.02
Japan	9.3%	23.8%	0.21	6.4%	21.8%	0.10	2.9%	2.0%	0.11
Netherlands	14.7%	20.0%	0.52	13.1%	19.0%	0.46	1.6%	1.0%	0.06
Norway	13.6%	24.9%	0.38	13.3%	25.9%	0.35	0.2%	−1.0%	0.02
Singapore	10.1%	31.7%	0.19	9.6%	24.3%	0.22	0.5%	7.3%	−0.03
Sweden	14.8%	24.7%	0.42	13.8%	24.9%	0.39	0.9%	−0.2%	0.04
Switzerland	11.0%	17.9%	0.38	13.5%	17.3%	0.53	−2.5%	0.6%	−0.16
United Kingdom	11.8%	19.8%	0.38	11.5%	17.7%	0.41	0.3%	2.1%	−0.03
United States	13.3%	19.1%	0.48	12.0%	15.2%	0.51	1.3%	3.9%	−0.04
Arithmetic average:	12.5%	22.6%	0.38	11.6%	21.8%	0.35	0.9%	0.8%	0.03
Full Sample, United States, 1926–2014									
United States	11.8%	27.2%	0.31	9.8%	18.4%	0.34	2.1%	8.7%	−0.03

Note: Within each country we split stocks into capitalization-weighted large and small portfolios. Following Fama and French (2012), the large stock portfolio comprises 90% of the national market, and the small stock portfolio, 10%. The returns shown are the geometric average returns of the small and large stock portfolios. The difference columns represent the simple differences of the geometric average return, volatility, and Sharpe ratios.

Source: Research Affiliates, using CRSP/Compustat and Worldscope/Datastream data.

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of small stocks was exceptional in the 1930s. Eugene Fama was one of Rolf Banz's professors at the University of Chicago; in fact, as a member of Banz's dissertation committee, he was intimately familiar with Banz's research on the small-cap anomaly.[3] Fama and Kenneth French included the size premium in their influential three-factor model, an analytical advance that opened the gate for empirical research into studying factors previously unexplained by then-existing theories. Riding on the popularity of the Fama–French theory, the size premium was soon entrenched in the pantheon of risk factors.

Berk (1997) argued that the size premium observed in the data is nothing more than a poor way of value investing. Value investing relies on buying cheaply priced companies as measured by a ratio of price to company fundamentals. Investing based on size, measured by company market capitalization, would use only the price side of the valuation measure. Because it would therefore use only a fraction of the relevant information, the strategy is significantly weaker than a value strategy that uses prices as they relate to company fundamentals. In our view, Berk's argument is, to date, the strongest explanation why the size premium is observed.

However, we go one step further. If Berk questioned the size premium as a separate factor, we question the size premium as a phenomenon. Today, more than 30 years after the initial publication of Banz's paper, the empirical evidence is extremely weak even before adjusting for possible biases. The return premium is not statistically significant in any of the international markets, whether taken alone or in combination. The U.S. long-term size premium is driven by the extreme outliers, which occurred three-quarters of a century ago. These extreme outliers confound the standard techniques of setting confidence bounds around the estimated premium. Finally, adjusting for biases, most notably the delisting bias, makes the size premium vanish. If the size premium were discovered today, rather than in the 1980s, it would be challenging to even publish a paper documenting that small stocks outperform large ones. All this evidence makes us question the existence of the size premium as such.

We are not arguing that investors should completely abandon small stocks. Small stocks are more volatile than large stocks, and they receive considerably less attention from sell-side analysts. Consequently, small stocks are more likely to be mispriced. The major anomalies are, in fact, stronger in the small-cap sector. Small stocks are more attractive as an alpha pool to be fished by skillful active managers and exploited by rules-based value and momentum strategies.

Endnotes

1. The authors argue further that "a newly discovered factor today should have a *t*-ratio that exceeds 3.0." Page 35.
2. This result relies on the central limit theorem, which says that, as the number of random observations increases, the arithmetic average converges to a normal distribution. If the observations include extreme outliers, the convergence can be either extremely slow or may not occur at all.
3. Fox (2009), page 204.

References

Banz, Rolf W. 1981. "The Relationship Between Return and Market Value of Common Stocks." *Journal of Financial Economics*, vol. 9, no. 1 (March):3-18.

Berk, Jonathan B. 1997. "Does Size Really Matter?" *Financial Analysts Journal*, vol. 53, no. 5 (September/October):12–18.

Black, Fischer. 1993. "Beta and Return." *Journal of Portfolio Management*, vol. 20, no. 1 (Fall):8–18.

Carhart, Mark M. 1997. "On Persistence in Mutual Fund Performance." *Journal of Finance*, vol. 52, no. 1 (March):57–82.

Fama, Eugene F., and Kenneth R. French . 1993. "Common Risk Factors in the Returns on Stocks and Bonds." *Journal of Financial Economics*, vol. 33, no. 1 (February):3–56.

———. 2012. "Size, Value, and Momentum in International Stock Returns." *Journal of Financial Economics*, vol. 105, no. 3 (September):457–472.

Fox, Justin. 2009. *The Myth of the Rational Market: A History of Risk, Reward, and Delusion on Wall Street*. HarperCollins e-books.

Harvey, Campbell R., Yan Liu, and Heqing Zhu. 2014. "…And the Cross-Section of Expected Returns." NBER Working Paper No. 20592. Available at SSRN: http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2513152## OR Available at nber.org/papers/w20592.

References (continued)

Hsu, Jason and Vitali Kalesnik. 2014. "Finding Smart Beta in the Factor Zoo." Research Affiliates (July).

Lo, Andrew W., and A. Craig MacKinlay. 1990. "Data-Snooping Biases in Tests of Financial Asset Pricing Models." *Review of Financial Studies*, vol. 3, no. 3 (Fall):431–467.

MacKinlay, A. Craig. 1995. "Multifactor Models Do Not Explain Deviations from the CAPM." *Journal of Financial Economics*, vol. 38, no. 1 (May):3–28.

Shumway, Tyler. 1997. "The Delisting Bias in CRSP Data." *Journal of Finance*, vol. 52, no. 1 (March):327-340.

Shumway, Tyler, and Vincent A. Warther. 1999. "The Delisting Bias in CRSP's Nasdaq Data and Its Implications for the Size Effect." *Journal of Finance*, vol. 54, no. 6 (December):2361–2379.

Stoll, Hans R. and Robert E. Whaley. 1983. "Transaction Costs and the Small Firm Effect." *Journal of Financial Economics*, vol. 12, no. 1 (June):57–79.

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Congressional Budget Office

Nonpartisan Analysis for the U.S. Congress

The 2022 Long-Term Budget Outlook

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JULY | 2022

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At a Glance

Each year, the Congressional Budget Office publishes a report presenting its budget projections and economic forecast for the next 30 years under the assumption that current laws governing taxes and spending generally do not change. This report is the latest in the series.

- **Deficits.** At 3.9 percent of gross domestic product (GDP), the projected deficit in 2022 is much smaller than those recorded in 2020 and 2021, because federal spending in response to the coronavirus pandemic has waned and revenues have risen sharply. Nevertheless, in CBO's projections, federal deficits over the 2022–2052 period average 7.3 percent of GDP (more than double the average over the past half-century) and generally grow each year, reaching 11.1 percent of GDP in 2052. That projected growth in total deficits is largely driven by increases in interest costs: Net interest outlays more than quadruple over the period, rising to 7.2 percent of GDP in 2052. Primary deficits—that is, deficits excluding net outlays for interest—grow from 2.3 percent of GDP in 2022 to 3.9 percent in 2052.
- **Debt.** By the end of 2022, federal debt held by the public is projected to equal 98 percent of GDP. The rapid growth of nominal GDP—which reflects both high inflation and the continued growth of real GDP (that is, GDP adjusted to remove the effects of inflation)—helps hold down the amount of debt relative to the nation's output in 2022 and 2023. In CBO's projections, debt as a percentage of GDP begins to rise in 2024, surpasses its historical high in 2031 (when it reaches 107 percent), and continues to climb thereafter, rising to 185 percent of GDP in 2052.

  Debt that is high and rising as a percentage of GDP could slow economic growth, push up interest payments to foreign holders of U.S. debt, heighten the risk of a fiscal crisis, elevate the likelihood of less abrupt adverse effects, make the U.S. fiscal position more vulnerable to an increase in interest rates, and cause lawmakers to feel more constrained in their policy choices.
- **Spending.** In CBO's projections, outlays in 2022 are 23.5 percent of GDP—less than last year's total—and they continue to decline in 2023 and 2024 as federal spending in response to the pandemic diminishes. Outlays then steadily increase, reaching 30.2 percent of GDP in 2052. Rising interest costs and growth in spending on the major health care programs and Social Security—driven by the aging of the population and growth in health care costs per person—boost federal outlays significantly over the 2025–2052 period.
- **Revenues.** In CBO's projections, revenues rise to 19.6 percent of GDP in 2022, one of the highest levels ever recorded, because of sizable increases in collections of individual income taxes. After falling in relation to the size of the economy for the next few years, revenues increase in 2026, largely because of scheduled changes in tax rules. They continue to rise after 2030 as an increasing share of income is pushed into higher tax brackets. In 2052, revenues reach 19.1 percent of GDP.

Future economic conditions are uncertain. But even if they were more favorable than CBO currently projects, debt in 2052 would probably be much higher than it is today. Moreover, according to CBO's analysis, if future paths for spending and revenues were more consistent with such paths in the past, debt in 2052 would probably be much higher than CBO projects.

In this year's projections, debt as a percentage of GDP is lower in most years than CBO projected last year. In the current projections, federal debt rises from 98 percent of GDP in 2022 to 180 percent in 2051. Those amounts are lower than CBO's previous projections—by 4 percentage points and 22 percentage points, respectively.

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Notes

The Congressional Budget Office's extended baseline projections follow the agency's 10-year baseline budget projections and then extend most of the concepts underlying those projections for an additional 20 years. In accordance with statutory requirements, CBO's projections reflect the assumptions that current laws generally remain unchanged, that some mandatory programs are extended after their authorizations lapse, and that spending on Medicare and Social Security continues as scheduled even if their trust funds are exhausted.

The budget projections in this report are based on CBO's economic projections and include the effects of legislation enacted through April 8, 2022. The economic projections reflect economic developments through March 2, 2022. The projections do not include budgetary or economic effects of subsequent legislation, economic developments, administrative actions, court rulings, or regulatory changes.

Unless this report indicates otherwise, all years referred to are federal fiscal years, which run from October 1 to September 30 and are designated by the calendar year in which they end. Budgetary values, such as the ratio of debt or deficits to gross domestic product (GDP), are calculated on a fiscal year basis; economic variables, such as GDP or interest rates on Treasury securities, are calculated on a calendar year basis.

When October 1 (the first day of the fiscal year) falls on a weekend, certain payments that ordinarily would have been made on that day are instead made at the end of September and thus are shifted into the previous fiscal year. In this report, budget projections have been adjusted to exclude the effects of those timing shifts.

Numbers in the text, tables, and figures may not add up to totals because of rounding.

Unless this report specifies otherwise, Medicare outlays are presented net of premiums paid by beneficiaries and other offsetting receipts, which reduce outlays for the program.

In this report, the term "additional cost growth" is used instead of "excess cost growth" (which was used in past reports) to describe the amount by which the growth rate of nominal health care spending per person (adjusted to remove the effects of demographic changes) exceeds the growth rate of potential GDP per person.

Detailed projections about the size of the U.S. population and its age and sex composition are presented in a companion report; see Congressional Budget Office, *The Demographic Outlook: 2022 to 2052* (July 2022), www.cbo.gov/publication/57975.

Supplemental information files—the data underlying the tables and figures in this report, supplemental budget projections, and the economic variables underlying those projections—are posted on CBO's website (www.cbo.gov/publication/57971#data). Previous editions of this report are also available on the website (https://go.usa.gov/xmezZ).

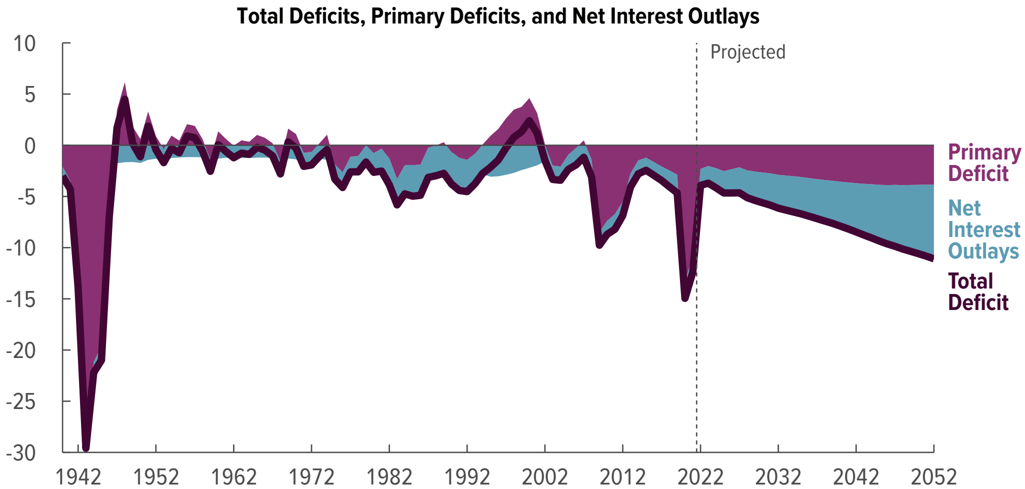
Visual Summary

In this report, the Congressional Budget Office describes its projections of what the federal budget would look like over the next 30 years if current laws generally remained unchanged; it also presents the economic forecast underlying those projections. The United States faces a challenging fiscal outlook according to those extended baseline projections, which show budget deficits and federal debt held by the public growing steadily in relation to gross domestic product (GDP) over the next three decades.

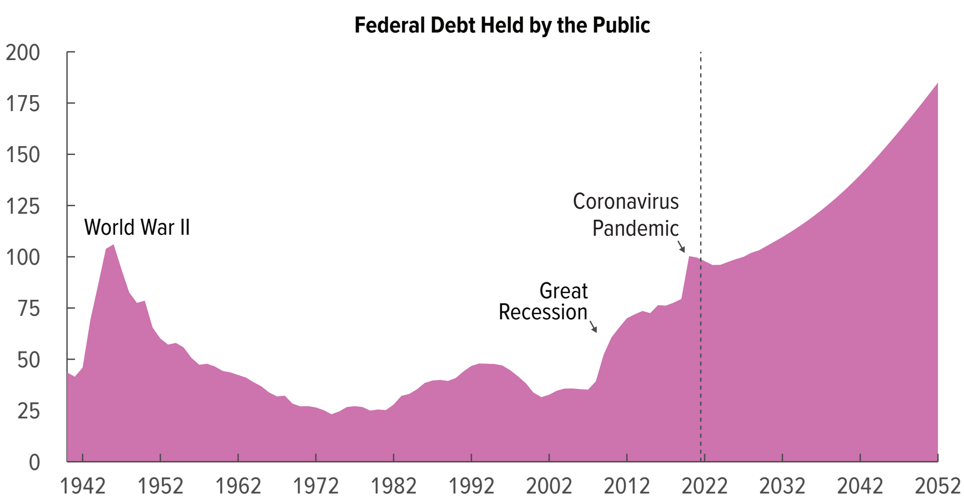
Deficits and Debt

Federal deficits are projected to nearly triple over the next 30 years, from 4 percent of GDP in 2022 to 11 percent in 2052. Such persistently growing deficits would cause federal debt held by the public, which is already high, to continue to rise even further. In CBO's projections, such debt reaches 185 percent of GDP in 2052.

Percentage of Gross Domestic Product



Net interest outlays more than quadruple as a percentage of GDP over the 2022–2052 period in CBO's projections, reaching 7.2 percent of GDP in 2052. Primary deficits (which exclude net interest costs) grow in most years and reach 3.9 percent of GDP at the end of the projection period; they exceed the 50-year average of 1.5 percent of GDP throughout the period.



Debt is projected to rise in relation to GDP over the 30-year period, and it is on track to grow even larger after 2052.

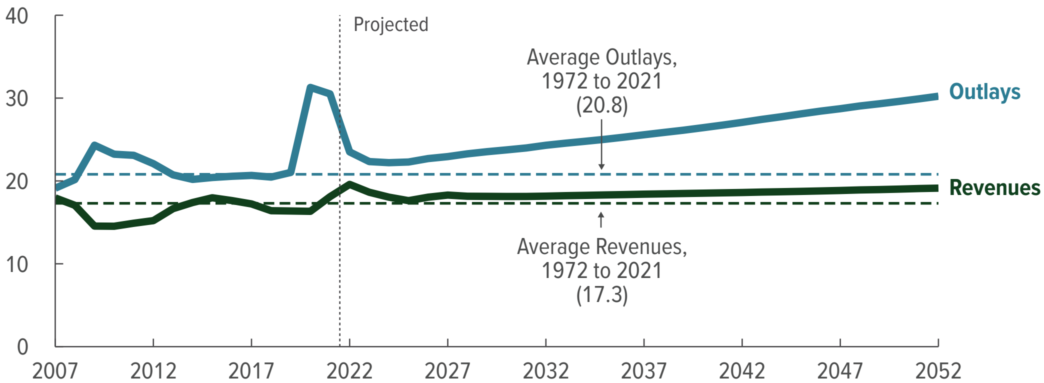
See Figure 1-1 on page 6

Spending and Revenues

In CBO's projections, federal spending grows from an average of 23 percent of GDP over the 2022–2032 period to an average of 29 percent of GDP over the 2043–2052 period. Federal revenues increase from an average of 18 percent of GDP over the 2022–2032 period to an average of 19 percent over the 2043–2052 period.

Total Outlays and Revenues

Percentage of Gross Domestic Product

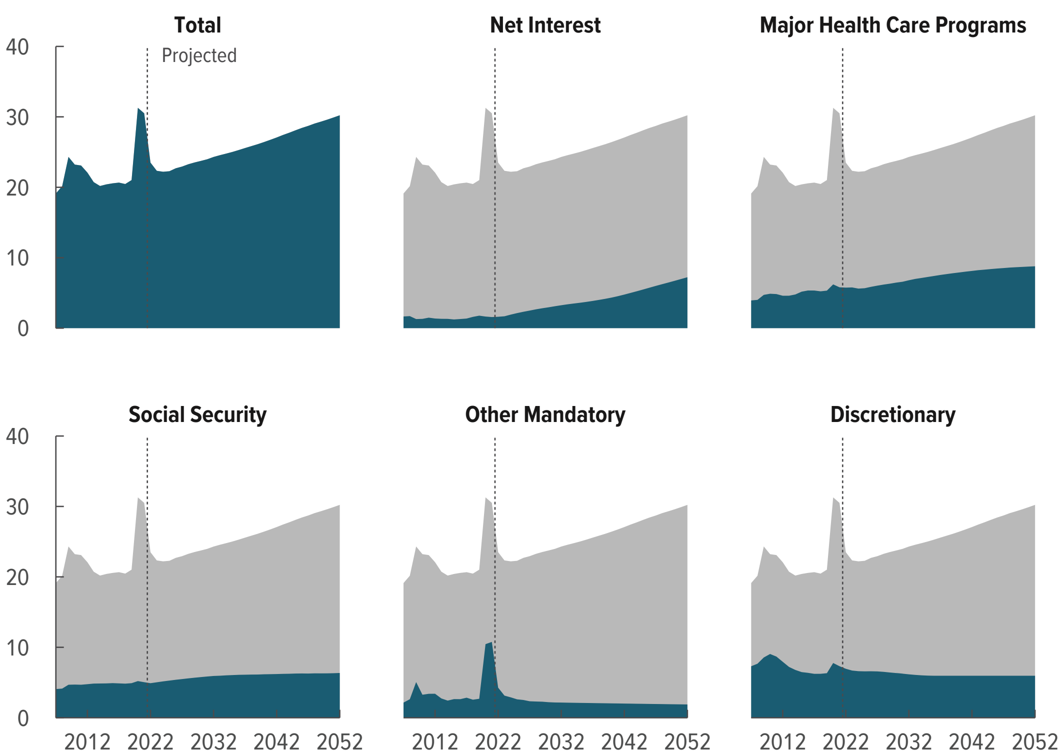


The gap between outlays and revenues widens over the long term. Outlays increase faster than revenues—mainly because of rising interest costs and growth in spending for Medicare and Social Security—resulting in ever-larger budget deficits.

See Figure 2-1 on page 16

Outlays, by Category

Percentage of Gross Domestic Product



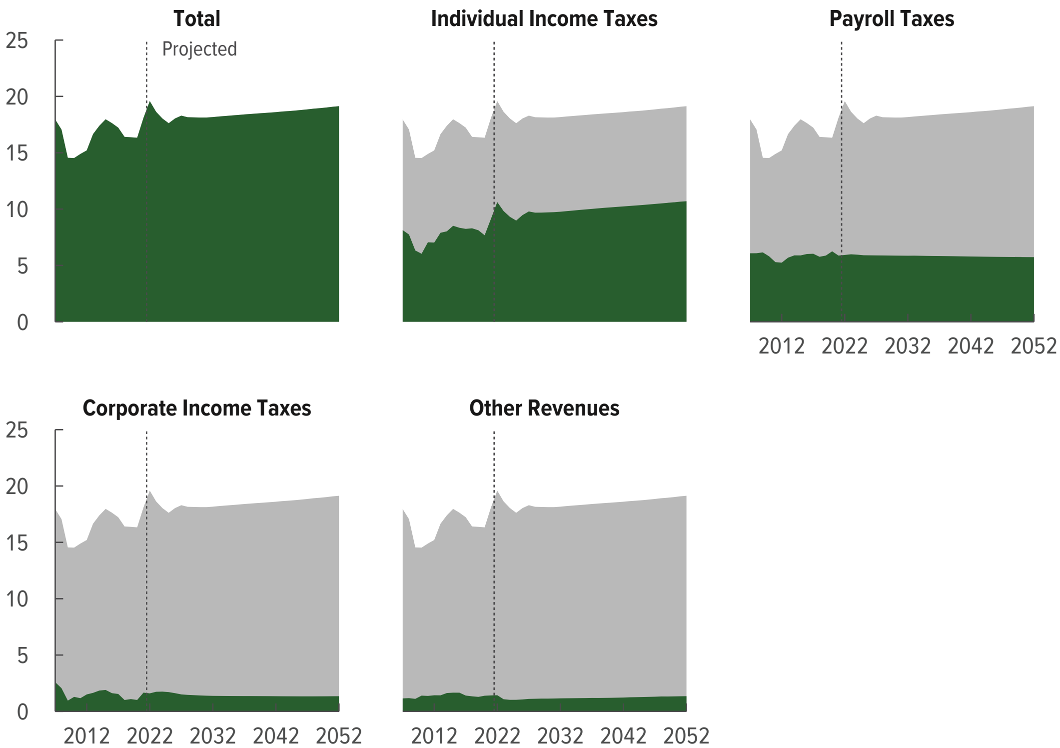
Rising interest rates and mounting debt cause net interest outlays to increase from 1.6 percent of GDP in 2022 to 7.2 percent in 2052 in CBO's projections. Outlays for the major health care programs also rise, from 5.8 percent of GDP in 2022 to 8.8 percent in 2052. Likewise, outlays for Social Security increase in almost every year of the period.

See Figure 2-2 on page 17

Spending and Revenues (Continued)

Revenues, by Source

Percentage of Gross Domestic Product

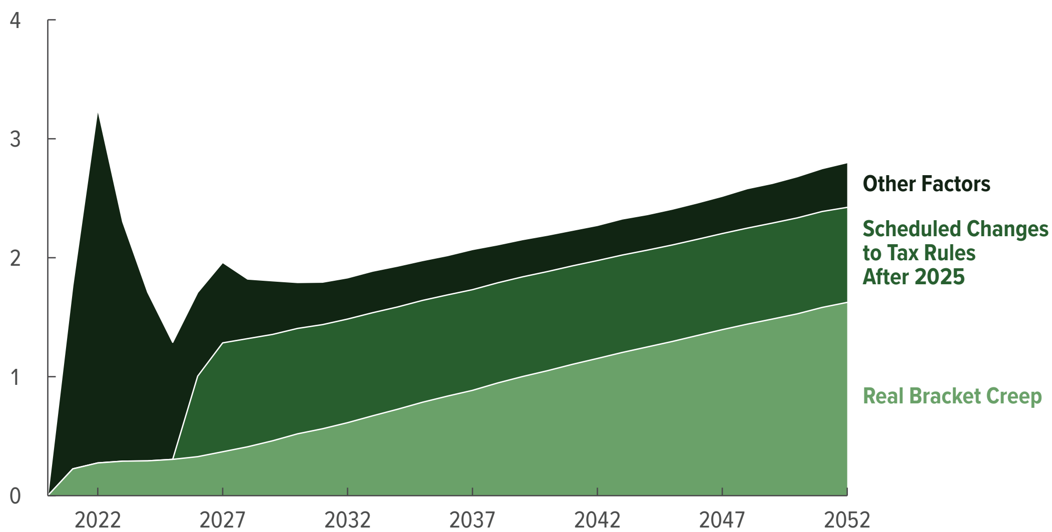


Measured as a percentage of GDP, revenues in 2022 are projected to be at one of the highest levels ever recorded. As temporary factors that boosted tax receipts fade, revenues fall in relation to the size of the economy for the next few years. They increase in 2026, largely because of the scheduled expiration of some provisions of the 2017 tax act.

See Figure 2-6 on page 22

Composition of Changes in Revenues, 2020 to 2052

Percentage of Gross Domestic Product



Over the long term, the largest source of growth in tax revenues is real bracket creep—the process in which, as income rises faster than prices, a larger proportion of income becomes subject to higher tax rates.

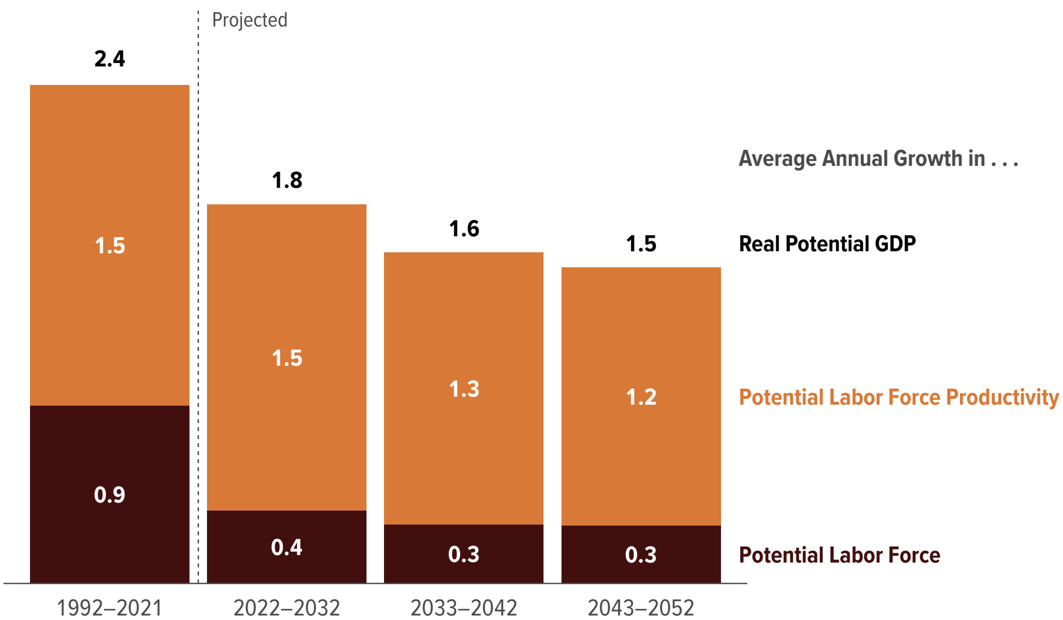
See Figure 2-7 on page 23

The Economy

In CBO's projections, real potential GDP (that is, the maximum sustainable output of the economy, adjusted to remove the effects of inflation) grows more slowly throughout the 2022–2052 period than it has, on average, over the past 30 years. Beginning in 2028 and continuing through the end of the projection period, potential output and actual output grow at the same rate, and the level of real GDP remains about 0.5 percent below the level of real potential GDP. That gap between real GDP and real potential GDP reflects the agency's assessment that actual output falls short of potential output by more and for longer during and after economic downturns than actual output exceeds potential output during economic booms.

The growth of real potential GDP is determined by the growth of the potential labor force (the labor force adjusted for fluctuations in the business cycle) and the growth of potential labor force productivity (potential output per member of the potential labor force).

Composition of the Growth of Real Potential GDP
Percent



Real potential GDP is projected to grow more slowly over the next 30 years than it did over the past 30 years, mostly because the potential labor force is projected to grow more slowly than it has in the past. From 2022 to 2052, the growth of real potential GDP slows primarily because the growth of potential labor force productivity slows in those years.

See Figure 3-2 on page 27

Chapter 1: Deficits and Debt

Overview

If current laws governing taxes and spending generally remained unchanged, the federal budget deficit, measured in relation to gross domestic product (GDP), would nearly triple over the next 30 years, the Congressional Budget Office projects. Those growing deficits are projected to drive up federal debt held by the public substantially. As a percentage of GDP, such debt in 2052 would far exceed any previously recorded level and be on track to increase further (see Figure 1-1).

As federal spending in response to the coronavirus pandemic wanes and revenues rise sharply, this year's budget deficit is set to be much smaller than those recorded in 2020 and 2021. Nevertheless, in CBO's projections, federal deficits are large by historical standards and generally grow over the next 30 years, reaching 11.1 percent of GDP in 2052.[1] In the past 100 years, the deficit has been that large only during World War II and during the pandemic in 2020 and 2021. The projected growth in deficits is largely driven by increases in interest costs. Over that period, deficits average 7.3 percent of GDP, more than double the average over the past half-century.

Those persistently increasing deficits generate high-and-rising debt in the agency's projections. Measured in relation to GDP, federal debt held by the public dips over the next two years but then rises, reaching 110 percent at the end of 2032—the highest it has ever been. Debt continues to climb thereafter and reaches 185 percent of GDP at the end of 2052.

Such high and rising debt could have significant economic and financial consequences. It could, among other things, slow economic growth, drive up interest payments to foreign holders of U.S. debt, elevate the risk of a fiscal crisis, increase the likelihood of less abrupt adverse effects, make the U.S. fiscal position more vulnerable to an increase in interest rates, and cause lawmakers to feel more constrained in their policy choices.

CBO estimated the size of changes in spending or revenues that would be needed if lawmakers wanted to achieve certain targets for federal debt held by the public. The size of those changes would depend on the level of debt that lawmakers wanted to achieve and when the changes were implemented. In addition, how and when lawmakers responded to high and rising debt would determine who bore the burden of the changes in spending or taxes and who realized the economic benefits of those changes.

Even if federal laws remained unchanged, CBO's budget projections would be subject to considerable uncertainty. Those projections depend on the agency's economic projections and many other factors, including the course of the ongoing pandemic. Developments that diverged from those underlying CBO's projections could lead to budgetary outcomes that were very different from those reported here. That uncertainty increases in later years of the projection period because changes in the economy, demographics, and a variety of other factors are more difficult to anticipate over longer time horizons.

This analysis does not account for some contingencies that could have significant effects on the budget—for example, an economic depression (such as the Great Depression of the 1930s), a catastrophe or major war, unexpectedly significant effects of climate change, or the development of a previously underused natural resource. Such occurrences could create conditions in the next 30 years that are substantially better or worse than those reflected in the historical data on which CBO based its analysis.

Deficits and Debt Through 2052

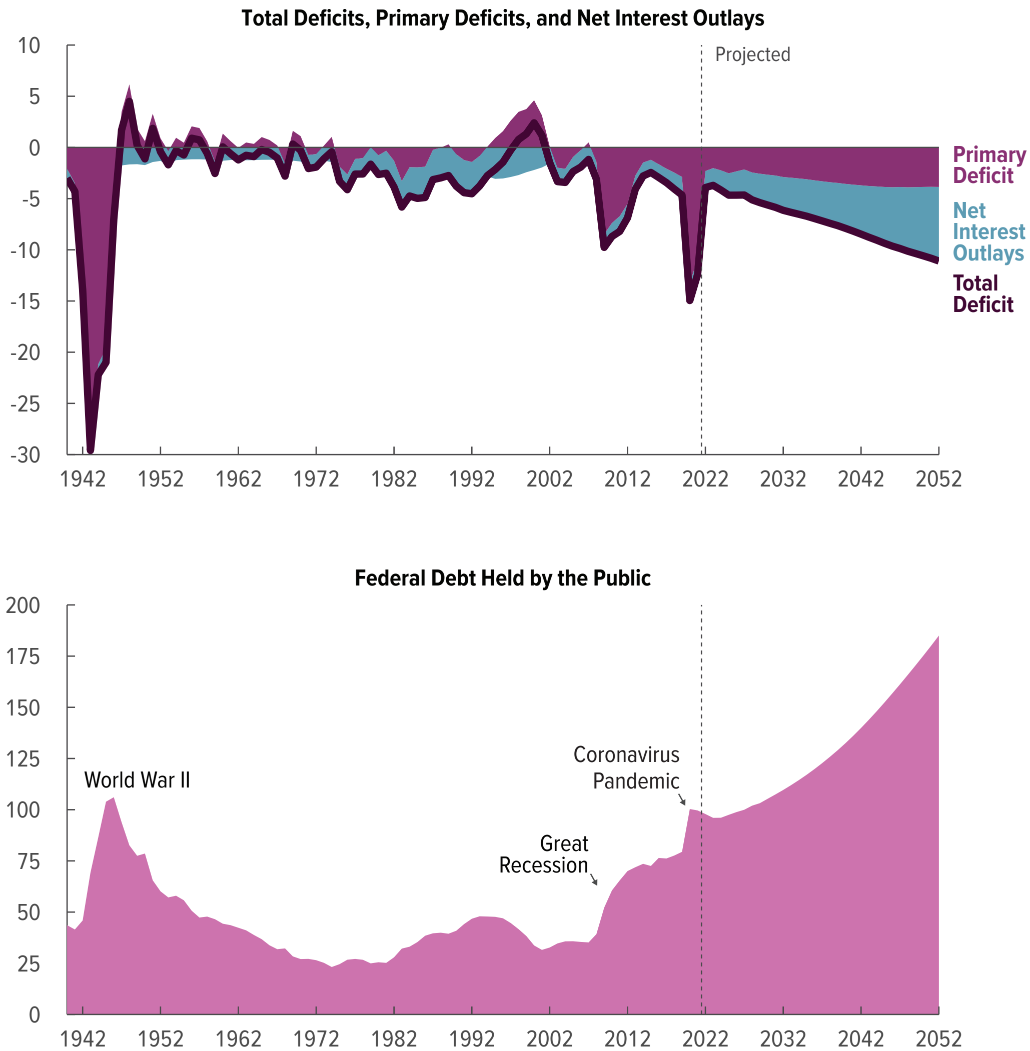
In CBO's projections, deficits drop below 4.0 percent of GDP for a few years and then generally rise again through 2052. Similarly, debt measured as a percentage of GDP dips for two years before increasing through 2052 as the federal government persistently incurs budget deficits that are large relative to the size of the economy.

1. The long-term projections of federal spending, revenues, deficits, and debt in this report are consistent with the baseline budget projections and the economic forecast for 2022 to 2032 that CBO published in May 2022. See Congressional Budget Office, *The Budget and Economic Outlook: 2022 to 2032* (May 2022), www.cbo.gov/publication/57950.

Figure 1-1.

Deficits and Debt

Percentage of GDP



In CBO's projections, primary deficits grow in most years and reach 3.9 percent of GDP in 2052; they exceed the 50-year average of 1.5 percent of GDP throughout the projection period. Driven up by large and sustained primary deficits and rising interest rates, net interest outlays more than quadruple as a percentage of GDP over the 2022–2052 period, reaching 7.2 percent of GDP in 2052.

Those persistently growing deficits push federal debt held by the public, which is already high, further up throughout the 30-year period—to 185 percent of GDP in 2052. Such debt would continue to rise thereafter.

Data source: Congressional Budget Office. See www.cbo.gov/publication/57971#data.

Primary deficits exclude net outlays for interest.

GDP = gross domestic product.

Deficits

The total deficit—that is, the deficit including net outlays for interest—in 2022 is estimated to be 3.9 percent of GDP, significantly smaller than it was in 2021. In CBO's projections, the total deficit declines to 3.7 percent of GDP in 2023 before increasing again. Over the 2043–2052 period, those deficits average 10.0 percent of GDP—almost three times the 3.5 percent of GDP they averaged over the past 50 years (see Table 1-1). Moreover, in years in the past half-century when unemployment was relatively low, as it is in CBO's projections, the average total deficit was even smaller.

Primary deficits—that is, deficits excluding net outlays for interest—capture the aspects of federal spending and revenues that policymakers can, in principle, affect directly through legislation; thus, they are the main mechanism through which lawmakers can influence the trajectory of federal debt and net interest outlays. In CBO's extended baseline projections, the primary deficit grows from 2.3 percent of GDP in 2022 to 3.9 percent of GDP in 2052, exceeding the 1.5 percent of GDP such deficits averaged over the past 50 years in every year of the projection period. Persistently large primary deficits increase federal debt and, in turn, net interest outlays in

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Table 1-1.

Key Projections in CBO's Extended Baseline

Percentage of Gross Domestic Product

		Projected Annual Average		
	2022	2023–2032	2033–2042	2043–2052
Revenues				
Individual income taxes	10.6	9.6	10.0	10.5
Payroll taxes	5.9	5.9	5.8	5.8
Corporate income taxes	1.6	1.5	1.4	1.3
Other[a]	1.4	1.1	1.2	1.3
Total Revenues	**19.6**	**18.1**	**18.4**	**18.9**
Outlays				
Mandatory				
Social Security	4.9	5.5	6.1	6.3
Major health care programs[b]	5.8	6.2	7.6	8.6
Other	4.3	2.4	2.1	1.9
Subtotal	14.9	14.2	15.8	16.8
Discretionary	7.0	6.5	6.0	6.0
Net interest	1.6	2.6	4.0	6.2
Total Outlays	**23.5**	**23.2**	**25.8**	**28.9**
Deficit	**-3.9**	**-5.1**	**-7.4**	**-10.0**
Debt Held by the Public at the End of the Period	**98**	**110**	**140**	**185**
Memorandum:				
Social Security				
Revenues[c]	4.5	4.6	4.6	4.5
Outlays[d]	4.9	5.5	6.1	6.3
Contribution to the Federal Deficit[e]	**-0.4**	**-1.0**	**-1.5**	**-1.8**
Medicare				
Revenues and offsetting receipts[c]	2.3	2.3	2.8	3.1
Outlays[d]	3.8	4.6	6.1	7.1
Contribution to the Federal Deficit[e]	**-1.5**	**-2.3**	**-3.4**	**-4.1**
Gross Domestic Product at the End of the Period (Trillions of dollars)	24.7	36.7	52.6	74.5

Data source: Congressional Budget Office. See www.cbo.gov/publication/57971#data.

This table provides the information specified in section 3111 of S. Con. Res. 11, the Concurrent Resolution on the Budget for Fiscal Year 2016.

The extended baseline projections, which generally reflect current law, follow CBO's 10-year baseline budget projections and then extend most of the concepts underlying those projections for an additional 20 years.

When October 1 (the first day of the fiscal year) falls on a weekend, certain payments that ordinarily would have been made on that day are instead made at the end of September and thus are shifted into the previous fiscal year. All projections have been adjusted to exclude the effects of those timing shifts.

a. Consists of excise taxes, remittances to the Treasury from the Federal Reserve System, customs duties, estate and gift taxes, and miscellaneous fees and fines.

b. Consists of outlays for Medicare (net of premiums and other offsetting receipts), Medicaid, and the Children's Health Insurance Program, as well as subsidies for health insurance purchased through the marketplaces established under the Affordable Care Act and related spending.

c. Includes all payroll taxes for the program except for those paid by the federal government on behalf of its employees; those payments are intragovernmental transactions. For Social Security, income taxes paid on Social Security benefits, which are credited to the trust funds, are included; for Medicare, premiums and other offsetting receipts are included. Amounts shown do not include any interest credited to the trust funds.

d. Does not include outlays related to the administration of the program, which are discretionary. For Social Security, outlays do not include intragovernmental offsetting receipts stemming from the employer's share of payroll taxes paid to the Social Security trust funds by federal agencies on behalf of their employees.

e. The contribution to the deficit shown here differs from the change in the trust fund balance for the program. It does not include intragovernmental transactions, interest earned on balances, or outlays related to the administration of the program.

later years of the projection period. Combined with rising interest rates, large and sustained primary deficits cause net interest outlays measured as a percentage of GDP to more than quadruple over the period: They rise from 1.6 percent of GDP in 2022 to 7.2 percent in 2052.

Federal Debt Held by the Public

Measured in relation to the size of the economy, debt dips from 100 percent of GDP at the end of 2021 to 96 percent in 2023 in CBO's projections. The rapid growth of nominal GDP over that period—which reflects both high inflation and the growth of real GDP (that is, GDP adjusted to remove the effects of inflation)—helps hold down debt measured as a percentage of GDP. After 2023, debt rises. It reaches 185 percent of GDP in 2052—far higher than its historical peak of 106 percent of GDP recorded in 1946, immediately after World War II—and is on track to rise higher still.

Consequences of High and Rising Federal Debt

If federal debt continued to rise in relation to GDP at the pace that CBO projects it would under current law, the economy would be affected in two significant ways in the long term:

- That debt path would raise borrowing costs throughout the economy, reduce private investment, and slow the growth of economic output over time.
- Rising interest costs associated with that debt would drive up interest payments to foreign holders of U.S. debt, decreasing the nation's net international income.

Persistently rising debt would also pose significant risks to the fiscal and economic outlook.[2] Such a debt path would have the following effects:

- It would elevate the risk of a fiscal crisis—that is, a situation in which investors lose confidence in the U.S. government's ability to service and repay its debt, causing interest rates to increase abruptly, inflation to spiral upward, or other disruptions to occur.
- It would increase the likelihood of less abrupt, but still significant, adverse effects, such as creating widespread expectations of higher rates of inflation, eroding confidence in the U.S. dollar as the dominant international reserve currency, or making it more difficult to secure financing for public and private activities in international markets.
- It would make the United States' fiscal position more vulnerable to an increase in interest rates because costs to service federal debt rise more for a given increase in interest rates when debt is higher than they do when it is lower.
- Policymakers might feel constrained from implementing deficit-financed fiscal policy to respond to unforeseen events or for other purposes, such as to promote economic activity or strengthen national defense.

Slower Economic Growth

High and rising federal debt such as that resulting from the federal borrowing in CBO's extended baseline projections would, over time, push up borrowing costs in all sectors of the economy, reduce private investment, and slow the growth of GDP, all else being equal.

Higher debt tends to lead to higher interest rates and thus increased borrowing costs in both the public and private sectors. When the government borrows, it does so from people and businesses whose savings would otherwise finance private investment in productive capital, such as housing and commercial structures. The portion of private savings used to buy Treasury securities is no longer available to fund such investment, so the borrowing costs of both the private and public sectors increase.

On net, an increase in government borrowing reduces private investment. The increases in borrowing costs reduce private investment, but at least three other effects tend to boost private investment and partially offset that reduction:

- Additional government borrowing strengthens the incentive to save—in part, by driving up interest rates—but the increase in private saving is not as large as the increase in government borrowing; national saving (or the amount of domestic resources available for investment) thus declines.[3]
- Higher interest rates tend to attract more foreign capital to the United States, and some of those funds become available for private investment.

2. For a fuller discussion of federal debt and the consequences of its growth, see Congressional Budget Office, *Federal Debt: A Primer* (March 2020), www.cbo.gov/publication/56165.

3. In CBO's assessment, another reason that an increase in government borrowing would strengthen the incentive to save is that some people would expect policymakers to raise taxes or cut spending in the future to cover the cost of paying interest on the additional federal debt. As a result, some of those people would increase their saving to prepare for paying higher taxes or receiving less in benefits. See Jonathan Huntley, *The Long-Run Effects of Federal Budget Deficits on National Saving and Private Domestic Investment*, Working Paper 2014-02 (Congressional Budget Office, February 2014), www.cbo.gov/publication/45140.

- Borrowing that supports increased high-quality and effective federal investment typically boosts private-sector productivity, investment, and output. However, the increasing deficits and debt in CBO's projections result primarily from increases in noninvestment spending. For instance, federal spending for Social Security, Medicare, and Medicaid for people age 65 or older, which accounts for less than 30 percent of all federal noninterest spending in 2022, accounts for more than 40 percent in 2052.

In CBO's assessment, the increases in private investment stemming from those three factors would not be as large as the reduction in private investment stemming from the additional government borrowing.

The reduction in private investment would slow economic growth. If investment in capital goods declined, workers would, on average, have less capital to use in their jobs. As a result, they would be less productive, their compensation would be lower, and they would thus be less inclined to work. Those effects would increase over time as federal borrowing grew.

Increased Interest Payments to Foreign Holders of U.S. Debt

If federal debt continued to rise, the government would spend more on interest payments. Larger outlays for interest would include an increase in payments to foreign investors, who currently hold roughly one-third of all federal debt held by the public (or 45 percent of such debt not held by the Federal Reserve). The increases in interest payments to foreign investors would, in turn, reduce the nation's net international income—the difference between the nation's income (as measured by its gross national product, or GNP) and its total production (as measured by GDP).[4] Typically, the nation's net international income is positive—that is, GNP exceeds GDP. When net international income falls, national income also declines, all else being equal.

Greater Risk of a Fiscal Crisis

The likelihood of a fiscal crisis increases as federal debt continues to rise, because mounting debt could erode investors' confidence in the U.S. government's fiscal position. Such an erosion of confidence would undermine the value of Treasury securities and drive up interest rates on federal debt as investors demanded higher yields to purchase those securities. Concerns about the government's fiscal position could lead to a sudden and potentially spiraling increase in people's expectations for inflation, a large drop in the value of the dollar, or a loss of confidence in the government's ability or commitment to repay its debt in full, all of which would make a fiscal crisis more likely.

A fiscal crisis could lead to a financial crisis. In a fiscal crisis, dramatic increases in Treasury rates would reduce the market value of outstanding government securities, and the resulting losses incurred by holders of those securities—including mutual funds, pension funds, insurance companies, and banks—could be large enough to cause some financial institutions to fail. Because the United States plays a central role in the international financial system, such a crisis could spread globally.

Risk Factors. The risk of a fiscal crisis depends on factors beyond the amount of federal debt. Ultimately, it is the cost of servicing the debt and the ability to refinance it as needed that matter. Among the factors affecting those two things are investors' assessment of the outlook for the budget and the economy, which can change over time, and their expectations about domestic and international financial conditions, including interest rates and exchange rates. The relationships between those factors and the risk of a crisis are uncertain and can shift—depending, in part, on the state of the economy.

Because the risk of a fiscal crisis depends on many uncertain and shifting factors, CBO cannot quantify the probability that a fiscal crisis would occur. In CBO's assessment, no tipping point exists at which the debt-to-GDP ratio would become so high that it made a crisis likely or imminent; nor is there a fixed point at which interest costs would become so high in relation to GDP that they were unsustainable.

Risk of a Crisis in the Near Term. The risk of a fiscal crisis in the near term appears to be low despite the larger deficits and higher debt stemming from the pandemic. The near-term risk is mitigated by certain characteristics of the U.S. financial system. For example, the Federal Reserve conducts independent monetary policy, government debt is issued in U.S. dollars, the dollar holds

4. Whereas GDP is the value of all final goods and services produced within the borders of the United States (whether the labor and capital used to produce them are supplied by residents or nonresidents), GNP is the value of all final goods and services produced by labor and capital supplied by residents of the United States, regardless of where that labor and capital are located.

a central place in the global financial system, and few investments can provide returns comparable to those of Treasury securities at similarly low levels of risk.

In addition, concern about a fiscal crisis in the near term is not currently apparent in financial markets. However, financial markets do not always fully reflect risks on the horizon, and the risk of a fiscal crisis could change suddenly in the wake of unexpected events. For example, a sudden rise in interest rates could cause investors to become concerned about the government's fiscal position over the long term as their uncertainty grew about whether the rise was temporary or signaled a long-term trend.

Options for Responding to Such a Crisis. If a fiscal crisis occurred, policymakers would have several options to respond, though choosing among them would involve difficult trade-offs. One policy option would be to dramatically cut noninterest spending or increase taxes, either of which could have adverse effects on the economy in the short run.

A second option would be for the Federal Reserve to fund deficits through the purchase of Treasury securities. That option, if pursued extensively, would raise inflation—and notably so, relative to the inflation expectations that were incorporated in the interest rates on existing debt—thereby reducing the real cost of financing outstanding debt. Such an action would also put downward pressure on the value of the dollar. High inflation over an extended period could, therefore, undermine the role of the dollar in international currency markets, depending on the attractiveness of other currencies. Such a development would lead to even higher inflation and declines in real wealth and in purchasing power.

A third option would be to restructure the debt (that is, modify the contractual terms of existing obligations) so that repayment was feasible. Restructuring the debt is, however, generally viewed as less likely than the other two options because it would undermine investors' confidence in the government's commitment to repay its debt in full.

Increased Likelihood of Less Abrupt Adverse Effects

Even in the absence of an abrupt fiscal crisis, high and rising debt could have persistent adverse effects on the economy beyond those incorporated in CBO's extended baseline projections, including a gradual decline in the value of Treasury securities and other domestic assets. High and rising debt could lead to increases in people's inflation expectations. Increases in federal borrowing could also lead to an erosion of confidence in the U.S. dollar as the dominant international reserve currency. Such developments would, among other things, make it more difficult to finance public and private activity.

Greater Vulnerability of U.S. Fiscal Position to an Increase in Interest Rates

A larger amount of debt makes the United States' fiscal position more vulnerable to an increase in interest rates than it would be if the amount was smaller. Debt of the amounts in CBO's extended baseline projections increases the risk that interest costs would be substantially greater than projected—even without a fiscal crisis—if interest rates were higher than those underlying the agency's projections. (The average interest rate on federal debt in CBO's projections increases from 1.8 percent in 2022 to 3.1 percent in 2032 and to 4.2 percent in 2052.) Conversely, lower interest rates would result in interest costs that were less than those in CBO's projections. (For further discussion of the potential effects of alternative interest rates on federal debt, see Chapter 4.)

Increased Perception of Fiscal Constraints Among Policymakers

The size of budget deficits and debt could influence policymakers' choices. Policies that increase spending or reduce revenues can provide support to the economy during challenging times, such as the current pandemic. Furthermore, increased high-quality and effective federal investment—which may require the federal government to borrow more and thus result in higher deficits and debt—would boost private-sector productivity and output (though it would only partially mitigate the adverse consequences of that additional borrowing).[5]

However, if policymakers perceived that debt was already very high, they could feel constrained from using deficit-financed fiscal policy to respond to unforeseen events, promote economic activity, or further other goals. They might not feel as hindered if debt was lower (or if the increases in deficits and debt that would result

5. See Congressional Budget Office, *Effects of Physical Infrastructure Spending on the Economy and the Budget Under Two Illustrative Scenarios* (August 2021), www.cbo.gov/publication/57327, and *The Macroeconomic and Budgetary Effects of Federal Investment* (June 2016), www.cbo.gov/publication/51628.

from policy changes were smaller). High debt could also undermine national security if it compromised the international geopolitical role of the United States or if policymakers felt constrained from increasing national security spending to prepare for or respond to an international crisis.

Other Consequences

Certain risks arise from the interaction of fiscal and monetary policy implemented in response to higher debt. For example, the Federal Reserve's large-scale purchases of Treasury securities and other financial assets in response to the pandemic pose risks to the outlook for interest rates. In CBO's baseline projections, the Federal Reserve begins reducing its holdings of Treasury securities in 2022, which puts modest upward pressure on long-term interest rates. There is, however, some risk that long-term interest rates would rise rapidly. It is also possible that concern about an adverse reaction by market participants could cause the Federal Reserve to delay reducing its holdings of Treasury securities, thereby causing long-term interest rates to remain lower for longer than CBO projects.

The Size and Timing of Policy Changes Needed to Meet Various Targets for Debt

CBO estimated the size of changes in spending or revenues (or both) that would be needed if lawmakers wanted to achieve certain targets for federal debt held by the public. The agency also assessed the extent to which the size of the necessary adjustments would change if the implementation of policies aimed at reducing deficits was delayed. Finally, it examined how waiting to resolve the long-term fiscal imbalance would affect the economy and different generations of the U.S. population.

The Size of Policy Changes

The size of policy changes necessary to achieve a given debt target would depend on the level of debt that lawmakers wanted to achieve. If lawmakers wanted debt in 2052 to remain at roughly its level at the end of this fiscal year (about 100 percent of GDP), they could, for example, cut noninterest spending or raise revenues (or do both) to reduce the deficit in each year beginning in 2027 by an amount equal to 2.8 percent of GDP, which would amount to $800 billion, or about $2,400 per person, in 2027.

The changes would need to be larger if lawmakers wanted to achieve a lower debt target. For example, to reduce debt to its approximate level in 2019 (80 percent of GDP) by 2052, lawmakers would need to increase revenues or cut noninterest spending (or adopt some combination of those two actions) to reduce the deficit by an amount equal to 3.5 percent of GDP each year starting in 2027.

In those examples, the projected effects on debt include both the direct effects of the policy changes and the feedback to the federal budget that would result from faster economic growth. The policy changes examined here are illustrative, and the results do not reflect any assumptions about specific changes. Any policy change could alter productivity growth or people's incentives to work and save, which would, in turn, affect overall economic output and feed back into the federal budget.

The Timing of Policy Changes

The longer policymakers waited to address high and rising debt, the greater the policy changes required to achieve long-term objectives would be. Reducing deficits sooner would result in a smaller accumulated debt and therefore less risk to long-term economic growth and stability. But reducing deficits sooner might also lead to economic and financial disruptions if people had insufficient time to plan for or to adjust to the new measures, or if such a reduction occurred when the economy was weak. In addition, there may be favorable effects of delaying deficit reduction. If the policies that resulted in large deficits supported the economy during challenging times or increased high-quality and effective federal investment, changing those policies to reduce deficits sooner would dampen those effects.

CBO estimated the extent to which the size of the necessary policy adjustments would change if deficit reduction was delayed until 2032 or 2037. If lawmakers sought to reduce debt as a share of GDP to 80 percent in 2052 and if the necessary policy changes did not take effect until 2032, the annual reduction in the primary deficit would need to amount to 4.3 percent of GDP rather than the 3.5 percent that would accomplish the same goal if the changes were made starting in 2027. If, instead, lawmakers chose to wait until 2037 to implement the policies, even larger changes would be necessary; in that case, the required annual reduction in the primary deficit would amount to 5.7 percent of GDP.

Effects on Different Generations

How lawmakers responded to high and rising debt would determine who bore the burden of the changes in spending or taxes and who realized the economic benefits of those changes.[6] In general, if policymakers postponed fiscal tightening and if debt as a share of GDP continued to rise, future generations—who CBO projects would have higher incomes, in aggregate and on average, than earlier generations—would bear more of the burden of the changes necessary to stabilize debt. Earlier generations—particularly people in those generations who have higher income and more wealth—would bear less of the burden. Within any given generation, who bore the brunt of the burden would depend on the specific policies implemented and on how long policymakers waited to implement those policies.

Uncertainty of CBO's Long-Term Projections

The long-term budget outlook is highly uncertain. CBO's budget projections depend on the agency's economic projections and demographic projections, both of which are themselves uncertain. Even if future tax and spending policies did not vary from those specified in current law, budgetary outcomes over the next 30 years would undoubtedly differ from those in CBO's extended baseline projections because of unexpected changes in the economy and demographics, among other factors. Not only can small changes in some factors, compounding over many years, greatly affect projected budgetary outcomes decades into the future, but the pandemic's effects on long-term trends are unknown. Furthermore, the effectiveness of monetary and fiscal policy and the response of global financial markets to the substantial projected increases in federal deficits and debt are also uncertain.

Uncertainty About Budgetary Outcomes

Developments that vary from what CBO projects could lead to budgetary outcomes that are very different from those in the baseline projections. (For a discussion of how changes in economic conditions, spending, or revenues could cause budgetary outcomes to differ from those in CBO's budget projections, see Chapter 4.) That uncertainty increases in later years of the projection period because changes in the economy, demographics, and a variety of other factors are more difficult to anticipate over longer time horizons.

Moreover, outcomes will depend on future legislative action, which could increase or decrease budget deficits. For example, CBO's baseline projections reflect the scheduled expiration of several individual income tax provisions contained in Public Law 115-97 (referred to as the 2017 tax act in this report). If the scheduled expirations did not occur and, instead, current tax policies continued, much larger deficits and greater debt would result. Also, in accordance with CBO's standard procedures for projecting discretionary spending, funding provided for 2022 by the Infrastructure Investment and Jobs Act (P.L. 117-58) continues each year with adjustments for inflation in the agency's baseline projections. If, instead, only the funding amounts stated in that law were included in CBO's projections, the deficit, including the associated debt-service costs, would be smaller, and debt would be lower.

Uncertainty About the Economic Outlook

CBO's economic projections are subject to a high degree of uncertainty, including uncertainty about the course of the ongoing pandemic. In particular, projections of economic output and labor market conditions are highly uncertain: Growth in the labor force or in labor force productivity could be faster or slower than expected. Other key sources of uncertainty are future monetary policy and the path of interest rates. For example, uncertainty about the path of interest rates contributes to the uncertainty of the agency's estimates of the impact that higher deficits and debt would have on the economy.

Uncertainty About the Demographic Outlook

CBO's demographic projections for the next 30 years are subject to significant uncertainty because, compounded over many years, even small changes in rates of fertility, mortality, or net immigration—such as any long-term effects that may result from the pandemic—could greatly affect outcomes later in the projection period.[7] For example, because many immigrants are of working age, if immigration rates were higher or lower than CBO projected, the size of the labor force would be larger or smaller than it is in CBO's projections. Changes in fertility rates would have larger effects in later years of the projection period, once members of the affected generations reach working age. Changes in mortality rates, which would probably most affect the size of the older population, would result in outlays for the major health care programs and Social Security that differed from those in CBO's projections.

6. See Congressional Budget Office, *The Economic Effects of Waiting to Stabilize Federal Debt* (April 2022), www.cbo.gov/publication/57867.

7. For the agency's latest demographic projections, see Congressional Budget Office, *The Demographic Outlook: 2022 to 2052* (July 2022), www.cbo.gov/publication/57975.

Potential Developments and Their Possible Effects on the Budget

The sources of uncertainty discussed above are not the only ones associated with long-term budget projections. Other plausible but unpredictable developments could also increase or decrease federal debt in relation to CBO's projections. Such contingencies include a severe economic depression; catastrophes, such as a major natural disaster or world war; effects of climate change that are more significant than expected; or the development of previously underused natural resources.

A Severe Economic Downturn

In general, when economic output rises or falls, the federal budget is affected. For example, economic downturns can reduce revenues significantly and raise some outlays, such as those for unemployment insurance and nutrition assistance. In addition, downturns have historically prompted policymakers to enact legislation that further reduces revenues and increases federal spending—as they did during the pandemic—to help people suffering from the weak economy, to bolster the financial position of state and local governments, and to stimulate additional economic activity and employment. For instance, federal debt measured relative to the size of the economy doubled from 35 percent of GDP in 2007 to 70 percent in 2012 as a result of the financial crisis and its aftermath.

Severe economic downturns—like the Great Depression of the 1930s—are rare; for that reason and others, their size and timing cannot be readily predicted, and CBO's projections do not account for their possibility. The agency's long-term projections of output and unemployment do, however, reflect economic trends from the end of World War II to the present, a period that included several economic downturns that were not fully offset by upturns of similar magnitude.

Catastrophes or Wars

The federal government also faces implicit obligations in the case of catastrophes and could spend large sums to fight a major war. However, because such events are rare and unpredictable, it is very difficult to estimate the probability of their occurring in the future and their possible effects on the budget.

Small-scale natural and manmade disasters occur fairly often in the United States; they may seriously damage local communities and economies, but they rarely have significant, lasting impacts on the national economy.

By contrast, a catastrophe—such as another pandemic or a massive earthquake—could affect budgetary outcomes by reducing economic growth over several years or by leading to substantial increases in federal spending. For instance, federal debt measured as a percentage of GDP rose by 20 percentage points from 2019 to 2021 in response to the pandemic and the recession it brought about.

The United States' involvement in a major war could also have a significant impact on the economy and federal budget. For example, federal debt held by the public measured in relation to GDP rose by about 60 percentage points during World War II. Geopolitical events, including Russia's invasion of Ukraine, add to the uncertainty of the economic outlook, particularly the outlook for inflation.

Climate Change

In CBO's assessment, climate change will reduce GDP, on net. Some aspects of climate change will have positive effects on output in some parts of the country—warmer temperatures will increase the productivity of agricultural land in some areas by extending growing seasons, for example—but the negative effects in other areas are projected to outweigh those positive effects. Similarly, the net effects of climate change on labor productivity, labor supply, and the private sector's production costs are all expected to reduce output, on net, even though some of those factors will be positive in some instances. Other aspects of climate change are entirely negative. For instance, wildfires, floods, hurricanes, and tropical storms reduce the nation's output of goods and services by damaging and destroying buildings, equipment, and inventory.[8]

In CBO's projections, real GDP in 2052 is 1.0 percent lower than it would have been if climatic conditions from 2022 to 2052 were the same as they were at the end of the 20th century.[9] Any projection that attempts to account for the impact of climate change on the economy or on the budget is highly uncertain. CBO's projection is in the middle of a range of likely outcomes,

8. See Congressional Budget Office, *Budgetary Effects of Climate Change and of Potential Legislative Responses to It* (April 2021), www.cbo.gov/publication/57019.

9. For additional information about the methods that CBO used to estimate the effects of climate change on GDP, see Evan Herrnstadt and Terry Dinan, *CBO's Projection of the Effect of Climate Change on U.S. Economic Output*, Working Paper 2020-06 (Congressional Budget Office, September 2020), www.cbo.gov/publication/56505.

reflecting a variety of economic and scientific uncertainties. The agency also expects climate change to have various effects on the United States that are not directly reflected in economic output.

Though CBO's extended baseline projections incorporate some effects of climate change, unexpected and significant changes to the climate still pose a sizable risk to the federal budget. In the future, if weather-related disasters increased in frequency or magnitude, lawmakers could respond by increasing funding above the amounts in CBO's projections.[10] For example, increased damage from storm surges might lead the Congress to pass additional emergency supplemental appropriations for disaster relief or to approve legislation providing funding to protect infrastructure that is vulnerable to rising sea levels. Conversely, lawmakers could amend existing laws to reduce federal spending on weather-related disasters. For instance, the Congress might decide to alter flood insurance or crop insurance programs in a way that provided insured parties with a greater incentive to avoid potential damage.

Because, on net, climate change has a negative effect on the budget and the economy, successful investments in mitigation or adaptation—those that reduce the extent of climate change or its adverse consequences—can generally be expected to yield future savings to the federal budget.[11] Such savings might stem from reductions in physical damage, increases in the productivity of land and outdoor labor, or lower health care costs. Some efforts to mitigate climate change or adapt to its effects would take a long time to implement but could provide long-lasting budgetary savings in the future or provide benefits to the private sector or other governments that would not be reflected in the federal budget. Ineffective policies could, however, impose costs on federal, state, and local governments without yielding budgetary savings or other benefits to justify those costs. Determining whether a particular policy would lower harmful emissions and improve the climate trajectory can be difficult.

Advances in the Development of Natural Resources

The future discovery and development of productive natural resources may increase federal receipts. For example, advances in combining two drilling techniques, hydraulic fracturing and horizontal drilling, have allowed access to large deposits of shale resources—that is, crude oil and natural gas trapped in shale and certain other dense rock formations. Virtually nonexistent 15 years ago, the production of oil and natural gas from shale has boomed in the United States since then. The primary budgetary impact of that increase in production is an increase in federal tax revenues.[12] Advances in the development of other resources might also contribute to federal receipts by bolstering the economy and making federally owned resources more valuable.

It is impossible to predict the discovery of new natural resources or ways to extract them—particularly discoveries that would have significant effects on the economy or the federal budget. Furthermore, the effects of any such discoveries on the federal budget would depend on the natural resource in question. The effects would also depend on the amount of private investment, government regulations, and the availability of the infrastructure necessary to access and transport those resources. As a result, CBO's projections do not account for the budgetary effects of the unexpected development of natural resources.

10. See Congressional Budget Office, *Potential Increases in Hurricane Damage in the United States: Implications for the Federal Budget* (June 2016), www.cbo.gov/publication/51518.

11. See Congressional Budget Office, *Budgetary Effects of Climate Change and of Potential Legislative Responses to It* (April 2021), www.cbo.gov/publication/57019.

12. See Congressional Budget Office, *The Economic and Budgetary Effects of Producing Oil and Natural Gas From Shale* (December 2014), www.cbo.gov/publication/49815.

Chapter 2: Spending and Revenues

Overview

Under current law, spending by the federal government is projected to represent a larger percentage of gross domestic product (GDP) in coming years than it did, on average, during the past 50 years. From 1972 to 2021, total federal outlays averaged 21 percent of GDP; over the 2022–2052 period, such outlays are projected to average 26 percent of GDP (see Figure 2-1).

In CBO's projections, outlays in 2022 are 23.5 percent of GDP—less than last year's total—as federal spending in response to the coronavirus pandemic wanes.[1] Outlays increase after 2022, reaching 24.3 percent of GDP in 2032. Subsequently, total spending rises relative to the size of the economy, reaching about 30 percent of GDP in 2052. Spending has exceeded that level only once, for a three-year period during World War II. In those years, when defense spending increased sharply, total federal spending topped 40 percent of GDP. (CBO develops its extended baseline projections according to certain assumptions that are specified in law. For a discussion of those assumptions and the methods underlying the projections, see Appendix A.)

Over the 2022–2052 period, revenues measured as a percentage of GDP are projected to be higher than they have been, on average, in recent decades. Revenues averaged about 17 percent of GDP over the past 50 years. Over the next 30 years, they are projected to average about 19 percent of GDP.

In CBO's projections, revenue growth is strong in 2022, following the sharp increase in revenues observed in 2021. That strong growth results mostly from large increases in receipts from individual income taxes. From 2023 to 2025, revenues decline as a percentage of GDP as the effects of temporary factors that had boosted tax receipts in recent years fade. In 2026 and 2027, by contrast, revenues rise in relation to GDP because of changes to rules governing the individual income tax that are scheduled to occur at the end of calendar year 2025. After 2032, revenues grow faster than GDP, reaching 19.1 percent of GDP in 2052.

1. The budget projections in this report include the effects of legislation enacted through April 8, 2022, and are based on the Congressional Budget Office's economic projections. Those economic projections reflect economic developments through March 2, 2022. The projections do not include budgetary or economic effects of subsequent legislation, economic developments, administrative actions, court rulings, or regulatory changes.

Spending

Total spending comprises spending on mandatory programs, discretionary spending, and net outlays for interest. In CBO's projections, federal outlays for mandatory programs, measured as a percentage of GDP, initially fall through 2025 as pandemic-related mandatory spending continues to decline. Such outlays then rise steadily to 17.0 percent of GDP in 2052. Much of that increase is driven by growth in spending on the major health care programs (see Figure 2-2).

The response to the global pandemic has also resulted in a near-term boost to discretionary spending, though pandemic-related discretionary spending will decrease this year and next. However, in CBO's projections, the effects of the Infrastructure Investment and Jobs Act (IIJA, Public Law 117-58) somewhat offset that decrease.[2] Discretionary outlays equal 6.2 percent of GDP in 2032 and 6.0 percent in 2052.

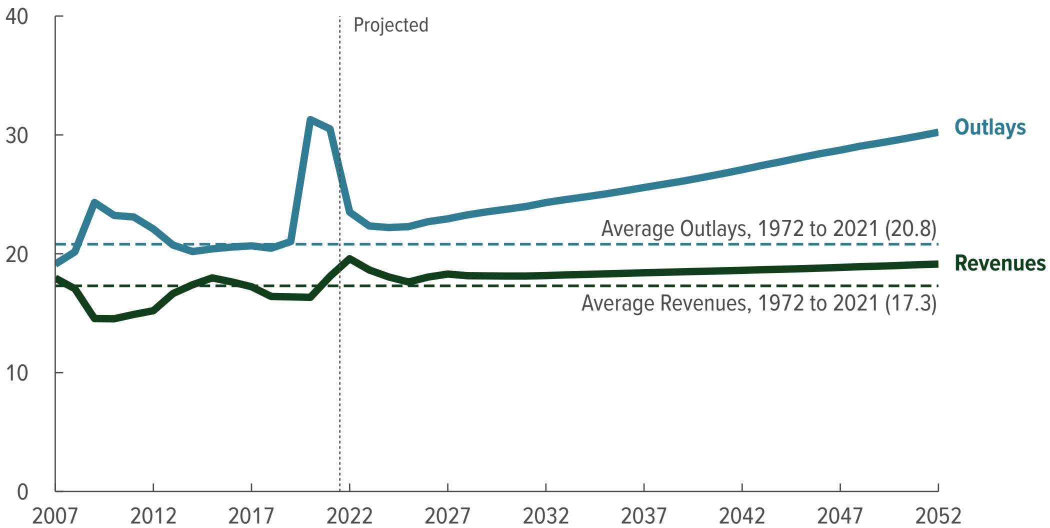
Finally, net interest costs measured as a percentage of GDP are projected to increase throughout the 2022–2052 period. Those costs increase nearly four and one-half times, from 1.6 percent of GDP in 2022 to 7.2 percent in 2052. If net interest costs followed their projected path, they would exceed all mandatory spending other than that for the major health care programs and Social Security by 2027, discretionary spending by 2047, and spending on Social Security by 2049.

2. For a discussion of how the IIJA affects CBO's projections of discretionary spending over the 2022–2032 period, see Congressional Budget Office, *The Budget and Economic Outlook: 2022 to 2032* (May 2022), pp. 76–77, www.cbo.gov/publication/57950.

Figure 2-1.

Total Outlays and Revenues

Percentage of Gross Domestic Product



In most years, growth in outlays is projected to outpace growth in revenues, resulting in widening budget deficits.

Data source: Congressional Budget Office. See www.cbo.gov/publication/57971#data.

Moreover, CBO projects that growth in spending on the major health care programs and on interest would reshape the spending patterns of the U.S. government by 2052 (see Figure 2-3). Net interest costs would account for a much greater portion of total federal spending in 2052 than they have historically, as would spending on the major health care programs.

Mandatory Spending

In CBO's extended baseline projections, the growth in spending on mandatory programs is driven primarily by increases in spending on the major health care programs and, especially in the first decade, by increases in spending on Social Security. Other mandatory spending is projected to decline in relation to GDP over the next 30 years, particularly in the first decade of that period.

Spending on the major health care programs climbs largely because, in CBO's estimation, health care costs per person will continue to rise. The aging of the population also contributes to the increases in spending on health care programs and on Social Security.

Major Health Care Programs. Outlays for the major health care programs consist of spending on Medicare, Medicaid, and the Children's Health Insurance Program (CHIP), as well as outlays to subsidize health insurance purchased through the marketplaces established under the Affordable Care Act and related spending.[3] Spending on Medicare, which provides health insurance to roughly 64 million people (about 85 percent of whom are at least 65 years old), will account for nearly half of that spending in 2022, CBO projects.

Over the past five decades, spending on the major health care programs has grown faster than the economy, and that trend persists in CBO's extended baseline projections.[4] In 2022, net federal spending on the major health care programs is estimated to equal 5.8 percent of GDP. In the agency's projections, net outlays for those programs increase to 8.8 percent in 2052.[5] Spending on

3. Federal subsidies for health insurance for low- and moderate-income households account for most of the outlays for subsidies for insurance purchased through the marketplaces and related spending. The related spending consists almost entirely of payments for risk adjustment (which are financed by funds collected from insurers with healthier enrollees and made to health insurers whose enrollees are in poorer health) and spending for the Basic Health Program (an optional state program that covers low-income residents outside the health insurance marketplaces).

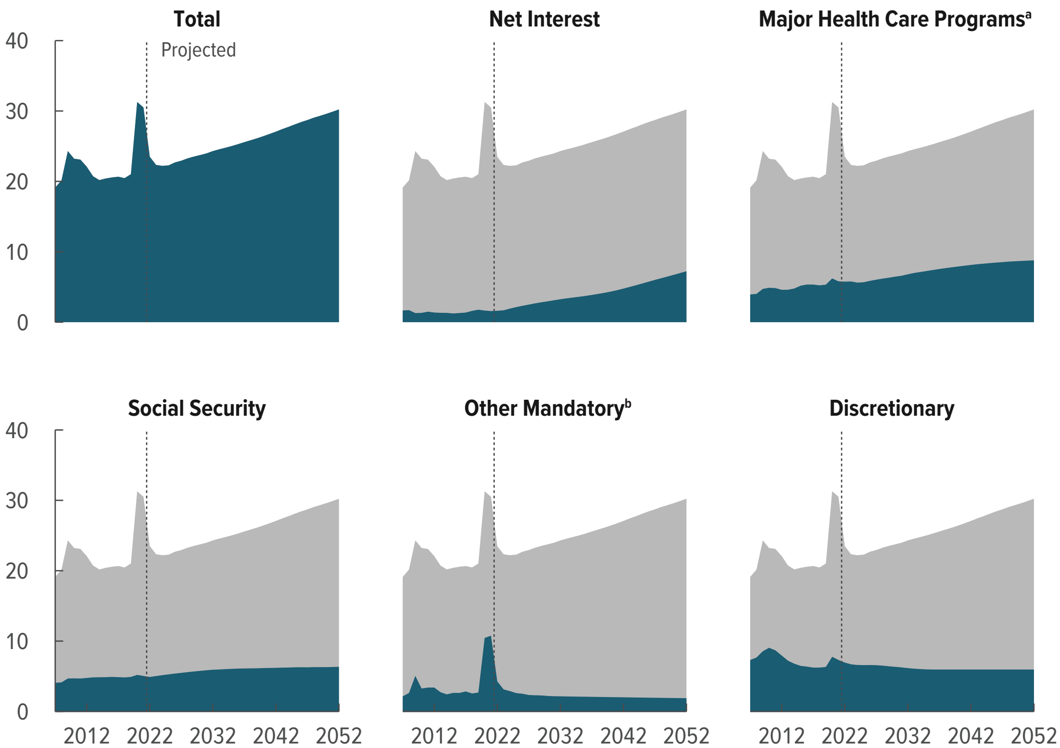
4. Since publishing *The 2021 Long-Term Budget Outlook*, CBO has refined its projections of the increase in health care costs per person. See Appendix D for a discussion of that change.

5. CBO assumes that Medicare will pay benefits as scheduled under current law (the same assumption it makes for Social Security), regardless of the amounts in the program's trust funds.

Figure 2-2.

Outlays, by Category

Percentage of GDP



Over the long term, net outlays for interest and spending on the major health care programs and Social Security are projected to rise in relation to GDP; other spending, in total, is projected to decline.

Data source: Congressional Budget Office. See www.cbo.gov/publication/57971#data.

GDP = gross domestic product.

a. Consists of spending on Medicare (net of premiums and other offsetting receipts), Medicaid, and the Children's Health Insurance Program, as well as outlays to subsidize health insurance purchased through the marketplaces established under the Affordable Care Act and related spending.

b. Consists of all mandatory spending other than that for Social Security and the major health care programs. "Other Mandatory" includes the refundable portions of the earned income tax credit, the child tax credit, and the American Opportunity Tax Credit.

Medicare (net of offsetting receipts, which are mostly premiums paid by enrollees) is the primary driver of that increase; such spending, measured as a percentage of GDP, grows by 2.9 percentage points. Spending on Medicaid and CHIP, combined with outlays to subsidize health insurance purchased through the marketplaces established under the Affordable Care Act and related spending, grows by 0.1 percentage point (see Figure 2-4).

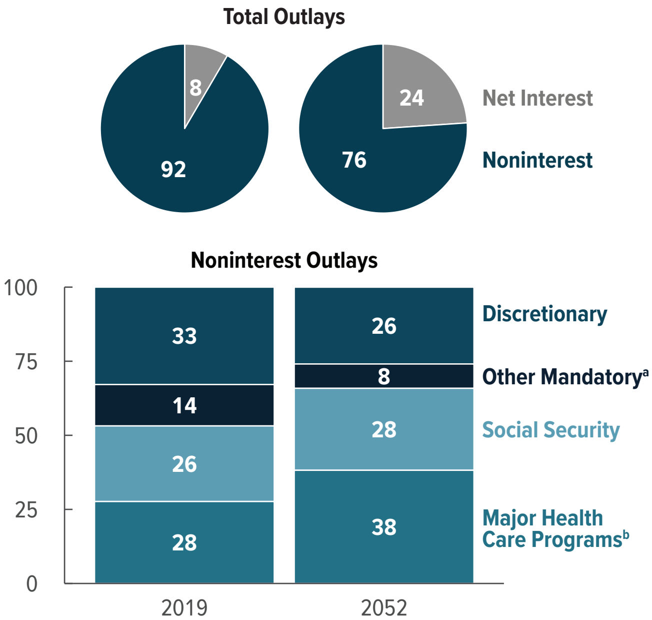
Social Security. In CBO's projections, spending on Social Security increases as a percentage of GDP over the next 30 years, continuing the trend of the past five decades. The number of Social Security beneficiaries rises from 66 million (or one-fifth of the population) in 2022 to 77 million in 2032 and then to 97 million (or over one-quarter of the projected population) in 2052. Spending on the program increases from 4.9 percent of GDP in 2022 to 5.9 percent in 2032. Spending continues to increase but slows along with the pace of population growth as members of the large baby-boom generation die; spending on Social Security reaches 6.4 percent of GDP in 2052.[6]

6. Those projections reflect the assumption that Social Security will continue to pay benefits as scheduled under current law, regardless of the status of the program's trust funds. That approach is consistent with a statutory requirement that CBO's 10-year baseline projections reflect the assumption that funding for such programs is adequate to make all payments required by law. See sec. 257(b)(1) of the Balanced Budget and Emergency Deficit Control Act of 1985, P.L. 99-177 (codified at 2 U.S.C. §907(b)(1) (2016)). The baby-boom generation comprises people born between 1946 and 1964.

Figure 2-3.

Composition of Outlays, 2019 and 2052

Percent



Net outlays for interest would, under current law, account for a greater portion of total federal outlays in 2052 than they did in 2019 (before the coronavirus pandemic), and spending on the major health care programs would account for a much larger share of all federal noninterest spending.

Data source: Congressional Budget Office. See www.cbo.gov/publication/57971#data.

a. Consists of all mandatory spending other than that for Social Security and the major health care programs. "Other Mandatory" includes the refundable portions of the earned income tax credit, the child tax credit, and the American Opportunity Tax Credit.

b. Consists of spending on Medicare (net of premiums and other offsetting receipts), Medicaid, and the Children's Health Insurance Program, as well as outlays to subsidize health insurance purchased through the marketplaces established under the Affordable Care Act and related spending.

The Social Security program is funded by dedicated tax revenues from two sources. Currently, 96 percent of the funding comes from a payroll tax; the rest is collected from income taxes on Social Security benefits. Revenues from the payroll tax and the income tax on benefits are credited to the Old-Age and Survivors Insurance (OASI) Trust Fund and the Disability Insurance (DI) Trust Fund, which finance the program's benefits. In CBO's extended baseline projections, dedicated tax revenues for the combined trust funds decline from 4.5 percent of GDP in 2022 to 4.4 percent in 2052.

A commonly used measure of Social Security's financial position is the dates by which the trust funds would be exhausted. CBO projects that the OASI trust fund would be exhausted in calendar year 2033 and that the DI trust fund would be exhausted in calendar year 2048. If their balances were combined, the Old-Age, Survivors, and Disability Insurance (OASDI) trust funds would be exhausted in calendar year 2033. CBO estimated the amounts that annual benefits would have to be reduced by for the trust funds' outlays to match their revenues in each year after the two funds were exhausted. Benefits would need to be reduced (in relation to the agency's baseline projections) by about 25 percent in 2034, an amount that would climb to about 30 percent in 2052.

Other Mandatory Programs. Before the pandemic, mandatory spending excluding that for the major health care programs and Social Security had generally remained between 2 percent and 4 percent of GDP since the mid-1960s (it was 2.7 percent of GDP in 2019, for example). Such spending includes outlays for the Supplemental Nutrition Assistance Program (SNAP), unemployment compensation, retirement programs for federal civilian and military employees, certain programs for veterans, Supplemental Security Income, and certain refundable tax credits.[7] That spending increased significantly in 2020 and 2021—to 10.4 percent and 10.8 percent of GDP, respectively—mainly because of policies enacted in response to the pandemic and the associated economic downturn.

In CBO's projections, other mandatory spending totals 4.3 percent of GDP in 2022. It then generally declines as a share of the economy, reaching 2.2 percent of GDP in 2032 and 1.9 percent in 2052.[8] The projected decline through 2032 occurs in part because the amounts of benefits for many of the programs are adjusted for inflation each year, and in CBO's economic forecast, inflation is projected to be less than the rate of growth in nominal GDP. The decline from 2032 to 2052 is partly attributable to growth in income, which decreases the number of people who qualify for refundable tax credits.

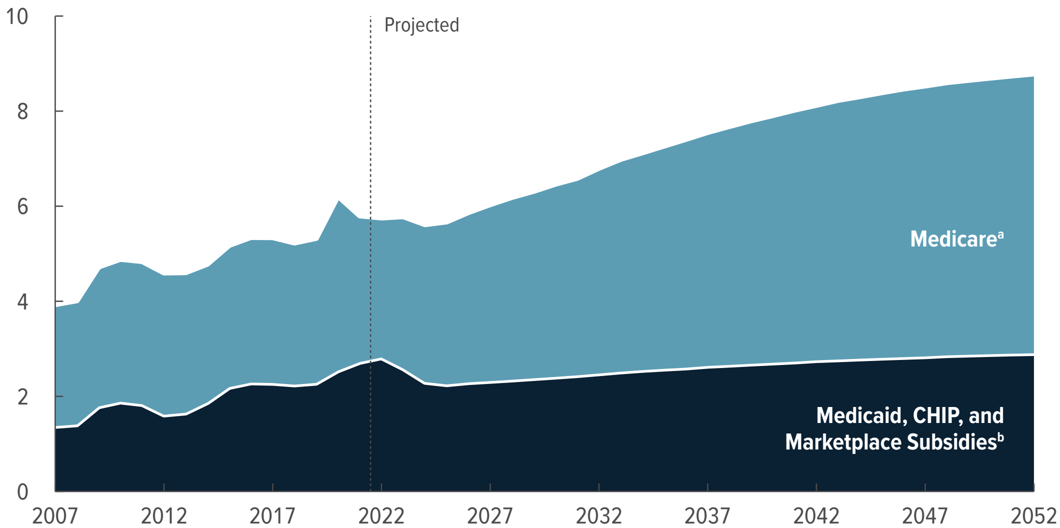
7. Refundable tax credits reduce a filer's overall income tax liability; if the credit exceeds the filer's income tax liability, the government pays all or some portion of that excess to the taxpayer (and the payment is treated as an outlay in the budget). See Congressional Budget Office, *Refundable Tax Credits* (January 2013), www.cbo.gov/publication/43767.

8. Sec. 257(b)(2) of the Deficit Control Act, which governs CBO's baseline projections, makes exceptions regarding current law for some programs, such as SNAP, that have expiring authorizations but that are assumed to continue as currently authorized.

Figure 2-4.

Composition of Outlays for the Major Health Care Programs

Percentage of Gross Domestic Product



Spending on Medicare is projected to account for more than four-fifths of the increase in spending on the major health care programs over the next 30 years.

Data source: Congressional Budget Office. See www.cbo.gov/publication/57971#data.

CHIP = Children's Health Insurance Program.

a. Net of premiums and other offsetting receipts.

b. "Marketplace Subsidies" refers to outlays to subsidize health insurance purchased through the marketplaces established under the Affordable Care Act and related spending.

Causes of Growth in Mandatory Spending. Rising health care costs per person and the aging of the population are the primary reasons for the sharp rise in projected spending on the major health care programs over the next 30 years. The aging of the population also contributes to the increase in spending on Social Security.

In CBO's estimation, if, over the 2022–2052 period, health care costs per person (adjusted for demographic changes) grew at the rate of potential GDP per person—which would mean that costs grew more slowly than the agency currently projects—and the population was not aging, then spending on the major health care programs would be 6.4 percent of GDP in 2052, or 0.3 percentage points lower than the agency currently projects for 2022.[9] And if the effects of the aging of the population alone were excluded, then spending on Social Security would be 4.9 percent of GDP in 2052, the same as the agency projects for 2022 (see Figure 2-5).

Rising Health Care Costs per Person. The growth of health care costs per person has recently slowed. However, in CBO's extended baseline projections, such costs, adjusted for demographic changes, continue to grow faster than potential GDP per person—0.9 percent faster for Medicare and 0.9 percent faster for Medicaid, on average—over the second and third decades of the projection period. That growth in health care costs per person accounts for about two-thirds of the increase in spending, measured as a percentage of GDP, on the major health care programs between 2022 and 2052.

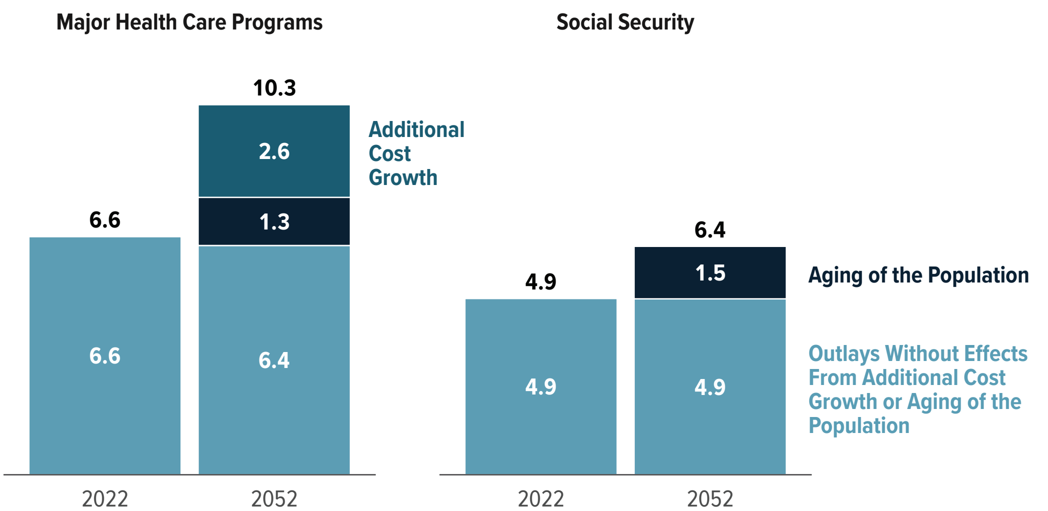
Aging of the Population. Over the 2022–2052 period, about one-third of the projected increase in total spending on the major health care programs, measured as a percentage of GDP, is attributable to the aging of the population and is mostly the result of increased spending on Medicare. (See Figure 3-1 on page 26 for CBO's projections of the population by age group.) That is because Medicare is the largest of the major health care programs, and most beneficiaries qualify for it at age 65. As the group of people who qualify for Medicare becomes larger and, on average, older, Medicare spending will increase, not only because the number of beneficiaries will rise but also because spending on health care tends to increase as people age.

9. Potential GDP is the maximum sustainable output of the economy. The analysis of the causes of the growth in spending on the major health care programs encompasses gross spending on Medicare and does not reflect receipts credited to the program from premiums and other sources.

Figure 2-5.

Composition of Growth in Outlays for the Major Health Care Programs and Social Security, 2022 to 2052

Percentage of GDP



Much of the growth in spending on the major health care programs and on Social Security results from the aging of the population. Growth in spending on the major health care programs is also driven by cost growth above and beyond that accounted for by demographic changes or the growth of potential GDP per person.

Data source: Congressional Budget Office. See www.cbo.gov/publication/57971#data.

The spending on the major health care programs examined here consists of gross spending on Medicare (which does not account for premiums or other offsetting receipts), Medicaid, and the Children's Health Insurance Program, as well as outlays to subsidize health insurance purchased through the marketplaces established under the Affordable Care Act and related spending.

Additional cost growth is the extent to which the growth rate of nominal health care spending per person (adjusted for demographic changes) exceeds the growth rate of potential GDP per person. Potential GDP is the maximum sustainable output of the economy.

GDP = gross domestic product.

From 2022 to 2052, all of the projected increase in spending on Social Security, measured as a percentage of GDP, is attributable to the aging of the population. The effects of the aging of the population, which push spending on Social Security up, are offset by scheduled increases in the full retirement age for Social Security, which reduce the lifetime benefits for affected beneficiaries and thus push spending down.[10]

Discretionary Spending

About 45 percent of all discretionary spending is dedicated to national defense, and the rest is for an array of federally funded activities and programs, including education, transportation, housing assistance, veterans' health care, health-related research and public health programs, the administration of justice, and international affairs.

In the half-century preceding the pandemic, discretionary outlays decreased significantly in relation to the size of the economy, from 11.5 percent of GDP in 1970 to 6.3 percent in 2019. In 2020 and 2021, such outlays were boosted to 7.8 percent and 7.3 percent of GDP, respectively, by policies put in place to counter the pandemic-related economic disruption. In the agency's projections, discretionary outlays generally decrease as a percentage of GDP, from 7.0 percent in 2022 to 6.0 percent in 2036. After that, CBO's extended baseline projections reflect the assumption that discretionary spending remains constant at 6.0 percent of GDP through 2052.

Net Interest Costs

Over the past 50 years, the government's net interest costs have ranged from 1.2 percent to 3.2 percent of GDP, averaging 2.0 percent of GDP. In CBO's projections, net interest costs are 1.6 percent of GDP in 2022. By 2032, those costs double, reaching 3.3 percent of GDP, as federal debt grows and interest rates rise.

10. For more details about the full retirement age for Social Security, see Zhe Li, *The Social Security Retirement Age,* Report R44670, version 14 (Congressional Research Service, July 6, 2022), https://go.usa.gov/xGnEx.

Net interest costs continue to rise thereafter, reaching 7.2 percent of GDP in 2052. They would be higher in that year than spending on Social Security, discretionary spending, or all mandatory spending other than that for the major health care programs and Social Security—and higher than at any time since at least 1940 (the first year for which the Office of Management and Budget reports such data).[11]

The increase in interest payments is the result of escalating interest rates and the rising level of debt. CBO estimates that the increase in interest rates accounts for about one-half of the projected growth in net outlays for interest over the 2022–2052 period.[12]

Revenues

In CBO's projections, revenues measured as a percentage of GDP are generally higher over the next 30 years than they have been, on average, in recent decades.[13] Revenues averaged 17.3 percent of GDP over the past 50 years, but they fluctuated between 14.5 percent and 20.0 percent of GDP over that period because of changes in tax laws and interactions between those laws and economic conditions. Over the 2022–2052 period, revenues average 18.6 percent of GDP in the agency's projections.

Projected Revenues

In CBO's projections, the strong growth in federal revenues seen in 2021 continues temporarily as revenues equal 19.6 percent of GDP in 2022, one of the highest levels ever recorded. That strong growth is largely the result of sizable increases in collections of individual income taxes. From 2023 to 2025, revenues decline as a percentage of GDP as the effects of temporary factors that boosted tax receipts in 2021 and 2022 fade. In 2026 and 2027, by contrast, revenues rise in relation to GDP because of changes to rules governing the individual income tax that are scheduled to occur at the end of calendar year 2025. In the agency's extended baseline projections, revenues grow faster than the economy after 2032, totaling 19.1 percent of GDP in 2052.

From 2022 to 2052, total revenues, measured as a percentage of GDP, fall by one-half of one percentage point in CBO's projections. Receipts from corporate income taxes and payroll taxes decline by a small amount over that period (by 0.3 percent of GDP and 0.2 percent of GDP, respectively). Receipts of individual income taxes increase slightly, on net, reflecting offsetting factors. Such receipts initially fall from their highs seen in 2022 as the effects of the temporary factors that had boosted tax receipts dissipate; but they then resume their growth, ending the 30-year projection period at 10.7 percent of GDP—one-tenth of one percentage point higher than their value in 2022 (see Figure 2-6).

Factors Affecting Revenues

The small projected decline in total revenues as a percentage of GDP over the next 30 years is the result of several factors whose effects are largely offsetting. Real bracket creep, scheduled changes to tax rules, and faster earnings growth for higher-earning people (who are taxed at higher individual income tax rates) are among the factors that cause revenues to increase. But the near-term boost to tax receipts dissipates, which, along with growing health care costs, causes revenues to decrease (see Figure 2-7).

End of the Temporary Boost to Tax Receipts. In CBO's estimation, some of the causes of the recent jump in individual income tax receipts will dissipate, reducing revenues as a percentage of GDP from 2023 to 2025. First, a pandemic-related tax provision allowed some taxes due in 2020 and 2021 to be deferred until 2022 and 2023. That provision boosts tax receipts in 2022 and 2023 but will have no effect thereafter, causing receipts to drop.

11. Since publishing *The 2021 Long-Term Budget Outlook*, CBO has refined its projections of the average interest rate on federal debt held by the public. See Appendix D for a discussion of that change.

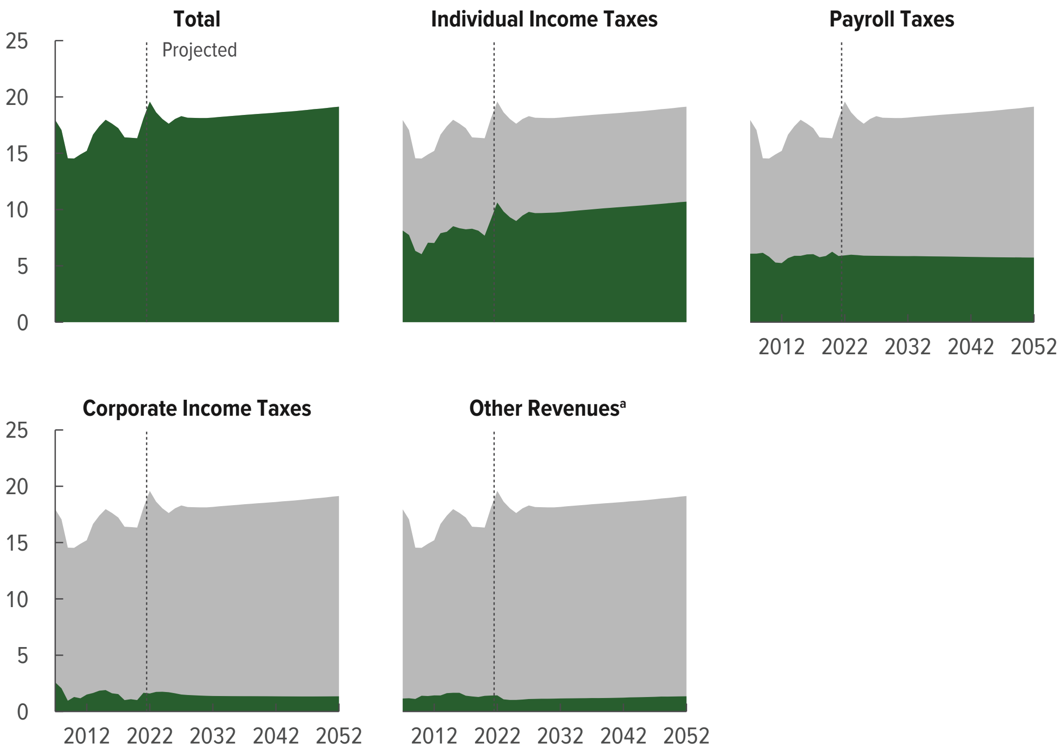
12. The agency estimated the contribution of rising interest rates to net interest costs by keeping interest rates on marketable debt held by the public at their values at the end of 2021. In that scenario, the average interest rate on federal debt gradually declines to 1.3 percent in 2052 instead of rising to 4.2 percent in 2052, as CBO currently projects. That analysis accounts only for the reduction in interest payments from lower interest rates and the reduction in federal borrowing from smaller deficits. It does not account for the economic effects that would result from lower interest rates or less federal borrowing.

13. CBO's revenue projections are based on the assumption that the rules for all tax sources (individual income taxes, corporate income taxes, payroll taxes, and other taxes) will change as scheduled under current law. The sole exception is expiring excise taxes dedicated to trust funds. The Deficit Control Act requires CBO's baseline to reflect the assumption that those taxes would be extended at their current rates. That law does not stipulate that the baseline include the extension of other expiring tax provisions, even if lawmakers have routinely extended them in the past.

Figure 2-6.

Revenues, by Source

Percentage of Gross Domestic Product



Taken together, receipts from all sources of revenues are projected to be about the same percentage of GDP in 2052 as they are today. Receipts from individual income taxes—the largest source of federal revenues—are projected to continue the strong growth observed in 2021 this year, mostly because of temporary factors whose effects quickly subside. Those receipts are projected to rise again after 2025 because of the scheduled expiration of some provisions of the 2017 tax act.

Data source: Congressional Budget Office. See www.cbo.gov/publication/57971#data.

a. Consists of excise taxes, remittances to the Treasury from the Federal Reserve System, customs duties, estate and gift taxes, and miscellaneous fees and fines.

Second, various types of taxable income are projected to decline as a percentage of GDP in the near term. The most notable declines are in the realizations of capital gains and in wages and salaries.

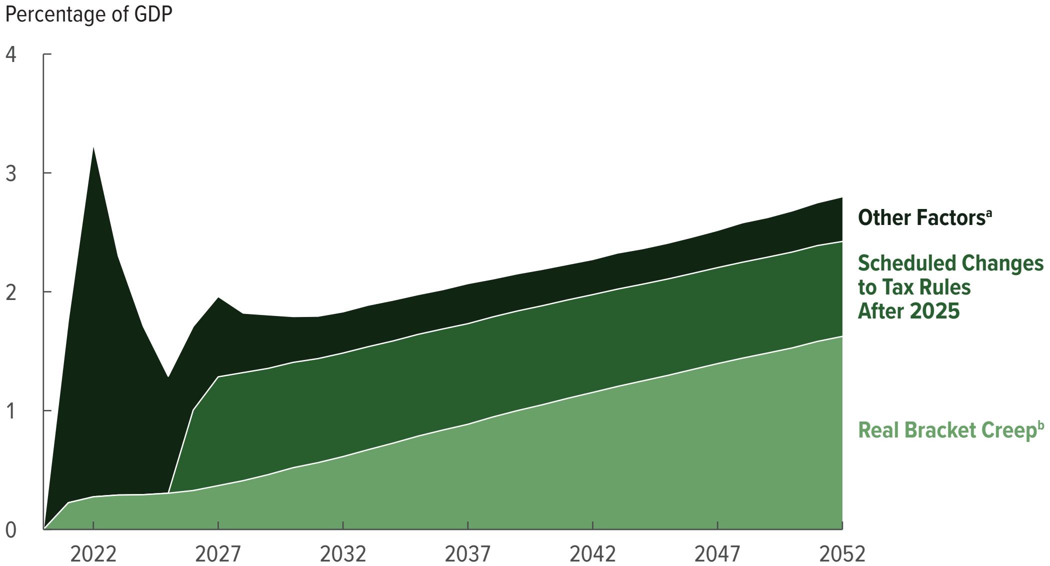
Third, and most significant, the strength of recent tax receipts is projected to gradually dissipate over the next few years, thus better reflecting the past relationship between tax revenues and the state of the economy. The source of that recent strength in individual income tax receipts is uncertain. Receipts in the past few years have been larger than expected given currently available data on economic activity. Those larger-than-anticipated receipts may result from several factors whose effects could persist, end, or reverse.

Real Bracket Creep. The income thresholds for the various tax rate brackets in the individual income tax are indexed to increase with inflation (as measured by the chained consumer price index for all urban consumers as published by the Bureau of Labor Statistics). If income grows faster than prices—as typically happens during economic expansions—more income is pushed into higher tax brackets, even when the underlying distribution of income remains unchanged. That process is known as real bracket creep and is the largest source of growth in total revenues over the next three decades. If current laws generally remained unchanged, real bracket creep would continue to gradually push up taxes in relation to income through 2052, CBO projects, thereby increasing tax receipts. From 2032 to 2052, the share of income taxed at the top rate of 39.6 percent would rise by 1 percentage point—and the share of income excluded from taxation would fall by 3 percentage points—because of real bracket creep (see Figure 2-8).[14]

14. For more details, see Congressional Budget Office, "How Income Growth Affects Tax Revenues in CBO's Long-Term Budget Projections" (June 2019), www.cbo.gov/publication/55368.

Figure 2-7.

Composition of Changes in Revenues, 2020 to 2052



Over the long term, the largest source of growth in tax revenues is real bracket creep. Revenues as a percentage of GDP are projected to rise in 2022 as a result of temporary increases in the collections of individual income taxes. Projected revenues rise sharply after certain temporary provisions of the 2017 tax act expire at the end of 2025.

Data source: Congressional Budget Office. See www.cbo.gov/publication/57971#data.

GDP = gross domestic product.

a. Other factors include an end to the temporary boost to tax receipts as recent strength in collections dissipates, as well as factors that affect revenues over the longer term, such as changes in the distribution of wages and growth in nontaxable compensation resulting from health care costs.

b. Real bracket creep is the process in which, as income rises faster than inflation, a larger proportion of income becomes subject to higher tax rates, even when the underlying distribution of income remains unchanged.

Scheduled Changes to Tax Rules. The most significant factor pushing up taxes in relation to income is the scheduled expiration, after calendar year 2025, of nearly all provisions of the 2017 tax act (P.L. 115-97) that affect individual income taxes. The expiring provisions include lower statutory tax rates, the higher standard deduction, the repeal of personal exemptions, and the expansion of the child tax credit. The scheduled changes to tax rules after 2025 would cause tax liabilities to rise in calendar year 2026. Those changes boost revenues as a share of GDP by 0.8 percentage points, on average, after 2025.

Other Factors. Two other factors affect revenues—but to a lesser extent—in CBO's projections. The first factor is the growth in health care costs, which is projected to reduce revenues as a percentage of GDP over the next three decades. The share of employees' compensation that is paid in the form of spending on fringe benefits, such as employment-based health insurance, is projected to increase, and those benefits are not taxable. Conversely, the share of employees' compensation that is paid in the form of wages and salaries, which are subject to income and payroll taxes, is projected to decline. That shift in compensation would decrease taxable income—and thus revenues from both income and payroll taxes—in relation to GDP.

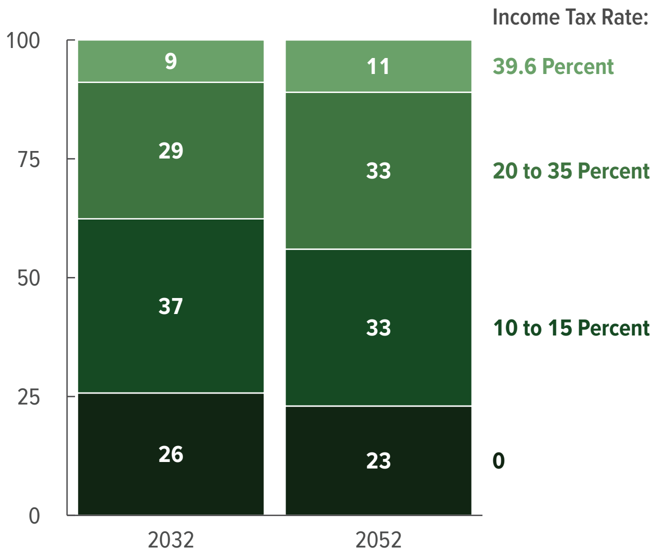
The second factor is the change in the distribution of earnings. Earnings are projected to grow faster for higher-earning people than for other people in the long term. That trend would cause a larger share of individual earnings to be taxed at higher rates. However, the resulting increase in individual income tax revenues would be largely offset by a decrease of nearly the same amount in payroll tax receipts, CBO projects, because the share of earnings above the maximum amount subject to Social Security payroll taxes would grow.[15]

15. For additional information, see Brooks Pierce, *How Changes in the Distribution of Earnings Affect the Federal Deficit*, Working Paper 2021-12 (Congressional Budget Office, October 2021), www.cbo.gov/publication/57217.

Figure 2-8.

Shares of Income Taxed at Different Rates Under the Individual Income Tax System

Percent



The largest contributor to growth in projected revenues over the long term is real bracket creep—the process in which, as income rises faster than inflation, a larger proportion of income becomes subject to higher tax rates. As the share of income taxed at higher rates grows, the share exempt from taxation shrinks.

Data source: Congressional Budget Office. See www.cbo.gov/publication/57971#data.

In this figure, income refers to adjusted gross income—that is, income from all sources not specifically excluded by the tax code, minus certain deductions. The income tax rate is the statutory rate specified under the individual income tax system. The lowest statutory tax rate is zero (because of deductions and exemptions).

Chapter 3: Long-Term Demographic and Economic Projections

Overview

Demographic and economic trends are key determinants of the long-term budget outlook. By the Congressional Budget Office's estimates, the population will grow more slowly over the next 30 years than it did over the past 30 years, and it will get older, on average. The economy is projected to grow more slowly over the next three decades than it has over the past three, and interest rates are expected to rise significantly.

Demographic Projections

The size and age profile of the U.S. population affects the nation's economy and the federal budget. For example, those two factors help determine the number of people in the labor force and thus affect both gross domestic product (GDP) and federal tax receipts. Those factors also help determine the number of beneficiaries of Social Security and other federal programs and thus federal outlays.

CBO estimates the population in future years by projecting rates of fertility, net immigration, and mortality. In the agency's projections, the population increases from 335 million people at the beginning of 2022 to 369 million people at the beginning of 2052—an average expansion of 0.3 percent per year (see Figure 3-1).[1] That rate is one-third the average annual rate of growth over the past 30 years (0.9 percent). Moreover, as fertility remains lower than necessary for a generation to replace itself, population growth is increasingly driven by immigration, which by 2043 accounts for all population growth.

The proportion of the population that is age 65 or older expands over the coming decades, continuing a long-standing historical trend. By 2052, 22 percent of the population will be 65 or older; today, that proportion is 17 percent.

Economic Projections

The state of the U.S. economy in coming decades will affect the federal government's budget deficits and debt. Key to the agency's long-term budget projections are its long-term projections of GDP, interest rates, and inflation. Among the factors incorporated into the agency's long-term economic forecast are the effects of projected deficits on private investment and the effects of marginal tax rates on the supply of labor and private saving.

Real Potential Gross Domestic Product

In CBO's extended baseline projections, the growth of real potential GDP (the maximum sustainable output of the economy, adjusted to remove the effects of inflation) over the next 30 years is slower than it has been over the past 30 years (see Figure 3-2).[2] From 2022 to 2052, real potential GDP increases at an average rate of 1.7 percent per year; from 1992 to 2021, it grew at an average annual rate of 2.4 percent.

That slower growth in real potential GDP is primarily attributable to slower growth in the potential labor force (that is, the labor force adjusted to account for fluctuations in the business cycle). Whereas the potential labor force grew by an average of 0.9 percent per year over the past 30 years, in CBO's projections it grows by an average of 0.3 percent per year through 2052. Slowing population growth and the aging of the population account for most of that slowdown.

An additional factor contributing to real potential GDP's slower growth is that potential labor force productivity (that is, potential output per member of the potential labor force) is projected to grow more slowly over the next 30 years than it did over the past 30 years. In CBO's projections, potential labor force productivity grows at an average annual rate of 1.3 percent from 2022 to 2052; over the past 30 years, it grew at an average annual rate of 1.5 percent.

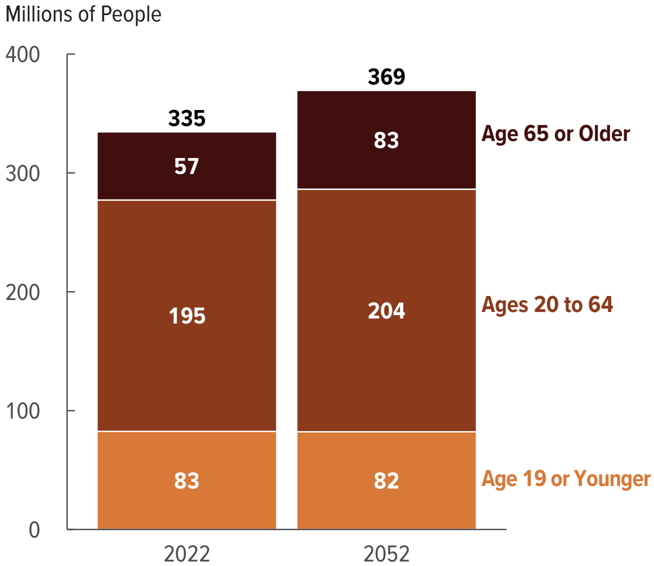
The slightly slower growth in potential labor force productivity is, in turn, driven by two key factors. First, the accumulation of capital—structures and equipment, intellectual property products (such as computer software), and residential housing, for example—per worker

1. See Congressional Budget Office, *The Demographic Outlook: 2022 to 2052* (July 2022), www.cbo.gov/publication/57975.

2. For a more detailed discussion of the economic projections, see Appendix B.

Figure 3-1.

U.S. Population, by Age Group



CBO projects that over the next three decades, the population will become older, on average, as the share of the population age 65 or older continues to grow.

Data source: Congressional Budget Office. See www.cbo.gov/publication/57971#data.

is projected to be slower over the next three decades than it has been in the past, in part because increased federal borrowing is projected to reduce private investment. (See Chapter 1 for details.)

The second reason the growth of potential labor force productivity slows is that total factor productivity (TFP)—that is, the real output per unit of combined labor and capital in the nonfarm business sector—is also expected to grow more slowly over the next 30 years than it did over the past 30 years (although the growth of TFP is projected to accelerate from its historically slow rate in recent years). That slower growth in TFP is attributable to several factors, including the following:

- A slowdown in the growth of workers' educational attainment,
- Reductions in federal investment relative to the size of the economy, and
- Climate change.[3]

Growth in real potential GDP slows over the three decades in the projection period, from an annual average of 1.8 percent over the 2022–2032 period to an average of 1.5 percent over the 2043–2052 period. That decrease is attributable primarily to falling potential labor force productivity. From 2022 to 2032, potential labor force productivity is expected to grow at an average annual rate of 1.5 percent. Over the third decade of the projection period, that growth is expected to average 1.2 percent. That slowing growth of potential labor force productivity stems primarily from increased federal borrowing, which is projected to reduce private investment below what it otherwise would be and lower the rate of capital accumulation.

Real Gross Domestic Product

In CBO's projections, real GDP grows slightly faster than real potential GDP, on average, over the next decade—at an average annual rate of 1.9 percent. The growth rate of real GDP converges with the growth rate of real potential GDP in the second half of the decade, and the level of real GDP stays about 0.5 percent below the level of real potential GDP thereafter. That gap reflects the agency's assessment that real GDP falls short of real potential GDP by a larger amount and for longer during and after economic downturns than actual output exceeds potential output during economic expansions.[4]

Nominal Gross Domestic Product

Nominal GDP grows by 9.3 percent this year in CBO's projections. After 2022, an easing of upward pressure on prices and the same factors that slow the growth of real GDP slow the growth of nominal GDP. From 2023 to 2026, the annual growth of nominal GDP averages 4.1 percent. As is the case with real GDP, the growth rate of nominal GDP converges with the growth rate of nominal potential GDP in the second half of the first decade of the projection period.

Interest Rates

CBO expects interest rates to rise throughout the projection period but to remain lower than they have been, on

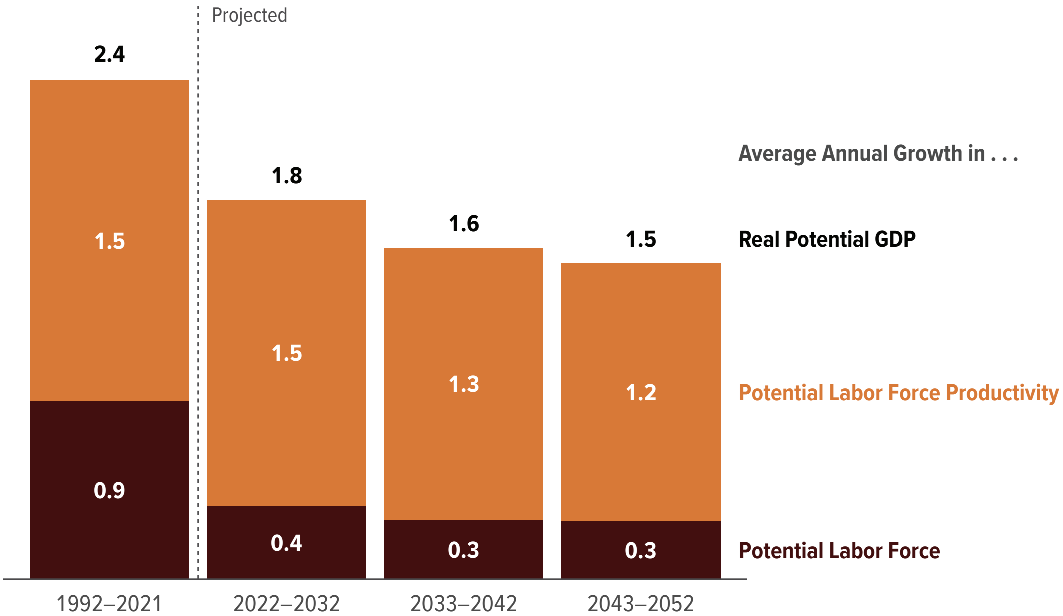
3. See Evan Herrnstadt and Terry Dinan, *CBO's Projection of the Effect of Climate Change on U.S. Economic Output*, Working Paper 2020-06 (Congressional Budget Office, September 2020), www.cbo.gov/publication/56505.

4. One recent study explains the existence of a persistent output gap by examining asymmetric fluctuations in the noncyclical rate of unemployment (that is, the rate that results from all sources except fluctuations in aggregate demand). See Stéphane Dupraz, Emi Nakamura, and Jón Steinsson, *A Plucking Model of Business Cycles*, Working Paper 748 (Banque de France, January 2020), https://tinyurl.com/1njkmzkf. CBO assessed the persistent output gap in an earlier report. See Congressional Budget Office, *Why CBO Projects That Actual Output Will Be Below Potential Output on Average* (February 2015), www.cbo.gov/publication/49890.

Figure 3-2.

Composition of the Growth of Real Potential GDP

Percent



Growth in real potential GDP is projected to be slower over the next 30 years than it was over the past 30 years. That slowdown occurs mostly because the potential labor force is projected to grow more slowly than it has in the past. Within the 2022–2052 period, the growth in real potential GDP slows primarily because the growth of potential labor force productivity slows in those years.

Data source: Congressional Budget Office. See www.cbo.gov/publication/57971#data.

Real values are nominal values that have been adjusted to remove the effects of inflation. Potential GDP is the maximum sustainable output of the economy. The potential labor force is the labor force (that is, the number of people in the civilian noninstitutionalized population who are age 16 or older and who have jobs or who are available for work and are actively seeking jobs) adjusted to remove the effects of fluctuations in the business cycle. Potential labor force productivity is the ratio of real potential GDP to the potential labor force. The sum of growth in the potential labor force and growth in potential labor force productivity is equal to growth in real potential GDP.

GDP = gross domestic product.

average, over the past three decades. In CBO's projections, the interest rate on 10-year Treasury notes rises to 3.8 percent in 2032 and to 4.6 percent in 2052—about one percentage point below the 5.4 percent average recorded over the 1995–2004 period.[5] Several factors, including slower growth of the labor force and of productivity than in the past, keep interest rates in the period below their historical averages; the effects of those factors outweigh the effects of rising federal debt and other factors that tend to push interest rates above historical rates.[6]

The average interest rate on all federal debt held by the public tends to be lower than the rate on 10-year Treasury notes. (Interest rates on shorter-term debt are generally lower than those on longer-term debt because shorter-term debt is less risky; the average term to maturity for federal debt has been less than 10 years since the 1950s.) In CBO's projections, the average interest rate on federal debt is 3.1 percent in 2032 and 4.2 percent in 2052. Over the 2022–2052 period, that rate is an average of 0.7 percentage points lower than the interest rate on 10-year Treasury notes. (Since last year, CBO has refined the methods that it uses to project the average

5. The 1995–2004 period was chosen for comparison for several reasons. In those years, expectations for inflation were stable, there were no severe economic downturns or significant financial crises, and monetary policy was, according to CBO's estimates, neutral, on average—that is, the real federal funds rate (the interest rate that financial institutions charge each other for overnight loans of their monetary reserves) was generally consistent with the economy's operating at full employment.

6. See Edward N. Gamber, *The Historical Decline in Real Interest Rates and Its Implications for CBO's Projections,* Working Paper 2020-09 (Congressional Budget Office, December 2020), www.cbo.gov/publication/56891.

interest rate on all federal debt. For a discussion of those changes, see Appendix D.)

Inflation

In CBO's extended baseline projections, inflation in the consumer price index for all urban consumers falls to its 30-year historical average of 2.3 percent in 2024. That decline is attributable to reduced disruptions in supply chains, slower growth in the prices of goods, and actions taken by the Federal Reserve to rein in inflation by reducing monetary accommodation.[7] After 2024, inflation remains roughly at its historical average through the rest of the 30-year projection period. Inflation in the GDP price index—a measure of prices in the overall economy (rather than just consumer prices) that is used to derive real GDP from nominal GDP—follows a similar path over the next few years, falling to 2.0 percent, its 30-year historical average, in 2025.

Effects of Fiscal Policy in CBO's Economic Projections

CBO's economic projections incorporate the effects of projected federal deficits under current law. In those budget projections, deficits grow, and as a result, the federal government borrows more. That increase in federal borrowing pushes up interest rates and thus reduces private investment in capital, causing output to be lower in the long term than it would be otherwise, especially in the last two decades of the projection period. Less private investment reduces the amount of capital per worker, making workers less productive and leading to lower wages, which reduces people's incentive to work and thus leads to a smaller supply of labor.

The agency's baseline projections also incorporate the economic effects of changes in federal tax policies scheduled under current law, including the effects of higher marginal tax rates. (The marginal tax rate is the percentage of an additional dollar of income from labor or capital that is paid in taxes.) Under current law, tax rates on individual income are scheduled to rise at the end of 2025. Moreover, as income rises faster than inflation and more income is pushed into higher tax brackets over the long term—a phenomenon referred to as real bracket creep—labor income and capital income are taxed at higher tax rates.[8] Higher marginal tax rates on labor income would reduce people's after-tax wages and thus weaken their incentive to work. Likewise, an increase in the marginal tax rate on capital income would lower people's incentives to save and invest, thereby reducing the stock of capital and, in turn, labor productivity; the reduction in labor productivity would put downward pressure on wages. All told, less private investment and a smaller labor supply decrease economic output and income in CBO's extended baseline projections.

7. Monetary accommodation refers to a central bank's lowering interest rates in an attempt to boost economic growth, thereby stabilizing or reducing unemployment.

8. See Congressional Budget Office, "How Income Growth Affects Tax Revenues in CBO's Long-Term Budget Projections" (June 2019), www.cbo.gov/publication/55368.

Chapter 4: The Long-Term Outlook Under Alternative Paths for the Economy and Budget

Overview

The extended baseline projections that the Congressional Budget Office describes in this report are based on the agency's economic forecast and reflect the assumption that current laws governing taxes and spending generally remain unchanged. To show how changes in economic conditions or in current law might affect budgetary outcomes, CBO analyzed four illustrative economic paths and three illustrative budgetary paths that differ from those underlying the agency's baseline projections.[1]

Illustrative Economic Paths

CBO's long-term budget projections depend on its forecasts of economic factors, including productivity growth and interest rates. If economic conditions differed from those in CBO's forecast, budgetary outcomes would diverge from those in the agency's extended baseline projections. To illustrate the effects of such differences, CBO analyzed how its budget projections would differ if productivity growth or interest rates were higher or lower than it anticipated (see Figure 4-1).

Paths for Growth of Total Factor Productivity

CBO examined the effects of changes in the growth rate of total factor productivity (TFP) in the nonfarm business sector on its projections of federal debt measured as a percentage of gross domestic product (GDP). The growth of TFP—the average real output (that is, output adjusted to remove the effects of inflation) per unit of combined labor and capital services—is a key contributor to growth in GDP.

The agency projected budgetary outcomes using rates of growth for TFP in the nonfarm business sector that were 0.5 percentage points higher and 0.5 percentage points lower than the rates underlying the extended baseline projections. That range reflects the variation of about one percentage point in average TFP growth over the 43 30-year periods between 1950 and 2021. After accounting for the effects of the alternative paths for TFP on capital and other macroeconomic factors, CBO made the following projections:

- If nonfarm business productivity grew 0.5 percentage points faster than CBO projects, federal debt held by the public would be 140 percent of GDP in 2052 rather than the 185 percent it amounts to in the extended baseline projections.
- If nonfarm business productivity grew 0.5 percentage points more slowly than projected, federal debt held by the public would be 234 percent of GDP in 2052.

Paths for Interest Rates on Federal Debt Held by the Public

CBO also examined the effects of changes in interest rates on its projections of federal debt as a percentage of GDP. The agency projected budgetary outcomes under two illustrative paths in which interest rates on federal debt were either higher or lower than the rates underlying the agency's extended baseline.

- For the first path, CBO boosted the average interest rate on federal debt above the baseline rate by a differential that starts at 5 basis points in 2022 and increases by 5 basis points each year (before macroeconomic effects, which are described below, are accounted for).[2] Under that path, federal debt held by the public equals 235 percent of GDP in 2052 rather than the 185 percent of GDP it equals in the extended baseline projections.

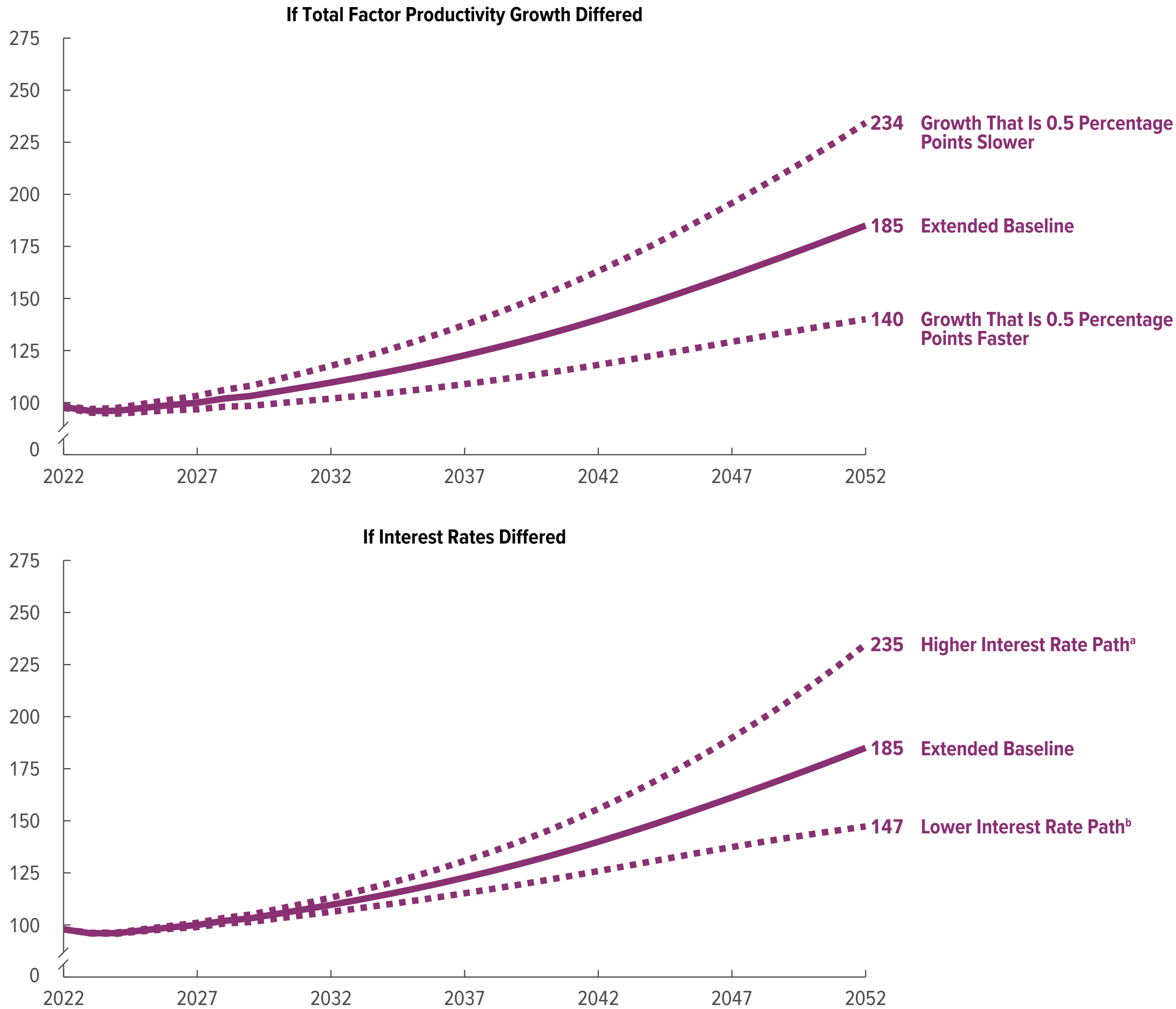
1. For detailed information about the illustrative paths, see the supplemental data for this report at www.cbo.gov/publication/57971#data.

2. That is, the interest rate was boosted above the baseline rate by 5 basis points in 2022, 10 basis points in 2023, 15 basis points in 2024, and so on. A basis point is one one-hundredth of a percentage point.

Figure 4-1.

Federal Debt If Total Factor Productivity Growth or Interest Rates Differed From the Values Underlying CBO's Extended Baseline Projections

Percentage of Gross Domestic Product



Data source: Congressional Budget Office. See www.cbo.gov/publication/57971#data.

Total factor productivity growth is the growth of real output (that is, output adjusted to remove the effects of inflation) per unit of combined labor and capital services in the nonfarm business sector. The interest rate is the average interest rate on federal debt.

The extended baseline projections, which generally reflect current law, follow CBO's 10-year baseline budget projections and then extend most of the concepts underlying those projections for an additional 20 years.

a. For this path, the average interest rate on federal debt was boosted above the rate underlying CBO's extended baseline by a differential that starts at 5 basis points in 2022 and increases by 5 basis points each year (before macroeconomic effects are accounted for)—that is, the interest rate is 5 basis points higher than the baseline rate in 2022, 10 basis points higher than the baseline rate in 2023, 15 basis points higher than the baseline rate in 2024, and so on. (A basis point is one one-hundredth of a percentage point.)

b. For this path, the average interest rate on federal debt was pushed below the rate underlying CBO's extended baseline by a differential that starts at 5 basis points in 2022 and increases by 5 basis points each year (before macroeconomic effects are accounted for)—that is, the interest rate is 5 basis points lower than the baseline rate in 2022, 10 basis points lower than the baseline rate in 2023, 15 basis points lower than the baseline rate in 2024, and so on.

- For the second path, the average interest rate on federal borrowing was pushed below the baseline rate by those same amounts each year. Under that path, federal debt held by the public equals 147 percent of GDP in 2052.

Under the first path, the boost to interest rates increases the government's interest costs and thus deficits. Larger deficits and the increased federal borrowing required to finance them decrease private investment. (For a discussion of why increased federal borrowing reduces private investment, see Chapter 1.) The decrease in private investment reduces the amount of capital and increases the return on investment because more workers make use of each unit of capital. When the return on capital grows, interest rates—including the rates that the federal government pays on debt held by the public—rise further. Thus, macroeconomic effects push interest rates above the initial boost that was built into the illustrative path.

The average interest rate on federal debt, which is 4.2 percent in 2052 in the extended baseline projections, reaches 6.0 percent that year under the path with higher interest rates. That rate reflects both the initial boost of the rate built into the path and the resulting effects of larger deficits, less investment and capital, and the additional increases in interest rates. About one-eighth of the 1.8 percentage-point difference between the rate in the illustrative path and that in the extended baseline in 2052 results from those macroeconomic effects rather than from the initial boost to borrowing rates.

The lower interest rates in the second illustrative path result in smaller interest payments and smaller deficits than those in CBO's baseline projections. Those smaller deficits increase private investment, making the amount of capital per worker grow and the return on capital—and, ultimately, interest rates—fall. The average interest rate on federal debt falls to 2.4 percent in 2052 under that path.

The budgetary effects of higher or lower interest rates are highly uncertain because those effects depend on the amount of debt that the interest rates are applied to and on the macroeconomic effects of the higher or lower rates. Also, this analysis does not explicitly account for the budgetary effects that might stem from the sources of the changes in interest rates.

Illustrative Budgetary Paths

The budget projections in this report show what federal spending, revenues, and deficits would be if current laws governing spending and taxes generally remained unchanged. Those projections are not intended to be a forecast of budgetary outcomes; rather, they are meant to provide a benchmark that policymakers can use to assess the potential effects of policy decisions.

When constructing its baseline projections of spending and revenues, CBO follows procedures specified in law as well as long-standing guidelines. For example, the Balanced Budget and Emergency Deficit Control Act of 1985 (Public Law 99–177) requires CBO to incorporate the assumption that future discretionary funding will match the amounts most recently provided, with adjustments for inflation, through the end of the 10-year baseline projection period. For later years in the projection period, CBO assumes that after a five-year transition period, discretionary spending would grow at the rate of nominal GDP. The agency's projections also reflect the assumptions that most laws governing mandatory spending will continue beyond their statutory expiration and that scheduled payments from trust funds, such as Social Security benefits, would be made even after the program's trust funds were exhausted and its annual revenues were inadequate to fund those payments. By contrast, CBO's projections of revenues reflect the assumption that certain provisions affecting the tax code—including changes in statutory tax rates—will expire as scheduled under current law.

Spending and revenues may, however, differ from the amounts in CBO's baseline projections. For example, lawmakers can—and do—set discretionary funding at amounts that differ from the projected amounts. To illustrate how changes in spending and revenues could affect the long-term budget outlook, CBO examined three illustrative budgetary paths that are more consistent with the past than are its extended baseline projections—one in which discretionary spending (measured as a percentage of GDP) deviates from the path underlying the agency's extended baseline projections and two in which both discretionary spending and revenues (measured as a percentage of GDP) differ from their baseline amounts.[3]

3. In 2019, CBO analyzed an additional illustrative scenario in which the Social Security Administration would no longer pay beneficiaries the full amount specified in law once the combined trust funds were exhausted and current revenues were insufficient to pay those amounts. CBO found that such a reduction in benefits would substantially reduce the amount of debt by the end of the projection period. See Congressional Budget Office, *The 2019 Long-Term Budget Outlook* (June 2019), pp. 41–44, www.cbo.gov/publication/55331.

Path for Discretionary Spending

For the first illustrative budgetary path, discretionary outlays were set to equal 7.0 percent of GDP—their value in 2022—over the entire projection period. That path for discretionary spending more closely resembles the past than does the path for such spending in CBO's extended baseline projections. Whereas discretionary outlays have averaged 6.9 percent of GDP over the past 10 years and 7.3 percent of GDP over the past 20 years, they average 6.2 percent of GDP over the 2022–2052 period in CBO's extended baseline projections. Discretionary spending of the amount specified in the illustrative path would exceed the amount in the extended baseline projections by 0.8 percentage points in 2032 and by 1.0 percentage point in 2052.[4]

Paths for Revenues

Following changes in tax law scheduled to take effect at the end of 2025, revenues generally increase as a share of the economy in CBO's projections; they reach 19.1 percent of GDP in 2052. That upward trend does not align with experience, however. Largely because of legislated changes, federal revenues have fluctuated around their 50-year average of 17.3 percent over the past five decades and have followed no apparent long-term trend.

In the second illustrative budgetary path, noninterest spending is about the same as it is in the illustrative path for discretionary spending throughout the projection period, and revenues are the same as they are in the agency's extended baseline projections through 2026. Thereafter, revenues remain at 18.0 percent of GDP through the rest of the projection period—much closer to their 50-year average than are the amounts in the extended baseline but still high by historical standards.

The third illustrative budgetary path is more consistent with past revenues than is the second budgetary path. Revenues in the third path are the same as they are in the extended baseline projections through 2025, and in 2026, they return to their 50-year average of 17.3 percent of GDP, where they remain through the rest of the projection period. Noninterest spending is about the same as it is in the illustrative path for discretionary spending throughout the projection period.

4. For a more detailed discussion of the budgetary effects of a similar illustrative path through 2032, see Congressional Budget Office, *The Budget and Economic Outlook: 2022 to 2032* (May 2022), pp. 101–104, www.cbo.gov/publication/57950.

Budgetary and Economic Outcomes Under the Illustrative Budgetary Paths

Different paths for spending and revenues would result in different paths for the deficit and debt and thus affect the amount of federal borrowing. That change in federal borrowing would, in turn, affect the economy, and those macroeconomic effects would feed back into the federal budget. To assess the effects of different budgetary paths on the long-term budget outlook, CBO analyzed deficits and debt under the three paths when those macroeconomic effects are taken into account (see Figure 4-2).

Path for Discretionary Spending. Under the first illustrative budgetary path, in which discretionary spending is set equal to 7.0 percent of GDP for the entire projection period, the primary deficit (which excludes interest costs) would be 1.2 percentage points higher in 2052 than it is in CBO's extended baseline projections (see Table 4-1). Once the rising costs of debt service are added, the total deficit in 2052 would be 2.8 percentage points higher than the baseline amount. Debt held by the public would reach 218 percent of GDP in 2052, 33 percentage points higher than it is in CBO's extended baseline projections.

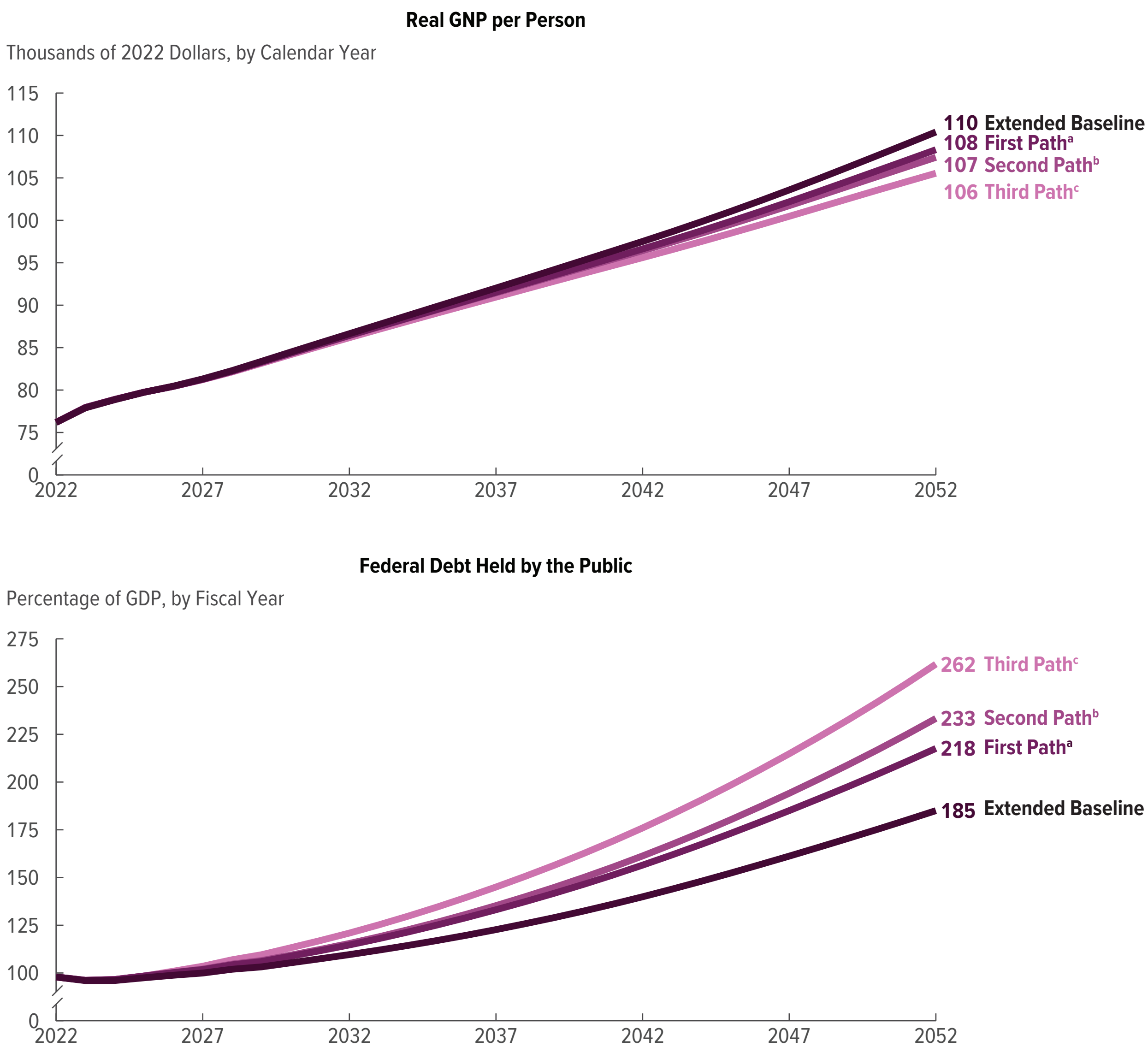
More discretionary spending and increased government borrowing would lead to a reduction in private investment and a smaller capital stock, thus causing output to be lower and interest rates to be higher in the long term than they are in the extended baseline projections. In 2052, for instance, real gross national product (GNP) under the illustrative path would be 1.9 percent lower than it is in CBO's baseline projections, and real GNP per person would be about $2,100 lower (in 2022 dollars).[5]

Paths for Revenues. Under the second illustrative budgetary path, in which discretionary spending is set equal to 7.0 percent of GDP for the entire projection period and revenues remain at 18.0 percent of GDP after 2026, the primary deficit in 2052 would be 2.3 percent of GDP larger than it is in CBO's extended baseline projections, and the total deficit for that year would be 4.7 percent of GDP larger than the baseline amount.

5. Whereas GDP, the more common measure of economic output, is the value of all final goods and services produced within the borders of the United States, GNP is the value of all final goods and services produced by labor and capital supplied by residents of the United States, regardless of where that labor and capital are located.

Figure 4-2.

Output per Person and Federal Debt Under Three Illustrative Budgetary Paths



Data source: Congressional Budget Office. See www.cbo.gov/publication/57971#data.

The extended baseline projections, which generally reflect current law, follow CBO's 10-year baseline budget projections and then extend most of the concepts underlying those projections for an additional 20 years.

The estimates of debt include macroeconomic feedback.

Whereas GDP, the more common measure of economic output, is the value of all final goods and services produced within the borders of the United States, GNP is the value of all final goods and services produced by labor and capital supplied by residents of the United States, regardless of where that labor and capital are located.

GDP = gross domestic product; GNP = gross national product.

a. In the first path, discretionary outlays equal 7.0 percent of GDP—their value in 2022—over the entire projection period.

b. In the second path, noninterest outlays are about the same as they are in the first path, and revenues are the same as they are in the extended baseline projections through 2026. Thereafter, revenues remain at 18.0 percent of GDP through the rest of the projection period.

c. In the third path, noninterest outlays are about the same as they are in the first path, and revenues are the same as they are in the extended baseline projections through 2025. In 2026, revenues return to their 50-year average of 17.3 percent of GDP, where they remain through the rest of the projection period.

Table 4-1.

Budget Projections in 2052 Under Three Illustrative Paths

Percentage of Gross Domestic Product

	Extended Baseline	First Path[a]	Second Path[b]	Third Path[c]
Revenues	19.1	19.1	18.0	17.3
Outlays, Excluding Interest Payments	23.0	24.1	24.2	24.3
Primary Deficit	3.9	5.0	6.2	7.0
Total Deficit	11.1	13.9	15.8	18.2
Federal Debt Held by the Public	185	218	233	262

Data source: Congressional Budget Office. See www.cbo.gov/publication/57971#data.

The extended baseline projections, which generally reflect current law, follow CBO's 10-year baseline budget projections and then extend most of the concepts underlying those projections for an additional 20 years.

The estimates of debt include macroeconomic feedback.

a. In the first path, discretionary outlays equal 7.0 percent of GDP—their value in 2022—over the entire projection period.

b. In the second path, noninterest outlays are about the same as they are in the first path, and revenues are the same as they are in the extended baseline projections through 2026. Thereafter, revenues remain at 18.0 percent of GDP through the rest of the projection period.

c. In the third path, noninterest outlays are about the same as they are in the first path, and revenues are the same as they are in the extended baseline projections through 2025. In 2026, revenues return to their 50-year average of 17.3 percent of GDP, where they remain through the rest of the projection period.

Debt held by the public would reach 233 percent of GDP in 2052, 48 percentage points higher than it is in CBO's extended baseline projections. That year, real GNP would be 2.7 percent lower than it is in the agency's baseline projections, and real GNP per person would be about $3,000 lower (in 2022 dollars).

Under the third illustrative budgetary path, in which discretionary spending equals 7.0 percent of GDP for the entire projection period and revenues remain at 17.3 percent of GDP after 2025, the primary deficit in 2052 would be 3.2 percent of GDP larger than it is in the agency's baseline projections, and the total deficit for that year would be 7.1 percent of GDP larger than the baseline amount. Debt held by the public would reach 262 percent of GDP in 2052, 77 percentage points higher than it is in CBO's extended baseline projections. That year, real GNP would be 4.4 percent lower than the baseline projection, and real GNP per person would be about $4,900 lower (in 2022 dollars).

How CBO Analyzed Outcomes Under the Illustrative Budgetary Paths

Fiscal policy underlying the three illustrative budgetary paths would differ significantly from fiscal policy under current law. For simplicity—and to avoid presuming which fiscal policies lawmakers might implement to alter the deficit—CBO analyzed the paths without specifying the tax and spending policies underlying them. In particular, CBO assumed that under all paths, transfer payments to people would be the same as they are under current law and that spending on federal investment would make the same contribution to future productivity and output that it makes in the agency's baseline projections. Under the paths in which revenues are lower, the effective marginal tax rates on labor and capital income are assumed to move proportionally for all households as revenues change to meet the specified targets.

Those changes in fiscal policy are projected to have effects on the economy that would feed back into the budget. CBO has not analyzed every way in which those changes would affect the economy in the long term. Instead, for the simplified analysis presented in this report, CBO considered these three effects:

- Effective marginal tax rates on labor income would be lower under the two paths in which revenues are reduced than they are in the extended baseline projections. Those lower rates would encourage people to work and save more and thus increase output.[6]

6. The effective marginal tax rate on labor income is the share (averaged among all taxpayers by assigning them weights proportional to their labor income) of an additional dollar of such income that is paid in federal individual income taxes and payroll taxes.

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- Effective marginal tax rates on income from most types of capital would also be lower under the paths in which revenues are reduced. Those lower rates would encourage saving and investment and thus further increase output.[7]
- Federal debt would be greater under all three paths than it is in the extended baseline projections. The increase in federal borrowing would draw money away from investment in capital goods and services and thus reduce the stock of private capital and output.

In addition to those three effects, CBO's analysis accounts for the short-term effects that the illustrative budgetary paths would have on the economy. Policies that increased spending or reduced revenues would boost overall demand for goods and services over the next few years, thereby increasing output and employment in the short term.

Changes to fiscal policy could also alter people's incentives in other ways, possibly resulting in significant long-term changes to the economy. For example, changes to tax policy might alter businesses' choices about how they were structured, and those choices might, in turn, alter the effective marginal tax rate on capital income. Similarly, changes in the tax treatment of mortgage debt would affect households' decisions about how much to save. Because this analysis is simplified, it does not account for any changes in individuals' or businesses' incentives or activities that might result from particular policy changes.

7. The effective marginal tax rate on capital income is the share of the return on an additional dollar of investment made in a particular year that will be paid in taxes over the life of that investment.

Appendix A: Assumptions and Methods Underlying CBO's Long-Term Budget Projections

The Congressional Budget Office's long-term budget projections are consistent with the baseline budget projections and economic forecast for 2022 to 2032 that the agency published in May 2022.[1] The long-term projections extend most of the concepts underlying the 10-year projections for an additional 20 years. Together, those projections constitute the agency's *extended baseline* projections.

CBO's extended baseline projections give lawmakers a point of comparison from which to measure the effects of policy options or proposed legislation. The projections are not predictions of budgetary outcomes. Rather, they represent the agency's assessment of future spending, revenues, deficits, and debt under these assumptions:

- Current laws affecting revenues and spending generally remain unchanged;
- Some programs—for example, the Supplemental Nutrition Assistance Program—are nevertheless extended after their authorizations lapse;
- Spending on Medicare and Social Security continues as scheduled regardless of the amounts in those programs' trust funds; and
- Discretionary spending follows the agency's baseline projection through 2032, after which time it transitions (over a five-year period) to grow at the rate of nominal gross domestic product. (For a summary of the assumptions about outlays and revenues that underlie CBO's extended baseline projections, see Table A-1.)

1. See Congressional Budget Office, *The Budget and Economic Outlook: 2022 to 2032* (May 2022), www.cbo.gov/publication/57950.

For years beyond 2032, the agency used a model with the following four components to integrate its demographic, economic, and long-term budget projections.[2]

- A *demographic model* was used to project the size of the population by age and sex.
- A *microsimulation model* was used to project annual changes in demographic characteristics and economic outcomes for a representative sample of the population and to project Social Security outlays beyond CBO's standard 10-year budget period.
- A *long-term budget model* was used to project federal outlays (except those for Social Security), revenues, deficits, and debt beyond CBO's standard 10-year budget period.
- A *model of economic growth* was used to simulate how demographic changes, economic factors, and fiscal policy would affect the U.S. economy and, in turn, the federal budget.[3]

Those four components interact in various ways. For example, the economic projections reflect how increases in spending and revenues in the extended baseline projections would affect the economy. In turn, the budget projections in the extended baseline projections reflect those economic effects.

2. See Congressional Budget Office, *An Overview of CBOLT: The Congressional Budget Office Long-Term Model* (April 2018), www.cbo.gov/publication/53667.

3. See Congressional Budget Office, "CBO's Policy Growth Model" (April 2021), www.cbo.gov/publication/57017; and Robert W. Arnold, Chief, Projections Unit, Macroeconomic Analysis Division, Congressional Budget Office, "CBO's 10-Year Economic Forecast and How It Is Produced" (presentation at a seminar of the Congressional Research Service, April 26, 2018), www.cbo.gov/publication/53792.

Table A-1.

Assumptions About Outlays and Revenues Underlying CBO's Extended Baseline Projections

	Assumption
	Outlays
Social Security	As scheduled under current law[a]
Medicare	As scheduled under current law through 2032; thereafter, projected spending depends on the estimated growth rates of the number of beneficiaries, health care costs per beneficiary, potential GDP per person, and additional cost growth for Medicare (which is projected separately for various parts of the program and, by 2052, moves smoothly up to a rate of 0.1 percent for Part A, down to a rate of 0.2 percent for Part B, and down to a rate of 0.6 percent for Part D)[a]
Medicaid	As scheduled under current law through 2032; thereafter, projected spending depends on the estimated growth rates of the number of beneficiaries, health care costs per beneficiary, potential GDP per person, and additional cost growth for Medicaid (which is projected to move smoothly down to a rate of 0.6 percent by 2052)
Children's Health Insurance Program	As projected in CBO's baseline through 2032; thereafter, projected spending remains constant as a percentage of GDP
Subsidies for Health Insurance Purchased Through the Marketplaces	As scheduled under current law through 2032; thereafter, projected spending depends on the estimated growth rates of the number of beneficiaries and potential GDP per person, an additional indexing factor for subsidies, and additional cost growth for subsidies for health insurance purchased through the marketplaces (which is projected to move smoothly down to a rate of 1.0 percent by 2052)
Other Mandatory Spending	As scheduled under current law through 2032; thereafter, refundable tax credits are estimated as part of revenue projections, and the rest of other mandatory spending is assumed to decline as a percentage of GDP at roughly the same annual rate at which it is projected to decline between 2026 and 2030 in the agency's baseline projections published in March 2020 (those projections are the most recent projections that exclude the effects of the coronavirus pandemic)
Discretionary Spending	As projected in CBO's baseline through 2032; beyond that year, after a five-year transition period, discretionary spending would grow at the rate of nominal GDP
	Revenues
Individual Income Taxes	As scheduled under current law
Payroll Taxes	As scheduled under current law
Corporate Income Taxes	As scheduled under current law
Excise Taxes	As scheduled under current law[b]
Estate and Gift Taxes	As scheduled under current law
Other Sources of Revenues	As scheduled under current law through 2032; constant as a percentage of GDP thereafter

Data source: Congressional Budget Office.

The extended baseline projections, which generally reflect current law, follow CBO's 10-year baseline budget projections and then extend most of the concepts underlying those projections for an additional 20 years.

For CBO's most recent 10-year baseline projections, see Congressional Budget Office, *The Budget and Economic Outlook: 2022 to 2032* (May 2022), www.cbo.gov/publication/57950.

Additional cost growth (which was called excess cost growth in CBO's past reports) is the amount by which the growth rate of nominal health care spending per person (adjusted to remove the effects of demographic changes) exceeds the growth rate of potential GDP per person. Potential GDP is the maximum sustainable output of the economy.

GDP = gross domestic product.

a. Assumes the payment of full benefits as scheduled under current law, regardless of the amounts in the program's trust funds.

b. The exception to the current-law assumption applies to expiring excise taxes dedicated to trust funds. The Balanced Budget and Emergency Deficit Control Act of 1985 requires CBO's baseline to reflect the assumption that those taxes would be extended at their current rates. That law does not stipulate that the baseline include the extension of other expiring tax provisions, even if they have been routinely extended in the past.

Appendix B: CBO's Projections of Economic Variables

Overview

The Congressional Budget Office develops its assessment of the long-term outlook for the federal budget on the basis of its projections of economic variables over the next three decades.[1] The projections presented in this report are consistent with the baseline budget projections and the economic forecast for the 2022–2032 period that CBO published in May 2022.[2] Those projections reflect the assumption that current laws governing federal taxes and spending generally remain unchanged. (The agency's annual projections of economic variables through 2052 are included in this report's supplemental data, which are available at www.cbo.gov/publication/57971#data.)

Projections of federal budgetary outcomes depend on many economic variables. In this appendix, CBO describes and explains the projected growth of gross domestic product (GDP); labor force participation and labor force growth; other labor market outcomes (unemployment, average weekly hours worked, total hours worked, earnings as a share of compensation, the growth of inflation-adjusted earnings per worker, and the distribution of earnings among workers); capital accumulation and productivity; inflation; and interest rates.

CBO's projections of those variables reflect the agency's assessment of various economic and demographic developments, as well as the effects of monetary and fiscal policy on economic activity.

1. In previous versions of this report, the agency discussed projections of demographic trends along with its discussion of economic trends. This year, the demographic trends are instead covered in a companion report. See Congressional Budget Office, *The Demographic Outlook: 2022 to 2052* (July 2022), www.cbo.gov/publication/57975.

2. See Congressional Budget Office, *The Budget and Economic Outlook: 2022 to 2032* (May 2022), www.cbo.gov/publication/57950.

Gross Domestic Product

In CBO's projections, the average annual growth of real GDP (that is, GDP adjusted to remove the effects of inflation) slows from 1.9 percent in the first decade of the projection period (2022 to 2032) to slightly less than 1.6 percent in the second decade (2033 to 2042) to just over 1.5 percent in the third decade (2043 to 2052); see Table B-1. Those rates of growth are 0.6 to 0.9 percentage points lower than the average growth rate of 2.5 percent that has occurred for the past three decades. In the agency's current projections, real GDP grows slightly faster over the 2021–2051 period than the agency projected last year.

Real GDP per person is expected to increase at an average annual rate of 1.4 percent over the 2022–2052 period. Over the past 30 years, that measure has increased at an average annual rate of 1.5 percent. In the agency's projections, the average annual growth of real GDP per person falls from 1.5 percent in the first decade of the projection period to 1.2 percent in the second decade. In the third decade, growth in real GDP per person rises to 1.3 percent as population growth slows more than real GDP.

Nominal GDP is projected to grow at an average annual rate of 3.9 percent over the next 30 years, which is slower than the average growth rate over the past 30 years (4.5 percent). CBO projects that average annual growth in nominal GDP will slow from 4.4 percent over the first decade to 3.7 percent in the second decade and to 3.5 percent in the third decade. In the agency's current projections, nominal GDP grows faster over the 2021–2051 period than the agency projected last year.

Real GDP

The long-term growth of real GDP in CBO's forecasts is driven by the growth rate of real potential GDP (that is, the maximum sustainable output of the economy, adjusted to remove the effects of inflation). Over the next three decades, the growth of real potential GDP is

Table B-1.

Average Annual Growth Rates for Economic Variables That Underlie CBO's Extended Baseline Projections, by Calendar Year

Percent

	1992–2021	2022–2032	2033–2042	2043–2052	Overall, 2022–2052
GDP					
Real GDP	2.5	1.9	1.6	1.5	1.7
Real potential GDP[a]	2.4	1.8	1.6	1.5	1.7
Potential labor force	0.9	0.4	0.3	0.3	0.3
Potential labor force productivity	1.5	1.5	1.3	1.2	1.3
Nominal GDP	4.5	4.4	3.7	3.5	3.9
Real GDP per person	1.5	1.5	1.2	1.3	1.4
Labor Force Participation Rate (Average annual value)	65.1	61.8	61.0	60.7	61.2
Labor Force	0.8	0.5	0.3	0.3	0.4
Unemployment (Average annual value)					
Unemployment rate	5.9	4.2	4.4	4.2	4.2
Noncyclical rate of unemployment[b]	5.0	4.4	4.1	3.9	4.1
Average Weekly Hours Worked	*	*	*	*	*
Total Hours Worked	0.8	0.6	0.3	0.3	0.4
Earnings as a Share of Compensation (Average annual value)	81	82	81	80	81
Real Earnings per Worker	1.4	0.7	0.9	0.9	0.8
Productivity					
Total factor productivity in the nonfarm business sector	1.3	1.0	1.1	1.1	1.1
Real GDP per hour worked	1.6	1.4	1.3	1.2	1.3
Inflation					
CPI-U	2.3	2.8	2.3	2.3	2.4
GDP price index	2.0	2.4	2.0	2.0	2.2
Interest Rates (Average annual value)					
Real rates					
On 10-year Treasury notes and the OASDI trust funds	1.7	0.7	1.7	2.3	1.5
Nominal rates					
On 10-year Treasury notes and the OASDI trust funds	4.0	3.4	4.0	4.5	4.0
On all federal debt held by the public (By fiscal year)[c]	4.2	2.5	3.4	4.0	3.3

Data source: Congressional Budget Office. See www.cbo.gov/publication/57971#data.

The extended baseline projections, which generally reflect current law, follow CBO's 10-year baseline budget projections and then extend most of the concepts underlying those projections for an additional 20 years.

Real values are nominal values that have been adjusted to remove the effects of inflation.

CPI-U = consumer price index for all urban consumers; GDP = gross domestic product; OASDI = Old-Age, Survivors, and Disability Insurance; * = between -0.05 percent and zero.

a. Potential GDP is the maximum sustainable output of the economy. The potential labor force is the labor force (that is, the number of people in the civilian noninstitutionalized population who are age 16 or older and who have jobs or who are available for work and are actively seeking jobs) adjusted to remove the effects of fluctuations in the business cycle. Potential labor force productivity is the ratio of real potential GDP to the potential labor force. The sum of growth in the potential labor force and growth in potential labor force productivity is equal to growth in real potential GDP.

b. The noncyclical rate of unemployment is the rate that results from all sources except fluctuations in aggregate demand. It reflects the normal turnover of jobs and mismatches between the skills of available workers and the skills necessary to fill vacant positions.

c. The interest rate on all federal debt held by the public equals net interest payments in the current fiscal year divided by debt held by the public at the end of the previous fiscal year.

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expected to decelerate. Starting from an average annual rate of growth of 1.8 percent in the first decade of the projection period, that measure declines to slightly less than 1.6 percent in the second decade and then inches down to just over 1.5 percent in the third decade.

That deceleration reflects a gradual slowing of growth in the potential labor force (the labor force adjusted for fluctuations in the business cycle) and slower growth in potential labor force productivity (potential output per member of the potential labor force). Growth in potential labor force productivity is, in turn, driven by two key factors; total factor productivity, or TFP (real output per unit of combined labor and capital inputs in the nonfarm business sector), and capital accumulation per worker. Over the 30-year projection period, capital accumulation slows, primarily owing to the effect of increased federal borrowing on private investment.

In CBO's current 10-year economic forecast, the level of real GDP exceeds that of real potential GDP for several years, but then the growth rate of real GDP slows in relation to the growth rate of real potential GDP, in part because monetary policy gradually tightens. As a result, the level of real GDP falls below that of real potential GDP in 2026. By 2028, real GDP is 0.5 percent below real potential GDP, and that gap persists through 2052 and beyond.[3] Therefore, over the second and third decades of the projection period, real GDP and real potential GDP grow at the same annual rate in the agency's projections.

Nominal GDP

In CBO's forecast, growth in nominal GDP is determined by the growth of the GDP price index and the growth of real GDP. Projected growth in the GDP price index falls from an average annual rate of 2.4 percent in the first decade to 2.0 percent in the second and third decades. Slower growth in both the GDP price index and real GDP leads to a significant decline in the average growth of nominal GDP from the first decade to the second decade. Slowing growth of GDP between the second and third decades, however, is almost entirely because of lower growth in real GDP.

Changes in Projections of GDP Since Last Year

CBO's current projections of real GDP and nominal GDP are higher than last year's projections throughout the 30-year period. Faster growth of real GDP over the first two decades results from levels of investment that are projected to be stronger than CBO previously estimated and that lead to a larger stock of productive capital. For the third decade, the agency's projections of growth in nominal GDP, real GDP, and real potential GDP are similar to last year's projections, because a slight decrease in the growth of TFP offsets the effect of the larger stock of capital. In 2031, the level of real GDP is projected to be 1.1 percent higher in this year's projections than in last year's. By 2051, the level of real GDP is projected to be 1.8 percent greater than CBO expected last year. Compared with last year's projections of real GDP per person over the next three decades, this year's projections grow faster because real GDP grows slightly faster and growth in the U.S. population is slightly slower.

The agency's projections of nominal GDP are higher this year because CBO now expects much higher inflation in the GDP price index over the next few years and slightly faster growth in real GDP and real potential GDP in the first and second decades of the projection period. (For details, see Box C-1 on page 53.) By 2051, the level of nominal GDP is projected to be 9.3 percent higher than the agency projected last year.

Factors Affecting Projections of Potential GDP

The growth of potential GDP is determined by the growth of the potential labor force and of potential labor force productivity. Projected growth in potential labor force productivity, in turn, is determined by projections of various trends. Among those economywide trends are average weekly hours worked; investment and the accumulation of capital, such as structures and equipment, intellectual property products, and residential housing; and the growth of TFP in the nonfarm business sector.

The Labor Force Participation Rate and Labor Force Growth

In CBO's projections, the size of the labor force depends on the rates at which people in different demographic

3. That output gap reflects the agency's assessment that GDP falls short of potential GDP to a greater extent and for longer periods during and after economic downturns than actual output exceeds potential output during economic expansions. A recent study explains the existence of a persistent output gap by examining asymmetric fluctuations of the unemployment rate around the noncyclical rate of unemployment (that is, the rate that results from all sources except fluctuations in aggregate demand). See Stéphane Dupraz, Emi Nakamura, and Jón Steinsson, *A Plucking Model of Business Cycles,* Working Paper 748 (Banque de France, January 2020), https://tinyurl.com/1njkmzkf. CBO assessed the persistent output gap in an earlier report; see Congressional Budget Office, *Why CBO Projects That Actual Output Will Be Below Potential Output on Average* (February 2015), www.cbo.gov/publication/49890.

groups participate in the labor market and on the number of people in those groups. Since the mid-2000s, the overall rate of labor force participation (the rate for people age 16 or older) in the United States has declined substantially, driven predominantly by the aging of the population.[4] Because that aging is likely to continue, CBO expects the decline in participation to persist during the first half of the 30-year projection period before stabilizing in the second half of the period. As a result, the labor force is expected to grow more slowly than the number of people age 16 or older for the first two decades of the period and at roughly the same pace as the number of people age 16 or older in the third decade.

The agency's projections of the labor force participation rate (LFPR) and the size of the labor force are important factors for CBO's projections of other economic outcomes. For example, faster growth of the labor force would directly boost GDP growth. It would also cause private capital to accumulate faster, which would further boost the growth of GDP.

The Labor Force Participation Rate

The labor force participation rate is generally projected to decline as the effects of the aging population become more prominent in relation to the short-term effects of the expanding economy. The LFPR falls from 61.8 percent, on average, in the first decade to 61.0 percent in the second decade. As demographic shifts slow over time, the LFPR is expected to stabilize, averaging 60.7 percent in the third decade of the projection period.

Labor Force Growth

The labor force is expected to grow from 164 million people in 2022 to 181 million people in 2052. Growth in the labor force slows in CBO's projections, from 0.5 percent per year, on average, from 2022 to 2032 to 0.3 percent per year, on average, from 2043 to 2052. That change represents a significant slowdown from the pace of growth in earlier periods: For example, the average annual growth rate of the labor force was 1.2 percent during the 1990–2006 period and 0.6 percent during the 2010–2019 period.

Changes in Projections of the Labor Force Participation Rate and Labor Force Growth Since Last Year

CBO's current projections of the labor force participation rate are higher than last year's throughout the projection period. Even though lingering health concerns and issues related to the availability of child care and other in-home care (stemming from the coronavirus pandemic) reduced the agency's labor force projections in the near term, upward revisions to the agency's projection of the size of the prime-age population (people ages 25 to 54) more than offset those factors throughout the projection period.[5]

As a result of those upward revisions to the agency's projections of the prime-age population and labor force participation rates, CBO's current projections of the size of the labor force are about 0.5 percent larger, on average, for 2023 to 2051, compared with last year's projections.

Factors Affecting Projections of the Labor Force Participation Rate and Labor Force Growth

The projected decline in the overall labor force participation rate in the coming decades stems mainly from the aging of the population: People age 65 or older tend to participate in the labor force at lower rates than younger people do. In 2019, for example, the average participation rate was 82.5 percent among the civilian noninstitutionalized population ages 25 to 54, and it was 20.1 percent among those age 65 or older.[6] As members of the baby-boom generation started to turn 65 in the early 2010s, the share of people age 65 or older in the civilian noninstitutionalized population increased rapidly, growing from 16.3 percent in 2010 to 20.4 percent in 2019. (The baby-boom generation encompasses people born between 1946 and 1964.) In CBO's projections, the percentage of people age 65 or older continues to rise (reaching 25.2 percent by 2032) and averages 27.1 percent during the third decade of the projection period.

4. The labor force participation rate is the share of the civilian noninstitutionalized population age 16 or older that is working or actively seeking work.

5. Technical adjustments made in response to recent data from the Census Bureau account for most of the revisions to the population ages 25 to 54 since last year. See Bureau of Labor Statistics, *Adjustments to Household Survey Population Estimates in January 2022* (February 2022), https://go.usa.gov/xuDH6 (PDF, 73 KB).

6. The civilian noninstitutionalized population excludes people who are younger than age 16, members of the armed forces on active duty, and people in penal or mental institutions or in homes for the elderly or infirm.

To assess the importance of aging in CBO's projections of the labor force participation rate, the agency calculated what the rate would be if the age-and-sex composition of the population remained the same in each year of the projection period as it was in 2022. That adjusted LFPR would increase from 61.9 percent in 2022 to 64.6 percent in 2052. Because the sex composition of the population is projected to change only slightly over the next three decades, the analysis implies that the effect of the aging of the population is roughly equal to the difference between the unadjusted and adjusted rates.[7] The adjusted LFPR in 2052 (64.6 percent) is 3.9 percentage points higher than the unadjusted LFPR in that year (60.7 percent). Therefore, aging accounts for more than the 1.2 percentage-point decline in the unadjusted rate that CBO projects from 2022 to 2052.

In contrast to the aging of the population, CBO expects two long-term trends to boost participation in the labor force:

- The population is becoming more educated, and people with more education tend to participate in the labor force at higher rates than do people with less education.
- Increasing longevity is expected to lead people to continue working until increasingly older ages.

CBO expects those two trends to be mostly offset by other trends that will put downward pressure on the labor force participation rate:

- Members of each generation (particularly men) that followed the baby boomers—Generation X and the Millennial Generation—tend to participate in the labor force at lower rates than their predecessors did at the same ages. (One notable exception in later generations is women age 34 or younger, who participate in the labor force at higher rates than did baby-boomer women at the same ages. However, as those later generations of women have aged, their participation rates have also fallen below those of their predecessors.)
- The marriage rate is projected to continue to fall, and unmarried men tend to participate in the labor force at lower rates than married men.

In addition to the effects of those demographic trends, as people's income rises faster than inflation, more of their income is pushed into higher tax brackets through a process known as real bracket creep, raising their effective tax rates. Scheduled increases in tax rates and real bracket creep are projected to decrease participation in the labor force because people would earn less return on their labor.

Other Labor Market Outcomes

In addition to the rate of labor force participation and the size and growth of the labor force, CBO projects the unemployment rate, the average and total number of hours that people work, and various measures of workers' earnings. CBO regularly updates those projections to reflect revisions in historical data, reassessments of economic and demographic trends, and changes to the agency's analytic methods.

Unemployment

The unemployment rate is projected to gradually rise over the next few years. By 2028, it is projected to reach 4.5 percent, surpassing the noncyclical rate of unemployment.[8] From 2029 to 2052, the unemployment rate is expected to remain roughly one-quarter of one percentage point above the noncyclical rate of unemployment, a difference that is consistent with both the average historical relationship between the two measures and the projected gap of one-half of one percent between actual and potential GDP.

In CBO's projections, the noncyclical rate of unemployment declines gradually to 4.3 percent in 2032 and to 3.9 percent in 2052. That slow decline reflects the continuing shifts in the composition of the workforce toward older workers, who tend to have lower rates of unemployment (when they participate in the labor force), and away from less-educated workers, who tend to have higher ones. As the noncyclical rate of unemployment decreases, the actual unemployment rate also declines. By 2052, the actual rate is projected to reach 4.1 percent.

Average Weekly Hours Worked

Workers tend to work a different number of hours each week depending on their industry: For example,

7. The share of men and women in the population in 2022 is 49 percent and 51 percent, respectively.

8. The noncyclical rate of unemployment is the rate that results from all sources except fluctuations in aggregate demand, including normal turnover of jobs and mismatches between the skills of available workers and the skills necessary to fill vacant positions.

workers in manufacturing put in more than 40 hours per week, on average, whereas those in service industries typically work about 32 hours per week. Over the past few decades, as the share of workers employed in manufacturing has decreased and the share employed in service industries has increased, the average number of hours worked per week has declined for the economy as a whole. During the past decade, the shares of workers in the manufacturing and service industries have been largely stable. In CBO's assessment, future changes in the employment shares of different industries are unlikely to substantially affect the economywide number of average hours worked.

Some incentives under current tax law are projected to influence the average number of hours worked. Higher tax rates on individual income are set to take effect when, under current law, certain provisions of the 2017 tax act expire at the end of 2025. In CBO's projections, those higher rates slightly reduce the average number of hours worked beginning in 2026. In addition, effective tax rates on individual income are projected to rise because of real bracket creep. Given economic trends and current laws, CBO expects the average number of hours worked to decline slightly over the next 30 years. By 2052, the average worker is projected to work roughly half an hour less per week than he or she does today.

Total Hours Worked

Total hours worked are calculated on the basis of projections of the growth of the labor force, average hours worked, and unemployment. CBO estimates that total hours worked will increase at an average annual rate of 0.4 percent between 2022 and 2052—about half the average annual increase in total hours worked over the past three decades (0.8 percent). The deceleration in the growth of total hours worked mainly occurs because the working-age population is expected to grow more slowly in the future than it has over the past 30 years.

In CBO's projections, the average growth in total hours worked is 0.6 percent in the first decade and 0.3 percent in the second and third decades.

Earnings as a Share of Compensation

Workers' total compensation consists of taxable earnings and nontaxable benefits (such as employers' contributions to health insurance and pensions). Since 1960, the share of total compensation paid in the form of wages and salaries has declined—from 91 percent in that year to 82 percent in 2021—mainly because health insurance premiums have risen more quickly than total compensation.[9] Because CBO expects that the cost of health insurance will continue to rise faster than wages and salaries, the portion of compensation that workers receive as earnings is projected to decline, on average, to 81 percent over the 2022–2052 period, reaching 80 percent in 2052.

Growth of Real Earnings per Worker

Projections of prices, capital services (that is, the flow of productive services from the stock of capital assets), TFP, the amount of nonwage compensation (such as employment-based health insurance), and the average number of hours worked imply that real earnings per worker will grow by an average of 0.8 percent annually over the 2022–2052 period. That rate is less than the 1.4 percent average annual growth of real earnings per worker over the past 30 years.

Distribution of Earnings

In CBO's projections, earnings grow faster for high earners than for low earners, but the rate of growth for high earners is projected to be slower than it was in the past. The share of earnings accruing to workers in the top 10 percent of the earnings distribution, for example, increases at an average rate of 0.1 percentage point per year from 2022 to 2052. That growth is less than it was between 1978 and 2019, when the share of earnings accruing to workers in the top 10 percent of the distribution increased by 0.2 percentage points per year.

The distribution of earnings affects revenues from income taxes and payroll taxes (particularly Social Security taxes). Income taxes are affected by the earnings distribution because of the progressive rate structure of the individual income tax: People with lower income pay a smaller share of their earnings in taxes than people with higher income.

Social Security payroll taxes are levied only on earnings up to a certain annual amount (called the taxable maximum, which is $147,000 in 2022), so they also are affected by the earnings distribution. Because earnings have grown more for higher earners than for others, the portion of covered earnings on which Social Security payroll taxes are paid has fallen from 90 percent in

9. For more details, see Congressional Budget Office, *How CBO Projects Income* (July 2013), www.cbo.gov/publication/44433.

1983 to 84 percent in 2019.[10] The portion of earnings subject to Social Security taxes is projected to be 82 percent, on average, in the first and second decades of the projection period. It falls to 81 percent, on average, in the third decade and equals 81 percent in 2052. That decline in the share of covered earnings below the taxable maximum reduces CBO's projection of Social Security payroll taxes.

Changes in Projections of Other Labor Market Outcomes Since Last Year

Some of this year's projections are close to last year's. CBO's current projections of real earnings per worker grow at roughly the same rate as last year's projections over the 2021–2051 period, for instance. Other projections differ. For the unemployment rate, for example, CBO's current projections are substantially lower over the next five years and higher over the following five years than the rates the agency projected last year.

The near-term revision to the unemployment rate is mainly because recent data on unemployment indicate a much stronger labor market recovery than the agency anticipated, driven primarily by stronger economic growth, compared with last year's projected pace of recovery. Revisions for the latter part of the first decade stem from CBO's expectation that GDP will return to its historical relationship with potential output sooner than the agency forecast last year. As a result, the unemployment rate is expected to rise toward CBO's estimate of the noncyclical rate of unemployment sooner as well. That earlier rise in the unemployment rate accounts for the slight upward revision to the average unemployment rate over the 2027–2032 period. For the second and third decades, the unemployment rate is projected to be roughly the same as CBO projected last year.

In CBO's current projections, earnings as a share of compensation are slightly higher for the 30-year period than the agency projected last year. The higher projection is because the increase in taxable earnings relative to last year's projected amount is larger than the increase in nontaxable benefits.

10. Covered earnings are those received by workers in jobs subject to Social Security payroll taxes. Most workers pay payroll taxes on their earnings, although a small number of workers—mostly in state and local government jobs or in the clergy—are exempt. Earnings above the taxable maximum are also exempt from payroll taxes, and no additional Social Security benefits accrue to people who have those excess earnings.

CBO's projections of the earnings distribution also differ from last year's. The top 10 percent of earners in 2051 are now projected to make 45.7 percent of all earnings, slightly more than last year's projected amount (45.0 percent). CBO refined its method for calculating the distribution of earnings, leading to a larger share of earnings accruing to high-wage earners over the long run.

Capital Accumulation and Productivity

In addition to the labor force, two other factors directly affect CBO's projections of output. One is the accumulation of capital, which contributes a flow of services to production over a given time period. The second factor is growth in total factor productivity. In CBO's projection method, economywide growth is driven mainly by growth in the nonfarm business sector; for that reason, the productivity measure that contributes the most to economywide growth is TFP in the nonfarm business sector.

The combined effect of capital accumulation and productivity on real GDP can be accounted for by measuring growth in real GDP that is not attributable to growth in total hours worked. Growth in real GDP per hour worked is a measure of economywide productivity that CBO uses to calculate the combined effect of growth in capital and growth in productivity on real GDP growth.

Capital Accumulation

Accumulation of private capital depends primarily on growth of the labor force, private saving, international flows of capital, and federal borrowing. Over the next 30 years, CBO projects, private saving as a percentage of GDP will rise. In the agency's projections, however, those effects are more than offset by an increase in federal borrowing, which rises as a percentage of GDP over the same period, pushing up interest rates, reducing growth in private investment, and slowing the growth of the stock of private capital. The average annual real interest rate on 10-year Treasury notes (calculated by subtracting the rate of increase in the consumer price index from the nominal yield on those notes) is projected to be 1.5 percent in 2032, rising to 2.4 percent by 2052.

Total Factor Productivity

The annual growth of TFP in the nonfarm business sector is projected to average 1.1 percent through 2052. That projected growth rate is about 0.3 percentage points slower than the average annual rate of growth since

1950 (1.4 percent) and 0.2 percentage point slower than the average rate since 1990.

Recent analysis of historical trends in TFP growth suggests that projections for the next few decades should place greater weight on recent slower growth than on faster growth in the more distant past. Thus, although CBO projects that growth in nonfarm business TFP will accelerate from its unusually slow recent rate, the agency expects the future rate of growth to be slower than its long-term historical average.

Real GDP per Hour Worked

Given the projected slowdown in growth of the capital stock and TFP, average annual growth in real GDP per hour worked is expected to fall from 1.4 percent over the first decade of the projection period to 1.3 percent over the second decade and to 1.2 percent over the third decade. Potential labor force productivity is expected to follow a similar decline over the next 30 years.

Changes in Projections of Capital Accumulation and Productivity Since Last Year

CBO's projections of capital accumulation and growth of capital services over the 2022–2051 period are higher this year than last year, reflecting the agency's upward revision, on average, to its projections of fixed investment in nearly all types of capital. That revision was the result of updates to historical data on investment made over the past year.

CBO's projections of TFP growth in the nonfarm business sector are slightly lower over the 30-year period, reflecting modest revisions to historical trends in TFP growth that influence the agency's judgment about the potential for future growth. Stronger projected growth of capital services outweighs weaker projected growth of TFP, though, so CBO's projection of real GDP per hour worked is higher over the 30-year period than it was last year.

Factors Affecting Capital Accumulation and Productivity

Over the long term, in CBO's view, growth of the nation's stock of private capital (or the flow of private investment) will be driven by the growth of the labor force, private saving, international flows of direct foreign investment and financial capital, and federal borrowing. Private saving tends to move in the same direction as growth in the labor force, and both private saving and international capital flows tend to move in tandem with the rate of return on investment—a rate that measures the extent to which investment in the stock of capital results in a flow of income. In the agency's view, increased federal borrowing reduces the amount of funds available for private investment and puts upward pressure on interest rates. CBO's projections of private investment and the rate of return on investment are consistent with its projections of federal borrowing and interest rates on 10-year Treasury notes, which increase over the 30-year period.

Several developments support CBO's projections of slower growth in nonfarm business TFP over the next 30 years (at an average annual rate of 1.1 percent) relative to its average growth rate over the past 30 years (1.3 percent). One is improvement in labor quality—an aggregate measure of workers' skills that accounts for educational attainment and work experience—which CBO expects to slow over the next three decades. That measure is implicitly included in CBO's measure of TFP. The slower improvement in labor quality is expected to be partly offset by improvements in health and increases in life expectancy that will lead people (particularly highly educated people) to continue working past the ages at which previous generations retired, thus boosting the total stock of experience in the workforce.

Another development that affects nonfarm business TFP is federal investment in physical capital (such as transportation infrastructure and water and power projects), education and training, and research and development; that investment produces income and other benefits (higher productivity and greater efficiency, for example) for private businesses. In CBO's projections, federal discretionary spending declines to a much smaller percentage of GDP over the next decade than it has constituted in past decades. If federal investment spending generally remained unchanged as a share of discretionary spending, and if discretionary spending declined as a percentage of GDP, then federal investment spending also would decrease as a share of GDP. In CBO's assessment, such a reduction in federal investment spending would dampen TFP growth.[11]

11. For more details about how CBO estimates the macroeconomic effects of federal investment, see Congressional Budget Office, *Effects of Physical Infrastructure Spending on the Economy and the Budget Under Two Illustrative Scenarios* (August 2021), www.cbo.gov/publication/57327, and *The Macroeconomic and Budgetary Effects of Federal Investment* (June 2016), www.cbo.gov/publication/51628.

A third development that underlies slower growth in CBO's projections of nonfarm business TFP is climate change. In at least two ways, climate change affects CBO's projections of economic growth in future decades. First, climate change has had an effect on recent productivity trends, in the agency's assessment. Because those recent trends are used to project future trends, CBO's projections thus account for a portion of the effects of climate change. Second, the agency explicitly estimates a certain amount of additional impact from future changes in climate, which are projected to affect the growth of nonfarm business TFP. By CBO's estimate, TFP growth over the 2022–2052 period will be lower by about 0.02 percentage points per year, on average, owing to climate change; as a result, TFP will be about 0.7 percent less and GDP about 0.5 percent less in 2052 than they would have been without those additional effects.[12]

Inflation

CBO projects rates of inflation for two categories: prices of consumer goods and services and GDP prices (the prices of all goods and services included in GDP). Those rates affect nominal interest rates and, consequently, nominal interest payments on federal debt. They also affect income and the indexation of income tax brackets, thereby influencing tax revenues and federal expenditures. In this year's projections, inflation is notably higher in the first few years of the forecast than it was in last year's projections, though inflation over the longer term is about the same.

Prices of Consumer Goods and Services

One measure of consumer price inflation is the annual rate of change in the consumer price index for all urban consumers (CPI-U). Over the 2022–2052 period, that measure of inflation averages 2.4 percent in CBO's projections. That long-term rate is roughly the same as the average rate of inflation since 1992.

Using a chained measure of CPI-U inflation, CBO projects that prices will grow at a rate that is about 0.3 percentage points less than the annual increase in the traditional CPI-U, on average. The chained consumer price index for all urban consumers tends to grow more slowly than the traditional CPI-U, for two reasons. First, it uses a formula that better accounts for households' tendency to substitute goods and services with similar but cheaper alternatives when prices go up. Second, unlike the CPI-U, the chained CPI-U is little affected by statistical bias related to the sample sizes that the Bureau of Labor Statistics uses to compute each index. Historically, inflation as measured by the chained CPI-U has been about 0.25 percentage points lower, on average, than inflation as measured by the CPI-U. CBO's projections reflect that difference between the two measures.

GDP Prices

Over the 2022–2052 period, inflation in GDP prices, as measured by the annual rate of increase in the GDP price index, is projected to average 2.2 percent. That rate is slightly higher than the average annual growth in the GDP price index over the past 30 years (2.0 percent). The increase is mainly attributable to higher projected price growth over the next few years. The GDP price index grows at a different rate than the CPI-U because it is based on the prices of a different set of goods and services and is calculated using a different method.

Changes in Projections of Inflation Since Last Year

Inflation, as measured by growth in either the CPI-U or the GDP price index, is projected to be considerably higher from 2022 to 2025 than CBO projected last year. Data show that prices have been increasing more rapidly in many sectors of the economy than the agency had expected—largely because the combination of strong demand and restrained supply has created tighter markets for goods, services, and labor than the agency anticipated—and CBO has revised its projections upward as a result. CBO did not significantly revise its projections for 2026 to 2051, though. After 2025, inflation is projected to remain close to its long-term average. From 2032 to 2051, CBO projects, the CPI-U and the GDP price index will grow at roughly the same rates as the agency projected last year.

Factors Affecting Inflation

The Federal Reserve sets an explicit goal for the long-run average rate of inflation: 2.0 percent for the personal consumption expenditures (PCE) price index. From 2025 to 2052, the PCE price index is projected to grow at rates that are consistent with that goal. In CBO's projections, other rates of inflation, such as the CPI-U

12. CBO has drawn on studies that relate differences in regional economic activity and growth to differences in regional weather patterns, as well as studies of the economic effects of more-intense storms and rising sea levels. For more information, see Evan Herrnstadt and Terry Dinan, *CBO's Projection of the Effect of Climate Change on U.S. Economic Output*, Working Paper 2020-06 (Congressional Budget Office, September 2020), www.cbo.gov/publication/56505.

and the GDP price index, maintain growth rates that are consistent with those indexes' long-run relationship with the PCE price index. Over the 2026–2052 period, in CBO's projections, inflation in the CPI-U returns to a rate of growth that is slightly higher than that of the PCE price index, and inflation in GDP prices is roughly the same as inflation in the PCE price index.

Interest Rates

CBO projects the interest rates that apply to federal borrowing, including the rates on 10-year Treasury notes and special-issue Social Security bonds. It also projects the average interest rates on federal debt held by the public and on the bonds held in the Social Security trust funds. Those rates influence the cost of the government's debt and the balances of the trust funds.

Interest Rates on Notes, Bonds, and Debt

In CBO's projections, real interest rates on federal borrowing are lower in the future than they were, on average, between 1995 and 2004.[13] That historical period was chosen for comparison because it was a time when expectations of inflation were stable, when there were no severe economic downturns or significant financial crises, and when, according to CBO's estimates, monetary policy was, on average, neutral (that is, the real federal funds rate, which is the interest rate that financial institutions charge each other for overnight loans of their monetary reserves, was, on average, consistent with the economy's operating at full employment during that period).

The agency expects several factors, including slower growth of the labor force and slower growth of TFP relative to its pace in that historical period, to continue to put downward pressure on interest rates through 2052. That downward pressure is expected to be partly mitigated by upward pressure on interest rates from other factors, such as federal debt that is rising in relation to GDP.

The nominal interest rate on 10-year Treasury notes is projected to average 4.0 percent over the 2022–2052 period and to reach 4.6 percent in 2052. The real interest rate on those notes has averaged 0.5 percent since 2009; it is projected to be 1.5 percent in 2032 and to rise thereafter, reaching 2.4 percent in 2052. That projection for 2052 is 0.6 percentage points below the average real interest rate on 10-year Treasury notes over the 1995–2004 period (3.0 percent).

For all federal debt held by the public, the nominal interest rate averages 3.3 percent over the 2022–2052 period, in CBO's projections, and reaches 4.2 percent in 2052. The rate on that debt tends to be lower than the rate on 10-year Treasury notes because many of the Treasury's other securities—which, in addition to the 10-year notes, constitute the securities used to finance federal debt—mature over a shorter period and thus often have a lower interest rate. For example, the rate on 3-month Treasury bills is projected to be 1.2 percentage points lower, on average, than the rate on 10-year Treasury notes over the next decade. The average nominal interest rate on federal debt over the 2022–2052 period is projected to be 0.7 percentage points lower, on average, than the rate on 10-year Treasury notes. That difference is smaller than the projected gap of 0.9 percentage points between the two rates over the 2022–2032 period. The difference between the rates is larger before 2032 because federal debt up to that time includes more Treasury securities that were issued in the wake of the 2020 recession, when the Federal Reserve kept interest rates low to support the economic recovery.

The Social Security trust funds hold special-issue bonds that generally earn interest at higher rates than the average rate of interest on federal debt. Because interest rates have been low for most of the past decade, CBO projects that the average interest rate earned by all bonds held by the Social Security trust funds will be lower than the interest rate on bonds issued over the next decade. The average interest rate on all bonds, which CBO uses to calculate the interest those bonds earn for the trust funds, is projected to average 2.4 percent from 2022 to 2032, which is the year before the combined Social Security trust funds are projected to be exhausted.

Changes in Projections of Interest Rates Since Last Year

CBO's projections of interest rates are higher this year than they were last year. For 10-year Treasury notes and for newly issued bonds held in the Old-Age, Survivors, and Disability Insurance trust funds, nominal interest rates are projected to average 4.0 percent over the 2022–2051 period, up from last year's projection of 3.6 percent; real interest rates are projected to average 1.5 percent over the same period, slightly higher than last

13. For further details on the factors affecting CBO's interest rate projections, see Edward N. Gamber, *The Historical Decline in Real Interest Rates and Its Implications for CBO's Projections*, Working Paper 2020-09 (Congressional Budget Office, December 2020), www.cbo.gov/publication/56891.

year's projected rate of 1.3 percent. For federal debt, the average nominal interest rate is projected to be 3.3 percent over that period, 0.2 percentage points higher than last year's projection. (For a description of the changes in the method CBO uses to calculate the average interest rate on debt, see Appendix D.)

For the first decade of the projection period, CBO expects real interest rates to be higher than it did last year. That is because the agency now anticipates that, in response to recent inflation that was higher than expected, the Federal Reserve will raise the target range for the federal funds rate more rapidly than CBO previously projected. Short-term interest rates will rise in response to that more aggressive tightening of monetary conditions. Long-term rates, which partly reflect the expected path of short-term rates, will also be higher than CBO projected last year.

Because the agency raised its projections of inflation less than it raised its projections of the rate on 10-year Treasury notes (particularly after 2025), this year's projections of real interest rates are significantly higher over the 2022–2031 period—averaging 0.8 percent, compared with last year's projection of 0.1 percent for the same period. The upward revision to CBO's projections of real interest rates falls off sharply after 2027 and is mostly eliminated by the middle of the second decade of the projection period. For the latter two decades of the projection period, real interest rates average 2.0 percent and are roughly the same, on average, as in last year's projections.

Factors Affecting Interest Rates

Interest rates are determined by many factors. To project those rates, CBO compares how the values of factors that affect them are expected to differ in the long term from their average values over the 1995–2004 period (the period CBO uses for historical comparisons).

In CBO's projections for the 2022–2052 period, several factors tend to reduce interest rates on government securities below their average from 1995 to 2004.

- The labor force is projected to grow more slowly than it did from 1995 to 2004. Slower growth in the number of workers tends to increase the amount of capital per worker in the long term, reducing the return on capital and, therefore, decreasing the return on government bonds and other investments.[14]
- The share of total earnings received by higher-earning households is expected to be larger in the future than it was during the 1995–2004 period. Higher-income households tend to save a greater portion of their income, so the difference in the distribution of earnings is projected to increase the total amount of savings available for investment, all other things being equal. As a consequence, the amount of capital per worker is projected to rise, and interest rates are expected to be lower.
- TFP in the nonfarm business sector is projected to grow more slowly in the future than it did from 1995 to 2004. For a given rate of investment, a lower rate of productivity growth reduces the return on capital and results in lower interest rates, all else being equal.

At the same time, in CBO's projections, several factors tend to boost interest rates on government securities above their average over the 1995–2004 period—but not enough to offset the factors pushing rates downward.

- In CBO's baseline projections, federal debt is much larger as a percentage of GDP than it was before 2004, reaching 110 percent by 2032 and 185 percent by 2052. The latter figure is nearly five times the average amount of debt over the 1995–2004 period. Greater federal borrowing tends to crowd out private investment in the long term, reducing the amount of capital per worker and increasing interest rates and the return on capital over time.
- Before the onset of the pandemic in 2020, the percentage of total income that is paid to owners of capital (known as capital's share of income) had been rising for the past three decades. That share is projected to decline from its current percentage over the next decade but to remain greater than its average in decades before 2020. The factors that appear to have contributed to capital's rising share of income (such as technological change and globalization) are likely to persist, keeping it above its average from 1995 to 2004. In CBO's estimation, a larger share of income accruing to owners of capital would directly

14. For more information about the relationship between the growth of the labor force and interest rates, see Congressional Budget Office, *How Slower Growth in the Labor Force Could Affect the Return on Capital* (October 2009), www.cbo.gov/publication/41325.

boost the return on capital and thus would increase interest rates.

- The ongoing retirement of members of the baby-boom generation and slower growth in the size of the labor force mean that there will be fewer workers in their prime saving years relative to the number of older people who are drawing down their savings. As a result, CBO estimates, the total amount of savings available for investment will be less than it otherwise would be (all else being equal), and that decrease is expected to reduce the amount of capital per worker and thereby push up interest rates. (CBO estimates that the effect of that decrease will only partially offset the positive effect of the larger share of earnings received by higher-income households, leaving a net increase in savings available for investment.)
- CBO anticipates that other countries will attract a greater share of global investment in coming decades than they did in the 1995–2004 period. As those countries recover from the global economic downturn caused by the pandemic, they will become increasingly attractive destinations for foreign investment. CBO projects that the increased appeal of investing in those countries will put upward pressure on interest rates in the United States.

Some of those factors are easier to quantify than others. For instance, the effects of labor force growth and rising federal debt on interest rates can be estimated from available data by using theoretical models and the findings of existing research. The extent to which other factors affect interest rates is more difficult to estimate. For example, the effect on interest rates of changes in the distribution of earnings is difficult to quantify.

In light of those sources of uncertainty, CBO relies not only on economic models and findings from the research literature but also on information from financial markets to guide its assessments of the effects of various factors on interest rates over the long term. The current rate on 30-year Treasury bonds, for example, reflects market participants' judgments about the path that interest rates on short-term securities will take 30 years from now.

Appendix C: Changes in CBO's Long-Term Budget Projections Since March 2021

Overview

The Congressional Budget Office's current budget projections for the 2022–2051 period differ from those it published in March 2021.[1] In both cases, the 30-year *extended baseline* projections follow the agency's 10-year projections and then extend most of the concepts underlying them for an additional 20 years. CBO's extended baseline projections are not predictions of budgetary outcomes. Rather, they give lawmakers a point of comparison from which to measure the effects of policy options or proposed legislation.

In CBO's current projections:

- *Spending* as a percentage of gross domestic product (GDP) is higher through 2034 and lower thereafter than it was in last year's projections.
- *Revenues* as a percentage of GDP are higher throughout the 2022–2051 period than they were in last year's projections.
- *Total deficits* as a percentage of GDP are generally larger through 2031 and smaller thereafter, compared with deficits in last year's projections. *Primary deficits* (that is, total deficits excluding net outlays for interest) as a percentage of GDP are now smaller throughout the projection period than they were last year.
- *Federal debt held by the public* rises from 98 percent of GDP in 2022 to 180 percent in 2051 (see Figure C-1). Such debt is lower in most years than the agency projected last year: 4 percentage points lower for 2022 and 22 percentage points lower for 2051.

CBO also changed its projections of amounts in the two Social Security trust funds—the Old-Age and Survivors Insurance (OASI) Trust Fund and the Disability Insurance (DI) Trust Fund. In current projections, those trust funds are exhausted later than was estimated last year.

Changes in Projected Spending

In CBO's extended baseline projections, total spending, which includes net outlays for interest, is higher as a percentage of GDP in 2022 than it was in last year's projections; such spending remains higher through 2034 but is lower from 2035 to 2051.[2] Noninterest spending as a percentage of GDP is higher in 2022 than it was in last year's projections but is generally the same thereafter.[3]

Projected spending on the major health care programs, measured as a percentage of GDP, is now less throughout most of the projection period than was estimated last year, mainly because of changes in the agency's method of developing projections for those programs and increases in its estimates of nominal GDP. Higher estimates of inflation account for most of the increase in CBO's current projections of nominal GDP (see Box C-1). (For a discussion of changes in the method underlying the agency's projections of federal spending on Medicare, see Appendix D.)[4]

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1. See Congressional Budget Office, *The 2021 Long-Term Budget Outlook* (March 2021), www.cbo.gov/publication/56977. Because most of last year's projections ended in 2051, this appendix generally makes comparisons only through that year. For changes in projections of economic factors since 2021, see Appendix B of this report. For changes in projections of demographic factors since 2021, see Congressional Budget Office, *The Demographic Outlook: 2022 to 2052* (July 2022), www.cbo.gov/publication/57975. For further information about budgetary projections for the next decade, see Congressional Budget Office, *The Budget and Economic Outlook: 2022 to 2032* (May 2022), www.cbo.gov/publication/57950.

2. For an additional discussion of recent changes in CBO's budget projections for the first decade of the projection period, see Congressional Budget Office, *The Budget and Economic Outlook: 2022 to 2032* (May 2022), pp. 109–123, www.cbo.gov/publication/57950.

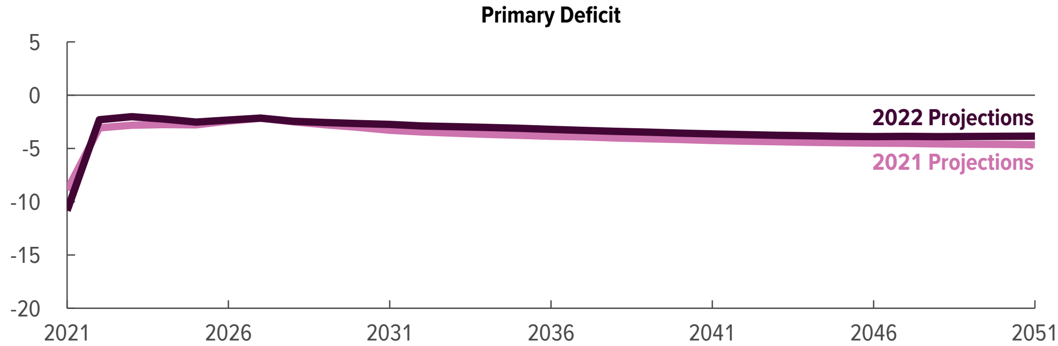
3. Noninterest spending, measured as a share of GDP, was 4 percentage points higher in 2021 than CBO projected it would be last year.

4. Spending on the federal government's major health care programs consists of spending on Medicare, Medicaid, and the Children's Health Insurance Program, as well as outlays to subsidize health insurance purchased through the marketplaces established under the Affordable Care Act and related spending.

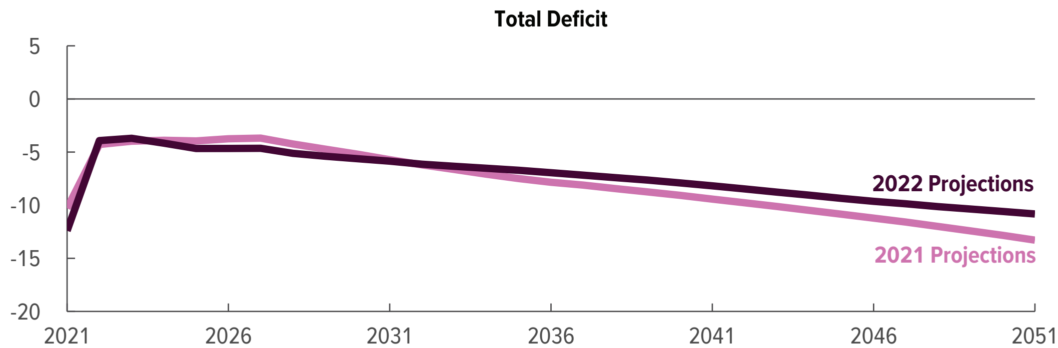
Figure C-1.

CBO's 2021 and 2022 Extended Baseline Projections of Deficits and Federal Debt Held by the Public

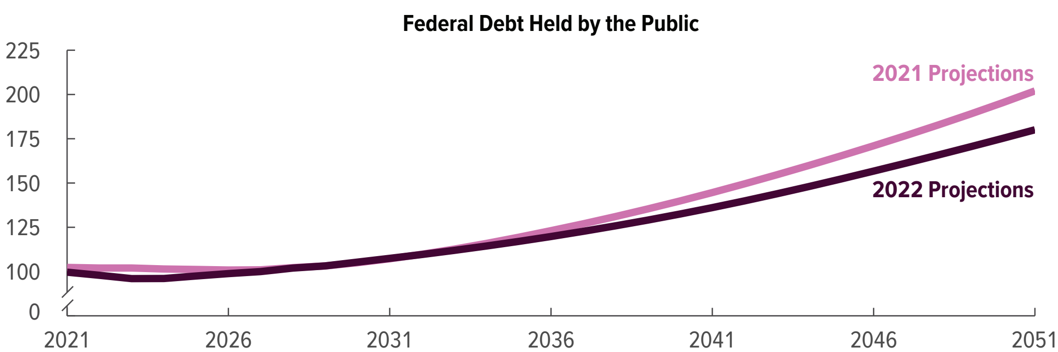
Percentage of GDP



In CBO's current projections, primary deficits as a percentage of GDP are smaller by 0.6 percentage points, on average, over the 2021–2051 period than in last year's projections.



CBO has generally increased its projections of total deficits in the near term but has decreased them over the long term.



Measured as a percentage of GDP, federal debt is now projected to be lower in most years than CBO projected last year.

Data source: Congressional Budget Office. See www.cbo.gov/publication/57971#data.

The extended baseline projections, which generally reflect current law, follow CBO's 10-year baseline budget projections and then extend most of the concepts underlying those projections for an additional 20 years.

Primary deficits exclude net outlays for interest.

GDP = gross domestic product.

Box C-1.

How Estimates of Inflation Affected CBO's Budget Projections

In the Congressional Budget Office's current projections, inflation, as measured by growth in either the consumer price index for all urban consumers or the gross domestic product (GDP) price index, is considerably higher from 2022 to 2025 than was projected last year. The agency did not significantly revise its projections of inflation for 2026 to 2051. Nevertheless, the effects of that higher projected inflation in the near term on the level of nominal GDP, revenues, and spending persist after 2025, when projected inflation falls to the rates in last year's projections. Those higher estimates of inflation also increased nominal interest rates paid on federal debt and net interest payments. (See Appendix B for an additional discussion of changes in CBO's economic forecast since last year.)

In CBO's assessment, higher projections of inflation in this year's economic forecast contributed to the decrease in current projections of federal debt held by the public, measured as a percentage of GDP, compared with last year's projections of such debt. It is, however, difficult to quantify the effects of those higher projections of inflation because inflation affects spending and revenues as well as the level of nominal GDP.

Higher estimates of inflation increased CBO's current projections of nominal GDP—which also reflect the faster growth of real GDP (that is, GDP adjusted to remove the effects of inflation)—compared with such projections last year. Nominal GDP is now 7.9 percent higher in 2022 and 9.3 percent higher in 2051 than was projected last year. For 2051, GDP prices (the prices of all goods and services included in GDP) are 7.2 percent higher, and real GDP is 1.8 percent higher, than the agency projected last year. The higher levels of nominal GDP mean that any amount of spending, revenues, deficits, or debt represents a smaller percentage of GDP than it otherwise would; thus, all else being equal, higher nominal GDP holds down the amount of debt in relation to the nation's output.

CBO did not analyze the effects of higher estimates of inflation on projections of revenues and spending for this report.[1] However, all else being equal, higher estimates of inflation led to higher estimates of taxable income, thereby increasing projections of tax revenues. Higher estimates of inflation also increased projections of noninterest spending, largely because the estimates raised cost-of-living adjustments for certain benefit programs (for example, Social Security) and increased prices for purchases of goods and services. Those changes counteract each other, but the net result is uncertain.

Higher estimates of inflation in the near term increased nominal interest rates in CBO's current projections, which elevated net interest costs in dollars. All else being equal, those higher net interest costs would result in larger budget deficits, pushing the debt that the Treasury issues to the public higher than was estimated in CBO's previous budget projections. Because higher inflation also increases nominal GDP, the effect of inflation on interest payments in relation to the size of the economy is unclear.

1. In March 2022, CBO provided an illustrative estimate of the effects of higher inflation and interest rates on its budget projections. In that example, revenue increases were roughly as large as increases in noninterest spending, so the resulting primary deficit did not differ much from the agency's projections. See Congressional Budget Office, *Budgetary Effects of Higher Inflation and Interest Rates* (March 2022), www.cbo.gov/publication/57868.

Projected discretionary spending, measured as a percentage of GDP, is now higher than estimated last year, mainly because newly enacted legislation increased the agency's projections of such spending over the first 10 years of the projection period; those projections establish the level of discretionary spending in the second and third decades of the projection period.[5]

In current projections, spending on Social Security as a percentage of GDP is slightly lower in all years than was projected last year. That lower spending is the result of offsetting factors: Although projections of nominal outlays are slightly higher than before (largely driven by the projected size of annual cost-of-living adjustments (COLAs) and increased projections of average wages), they are more than offset by higher projections of GDP. Projections of other mandatory spending are now slightly higher over the short term. Neither the projections of spending on Social Security nor the projections of other

5. Discretionary spending encompasses an array of federal activities that are funded through or controlled by appropriation acts. That category includes most defense spending, outlays for highway programs, and spending for many other nondefense activities, such as elementary and secondary education, housing assistance, international affairs, and the administration of justice.

Table C-1.

CBO's 2021 and 2022 Projections of Revenues, Outlays, Deficits, and Federal Debt Held by the Public in Selected Years

Percentage of Gross Domestic Product

	2022	2033	2043	2051
Revenues				
Individual income taxes				
2021 projections	8.8	9.4	9.9	10.3
2022 projections	10.6	9.8	10.3	10.7
Payroll taxes				
2021 projections	5.9	5.8	5.7	5.7
2022 projections	5.9	5.9	5.8	5.7
Corporate income taxes				
2021 projections	1.1	1.2	1.2	1.2
2022 projections	1.6	1.4	1.3	1.3
Other[a]				
2021 projections	1.5	1.1	1.2	1.3
2022 projections	1.4	1.2	1.3	1.3
Total Revenues				
2021 projections	**17.3**	**17.6**	**18.0**	**18.5**
2022 projections	**19.6**	**18.2**	**18.7**	**19.1**
Outlays				
Mandatory				
Social Security				
2021 projections	5.2	6.1	6.3	6.3
2022 projections	4.9	6.0	6.2	6.3
Major health care programs[b]				
2021 projections	5.7	7.3	8.6	9.4
2022 projections	5.8	7.0	8.2	8.7
Other[c]				
2021 projections	2.5	2.1	2.0	1.9
2022 projections	4.3	2.1	2.0	1.9
Subtotal, Mandatory				
2021 projections	13.4	15.6	16.9	17.6
2022 projections	14.9	15.1	16.5	17.0
Discretionary				
2021 projections	7.0	5.6	5.5	5.5
2022 projections	7.0	6.1	6.0	6.0
Net interest				
2021 projections	1.2	3.1	5.8	8.6
2022 projections	1.6	3.4	5.0	7.0
Total Outlays				
2021 projections	**21.6**	**24.2**	**28.2**	**31.8**
2022 projections	**23.5**	**24.6**	**27.4**	**29.9**

Continued

mandatory spending changed significantly over the long term since last year (see Table C-1).[6]

Net outlays for interest total 1.6 percent of GDP in 2022—0.4 percentage points higher than in last year's projections because of higher interest rates. Such outlays remain higher through 2034 than was estimated last year, but for most of the second decade and for all of the third decade of the projection period, they are lower than previously estimated (see Figure C-2). That is because primary deficits are now projected to be smaller

6. Other mandatory spending includes outlays for retirement programs for federal civilian and military employees, certain programs for veterans, certain refundable tax credits, the Supplemental Nutrition Assistance Program, and all other mandatory programs aside from Social Security and the health care programs described above.

Table C-1. Continued

CBO's 2021 and 2022 Projections of Revenues, Outlays, Deficits, and Federal Debt Held by the Public in Selected Years

Percentage of Gross Domestic Product

	2022	2033	2043	2051
Deficit				
2021 projections	**-4.3**	**-6.6**	**-10.1**	**-13.3**
2022 projections	**-3.9**	**-6.3**	**-8.8**	**-10.8**
Federal Debt Held by the Public				
2021 projections	**102**	**113**	**155**	**202**
2022 projections	**98**	**112**	**144**	**180**
Memorandum:				
Noninterest Spending				
2021 projections	20.4	21.1	22.4	23.2
2022 projections	21.9	21.2	22.4	22.9
Primary Deficit[d]				
2021 projections	-3.1	-3.6	-4.4	-4.6
2022 projections	-2.3	-3.0	-3.8	-3.8

Data source: Congressional Budget Office. See www.cbo.gov/publication/57971#data.

a. Consists of excise taxes, remittances to the Treasury from the Federal Reserve System, customs duties, estate and gift taxes, and miscellaneous fees and fines.

b. Consists of spending on Medicare (net of premiums and other offsetting receipts), Medicaid, and the Children's Health Insurance Program, as well as outlays to subsidize health insurance purchased through the marketplaces established under the Affordable Care Act and related spending.

c. Includes the refundable portions of the earned income tax credit, the child tax credit, and the American Opportunity Tax Credit.

d. Excludes net outlays for interest.

throughout the period, and the average interest rate on federal debt is now projected to be lower after 2034. (For a discussion of changes in the method underlying the agency's projections of the average interest rate on federal debt, see Appendix D.)

Changes in Projected Revenues

Compared with revenues in last year's projections, current projections of federal revenues as a percentage of GDP are higher throughout the 2022–2051 period—by 2.3 percentage points in 2022 and 0.6 percentage points in 2051 (see Figure C-3).[7] The major sources of revenues—that is, individual income taxes, payroll taxes, and corporate income taxes—are generally higher as a percentage of GDP throughout the projection period than was estimated last year.

The largest increases are in the projections of receipts of individual and corporate income taxes, whereas payroll tax receipts increased only slightly. In the near term, the increase in projected individual income tax receipts is a result of their recent unexplained strength; in the longer term, the increase is attributable to an upward revision to the agency's estimate of corporate business income taxed at the individual level. Projected receipts of individual income taxes and payroll taxes were also higher measured in nominal dollars, because of increases in estimates of factors that affect the size of the economy, including wages and salaries, proprietors' income, and corporate profits. Those changes had less of an impact on receipts as a percentage of GDP because they affected both revenues and GDP. CBO also increased its estimate of revenues from corporate income taxes as a percentage of GDP in the near term because recent receipts from such taxes have been stronger than expected. The agency has updated its modeling of corporate taxes and now estimates that a greater share of corporate profits will be taxable over the longer term.

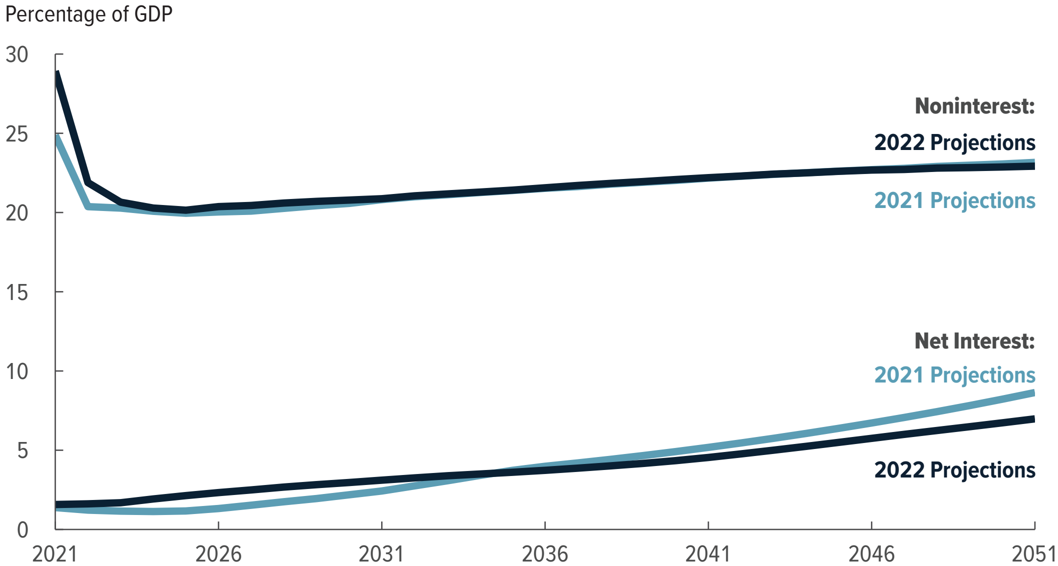
Changes in Projected Deficits and Debt

As a result of the changes to CBO's projections of spending, revenues, and GDP, projections of primary deficits as a percentage of GDP are now smaller throughout the projection period than they were last year. The current estimate of the primary deficit for 2022 is 2.3 percent of

7. Revenues, measured as a percentage of GDP, were 2.1 percentage points higher in 2021 than CBO projected they would be last year.

Figure C-2.

CBO's 2021 and 2022 Extended Baseline Projections of Outlays



In CBO's current projections, noninterest spending as a percentage of GDP is higher for 2022 than it was in last year's projections. Projections of such spending are generally the same after that year.

Net outlays for interest, measured as a percentage of GDP, are higher through 2034 than in last year's projections and lower thereafter.

Data source: Congressional Budget Office. See www.cbo.gov/publication/57971#data.

The extended baseline projections, which generally reflect current law, follow CBO's 10-year baseline budget projections and then extend most of the concepts underlying those projections for an additional 20 years.

GDP = gross domestic product.

GDP, which is 0.8 percentage points smaller than it was last year.[8] Projected primary deficits now average 2.4 percent of GDP from 2023 to 2031 and 3.5 percent from 2032 to 2051—0.3 percentage points and 0.6 percentage points smaller, respectively, than their averages over those periods in last year's projections.

However, as a result of upward revisions to net interest costs in the first decade of the projection period, projected total deficits as a percentage of GDP are now generally larger through 2031. Although the current estimate of the total deficit for 2022—3.9 percent of GDP—is 0.4 percentage points smaller than was projected last year, total deficits now average 4.9 percent of GDP from 2023 to 2031—0.5 percentage points larger than their average over that period in last year's projections.

As a percentage of GDP, total deficits are smaller in the long term than was projected last year. They average 8.4 percent of GDP over the 2032–2051 period—1.3 percentage points less than in last year's projections. The total deficit in 2051 is now estimated to be 10.8 percent of GDP—2.5 percentage points less than last year's estimate. Higher revenues, less spending on the major health care programs, and lower net outlays for interest in relation to the size of the economy pushed deficits lower as a percentage of GDP. Those changes were partially offset by higher projections of discretionary spending.

The factors that caused smaller projected deficits than CBO estimated last year also caused lower projections of federal debt held by the public. Measured as a percentage of GDP, such debt is projected to be lower in most years than the agency estimated last year. In current projections, that debt rises from 98 percent of GDP in 2022 to 180 percent in 2051; last year, CBO projected it would rise from 102 percent of GDP in 2022 to 202 percent in 2051.

Changes in Projected Amounts in the Social Security Trust Funds

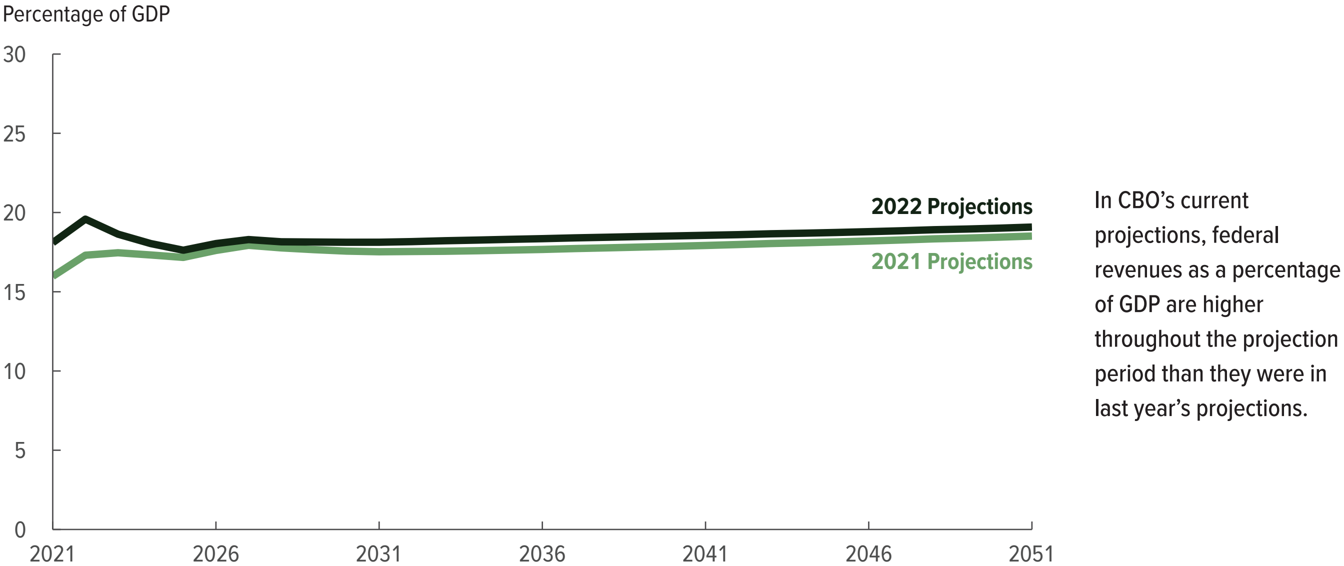
CBO projects that if current laws governing the Social Security program's taxes and benefits did not change, the OASI trust fund would be exhausted in calendar year 2033, and the DI trust fund would be exhausted in calendar year 2048.[9] Those dates are later than the

8. The primary deficit, measured as a share of GDP, was 1.9 percentage points larger in 2021 than CBO projected last year.

9. CBO expects to publish information later this year about its projections of outlays and income for Social Security over the next 75 years. For last year's long-term projections for Social Security, see Congressional Budget Office, *CBO's 2021 Long-Term Projections for Social Security: Additional Information* (July 2021), www.cbo.gov/publication/57342.

Figure C-3.

CBO's 2021 and 2022 Extended Baseline Projections of Revenues



Data source: Congressional Budget Office. See www.cbo.gov/publication/57971#data.

The extended baseline projections, which generally reflect current law, follow CBO's 10-year baseline budget projections and then extend most of the concepts underlying those projections for an additional 20 years.

GDP = gross domestic product.

agency projected last March, by 1 year and by 13 years, respectively. If the balances of the OASI and DI trust funds were combined, the funds would be exhausted in calendar year 2033, which is 1 year later than CBO projected last year.

Since last year, CBO has increased its estimates of income credited to the OASI trust fund from 2022 until the fund is exhausted in 2033 by 8.6 percent. The increase is mainly attributable to higher revenues from payroll taxes associated with the higher nominal GDP reflected in current projections. However, the agency has also increased its estimated expenditures from the fund, which are now 5.8 percent higher over that period than they were last year. Those increases stem primarily from higher estimates of COLAs through 2024 and higher projections of average wages, which offset the effect of fewer OASI beneficiaries through 2033 and slightly lower estimates of COLAs, on average, from 2025 through 2031. The projected increases in income more than offset the projected increases in expenditures from the fund, leading to the later estimate of when it would be exhausted.

In this year's projections, income credited to the DI trust fund from 2022 until it is exhausted in 2048 is higher by 12.7 percent than was estimated last year, mainly because of higher projected revenues from payroll taxes. The projected expenditures from the DI trust fund are lower from 2022 through 2035—in total, by 3.3 percent—than the agency estimated last year. CBO revised those projections of expenditures downward mainly because far fewer new beneficiaries have begun receiving benefits in recent months than was previously estimated, which led the agency to reduce its projection of the number of DI beneficiaries. Those downward revisions to the number of beneficiaries are partially offset by higher estimates of COLAs through 2024 and higher projections of average wages. The projections of increased income and (through 2035) decreased expenditures led to the later estimate of when the DI trust fund would be exhausted.

All told, CBO's projections of income credited to the combined OASDI trust funds are 8.8 percent higher over the 2022–2033 period than they were last year, and projections of expenditures are 4.6 percent higher. Because the increases in estimated income exceed the increases in estimated expenditures, CBO now anticipates that the combined trust funds would be exhausted 1 year later than the agency projected last year.

Appendix D: Changes to Methods Underlying Selected Long-Term Budget Projections

Overview

The Congressional Budget Office has changed the way it develops its long-term budget projections since publishing them last year; two of the changes have significant budgetary effects.[1] CBO refined its method of projecting the average nominal interest rate on federal debt held by the public in the second and third decades of the 30-year projection period. The agency also refined its estimate of a key parameter in the final year of the projection period (2052, in this year's projections)—specifically, the estimate of the growth in federal spending on health care above and beyond that attributable to demographic changes and to growth in potential gross domestic product (GDP) per person.[2] Refining that estimate, in turn, affects projections of federal spending on health care throughout the second and third decades of the projection period.

Projections of the Average Interest Rate on Federal Debt

In the past, to project the average nominal interest rate on federal debt beyond the standard 10-year budget period, CBO first calculated the difference between that rate and the nominal interest rate on 10-year Treasury notes as projected within that period. That difference was then held constant in the second and third decades of the projection period. (In last year's projections, the difference was calculated using rates from the 10th year of the projections underlying the agency's January 2020 forecast—the most recent forecast that excludes the effects of the coronavirus pandemic.) CBO developed that method when its only projections of nominal interest rates beyond the 10-year budget period were of rates for 10-year Treasury notes; now, the agency also projects nominal interest rates for Treasury securities of other maturities in the second and third decades of the projection period.

The method CBO now uses to project the average nominal interest rate on federal debt held by the public beyond the 10-year budget period is similar to the method it uses for projections within that period, but with some notable differences.[3] Projected interest payments *within* the 10-year budget period are generally based on the agency's projections of primary deficits (which exclude net outlays for interest), the stock of outstanding Treasury securities at the beginning of the projection period, the issuance of Treasury securities of different maturities, and interest rates on those securities of different maturities. Those projections reflect monthly calculations of interest rates. In the new method, projections of the average interest rate on federal debt *beyond* the 10-year budget period are based on those factors as well, but they reflect calculations that use quarterly data, and they do not include details about Treasury securities that mature in less than three months.

Using similar methods to project the average interest rate on federal debt for all three decades of the projection period makes the agency's projections of that rate more consistent throughout the period. Furthermore, when using the new method, debt issued in the past, which is subject to certain interest rates, is replaced with debt subject to the interest rates that CBO projects would occur in the future—a factor that the old method did not account for, because it used information from the 10th year of the projection period as the basis for the average interest rate on federal debt over the longer term.

1. For last year's projections, see Congressional Budget Office, *The 2021 Long-Term Budget Outlook* (March 2021), www.cbo.gov/publication/56977.

2. Potential GDP is the maximum sustainable output of the economy.

3. For a discussion of CBO's method of projecting interest rates in the first 10 years of the projection period, see Congressional Budget Office, *Federal Net Interest Costs: A Primer* (December 2020), www.cbo.gov/publication/56780.

Compared with the old method (all else being equal), the new one has resulted in lower projections of the average interest rate on federal debt, lower net outlays for interest as a share of GDP, and lower debt as a share of GDP in the second and third decades of the projection period. Accordingly, in CBO's current projections, the average interest rate on federal debt reaches 4.2 percent in 2052 instead of 4.6 percent using the old method. Net outlays for interest are 7.2 percent of GDP that year instead of 7.9 percent (see Figure D-1). And the current projection of federal debt held by the public in 2052 is 185 percent of GDP instead of 194 percent.

Projections of Federal Spending on Health Care

CBO's new method of projecting federal spending on health care beyond the 10-year budget period is similar to the method used last year but with two important differences. First, the agency refined its estimate of additional cost growth at the end of the 30-year projection period. (Additional cost growth is the amount by which the growth rate of nominal health care spending per person, adjusted to remove the effects of demographic changes, exceeds the growth rate of potential GDP per person.)[4] That new estimate consequently leads to changes in projections of spending on Medicare in the last two decades of the projection period. Second, the estimates of additional cost growth in Medicare Parts A and B now reflect CBO's expectation that certain provisions of current law will cause federal spending on various aspects of health care to grow more slowly than spending on health care overall.[5]

Estimate of Additional Cost Growth in Health Care Overall

In the past, CBO developed its long-term estimate of additional cost growth in the health care sector, and in Medicare more narrowly, by choosing a parameter for the end of the projection period on the basis of historical patterns. To refine that estimate in this year's projections, the agency separately assessed the effects of three factors on the growth of spending on health care in 2052.[6]

- *Growth in real national income per person* has been—and, in CBO's estimation, will continue to be—the most significant factor in the growth of spending on health care. The agency projects that in 2052, that factor would account for just over half of the rate of additional cost growth in the health care sector.
- *Increasing medical prices* have been—and, in CBO's estimation, will continue to be—another significant factor in the growth of spending on health care. The agency projects that in 2052, such increases would account for slightly less than half of the rate of additional cost growth in the health care sector.
- *Changes in out-of-pocket spending for health care* have historically been an important factor in the growth of spending on health care. However, CBO projects that, under current law, the out-of-pocket share of national health expenditures would not change over the 30-year projection period. That is, the agency does not expect changes in out-of-pocket spending to affect additional cost growth in the health care sector in 2052.

CBO reduced its estimate of additional cost growth in federal spending on health care because of its assessment of historical trends. Although the growth rate of such spending has varied over time, it has generally been declining, the agency estimates. For instance, additional cost growth in Medicare averaged 1.1 percent from 1985 to 2017 but averaged about –0.1 percent from 2005 to 2017.

CBO's estimate of additional cost growth in health care overall in 2052 is 0.6 percent. As they have in previous extended baseline projections, the rates of additional cost growth in this year's projections move linearly from their rates at the end of the 10-year budget period—as determined by the method of projecting such rates in

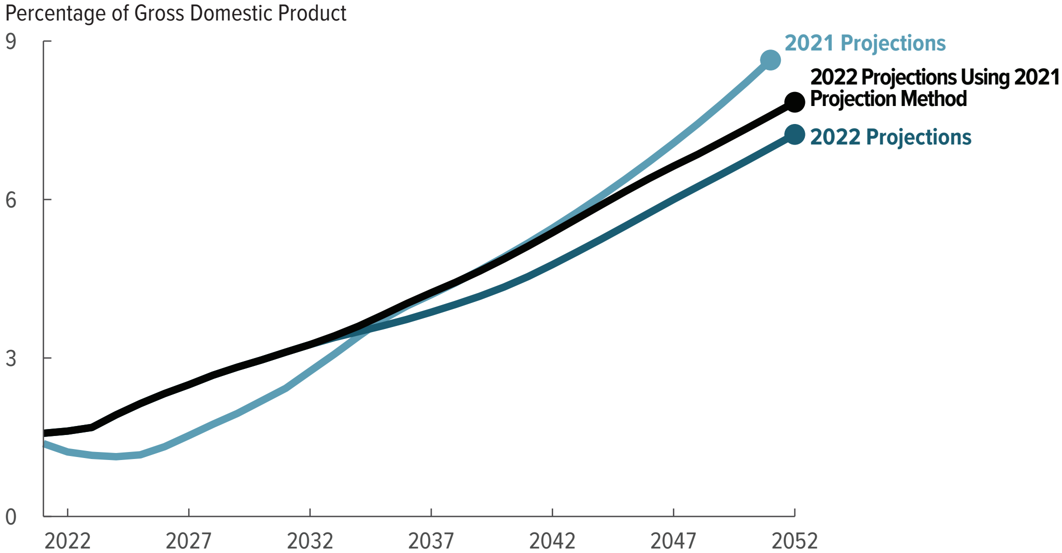
4. In CBO's past reports, the term "excess cost growth" was used instead of "additional cost growth" to describe the increase in such spending.

5. Medicare is the most significant contributor to growth in federal spending on health care over the 2032–2052 period. This appendix therefore focuses on a discussion of the agency's projections of spending on Medicare in the extended baseline. For a discussion of the agency's projections of spending on Medicare for the 2022–2032 period, see Congressional Budget Office, *The Budget and Economic Outlook: 2022 to 2032* (May 2022), www.cbo.gov/publication/57950.

6. To refine its estimate of additional cost growth in the health care sector, CBO essentially used the same method that the Centers for Medicare & Medicaid Services' Office of the Actuary uses to produce the 75-year projections of Medicare spending for its annual report to the Congress. Within that framework, CBO uses its own estimates of key parameters: an income-technology elasticity of 1.27, an insurance elasticity of –0.20, and a price elasticity of –0.55. For a discussion of the methods underlying projections by the Office of the Actuary, see Centers for Medicare & Medicaid Services, "The Long-Term Projection Assumptions for Medicare and Aggregate National Health Expenditures" (accessed May 16, 2022), https://tinyurl.com/msfjx6te (PDF, 1.3 MB).

Figure D-1.

Net Outlays for Interest Using Old and New Projection Methods, 2021 to 2052



CBO's current projection of net outlays for interest in 2052 is 0.6 percentage points lower than it would have been had the agency not refined its method of estimating the average nominal interest rate on federal debt held by the public.

Data source: Congressional Budget Office. See www.cbo.gov/publication/57971#data.

that period—to estimated rates at the end of the 30-year projection period.

Estimate of Additional Cost Growth in Medicare Part D

In CBO's current projections for 2052, additional cost growth in Medicare Part D is 0.6 percent; in last year's projections, it was 1.0 percent at the end of the 30-year projection period.[7] (CBO estimates that additional cost growth in spending on Medicaid and private health insurance premiums would also be 0.6 percent in 2052, down from 1.0 percent in last year's projections.) In the agency's view, additional cost growth in Medicare Part D at the end of the 30-year projection period would be the same as such growth in the health care sector overall. That is because the health care system in the United States will be integrated to such a degree over the long term that spending growth in most parts of the system will be affected by common factors, such as changes in physicians' practices and the development and diffusion of new medical technologies.

7. Part D is Medicare's optional prescription drug benefit, which is delivered through private-sector companies. Part A primarily covers services provided by hospitals and other facilities, and Part B covers physicians' and other outpatient services. Part C of Medicare (known as Medicare Advantage) specifies the rules under which private health care plans can assume responsibility for, and be paid for, providing benefits covered under Parts A and B.

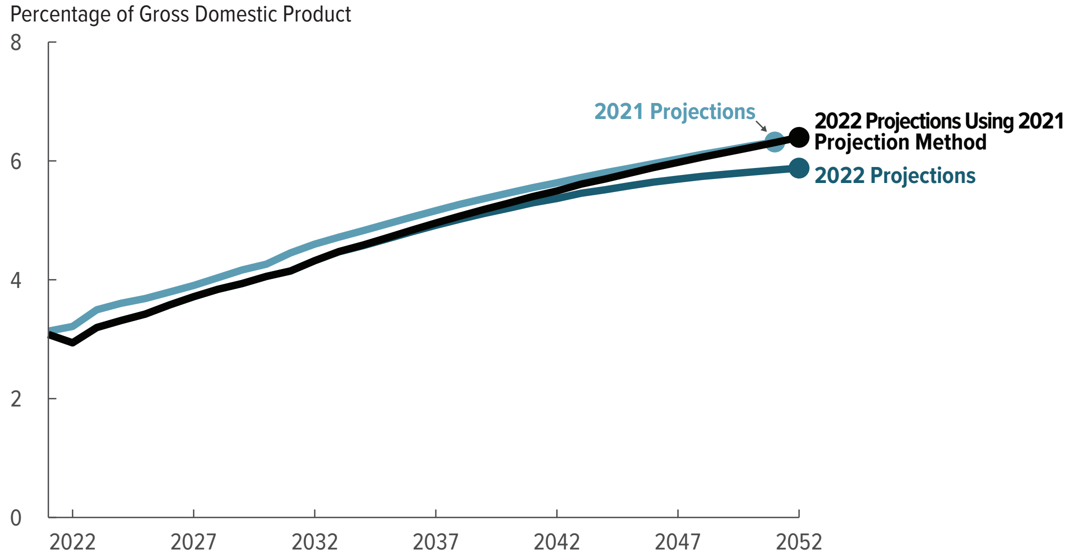
Estimates of Additional Cost Growth in Medicare Parts A and B

In CBO's current projections for 2052, additional cost growth is 0.1 percent in Medicare Part A and 0.2 percent in Medicare Part B. In last year's projections, both of those rates were 1.0 percent at the end of the 30-year projection period. The agency expects additional cost growth in Medicare Parts A and B to be lower than such growth in Medicare Part D (and in the health care sector overall) because of the way prices in Parts A and B are determined. Under current law, in Parts A and B, the prices of labor, goods, and services are adjusted to account for gains in private nonfarm business productivity (the ability to produce the same output using fewer inputs, such as hours of labor).[8] Because of expected gains in productivity, the increase in prices paid to providers would be less than they otherwise would be. In the agency's assessment, that would slow the growth in the use of Medicare Parts A and B by a small amount. That expectation largely stems from the agency's view that lower prices would cause some providers to decline to treat patients insured under Parts A or B and might also reduce providers' incentives to adopt innovative services and technologies.

8. See Centers for Medicare & Medicaid Services, *Methodology for Projecting Total Factor Productivity for the Private Nonfarm Business Sector* (March 2022), https://tinyurl.com/36c4d654 (PDF, 144 KB).

Figure D-2.

Spending on Medicare Using Old and New Projection Methods, 2021 to 2052



CBO's current projection of federal spending on Medicare in 2052 is about one-half of one percentage point lower than it would have been had the agency not refined its method of estimating additional cost growth in that program.

Data source: Congressional Budget Office. See www.cbo.gov/publication/57971#data.

Additional cost growth is the amount by which the growth rate of nominal health care spending per person (adjusted to remove the effects of demographic changes) exceeds the growth rate of potential gross domestic product (GDP) per person. Potential GDP is the maximum sustainable output of the economy.

Changes in Projections Attributable to the New Method of Estimating Additional Cost Growth

The new method of estimating additional cost growth in Medicare results in lower projections of federal spending on that program from 2037 to 2052 (see Figure D-2). In CBO's current projections, spending on Medicare (net of offsetting receipts, which are mostly premiums paid by enrollees) reaches 5.9 percent of GDP in 2052—lower than the 6.4 percent that would have resulted from using estimates of additional cost growth derived from the agency's previous method. The current projection of federal spending on health care overall in 2052 (comprising spending on Medicare, net of premiums and other offsetting receipts, and on Medicaid, the Children's Health Insurance Program, and marketplace subsidies) is also lower than it would have been using the previous estimate of additional cost growth: 8.8 percent of GDP instead of 9.4 percent. And the current projection of federal debt held by the public in 2052 is 185 percent of GDP instead of 190 percent that would result from using the previous method.

List of Tables and Figures

Tables

Figures

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About This Document

This volume is one of a series of reports on the state of the budget and the economy that the Congressional Budget Office issues each year. It builds on the 10-year budget and economic projections that CBO released on May 25, 2022. In keeping with CBO's mandate to provide objective, impartial analysis, the report makes no recommendations.

Overseen by Molly Dahl and prepared with guidance from Devrim Demirel, Edward Harris, John Kitchen (formerly of CBO), John McClelland, Julie Topoleski, and Jeffrey Werling, the report represents the work of many analysts at CBO. Jordan Trinh prepared the visual summary. Molly Dahl wrote Chapter 1 in collaboration with John Kitchen and with contributions from Daniel Crown, Sebastien Gay, Joseph Kile, Kerk Phillips, and John Seliski. Molly Dahl wrote Chapter 2 in collaboration with Kathleen Burke and with contributions from Xinzhe Cheng and Jordan Trinh. Aaron Betz wrote Chapter 3 with contributions from Edward Gamber, Chandler Lester, Jeffrey Schafer, and Robert Shackleton (formerly of CBO). Daniel Crown wrote Chapter 4 in collaboration with Kerk Phillips and with a contribution from Damir Cosic. Molly Dahl compiled Appendix A. Aaron Betz authored Appendix B with contributions from Damir Cosic, Daniel Crown, Edward Gamber, Chandler Lester, Jeffrey Schafer, and Robert Shackleton. Molly Dahl and Charles Pineles-Mark prepared Appendix C. Molly Dahl prepared Appendix D with contributions from Yiqun Gloria Chen (formerly of CBO), Michael Cohen, Grace Hwang, Kyoung Mook Lim, Michael McGrane, and Jordan Trinh.

Barry Blom, Stuart Hammond, Noah Meyerson, Eamon Molloy, Lisa Ramirez-Branum (formerly of CBO), Dan Ready, Sarah Sajewski, Emily Stern, and Robert Stewart contributed to the analysis in this report with guidance from Christina Hawley Anthony, Chad Chirico, Elizabeth Cove Delisle, Theresa Gullo, Leo Lex, Paul Masi, and Sam Papenfuss.

The long-term budget simulations were coordinated and prepared by Charles Pineles-Mark along with Xinzhe Cheng, Damir Cosic, Daniel Crown, Kyoung Mook Lim, and Jordan Trinh.

Edward Harris and Joshua Shakin coordinated the revenue simulations, which were prepared by Kathleen Burke, Paul Burnham, Madeleine Fox, Nathaniel Frentz, Shannon Mok, Omar Morales, Tess Prendergast, Kurt Seibert, Jennifer Shand, Ellen Steele, and James Williamson.

Robert Arnold coordinated the macroeconomic projections, which were prepared by Grace Berry, Aaron Betz, Yiqun Gloria Chen, Daniel Fried, Edward Gamber, Ron Gecan, Mark Lasky, Junghoon Lee, Chandler Lester, Vinay Maruri, Michael McGrane, Christine Ostrowski, Jeffrey Schafer, John Seliski, and Robert Shackleton.

Daniel Crown developed the population projections.

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Jordan Trinh coordinated the fact-checking process, which he contributed to along with Nicholas Abushacra, Grace Berry, Kyoung Mook Lim, Omar Morales, and Lucy Yuan.

Mark Doms, Jeffrey Kling, and Robert Sunshine reviewed the report. Christine Bogusz, Scott Craver, and Bo Peery edited it, and Casey Labrack and R. L. Rebach prepared the text for publication and created the graphics. Nicholas Abushacra, Grace Berry, Daniel Crown, Kyoung Mook Lim, Jordan Trinh, and Lucy Yuan prepared the supplemental information files. The report is available at www.cbo.gov/publication/57971.

CBO seeks feedback to make its work as useful as possible. Please send comments to communications@cbo.gov.

Phillip L. Swagel
Director
July 2022

About Teaching Writing Data Tools Blog YouTube Podcasts

Follow @AswathDamodaran

My name is Aswath Damodaran, and I teach corporate finance and valuation at the Stern School of Business at New York University. I am a teacher first, who also happens to love untangling the puzzles of corporate finance and valuation, and writing about my experiences. As a result, I am at the intersection of three businesses, education, publishing and financial services, that are all big, inefficiently run and deserve to be disrupted. I may not have the power to change the status quo in any of these businesses, but I can stir the pot, and this website is my attempt to do so.

Broadly speaking, the website is broken down into four sections. The first, teaching, includes all of my classes, starting with the MBA classes that I teach at Stern and including the shorter (2-day to 3-day) executive sessions I have on corporate finance and valuation. You will find not only the material for the classes (lecture notes, quizzes), but also webcasts of the classes that you can access on different platforms. I also have classes specifically tailored to an online audience on valuation, corporate finance and investment philosophies, as well as my quirky versions of accounting and statistics classes. The second, writing, includes links to almost everything I have written and continue to write, starting with my books and extending to my practitioner papers (on equity risk premiums, cash flows and other things valuation-related). The third, data, contains the annual updates that I provide on industry averages, for US and global companies, on both corporate finance and valuation metrics (including multiples). It is also where I provide my estimates of equity risk premiums and costs of capital. The fourth, tools, incorporates the spreadsheets that I have developed over time to value and analyze companies and short in-practice webcasts on how to analyze companies.

I have been told that my website is ugly, and I apologize for its clunky look and feel. While some of you have offered to make it look better for me, and I thank you for your kindness, I need to be able to tweak, modify and adapt the website as I go along and to do that, I have to work with what I know about website design, which is not much. You can try the search engine below and if that does not work, try this guide to the site.

Implied Equity Risk Premium Update

Implied ERP on May 1, 2023= 4.77% (Trailing

Other Updates

Teaching: The Spring 2022 Corporate Finance class, now fully archived, can be found here and the archived Spring 2022 Valuation class is linked here. The online versions of these classes can be found here and NYU is offering certificate versions here. In spring 2023, I will be teaching all three classes again and you can find the links to them here. If you need a short brush up on the basics of finance, I have added a class on the

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12 month, with adjusted payout); 5.30% (Trailing 12 month cash yield); 5.57% (Average CF yield last 10 years); 5.09% (Net cash yield); 4.52% (Normalized Earnings & Payout)

Implied ERP in previous month =4.88% (Trailing 12 month, with adjusted payout); 5.44% (Trailing 12 month cash yield); 5.72% (Average CF yield last 10 years); 5.19% (Net cash yield); 4.64% (Normalized Earnings & Payout)

Downloadable datasets (For more data, go here)

1. Implied ERP by month for previous months
2. Implied ERP (annual) from 1960 to Current
3. Implied ERP (daily) from February 14, 2020 - December 31, 2020
4. My annual update paper on ERP (March 2022)
5. My annual update paper on Country Risk (July 2021)
6. My data on ERP & CRP by country (January 2022)

Downloadable spreadsheets (For more spreadsheets, go here)

1. Spreadsheet to compute current ERP for current month
2. Spreadsheet to value the S&P 500 (January 1, 2023)
3. Valuation Spreadsheet for non-financial service firms with video guidance

foundations of finance as well as a minimalist accounting class to my online list. In 2021, I added a statistics class to the mix, again taught from the perspective of someone who uses statistics rather than a statistical expert.

Writing: This paper on valuing Tesla (with Brad Cornell) won readers' award (Bernstein-Levy) in Journal of Portfolio Management. Download the latest version of my annual equity risk premium update by clicking here and the latest version of my annual country risk update by clicking here. I also have a paper on valuing users, subscribers and members. My book on Narrative and Numbers, from Columbia University Press, should be in bookstores and the third edition of The Dark Side of Valuation came out in 2018. Finally, Brad and I have written a new paper on what we call the big market delusion, on how the allure of big markets coupled with overconfident entrepreneurs/investors can create over pricing across companies. In 2020, we added a paper on ESG, a concept that has been oversold and overhyped by its proponents, as well as a paper on value investing's travails in the last decade. In 2020, I also wrote a series of fourteen posts on the COVID crisis, with the emphasis on markets, in real time, which I put together as a paper (way too long) on what I learned and unlearned. I also added a paper on the disruption coming to the IPO process.

Data: The latest overall data update was on January 5, 2023; my next one will be in January 2024. My country risk premiums also get updated midyear; my next update will be in July 2023. Check under data for downloads and links, as well as archived data from prior years.

Tools: Check under tools for additions to spreadsheets and webcast. uValue is available at the iTunes store.

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DEPRECIATION OF GROUP PROPERTIES

By Robley Winfrey

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APPENDIX

The 18 Type Curves

The following section contains supplementary information on the 18 type curves. The final equations of the type frequency curves give the numerical values of the coefficients and constants in the general equations below. In Table 42 (page 124) is tabulated the percent surviving and probable lives corresponding to the 18 type curves which are illustrated in Figs. 28, 29, and 30.

General Equations of the 18 Type Frequency Curves

Left Mode Nos. 0 and 1

$$\left| \begin{array}{ll} y_x = y_0 \left(1 - \frac{(x \pm d_m)^2}{a^2}\right)^m & \text{for } x \text{ values to left of mode.} \\ y_x = y_0 \left(1 - \frac{(x \pm D_m)^2}{A^2}\right)^M & \text{for } x \text{ values to right of mode.} \end{array} \right.$$

Left Mode Nos. 2, 3, and 5 and
Right Mode Nos. 1, 2, 3, 4, and 5

$$\begin{aligned} y_x = {} & Y_e \left(1 + \frac{x \pm D_m}{A_1}\right)^{M_1} \left(1 - \frac{x \pm D_m}{A_2}\right)^{M_2} \\ & + y_e \left(1 + \frac{x \pm d_m}{a_1}\right)^{m_1} \left(1 - \frac{x \pm d_m}{a_2}\right)^{m_2} \end{aligned}$$

Left Mode No. 4

$$\left| \begin{array}{ll} y_x = Y_e \left(1 - \left[\frac{x+D}{A_1}\right]^2\right)^{M_1} + y_e \left(1 - \left[\frac{x+d}{a_1}\right]^2\right)^{m_1}, & -10 \leqq x \leqq -D \\ y_x = Y_e \left(1 - \left[\frac{x+D}{A_2}\right]^2\right)^{M_2} + y_e \left(1 - \left[\frac{x+d}{a_1}\right]^2\right)^{m_1}, & -D \leqq x \leqq -d \\ y_x = Y_e \left(1 - \left[\frac{x+D}{A_2}\right]^2\right)^{M_2} + y_e \left(1 - \left[\frac{x+d}{a_2}\right]^2\right)^{m_2}, & -d \leqq x \leqq (A_2 - D) \end{array} \right.$$

Symmetrical Nos. 0, 1, 2, 3, 4, 5, and 6

$$y_x = y_0 \left(1 - \frac{x^2}{a^2}\right)^m$$

in which
y_x = ordinate to the frequency curve at age x (origin at the mean age).
y_0 = ordinate to the frequency curve at its mode.
Y_e = ordinate to the major constituent curve at its mean.
y_e = ordinate to the minor constituent curve at its mean.

x = age (in units equal to 10 percent of average life), measured from the average-life ordinate.

D_m, d_m = x distance from the mean of the type curve to the mean of the constituent curve.

$A, A_1, A_2, a, a_1, a_2, M, M_1, M_2, m, m_1, m_2$ are parameters.

Final Equations of the 18 Type Frequency Curves

In the following 18 equations, x is measured from the mean, or average life, negative values of x being to the left of 100 percent of average life and positive values to the right. An age interval of 10 percent of average life is equal to x. Therefore, if $x = -2.5$ the equivalent age is 75 percent of average life. When $x = +4.2$ the equivalent is 142 percent of average life.

Left Mode No. 0

$$y_x = 6.24256418\left(1 - \frac{(x+5.06)^2}{24.60758105}\right)^{0.4411811}$$ for x values to left of 49.4 percent of average life, and

$$y_x = 6.24256418\left(1 - \frac{(x+5.06)^2}{1569.183739}\right)^{7.73906308}$$ for x values to right of 49.4 percent of average life.

Left Mode No. 1

$$y_x = 7.45095687\left(1 - \frac{(x+4)^2}{85.49500000}\right)^{4.77742941}$$ for x values to left of 60 percent of average life, and

$$y_x = 7.45095687\left(1 - \frac{(x+4)^2}{697.8983268}\right)^{4.74147112}$$ for x values to right of 60 percent of average life.

Left Mode No. 2

$$y_x = 6.2\left(1 + \frac{x-0.56632298}{10.56632298}\right)^{2.00691507}\left(1 - \frac{x-0.56632298}{18.11962398}\right)^{4.15639655}$$
$$+4.03141046\left(1 + \frac{x+1.98831766}{4.90258200}\right)^{2.73360630}\left(1 - \frac{x+1.98831766}{12.07825433}\right)^{6.10831822}$$

Left Mode No. 3

$$y_x = 6.12\left(1 + \frac{x-0.69997304}{9.94997304}\right)^{2.31767682}\left(1 - \frac{x-0.69997304}{13.35543784}\right)^{3.72162230}$$
$$+8.19722280\left(1 + \frac{x+1.22119072}{6.98766177}\right)^{18.13754029}\left(1 - \frac{x+1.22119072}{16.85048078}\right)^{23.90598437}$$

Left Mode No. 4

$$y_x=10.811999434\left(1-\frac{(x+0.600)^2}{51.8400}\right)^{25.300}$$
$$+9.901828065\left(1-\frac{(x+0.300)^2}{56.2500}\right)^{3.650} \quad \text{Limits:} \quad -10 \leqq x \leqq -0.6$$

$$y_x=10.811999434\left(1-\frac{(x+0.600)^2}{184.9600}\right)^{62.000}$$
$$+9.901828065\left(1-\frac{(x+0.300)^2}{56.2500}\right)^{3.650} \quad \text{Limits:} \quad -0.6 \leqq x \leqq -0.3$$

$$y_x=10.811999434\left(1-\frac{(x+0.600)^2}{184.9600}\right)^{62.000}$$
$$+9.901828065\left(1-\frac{(x+0.300)^2}{176.8900}\right)^{5.350} \quad \text{Limits:} \quad -0.3 \leqq x \leqq (13.6-0.6)$$

Left Mode No. 5

$$y=12.76925713\left(1+\frac{x-0.088051975}{5.9500}\right)^{4.7715}\left(1-\frac{x-0.088051975}{10.7500}\right)^{9.4275}$$
$$+16.28938438\left(1+\frac{x+0.161460055}{4.0000}\right)^{11.8000}\left(1-\frac{x+0.161460055}{5.7000}\right)^{17.3400}$$

Symmetrical No. 0

$$y_x=6.95219904\left(1-\frac{x^2}{100}\right)^{0.74857140}$$

Symmetrical No. 1

$$y_x=9.08025966\left(1-\frac{x^2}{100}\right)^{1.82839970}$$

Symmetrical No. 2

$$y_x=11.91103882\left(1-\frac{x^2}{100}\right)^{3.70009374}$$

Symmetrical No. 3

$$y_x=15.61048797\left(1-\frac{x^2}{100}\right)^{6.9015918}$$

Symmetrical No. 4

$$y_x=22.32936082\left(1-\frac{x^2}{81}\right)^{11.53537940}$$

Symmetrical No. 5

$$y_x = 33.22051575\left(1-\frac{x^2}{64}\right)^{21.43792170}$$

Symmetrical No. 6

$$y_x = 52.47259169\left(1-\frac{x^2}{49}\right)^{41.63414220}$$

Right Mode No. 1

$$y_x = 4.87234751\left(1+\frac{x+2.1173}{19.08200310}\right)^{2.16036988}\left(1-\frac{x+2.1173}{12.2}\right)^{1.03956945}$$

$$+2.95921394\left(1+\frac{x-2.03848}{9.25013197}\right)^{2.69374074}\left(1-\frac{x-2.03848}{6.76380495}\right)^{1.89831583}$$

Right Mode No. 2

$$y_x = 6.89465710\left(1+\frac{x+0.470}{30.05448169}\right)^{9.16816044}\left(1-\frac{x+0.470}{9.05171312}\right)^{2.06741419}$$

$$+3.34428110\left(1+\frac{x-0.470}{91.60465100}\right)^{100.000}\left(1-\frac{x-0.470}{7.80000000}\right)^{7.800}$$

Right Mode No. 3

$$y_x = 9.4035297069\left(1+\frac{x+0.235}{17.61801370}\right)^{7.950}\left(1-\frac{x+0.235}{7.18500000}\right)^{2.850}$$

$$+5.5945716839\left(1+\frac{x-0.698}{17.31323077}\right)^{27.900}\left(1-\frac{x-0.698}{6.25200000}\right)^{9.400}$$

Right Mode No. 4

$$y_x = 15.20129316\left(1+\frac{x+0.11}{17.92683200}\right)^{14.05850000}\left(1-\frac{x+0.11}{5.41801100}\right)^{3.58117010}$$

$$+5.85667821\left(1+\frac{x-0.70}{2.56783700}\right)^{3.88879450}\left(1-\frac{x-0.70}{3.45398750}\right)^{5.37997731}$$

Right Mode No. 5

$$y_x = 14.99330391\left(1+\frac{x+0.12869}{7.0000000}\right)^{5.79473320}\left(1-\frac{x+0.12869}{3.8764409}\right)^{2.76276090}$$

$$+15.44614441\left(1+\frac{x-0.2086}{4.2350000}\right)^{6.80833400}\left(1-\frac{x-0.2086}{2.4150000}\right)^{3.02300040}$$

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TABLE 42. PERCENT SURVIVING AND PROBABLE LIVES OF THE 18 TYPE CURVES

Age, percent of average life	Type curve L_0		Type curve L_1		Type curve L_2		Type curve L_3	
	Percent surviving	Probable life, percent of average life	Percent surviving	Probable life, percent of average life	Percent surviving	Probable life, percent of average life	Percent surviving	Probable life, percent of average life
0	100.00000	99.99089	100.00000	100.00591	100.00000	100.00021	100.00000	99.99845
5	98.88950	101.08161	99.63717	100.36022	99.98516	100.01451	100.00000	99.99845
10	97.09985	102.80445	99.05431	100.90562	99.88940	100.10272	99.99965	99.99876
15	94.91674	104.88011	98.19015	101.68240	99.64732	100.31485	99.98571	100.01084
20	92.45291	107.20770	96.98956	102.72297	99.20930	100.67961	99.92003	100.06478
25	89.77813	109.73060	95.40973	104.04967	98.53977	101.20981	99.75597	100.19186
30	86.94316	112.41127	93.42541	105.67380	97.61555	101.90662	99.44760	100.41659
35	83.98860	115.22191	91.03275	107.59530	96.41791	102.76752	98.95304	100.75520
40	80.94912	118.12988	88.25133	109.80281	94.85656	103.83997	98.23067	101.21934
45	77.85578	121.14498	85.12428	112.27369	92.77820	105.21155	97.22244	101.82694
50	74.73764	124.21747	81.71638	114.97383	90.07728	106.93905	95.82788	102.61565
55	71.61815	127.34132	78.11040	117.85716	86.72860	109.03813	93.88854	103.64810
60	68.50728	130.51283	74.40204	120.86522	82.78113	111.49302	91.20570	105.00185
65	65.41269	133.73056	70.67865	123.93996	78.33876	114.26913	87.59534	106.74906
70	62.34191	136.99308	66.96789	127.06747	73.53626	117.32197	82.95922	108.93749
75	59.30233	140.29892	63.28229	130.24572	68.51802	120.60387	77.33872	111.58081
80	56.30114	143.64678	59.63417	133.47273	63.42158	124.06752	70.92603	114.65849
85	53.34528	147.03529	56.03557	136.74659	58.36648	127.66832	64.02853	118.12085
90	50.44141	150.46313	52.49810	140.06547	53.44823	131.96588	57.00209	121.89571
95	47.59585	153.92902	49.03293	143.42757	48.73660	135. 2517	50.17916	125.89546
100	44.81457	157.43172	45.65060	146.83119	44.27685	139.12062	43.81420	130.02557
105	42.10313	160.97001	42.36105	150.27465	40.09316	142.72029	38.05845	134.19523
110	39.46668	164.54272	39.17345	153.75638	36.19309	146.51867	32.96348	138.32936
115	36.90985	168.14870	36.09618	157.27483	32.57226	150.30366	28.50446	142.37842
120	34.43690	171.78684	33.13678	160.82853	29.21877	154.07214	24.61134	146.32230
125	32.03150	175.45606	30.30187	164.41606	26.11696	157.82557	21.19779	150.16671
130	29.73687	179.15533	27.59711	168.03608	23.25024	161.56858	18.18205	153.93424
135	27.55568	182.88363	25.02721	171.68727	20.60301	165.30763	15.49809	157.65409
140	25.45004	186.63999	22.59584	175.36838	18.16176	169.04987	13.09864	161.35412
145	23.44162	190.42346	20.30567	179.07823	15.91540	172.80201	10.95300	165.03640
150	21.53151	194.23313	18.15836	182.81566	13.85508	176.56980	9.04239	168.77609
155	19.72030	198.06812	16.15455	186.57958	11.97372	180.35762	7.35497	172.52213
160	18.00807	201.92758	14.29388	190.36894	10.26528	184.16841	5.88180	176.29886
165	16.39443	205.81068	12.57504	194.18272	8.72414	188.00373	4.61406	180.10757
170	14.87851	209.71662	10.99580	198.01998	7.34447	191.86401	3.54138	183.94776
175	13.45001	213.64466	9.55804	201.87979	6.11983	195.74884	2.65120	187.81802
180	12.13419	217.59403	8.24285	205.76127	5.04292	199.65727	1.92866	191.71659
185	10.90195	221.56404	7.06053	209.66358	4.10549	203.58804	1.35694	195.64161
190	9.75982	225.55399	6.00073	213.58591	3.29841	207.53973	0.91776	199.59130
195	8.70500	229.56322	5.05748	217.52750	2.61180	211.51131	0.59797	203.56400
200	7.73441	233.59110	4.22433	221.48760	2.03322	215.50128	0.36025	207.55823
205	6.84472	237.63700	3.49436	225.46551	1.55792	219.50960	0.20376	211.57267
210	6.03238	241.70032	2.86034	229.46057	1.16899	223.53224	0.10480	215.60622
215	5.29368	245.78052	2.31483	233.47212	0.85763	227.57124	4.7892*	219.65804
220	4.62473	249.87702	1.85022	237.49955	0.61326	231.62472	1.7822*	223.72779
225	4.02158	253.98930	1.45887	241.54228	0.42576	235.69196	5.0302*	227.81621
230	3.48020	258.11684	1.13320	245.59973	0.28560	239.77193	8.5317*	231.92737
235	2.99653	262.25915	0.86577	249.67127	0.18296	243.86424	4.5269*	236.08217
240	2.56649	266.41576	0.64937	253.75670	0.11287	247.96820	1.2840*	240.50000
245	2.18608	270.58622	0.47711	257.85521	6.5248*	252.08326	Absolute zero at age 256.29	
250	1.85133	274.76467	0.34247	261.96644	3.5011*	256.20897		
255	1.55837	278.96689	0.23940	266.08993	1.7066*	260.34501		
260	1.30345	283.17628	0.16234	270.22531	7.3138*	264.49126		
265	1.08297	287.39783	0.10626	274.37212	2.6141*	268.64793		
270	0.89347	291.63117	6.6733*	278.53000	7.0921*	272.81610		

L_0, L_1 and L_2 concluded on page 128.

*Condensed from the usual form 6.6733x10[illegible].

TABLE 42. PERCENT SURVIVING AND PROBABLE LIVES OF THE 18 TYPE CURVES, CONT.

Age, percent of average life	Type curve L_4		Type curve L_1		Type curve S_2		Type curve S_1	
	Percent surviving	Probable life, percent of average life	Percent surviving	Probable life, percent of average life	Percent surviving	Probable life, percent of average life	Percent surviving	Probable life, percent of average life
0	100.00000	100.00000	100.00000	100.00000	100.00000	100.00000	100.00000	100.00000
5	100.00000	100.00000	100.00000	100.00000	99.64988	100.34029	99.97697	100.02220
10	100.00000	100.00000	100.00000	100.00000	98.83704	101.10222	99.84198	100.14667
15	100.00000	100.00000	100.00000	100.00000	97.66579	102.16343	99.51978	100.42955
20	100.00000	100.00000	100.00000	100.00000	96.18813	103.46285	98.95479	100.90197
25	99.99994	100.00004	100.00000	100.00000	94.44074	104.95979	98.10590	101.57914
30	99.99491	100.00364	100.00000	100.00000	92.45276	106.62440	96.94397	102.46569
35	99.95590	100.02975	100.00000	100.00000	90.24878	108.43369	95.45014	103.55926
40	99.81999	100.11433	100.00000	100.00000	87.85047	110.36934	93.61458	104.85306
45	99.49620	100.30086	99.99994	100.00003	85.27737	112.41641	91.43547	106.33757
50	98.87810	100.62951	99.99221	100.00400	82.54749	114.56250	88.91814	108.00180
55	97.85817	101.12929	99.91441	100.04046	79.67768	116.79719	86.07422	109.83412
60	96.33923	101.81492	99.59667	100.17473	76.68385	119.11161	82.92102	111.82276
65	94.23074	102.69191	98.77418	100.48580	73.58124	121.49814	79.48078	113.95626
70	91.40395	103.77670	97.15592	101.03139	70.38447	123.95016	75.78013	116.22361
75	87.59685	105.13067	94.48180	101.83388	67.10774	126.46193	71.84942	118.61443
80	82.36077	106.87908	90.38589	102.92896	63.76487	129.02839	67.72221	121.11903
85	75.23825	109.17620	84.04805	104.45606	60.36941	131.64506	63.43466	123.72839
90	66.20766	112.12265	74.41381	106.63177	56.93468	134.30800	59.02506	126.43422
95	56.03106	115.68281	61.43633	109.59890	53.47385	137.01367	54.53322	129.22889
100	45.92922	119.68734	46.93011	113.33734	50.00000	139.75891	50.00000	132.10541
105	36.82670	123.94999	33.60849	117.66137	46.52615	142.54088	45.46678	135.05737
110	29.20175	128.26669	23.25529	122.24209	43.06532	145.35702	40.97494	138.07892
115	23.07537	132.47687	15.99530	126.73263	39.63059	148.20503	36.56533	141.16474
120	18.18649	136.52655	11.00040	130.99004	36.23513	151.08280	32.27779	144.30994
125	14.22200	140.45874	7.44648	135.10346	32.89226	153.98841	28.15058	147.51008
130	10.93212	144.35016	4.86568	139.20493	29.61552	156.92014	24.21987	150.76110
135	8.24831	148.26014	3.02930	143.35703	26.41876	159.87638	20.51922	154.05930
140	6.04296	152.21838	1.78262	147.56930	23.31614	162.85568	17.07898	157.40128
145	4.28876	156.23332	0.98432	151.83527	20.32232	165.85670	13.92578	160.78396
150	2.93706	160.30381	0.50537	156.14726	17.45251	168.87822	11.08186	164.20450
155	1.93260	164.42557	0.23884	160.49880	14.72263	171.91911	8.56453	167.66033
160	1.21574	168.56388	0.10227	164.88452	12.14953	174.97833	6.38542	171.14910
165	0.72670	172.80433	3.8912*[illegible]	169.29999	9.75122	178.05493	4.54986	174.66867
170	0.40962	177.05300	1.2804*[illegible]	173.74155	7.54724	181.14740	3.05603	178.21712
175	0.21565	181.33633	3.5047*[illegible]	178.20627	5.55926	184.23699	1.89410	181.79275
180	0.10473	185.65127	7.5300*[illegible]	182.69165	3.81187	187.38111	1.04521	185.39421
185	4.6162*[illegible]	189.99496	1.1578*[illegible]	187.19677	2.33421	190.52013	0.48022	189.02070
190	1.8062*[illegible]	194.36507	1.0862*[illegible]	191.71987	1.16296	193.67471	0.15802	192.67315
195	6.0815*[illegible]	198.75944	4.5333*[illegible]	196.26370	0.35012	196.85095	2.3033*[illegible]	196.36205
200	1.6836*[illegible]	203.17626	3.8160*[illegible]	200.83860	0.00000	200.00000	0.00000	200.00000
205	3.5703*[illegible]	207.61398	2.4958*[illegible]	205.51817	Absolute zero at age 200.0		Absolute zero at age 200.0	
210	5.1468*[illegible]	212.07250	Absolute zero at age 208.35					
215	4.0358*[illegible]	216.55238						
220	1.0376*[illegible]	221.05924						
225	1.6690*[illegible]	225.62577						
230	0.00000	230.00000						
	Absolute zero at age 230.0							

*Condensed from the usual form 4.6162x10[illegible].

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TABLE 42. PERCENT SURVIVING AND PROBABLE LIVES OF THE 18 TYPE CURVES, CONT.

Age, percent of average life	Type curve S_2		Type curve S_3		Type curve S_4		Type curve S_5	
	Percent surviving	Probable life, percent of average life	Percent surviving	Probable life, percent of average life	Percent surviving	Probable life, percent of average life	Percent surviving	Probable life, percent of average life
0	100.00000	100.00000	100.00000	100.00000	100.00000	100.00000	100.00000	100.00000
5	99.99977	100.00022	100.00000	100.00000	100.00000	100.00000	100.00000	100.00000
10	99.99438	100.00516	99.99998	100.00002	100.00000	100.00000	100.00000	100.00000
15	99.96508	100.03065	99.99955	100.00039	100.00000	100.00000	100.00000	100.00000
20	99.87562	100.10424	99.99626	100.00308	100.00000	100.00000	100.00000	100.00000
25	99.67336	100.26114	99.98158	100.01439	100.00000	100.00000	100.00000	100.00000
30	99.29313	100.53891	99.93451	100.04835	99.99994	100.00004	100.00000	100.00000
35	98.66236	100.97274	99.81435	100.12926	99.99925	100.00050	100.00000	100.00000
40	97.70666	101.59210	99.55468	100.29188	99.99445	100.00347	100.00000	100.00000
45	96.35547	102.41897	99.06070	100.57890	99.97162	100.01648	100.00000	100.00000
50	94.54722	103.46727	98.21171	101.03602	99.89006	100.05898	99.99986	100.00007
55	92.23368	104.74343	96.86915	101.70637	99.65644	100.16958	99.99803	100.00092
60	89.38348	106.24745	94.88992	102.62546	99.09588	100.40916	99.98250	100.00741
65	85.98454	107.97422	92.14318	103.81800	97.93460	100.85553	99.89372	100.04020
70	82.04539	109.91488	88.52850	105.29680	95.81171	101.58974	99.52630	100.15847
75	77.59546	112.05802	83.99263	107.06350	92.33195	102.67936	98.36721	100.47958
80	72.68428	114.39069	78.54224	109.11051	87.15666	104.16613	95.47669	101.16553
85	67.37969	116.89776	72.25073	111.42339	80.11173	106.06203	89.62837	102.36758
90	61.76531	119.56988	65.25765	113.98329	71.27816	108.35315	79.84902	104.16759
95	55.93719	122.38914	57.76056	116.76904	61.03029	111.00766	66.16179	106.56047
100	50.00000	125.34413	50.00000	119.75873	50.00000	113.98428	50.00000	109.47834
105	44.06281	128.42274	42.23944	122.93091	38.96971	117.23965	33.83821	112.82728
110	38.23469	131.61370	34.74235	126.26526	28.72184	120.72977	20.15098	116.51424
115	32.62031	134.90665	27.74927	129.74306	19.88827	124.41839	10.37163	120.45988
120	27.31572	138.29213	21.45776	133.34737	12.84334	128.27194	4.52331	124.60162
125	22.40454	141.76152	16.00737	137.06307	7.66805	132.26238	1.63279	128.89201
130	17.95461	145.30705	11.47150	140.87676	4.18829	136.36718	0.47370	133.29585
135	14.01546	148.92167	7.85682	144.77670	2.06340	140.56669	0.10628	137.78717
140	10.61652	152.59905	5.11008	148.75260	0.90412	144.84554	1.7505*[illegible]	142.34693
145	7.76632	156.33349	3.13085	152.79550	0.34356	149.19105	1.9664*[illegible]	146.96095
150	5.45278	160.11988	1.78829	156.89763	0.10994	153.59287	1.3506*[illegible]	151.61900
155	3.64433	163.95363	0.93930	161.05224	2.8348*[illegible]	158.04267	4.7901*[illegible]	156.31432
160	2.29334	167.83069	0.44532	165.25372	5.5528*[illegible]	162.53372	6.5855*[illegible]	161.04285
165	1.33764	171.74735	0.18565	169.49639	7.4641*[illegible]	167.06104	2.0400*[illegible]	165.80504
170	0.70687	175.70044	6.5494*[illegible]	173.77668	5.8485*[illegible]	171.62133	4.1151*[illegible]	170.61149
175	0.32664	179.68720	1.8423*[illegible]	178.09091	1.9628*[illegible]	176.21146	8.2575*[illegible]	175.50523
180	0.12438	183.70544	3.7393*[illegible]	182.43656	1.3859*[illegible]	180.84334	0.00000	180.00000
185	3.4916*[illegible]	187.75240	4.5414*[illegible]	186.81280	2.0195*[illegible]	185.55252	Absolute zero at age 180.0	
190	5.6228*[illegible]	191.83362	2.1671*[illegible]	191.22341	0.00000	190.00000		
195	2.3407*[illegible]	195.96419	1.0622*[illegible]	195.69685	Absolute zero at age 190.0			
200	0.00000	200.00000	0.00000	200.00000				
205								
	Absolute zero at age 200.0		Absolute zero at age 200.0					

*Condensed from the usual form 3.4916x10[illegible].

TABLE 42. PERCENT SURVIVING AND PROBABLE LIVES OF THE 18 TYPE CURVES, CONT.

Age, percent of average life	Type curve S_6		Type curve R_1		Type curve R_2		Type curve R_3	
	Percent surviving	Probable life, percent of average life	Percent surviving	Probable life, percent of average life	Percent surviving	Probable life, percent of average life	Percent surviving	Probable life, percent of average life
0	100.00000	100.00000	100.00000	100.00180	100.00000	100.00000	100.00000	100.00000
5	100.00000	100.00000	98.67319	101.31249	99.49224	100.49725	99.90802	100.08061
10	100.00000	100.00000	97.25548	102.67964	98.89359	101.05981	99.77155	100.21605
15	100.00000	100.00000	95.74739	104.09966	98.19184	101.69229	99.57488	100.38902
20	100.00000	100.00000	94.14988	105.56867	97.37369	102.39917	99.29891	100.61902
25	100.00000	100.00000	92.46439	107.08232	96.42474	103.18491	98.92096	100.91703
30	100.00000	100.00000	90.69276	108.63674	95.32938	104.05390	98.41475	101.29408
35	100.00000	100.00000	88.83506	110.22850	94.07074	105.01055	97.75059	101.76079
40	100.00000	100.00000	86.88515	111.86026	92.63063	106.05929	96.89555	102.32699
45	100.00000	100.00000	84.83266	113.53789	90.98955	107.20471	95.81369	103.00148
50	100.00000	100.00000	82.66538	115.26865	89.12668	108.45158	94.46607	103.79204
55	100.00000	100.00000	80.37080	117.06003	87.02008	109.80490	92.81029	104.70369
60	100.00000	100.00000	77.93721	118.91905	84.64706	111.26991	90.79948	105.74941
65	99.99996	100.00001	75.35462	120.85185	81.98479	112.85214	88.38061	106.93117
70	99.99835	100.00052	72.61549	122.86346	79.01133	114.55726	85.49283	108.26067
75	99.96681	100.00880	69.71522	124.95773	75.70717	116.39102	82.06693	109.75071
80	99.64167	100.07979	66.65272	127.13731	72.05731	118.35898	78.02814	111.41669
85	97.71260	100.41499	63.43064	129.40376	68.05401	120.46632	73.30461	113.27609
90	90.70910	101.37934	60.05571	131.75762	63.70000	122.71746	67.84370	115.34643
95	74.50386	103.25562	56.53884	134.19860	59.01217	125.11571	61.63582	117.64274
100	50.00000	106.03512	52.89519	136.72566	54.02510	127.66288	54.74208	120.17476
105	25.49614	109.51346	49.14416	139.33722	48.79396	130.35885	47.31746	122.94437
110	9.29090	113.46679	45.30930	142.03122	43.39604	133.20108	39.61801	125.94491
115	2.28740	117.72763	41.41805	144.80529	37.92987	136.18423	31.98179	129.15823
120	0.35833	122.18704	37.50154	147.65686	32.51131	139.29962	24.78027	132.55624
125	3.3192*[illegible]	126.77740	33.59414	150.58322	27.26603	142.53491	18.34962	136.10033
130	1.6488*[illegible]	131.43697	29.73303	153.58167	22.31913	145.87373	12.92397	139.74426
135	3.8261*[illegible]	136.19977	25.95763	156.76125	17.78236	149.29606	8.59763	143.44083
140	3.3790*[illegible]	140.99968	22.30885	159.78422	13.74239	152.77927	5.33296	147.15341
145	8.2277*[illegible]	145.81720	18.82834	162.98331	10.25227	156.30063	3.00720	150.86725
150	3.1928*[illegible]	150.67814	15.55747	166.24450	7.32937	159.84111	1.46794	154.59151
155	6.9354*[illegible]	155.57400	12.53623	169.56552	4.96076	163.38985	0.56322	158.34810
160	7.5148*[illegible]	160.51341	9.80182	172.94384	3.11416	166.94690	0.13838	162.15692
165	1.0945*[illegible]	165.50004	7.38707	176.37592	1.74922	170.52220	9.9847*[illegible]	166.03924
170	0.00000	170.00000	5.31832	179.85535	0.82235	174.13012	Absolute zero at age 169.50	
175	Absolute zero at age 170.0		3.61284	183.36828	0.28040	177.78322		
180			2.27508	186.88301	4.5610*[illegible]	181.49239		
185			1.29047	190.33183	6.5960*[illegible]	185.50000		
190			0.61061	193.65695	Absolute zero at age 185.82			
195			0.17949	196.99140				
200			4.5573*[illegible]	200.50000				
205			0.00000	205.00000				
210			Absolute zero at age 200.83					

*Condensed from the usual form 3.3192x10[illegible].

TABLE 42. PERCENT SURVIVING AND PROBABLE LIVES OF THE 18 TYPE CURVES, CONT.

Age, percent of average life	Type curve R_4: Percent surviving	Type curve R_4: Probable life, percent of average life	Type curve R_5: Percent surviving	Type curve R_5: Probable life, percent of average life
0	100.00000	100.00049	100.00000	99.99926
5	99.99432	100.00601	100.00000	99.99926
10	99.98287	100.01658	100.00000	99.99926
15	99.96090	100.03575	100.00000	99.99926
20	99.92061	100.06894	100.00000	99.99926
25	99.84968	100.12389	100.00000	99.99926
30	99.72952	100.21114	100.00000	99.99926
35	99.53311	100.34439	99.99998	99.99929
40	99.22264	100.54048	99.99916	99.99978
45	98.74727	100.81910	99.99100	100.00441
50	98.04124	101.20197	99.95114	100.02514
55	97.02304	101.71159	99.82089	100.08662
60	95.59608	102.36961	99.49249	100.22610
65	93.65182	103.19487	98.80274	100.48760
70	91.07392	104.20140	97.53312	100.91344
75	87.75801	105.39634	95.39673	101.54592
80	83.60532	106.77778	91.94655	102.44128
85	78.55460	108.33401	86.55794	103.67210
90	72.42015	110.09162	78.54068	105.30800
95	64.75218	112.16416	67.46899	107.39151
100	55.28004	114.63438	53.61839	109.92926
105	44.79042	117.49630	38.27696	112.90149
110	33.97478	120.68231	23.63847	116.27958
115	24.00508	124.10272	12.08071	120.03018
120	15.65853	127.67001	4.95374	124.03417
125	9.26945	131.31951	1.61653	127.91040
130	4.80402	135.02206	0.29276	131.69264
135	2.02527	138.78538	5.3622*	135.63094
140	0.39682	142.63490	0.00000	140.00000
145	8.5247*	146.38440	Absolute zero at age 137.48	
150	1.2803*	150.66575		
155	0.00000	155.00000		
	Absolute zero at age 153.08			

Continuation of Left Mode Types

Age	Type curve L_1: Percent surviving	Type curve L_1: Probable life	Type curve L_2: Percent surviving	Type curve L_2: Probable life
275	3.9886*	282.69860	1.2032*	276.99972
280	2.2451*	286.87760	7.6517*	281.21830
285	1.1731*	291.06671	2.6860*	285.92339
290	5.5762*	295.26570	Absolute zero at age 286.86	
295	2.3399*	299.47442		
300	8.2700*	303.69289		
305	2.2723*	307.92147		
310	4.1607*	312.16147		
315	3.5342*	316.41771		
320	3.8022*	320.71968		
	Absolute zero at age 324.18			

Type curve L_0, concluded		
Age, percent of average life	Percent surviving	Probable life, percent of average life
275	0.73167	295.87393
280	0.59449	300.13177
285	0.47904	304.39833
290	0.38262	308.67529
295	0.30276	312.96234
300	0.23719	317.25916
305	0.18385	321.56548
310	0.14089	325.88100
315	0.10666	330.20545
320	7.9690*	334.53858
325	5.8700*	338.88014
330	4.2579*	343.22988
335	3.0373*	347.58758
340	2.1275*	351.95302
345	1.4608*	356.32609
350	9.8117*	360.70651
355	6.4322*	365.09417
360	4.1041*	369.48663
365	2.5406*	373.88746
370	1.5200*	378.29492
375	8.7481*	382.70842
380	4.8165*	387.12827
385	2.5188*	391.55417
390	1.2402*	395.98600
395	5.6842*	400.42364
400	2.3894*	404.86698
405	9.0286*	409.31583
410	2.9812*	413.77071
415	8.2579*	418.23126
420	1.8014*	422.69784
425	2.7860*	427.17126
430	2.5148*	431.65335
435	8.6258*	436.14968
440	2.9807*	440.68596
445	1.1790*	445.50000
Absolute zero at age 445.53		

*Condensed from the usual form 8.5247x10[illegible].

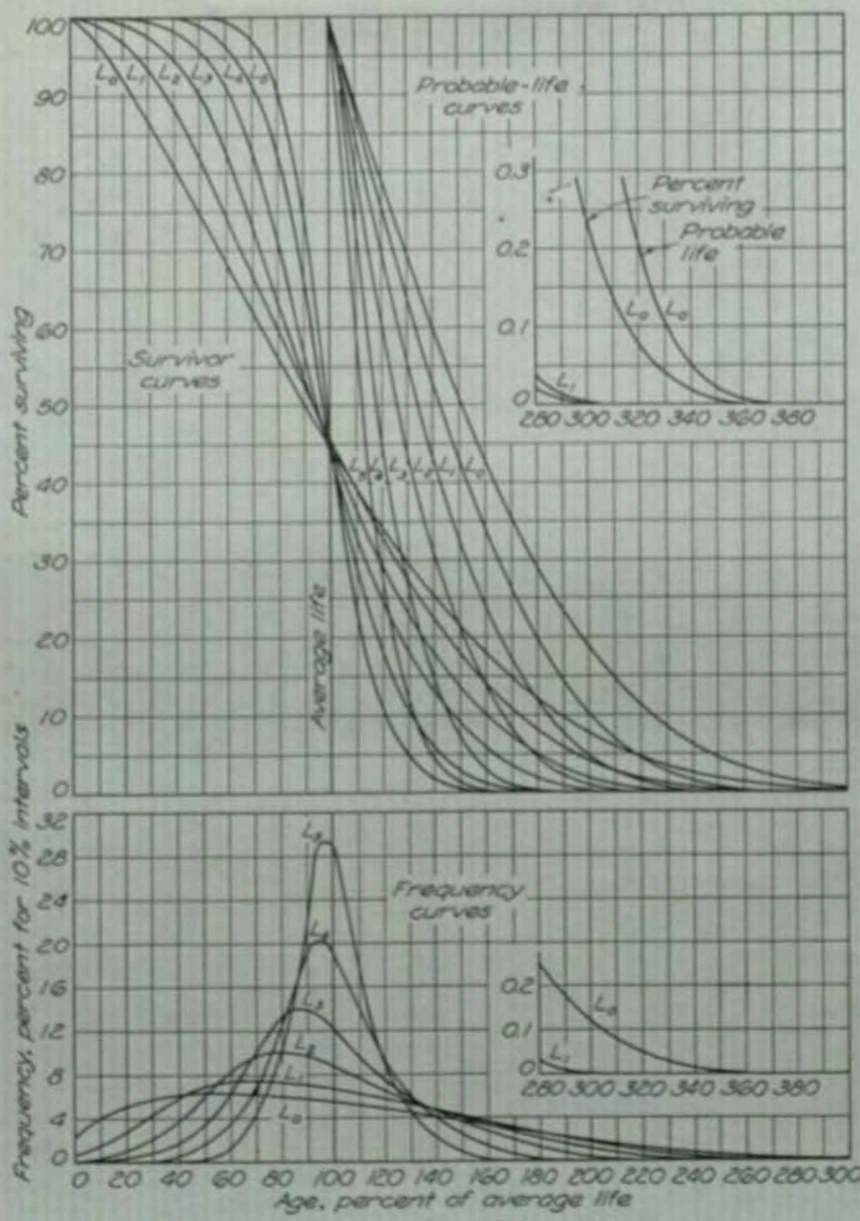

Fig. 28. Left mode type survivor, probable life, and frequency curves.

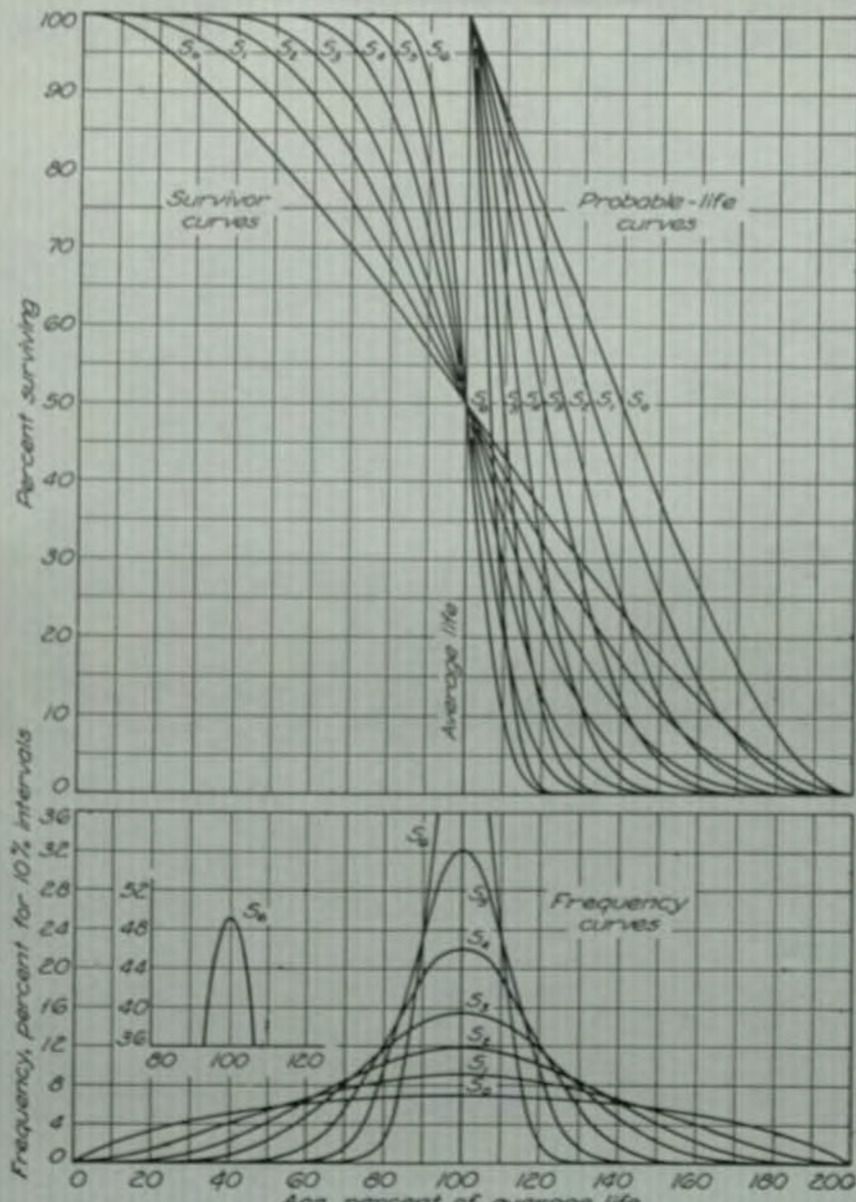

Fig. 29. Symmetrical type survivor, probable life, and frequency curves.

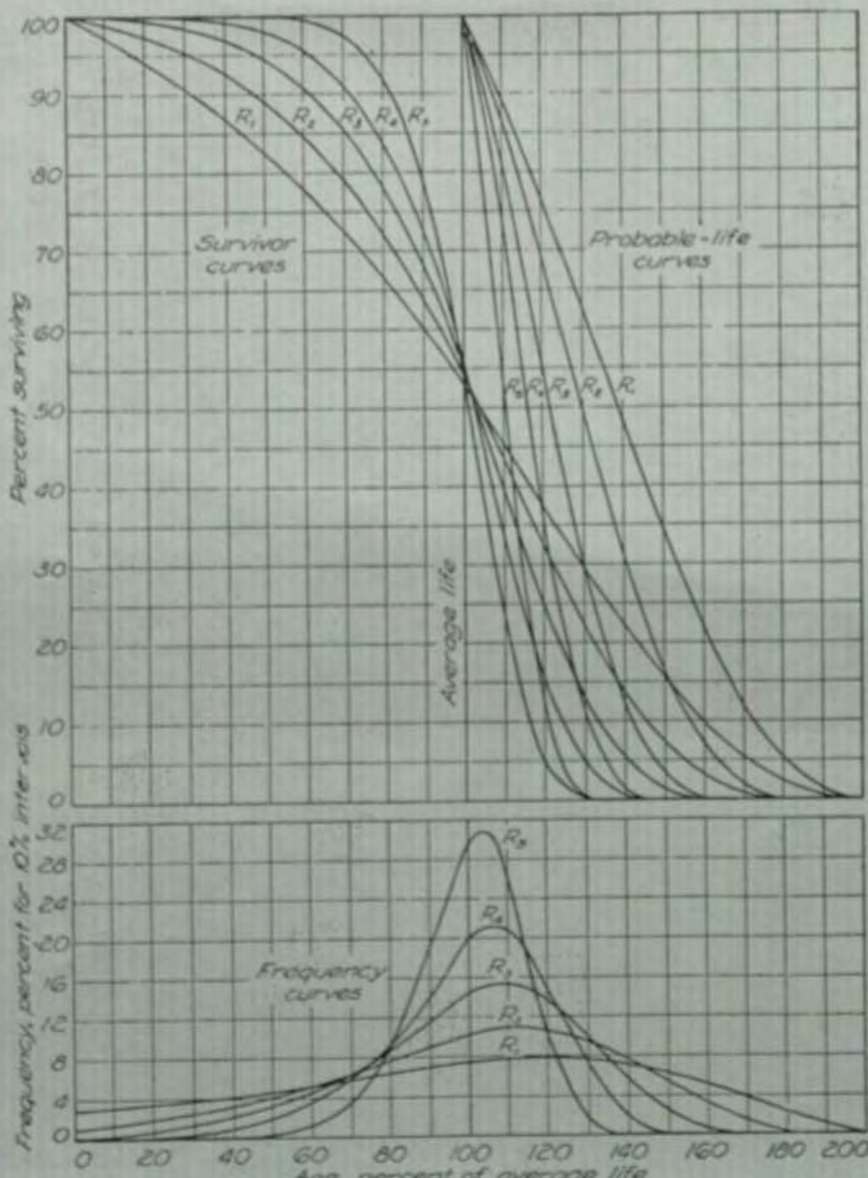

Fig. 30. Right mode type survivor, probable life, and frequency curves.

Depreciation Systems

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sary to define the average life, probable life, equal life groups, expectancy, and other values required by various systems of depreciation.

CHARACTERISTICS OF SERVICE LIFE

To develop a simple example showing the life characteristics of industrial property, imagine a group of 100 identical units of property that are placed in service during the same year. This is a *homogeneous group of property* because all units in the group are identical.

Age Intervals

The life has been defined as the time from installation to retirement from service, but conventions must be adopted to obtain consistent statistics. A common convention is to assume that any unit installed during the year is installed on July 1, that is, at the middle of the calendar year. This is equivalent to assuming the units are installed uniformly during the year, because this would result in an average installation date at midyear. Units retired before the end of the first year will be said to have been retired at age 0.25 years, while units remaining in service at the end of the year will have reached an age of 0.5 years. These assumptions are called the *half-year convention.* Adoption of the half-year convention leads to definition of age intervals 0–0.5 years, 0.5–1.5 years, 1.5–2.5 years, through the interval containing the maximum life, with midpoints of 0.25 years, 1 year, 2 years, and extending to the maximum life. These age intervals are stated in general terms and are shown below. It is convenient to use the index i to refer to an age interval and to denote the midpoint of age interval i by the value x(i). Any unit retired during the age interval i will be assumed to have a life equal to x(i). The symbol ML represents the midpoint of the final age interval and is called the *maximum life.*[1] Use of the half-year convention results in, with the exception of the first age interval, midpoints equal to integer values with the index for an age interval equal to the midpoint.

Index i	Age Interval i	Midpoint x(i)
0	$0 \leq$ Service Life < 0.5	0.25
1	$0.5 \leq$ Service Life < 1.5	1
2	$1.5 \leq$ Service Life < 2.5	2
•		•
•		•
ML	$(ML - 0.5) \leq$ Service Life $< (ML + 0.5)$	ML

This convention will be used throughout this text unless otherwise noted.

Frequency Curves

The *retirement frequency curve* is a graph of the frequency of retirements as a function of age. The retirement frequency is the ratio of retirements during an age interval to the original number of units installed and can be expressed as either a fraction or as a percentage. Of the 100 units in our example, suppose that the longest-lived unit was retired during the age interval 2.5–3.5 years so that the following four age intervals are necessary to describe the life characteristics.

Index i	Age Interval i	Midpoint x(i)
0	$0 \leq$ Service Life < 0.5	0.25
1	$0.5 \leq$ Service Life < 1.5	1
2	$1.5 \leq$ Service Life < 2.5	2
3	$2.5 \leq$ Service Life < 3.5	3

Of the 100 units, 20 were retired during the age interval 0, 40 during age interval 1, 30 during age interval 2, and the final 10 during age interval 3. The calculations for the retirement frequency curve, Figure 3.1, are

$$\begin{aligned} f(0) &= (20/100)100\% = 20\% \\ f(1) &= (40/100)100\% = 40\% \\ f(2) &= (30/100)100\% = 30\% \\ f(3) &= (10/100)100\% = \underline{10\%} \\ & \qquad\qquad\qquad\quad 100\% \end{aligned}$$

where f(i) = the retirement frequency during age interval i. Each frequency must be positive and the sum of the frequencies must total 100%, because all property must be accounted for.

Survivor Curves

The *survivor curve* is a graph of the percent of units remaining in service expressed as a function of age. The survivor curve could be obtained by recording the number of units remaining in service at the start of each age interval. Or, if the frequency curve is known, the percent surviving at the end of an age interval can be obtained by subtracting the percent retired during the interval from the percent surviving at the start of the interval. Conversely, the frequency curve can be obtained by subtracting successive reported in units, as in our example, their importance is often measured in dollars. Then the percent surviving is determined by the ratio of dollars retired divided by the total dollars installed. It can be argued that because it is dollars, not units, that are the object of capital recovery, dollars are a better measure than units for use in depreciation calculations.

Average Service Life

The average life of the property can be calculated from the frequency curve by using the equation

$$AL = E(\text{life}) = \Sigma x(i)f(i)/100\% \qquad \text{for } i = 1,2,3, \ldots, ML$$

Notice that the frequency is divided by 100% to convert it to a fraction. For our example the average life is AL = (.25)(.20) + (1)(.40) + (2)(.30) + (3)(.10) = 1.35 years.

The symbol E(life) is read *expected life* and is also called the *average life*. There is a direct relationship between the survivor curve and the average life; the area beneath the survivor curve equals the average life. Because the survivor curve is measured in percent, the area under the curve must be divided by 100% to convert it from percent-years to years. The area under the survivor curve and between ages of 0–0.5 years is calculated by multiplying the average height of the curve times the width of the interval or [(1.0 + 0.8)/2](0.5) = 0.45 years. The areas under the next three intervals, each of which has a width of 1 year, are (0.8 + 0.4)/2 = 0.6, (0.4 + 0.1)/2 = 0.25, and (0.1 + 0)/2 = 0.05 respectively. Their sum is 0.45 + 0.6 + 0.25 + 0.05 = 1.35 years, which is consistent with the previous calculation. These calculations are shown graphically in Figure 3.4.

Another average that is used frequently in depreciation calculations is the expectancy, which is a function of the age. The expectancy of a unit of property is simply its remaining years of service. The expectancy of a group of units is the average number of remaining years of service of the surviving property and is calculated as follows

$$RL(i) = E(\text{remaining life at beginning of age interval } i)$$
$$RL(i) = \Sigma\,[(x(k) - i)f(x)/100\%]/S(i) \qquad \text{for } k = i, i + 1, \ldots, ML$$

At age 1.5 years, the expectancy of our example property is [(2 − 1.5)(0.3) + (3 − 1.5)(0.1)]/0.4 = 0.75 years. This is also equal to the area under the survivor curve and to the right of the age 1.5 years divided by the percent surviving at age 1.5 years. Thus, this remaining service (i.e., the area under the curve) is allocated among the units remaining in service (i.e., the per-

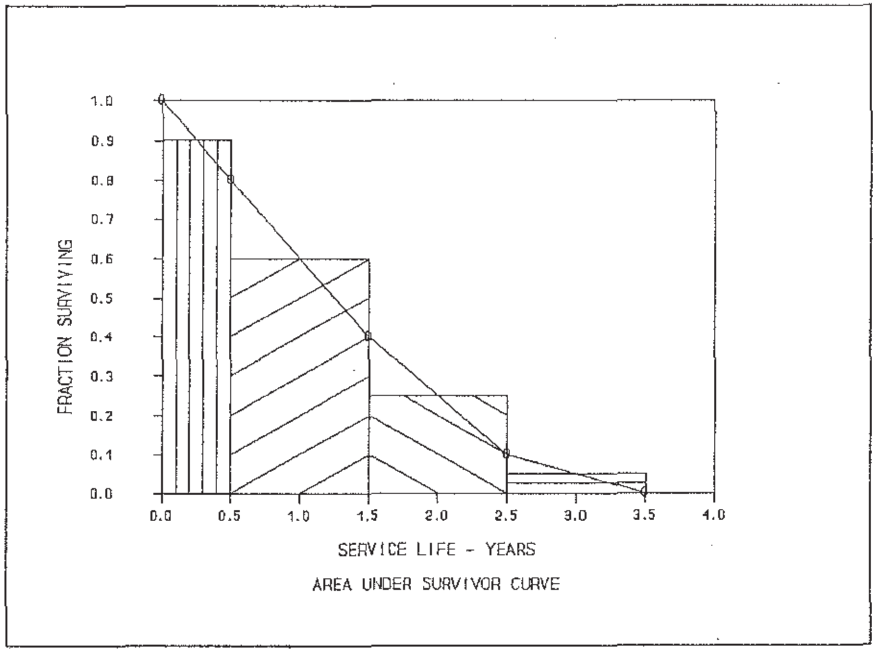

Figure 3.4. The area under the survivor curve.

cent surviving). This calculation is [(0.4 + 0.1)/2 + (0.1 + 0)/2]/0.4 = 0.3/0.4 = 0.75 years.

The terms *expectancy* and *average remaining life* are both commonly used to describe the E(remaining life at beginning of age interval i). The term average remaining life, which is often shortened to *remaining life*, is more descriptive and will be used in this text. The term *probable life* is used to describe the total life expectancy of the property surviving at any age and is equal to the remaining life plus the current age. The remaining life and probable life of the example property are shown below.

Age	% Surviving	Area under Curve, Right of Age, % – Years	Remaining Life, Years	Probable Life, Years
0.0	100%	135	1.35	1.35
0.5	80%	90	1.125	1.625
1.5	40%	30	0.75	2.25
2.5	10%	5	0.50	3.00
3.5	0%	0	. . .	. . .

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widely used in science and engineering. This involves the gathering of data that contain information about the subject under investigation. These data are analyzed and generalizations made. Models, based on observed data, are then developed. The system of Iowa curves is the result of this approach.

In either case, the resulting model must be verified by its application and proven usefulness. Both the Iowa curve and Gompertz-Makeham models have been used successfully over long periods of time.

The Iowa Curves

Development of the curves began at the Iowa Engineering Experiment Station at what was then Iowa State College, in Ames, Iowa. Much of the early work was done under the supervision of Anson Marston, director of the Experiment Station and dean of the College of Engineering. In 1916, Edwin Kurtz, then a student at the University of Wisconsin, began to assemble data relating to the life characteristics of industrial property. He started work for the Experiment Station in 1921 and a year later was joined in his research by Robley Winfrey, a young civil engineer also employed by the Experiment Station. In 1924 the Experiment Station published in *Bulletin 70* observed life data from 52 groups of utility property. Next Kurtz (1930) described methods of compiling mortality tables and frequency curve models.

Bulletin 103 by Winfrey and Kurtz (1931) contained the forerunners to the Iowa type curves. The purpose of *Bulletin 103* is described as follows:

> The following pages present a method of calculating the mortality curve, the probable life curve, and the rate of renewals of particular examples and types of physical equipment. The method has been applied to 65 sets of original life data for property found in the following industries. . . .
>
> The 13 type curves can be used as valuable aids in forecasting the probable future service lives of individual items and of groups of items of different kinds of physical equipment. (P. 5)

The use of these curves, although recommended, was qualified:

> In view of these natural changes in character of equipment and conditions of service, mortality curves based upon the records of equipment may not give accurate pictures of what may be expected in the way of service of new equipment—equipment used to replace old items as they are retired. However, these records should indicate the general trends and when *intelligently interpreted*, offer an exceedingly valuable aid in the estimation of the service which may be expected from physical equipment. (P. 6, emphasis added.)

A total of 65 property groups were examined, and frequency and survivor curves were calculated for each group. Because the average and maximum life of the curves vary over a wide range of years, the average life was set to 100% and the curves were redrawn using an abscissa of age as a percent of average life, rather than years. The curves were then compared and the following observations were made.

> The comparison brought out three distinguishing characteristics: location of the mode (the point of maximum ordinate) relative to the average life, magnitude of the mode, and the maximum age in percent of the average life. The curves were then classified into three groups according to whether the mode was to the left, approximately coincident, or to the right of the average life, and these three groups subclassified according to the magnitude of the mode. The classification, which was almost wholly by inspection, resulted in 13 groups or types, four groups having the mode to the left of the average life, five groups having the mode to the right of the average life. (P. 27)

Next the data were smoothed by fitting them to one of the 12 types of curves developed by the statistician Karl Pearson. Professor A. W. Snedecor, who became well known for his statistical work, provided assistance with the fitting.

Winfrey continued to gather and analyze data. In 1935 he wrote *Statistical Analysis of Industrial Property Retirements, Bulletin 125,* and it became the basic reference on life analysis. Additional data, the bulk of which was from public utilities, had been gathered and the number of property groups examined now totaled 176. Types of property included waterworks boilers, telegraph and electrical poles, railroad cars, various types of crossties, farm equipment, trucks, watt hour meters, gas meters, power transformers, pavement, and culverts. Analysis of this additional data resulted in the addition of 6 more curves to bring the total to 6 left modal curves, 7 symmetrical curves, and 5 right modal curves. These became known as the Iowa curves. The *Bulletin* contains the source of the original data, the equations of the curves, tables of the curves, and a discussion of the procedures used to analyze this data. Winfrey also discussed five methods of constructing survivor curves from aged data. In the following years, Winfrey applied the Iowa curves to a wide variety of property in many parts of the world.

Harold A. Cowles (1967) revised *Bulletin 125* to correct some errors in the tables and to add 4 origin modal models, or O type curves, developed by a graduate student, Frank Couch. The original data suggested the presence of these types of curves, but they were not included in earlier publications because of the low frequency with which they occurred. Couch fit these curves and they form Appendix D of the revised *Bulletin 125.*

Winfrey (1942) also wrote *Depreciation of Group Properties, Bulletin 155*. In it he discussed various methods of group depreciation and introduced the unit summation method of depreciation, which later became known as the equal life group (ELG) method.

The appendix of *Bulletin 155* contains tables of the percent surviving and probable life of the 18 Iowa type curves. These tables contain revisions to the tables in *Bulletin 125*. Although these changes are small and mostly in the R2 and R3 type curves, only the appendix of *Bulletin 155* contains the correct values of the 18 original Iowa type curves; these tables are tabulated at 5% intervals. The appendix of this book contains tables in which the equations are evaluated at 1% intervals. These tables are reproduced from original notes of Winfrey and are consistent with the values from *Bulletin 155*. Tables of the four O type curves in the same appendix are reproduced from the revised edition of *Bulletin 125*.

Attempts to reproduce the tables using an electronic computer and the equations for the curves have been made, but the results do not exactly match the 5 decimal place figures of Winfrey's revised calculations, with the major differences occurring as the percent surviving approaches zero. The original tables were the result of tedious calculations using mechanical calculators and tables of 10-place logarithms. Winfrey needed this high degree of accuracy to achieve consistency in the renewal calculations discussed in *Bulletin 125*. There are usually differences between the original tables and the computer-generated tables. These can result from procedures used to round calculations. Also, a function evaluated from tables of logarithms and the same function evaluated from an algorithm used by the computer may have different values. Current computer programs requiring values from the Iowa curves often store the table values of the 22 curves and use a table lookup procedure to obtain points on the survivor curve and the probable life curve.

The estimates made by the life forecaster do not require the five-decimal accuracy found in these tables. Yet, two identical sets of calculations, both using the same input data but done on different computers, should generate identical results. Failure to obtain this consistency may create undeserved reservations about the results of the computer programs. This, in turn, could cast doubts on the soundness of a report that uses the output in question.

The Iowa curves have become widely used models of the life characteristics of industrial property and they have been used for a variety of applications other than depreciation. Their repeated use has served as a test of their validity as a model. One criticism of the Iowa curves was that because their roots were in property installed near the turn of the century, new technology might bring with it new types of curves. The counterargument was that the Iowa type curves represent a wide range of patterns and

though the technology may change, the underlying patterns of retirements remain relatively constant and can be adequately described by the 22 Iowa type curves.

In the late 1970s, John Russo, a graduate student working under the supervision of Cowles, conducted research that reproduced the original development of the Iowa curves. Data from more than 2000 property accounts reflecting observations during the period 1965 to 1975 were collected. From these, 490 accounts from a wide range of industries were selected for analysis. They were grouped into 33 clusters that were designated as the Russo curves. Then 56 accounts, selected from the approximately 1500 remaining, were fit to both the Russo curves and to the Iowa curves. Russo (1980, 15) drew three major conclusions:

> 1. No evidence was produced to conclude that the Iowa curve set, as it stands, is not a valid set of standard curves.
>
> 2. No evidence was found to conclude that new curve shapes could be produced at this time that would add to the validity of the Iowa curve set. A very small percentage of industrial account data could not be well fit by Iowa curves. The vast majority were well fit within the current Iowa array. The occasional data not easily fit appeared to reflect random management or economic situations that did not fall within the norms of any particular industry. No pattern, however, was evident that would lead to an expansion of the array that would be useful over a large amount of data experience. By far the most prevalent types of accounts that could not be easily fit were multi-modal accounts. Because of the diversity of locations and magnitudes of the modes between multi-modal accounts, no discernable patterns were found that might lead to an expansion of the Iowa set or the formulation of a set of standard multi-modal curves. Additional study and research in this area is definitely in order.
>
> 3. No evidence was found to suggest that the number of curves within the Iowa set should be reduced, although some Iowa curves, especially in the symmetrical modal group, were not utilized to produce fits during the fitting of the 56 curves. A general review of the overall account data used in the study suggested that these curves would be utilized to some degree with larger sample sizes. Because some reasonably substantial usage of these curves can be expected in industrial practice and because the elimination of selected curves would interrupt a well-spaced existing array, it appeared unreasonable to reduce the number of curves within the Iowa set based on the evidence produced in this study.

The System of Iowa Curves

The equations for the frequency curves of the 22 Iowa curves and tables of the percent surviving and probable average service lives can be found in *Bulletin 125*. The curves are classified by three variables: the average life, the location of the mode, and the variation of the life. Because

$$Y = a_0 + a_1X + a_2X^2 + a_3X^3 + \ldots + a_nX^n$$

Standard regression techniques and computer programs can be used to find the regression coefficients a_i. Although this technique works well for smoothing, the polynomial function should only be used with great care to extrapolate data. In *Statistical Theory with Engineering Applications* Abraham Hald (1952:559) states, "From a purely statistical point of view the regression curve provides a description of the interrelation between the two variables within the limited range of the observations, and extrapolations, i.e., computations or values outside this range are in principle not justifiable as perhaps it is not possible to represent the interrelation outside the observed range by the function utilized. It is therefore absolutely necessary that extrapolation be firmly based on professional knowledge concerning the data." A polynomial curve may not be a good function to use for the difficult task of extrapolation.

If the Iowa curves are adopted as a model, an underlying assumption is that the process describing the retirement pattern is one of the 22 processes described by the Iowa curves. The problem is then to decide which specific type of Iowa curve "best" fits the observed data. *Best* can take on different meanings, each with subtle differences; here it will refer to the curve that most accurately represents the observed data.

One method is to fit the data visually. Until recently, this required a set of curves printed on translucent paper. Printed on each sheet is a family of a specific type Iowa curve. Each member of the family represents a different average life, typically running from 10 to 50 years in steps of 2 years. Traditionally these curves were scaled to 4 years/inch and 10% surviving/inch, but sets of curves scaled to one-half or double this size were also common. These scales can be multiplied or divided by a constant to accommodate observed data with very long or very short lives. If, for example, the observed curve had an average life of about 80 years, the scale could be doubled so that the curves would run from 20 to 100 years. The observed curve was plotted on graph paper using the same scale, and a translucent sheet of paper with the printed curves was then placed over the observed curve, allowing the analyst to compare visually the empirical and observed curves.

After plotting the observed curve, the analyst should first visually examine the plotted data to make an initial judgment about the type curves that may be good fits. The analyst also must decide which points or sections of the curve should be given the most weight. Points at the end of the curve are often based on fewer exposures and may be given less weight than points based on larger samples. The weight placed on those points will depend on the size of the exposures. Often the middle section of the curve (that section ranging from approximately 80% to 20% surviving) is given

more weight than the first and last sections. This middle section is relatively straight and is the portion of the curve that often best characterizes the survivor curve.

Begin fitting with the left modal curves and identify the two or three curves that appear to best fit the data. Note the curve type and the corresponding average life, which is typically estimated to the nearest year. Continue with the symmetrical, right modal, and origin modal curves. Some groups may not give a suitable fit.

Continue by reexamining the contenders selected during the first pass. Often the choice between two or three tentative selections is difficult to make. The conservative choice is toward the lower life and right modal curve.

An alternative to visual fitting is mathematical fitting. Usually the least squares method is used. This method is time consuming if done by hand, and is not practical unless a computer is used. Typical logic for a computer program is as follows. First a type curve is arbitrarily selected. If the observed curve goes to zero percent surviving, calculate the area under the curve and designate this the average life.

If the observed curve is a stub curve (i.e., if it does not go to zero), calculate the area under the curve and up to the age at final data point. Call this area the *realized life*. Then systematically vary the average life of the theoretical survivor curve and calculate its realized life at the age corresponding to the study date. This trial and error procedure ends when you find an average life such that the realized life of the theoretical curve equals the realized life of the observed curve. Call this the *average life*.

Once the average life is found, calculate the difference between each percent surviving point on the observed survivor curve and the corresponding point on the Iowa curve. Square each difference and sum them. The sum of squares is used as a measure of goodness of fit for that particular Iowa type curve. This procedure is repeated for the remaining 21 Iowa type curves. The "best fit" is declared to be the type of curve that minimizes the sum of differences squared.

On the surface, the removal of judgment from the fitting process may appear to be an advantage, but blind acceptance of mechanical fitting processes will occasionally but consistently result in poor results. A better procedure is to use the least squares method to select candidates for the best fit. Comparison of the sum of squares will reveal situations where the difference between the best choices is small. The analyst should then visually examine the observed data and compare them to the theoretical curves. This can be done quickly on a computer with graphic capabilities so that the analyst need not use time to plot the observed curve by hand. The analyst can consider single points that may contribute significantly to the sum of squares but that may deserve less weight than other points. Fits at various sections on the curve can be evaluated and weighted using the judgment of an experienced analyst.

The results of mathematical curve fitting serve as a guide for the analyst and speed the visual fitting process. But the results of the mathematical fitting should be checked visually and the final determination of the best fit be made by the analyst. Computer systems that will mathematically fit the data and provide graphical curve fitting are widely available. Wisely used, these systems are a powerful tool for the depreciation professional.

Fitting Stub Survivor Curves

Stub curves, which are survivor curves for which the data end before the curve reaches 0% surviving, are frequently encountered. The realized life of the property is the area under the curve and to the left of the final point on the survivor curve as shown in Figure 3.6. Unless the remaining property is retired immediately, the average life of the property will exceed the realized life.

The process of fitting stub survivor curves to an Iowa or other type curve is essentially the same as fitting a complete survivor curve. The obvious and critical difference is that the fit is valid only to the final observed point on the survivor curve.

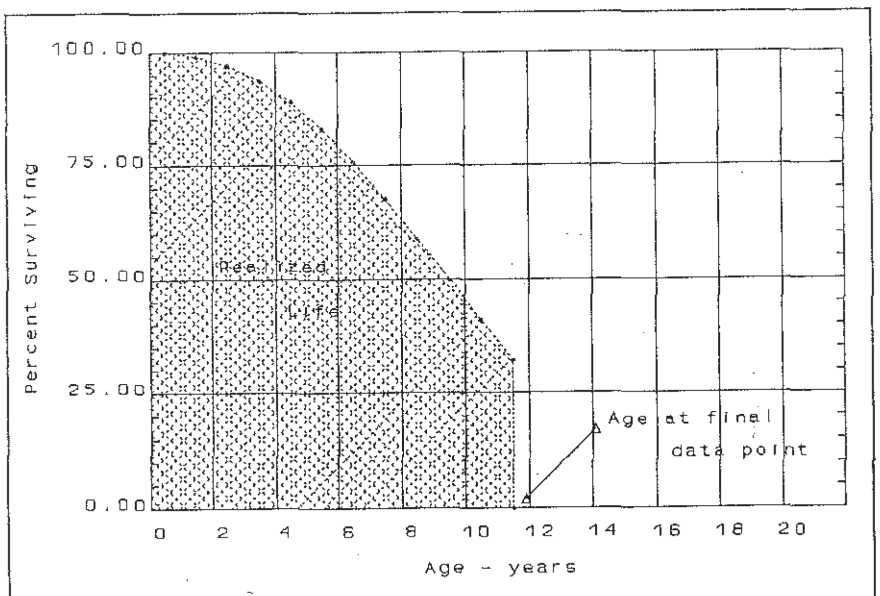

Figure 3.6. A stub survivor curve and the realized life.

Stub curves contain the essence of the shape of the curve and can be accurately fit even though the curve is not complete. To clarify this, think about conducting the following experiment. Gather several complete survivor curves and fit them to Iowa curves. Then truncate the observed survivor curves and fit the truncated curves to Iowa curves. Compare the fits of the complete and stub curves to see if consistent results are obtained. The experiment could be repeated by successively truncating more of the curve until only a short stub remained. Analysis of the results of the experiment will reveal how short the stub curve can be before the results of fitting the stub curves differ from the fit to the entire curve. This experiment was performed by Cowles (1957), who concluded that reasonably good fits were obtained for stub curves that ended at a point as high as 70% surviving. Longer stub curves (i.e., those with 40% or less surviving) were fit with a high degree of accuracy. This shows that the upper portions of the various types of Iowa curves are distinctive enough to identify the curve.

When reporting the results of fitting a stub curve, the analyst must be careful to say the fit applies to the observed data and not beyond. This can be done by appending the length of the stub to the results of the fit. This notation is suggested:

$$\text{Type} - \text{life } [S(b) - S(e)]$$

where S(b) = largest observed percent surviving (usually 100%) and S(e) = the smallest, or ending, percent surviving.

An example of the use of this notation is R2 – 30 [100% – 46%], which would show that the analyst believed the best fit of the observed data was an Iowa R2 type curve with a 30-year average life, and that the observed curve started at 100% surviving but was a stub curve whose final point was at 46% surviving. The average life of the property will be 30 years only if the future forces of retirement continue to follow those of a R2–30 Iowa curve.

A method of estimating the parameters g, s, and c of the Gompertz-Makeham equation to observed data has been described and can be applied to either a stub or complete curve. Once the parameters are estimated, the complete, smoothed curve can be plotted, and the area under the curve (i.e., the average life) can be calculated. Again, the reports of the results of the analysis must include the length of the stub curve. Only the realized life (i.e., the area under the stub curve) is known, and the estimate of life obtained from extrapolation of the survivor curves assumes that future forces of retirement will remain unchanged. The forces can be thought of as measured by the constants in the hazard function

$$h(x) = A + Bc^x$$

Table 4.10. Conversion of salvage in Table 4.9 to 1982 dollars.

	Experience year						
	82	83	84	85	86	87	88
Gross salvage	94	337	418	645	834	890	720
Cost of retiring	27	106	163	276	437	539	553
Annual retirements	157	627	941	1568	2508	3135	3292
Gross salvage ratio	.60	.54	.44	.41	.33	.28	.22
Cost of retiring ratio	.17	.17	.17	.18	.17	.17	.17
Net salvage ratio	.43	.37	.27	.23	.16	.11	.05

5 Depreciation Systems

THE recovery of capital through depreciation accruals may be thought of as a dynamic system. A system is an arrangement of things that are connected to form a complete organization of integrated parts. The state of the system at any time is defined by current values of the characteristics that define the system. A dynamic system is one where the state of the system depends on the history of the input variables. To define and study a system is to better understand the system so that more efficient methods of control can be designed to accomplish the desired ends.

There are two methods of controlling a system. One is to select an input and wait for the result or final output. If a different output is desired, the input is changed and the new output is obtained. The other method of control is to select an initial input, monitor the process, and when necessary, alter the input to achieve the desired goal. The first method is called an open control loop and the second a closed control loop. A necessary feature of the closed control loop is the feedback resulting from the monitoring of the system. A home heating system is a common and simple example of a dynamic system with a closed feedback loop. The parts of the system are a furnace and a thermostat. The thermostat monitors the room temperature and creates feedback, in the form of electrical signals, when the room temperature rises above or falls below the desired temperature. The electrical signals turn the furnace off or on to achieve the desired goal, a constant, predetermined room temperature.

Think of a depreciation accounting system as a dynamic system controlled with a closed feedback loop. Estimates of life and salvage and the

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amount of plant in service are inputs to the system, and the accumulated provision for depreciation is a measure of the state of the system at any time. The process of calculating the accumulated provision for depreciation is determined by the factors needed to define the system. The initial input to the system is estimates of the life and salvage, which are combined in an accrual rate. Dynamic forces affect the life and salvage, and revision of the original life and salvage estimates are the result of the monitoring process. These revisions to the initial input initiate feedback in the form of adjustments to the accumulated provision for depreciation. The goal of the system is recovery of capital in a timely manner.

One consideration that complicates this discussion is that many options can be combined to form many different depreciation systems. Whether the depreciation is for book, tax, valuation, or other purposes, each of these factors must be considered when discussing and defining a depreciation system.

DEFINING A DEPRECIATION SYSTEM

Below is a list of the factors needed to define a depreciation system. Each factor contains two or three options, and the complete definition of a system requires the selection of one option from each factor. The order of the list is arbitrary, but the last four factors are those whose options are varied when discussing depreciation systems commonly used to calculate book depreciation.

1. The depreciation concept, including (a) physical condition, (b) decrease in value, or (c) cost of operation
2. Depreciation over (a) time or (b) units of production
3. Depreciation of (a) a unit of property or (b) a group of property
4. Methods of allocation, including (a) the straight line method, (b) an accelerated method, or (c) a decelerated method
5. Procedures for applying the method of allocation including (a) the average life procedure, (b) the equal life group procedure, or (c) the probable life procedure
6. Adjustment using (a) the amortization method or (b) the remaining life method
7. Use of (a) the broad group model or (b) the vintage group model

The mathematically astute reader who multiplies the number of options in each factor will find that there are 432 combinations of options, each of which is a potential depreciation system. However, not all of these combinations are feasible, and some are unimportant. Only a few of these

combinations are of major interest when considering systems of book depreciation currently being used.

Concepts of Depreciation

Three options are available when defining the concept of depreciation. These include (a) physical condition, (b) decrease in value, or (c) cost of operation. Though all have been used by utilities to determine book value, the cost of operation is, with few exceptions, the concept in current use.

Physical condition is, perhaps, the first option a lay person would think of if asked to define depreciation. An early reference to the relationship between depreciation and physical condition is from the 1588 textbook by John Mellis who referred to a debit to the profit and loss account because "implements of householde I doe find at this day to be consumed and worn." A later reference is in the 1833 annual report of the Baltimore and Ohio Railroad, which reported that an annuity was established "to provide for the replacement of oak sills and sleepers and yellow pine stringpieces."

Two problems arise when using the concept of physical condition as a measure of depreciation. First, wear and tear do not account for all retirements; in fact, they are often a minor reason for the retirement of property. Second, physical condition can be difficult to measure. Though it is possible to measure directly the wear of railroad track and the corrosion of cast iron pipe, easily measurable wear is not characteristic of most industrial property.

The concept of loss of value is also a common depreciation concept, and the lay person often uses it to explain the difference between the purchase price and the current market value of an automobile or major household appliance. The definition from the Supreme Court case *Lindheimer v. Illinois Bell Telephone* (1934) is often quoted: "Broadly speaking, depreciation is the loss, not restored by current maintenance, which is due to all the factors causing the ultimate retirement of the property. These factors embrace wear and tear, decay, inadequacy, and obsolescence."

In contrast to the concept of physical depreciation, the Lindheimer definition recognizes that factors other than wear and tear cause or contribute to the retirement of property. The definition refers to the "loss" but does not clearly state what is "lost" or how the "loss" should be measured. A 1935 definition by the Federal Communications Commission was similar to the Lindheimer definition but referred to "loss in service value," where service value is equated to the original cost less salvage.

Use of the concept of loss of value to determine annual depreciation charges might imply the need for an annual valuation of the property owned by the organization, particularly if the rate of loss in value was not uniform or readily defined. The process of determining a value is complex, depending on the purpose of the valuation and type of property. Thus, an annual valuation of a utility could be such an expensive and time-consuming process that it would not be a practical approach to use in determining annual depreciation.

Many types of property provide a constant level of service until they are retired. The intrinsic physical value of this type of property is only that it functions. A gas meter is a common example of a type of property that may provide a constant level of service throughout its life. If value is measured by the level of service provided, the meter would retain full value until retirement because its value to the utility would depend on its function rather than its age. This concept ignores the consumption of future service and would result in an annual depreciation charge that would be zero until the final year of service. Then the charge would equal the full value and would result in deferring all depreciation charges until the final year of service. A concept that better matches depreciation to service rendered and weighs it in relation to the total service potential might be preferable for purposes of both book and valuation depreciation. That is, a quantitative measure of value, such as service-years, is generally preferable to a functional measure.

The third concept is that depreciation represents an allocated cost of capital to operation. This concept recognizes that depreciation is a cost of providing service and that an organization should recover the capital invested in equipment and other property needed to provide the required service. In fact, the term *capital recovery* is often used in connection with depreciation. An early reference to depreciation is by the Roman Marcus Vitrurius Pollio, who in 27 B.C. wrote of "walls which are built of soft and smooth-looking stone, that will not last long." He calculated that the walls would not last more than eighty years and suggested that, for purposes of valuation, one-eightieth part of their original cost be deducted each year. Pollio not only raised several issues concerning depreciation but seemed to be equating depreciation to a cost of operation.

The definition of *depreciation accounting* by the American Institute of Certified Public Accountants (1961, par. 56) reflects the concept of depreciation as a cost: "Depreciation accounting is a system of accounting that aims to distribute cost or other basic value of tangible capital assets, less salvage (if any), over the estimated useful life of the unit (which may be a group of assets) in a systematic and rational manner. It is a process of allocation, not of valuation." This definition does not use the term *loss of service value* because it is defining depreciation accounting rather than depreciation itself. The definition emphasizes that the purpose of depreciation accounting is a means of distributing cost in a rational manner during the service life, in turn providing for the systematic recovery of capital. By use of the term *useful life*, the definition encompasses all causes of retirement. By referring to the distribution of cost less salvage, this definition recognizes that salvage should be considered when developing depreciation charges.

Historically, all three concepts of depreciation have been used by utilities to determine the book value of industrial property. Of these, the concept of depreciation as the allocation of cost has proven to be the most useful and most widely used concept.

Time versus Unit of Production

Useful life can be measured in units of time or units of production (also called units of service). Measurement of life in years is a common and familiar concept. Measurement of life in units of production can be applied to some types of property such as a truck, whose life can be measured in miles (e.g., a useful life of 100,000 miles). A feeder pipeline connecting an oil field to a transmission line will be in service until the field is no longer productive. If the only function of the feeder line is to transport oil from the field to the transmission line, the life of the feeder line is determined by the reserves of the oil field that must eventually pass through the pipeline. Annual depreciation could be measured in units of production, such as barrels of oil. A railroad might depreciate rail as a function of the accumulated weight that the rail has carried.

Suppose a truck is to be depreciated over its life as measured in miles. First, the life must be estimated, say 100,000 miles. Second, the number of miles the truck will be driven during the next year, say 27,000 miles, must be forecast to have sufficient information to budget the annual depreciation charge. Third, at the end of the year when the budgeted annual depreciation becomes an accounting entry, the amount would be calculated to reflect the actual miles driven.

The most common measure of life is in units of time rather than units of production. Most types of property (e.g., poles, buildings, wire) do not have a measure of production associated with them. If the life can be measured in some unit of production and the rate of production is constant from year to year, measurement of life in either units of time or production will result in the same annual accruals. The unit of production has strong appeal in situations where use varies significantly over time and the life can be measured in units of production. But these two conditions are not often met, and usually life is measured over time.

Depreciation of an Individual Unit versus a Group

Accounting records of transactions relating to depreciable property can be kept on either a unit or a group basis. An individual unit of property has a single life, while the units in a group of property display a range, or dispersion, of lives. Grouping many units of property into a single account simplifies the accounting system but also creates a complexity not encountered in the depreciation of an individual unit. The resulting complications provide a major challenge to the depreciation analyst.

A vintage group refers to a group of property placed in service during the same year. The plant in service decreases until all units are retired from service. The individual unit and the vintage group are similar because each has well-defined life characteristics. The life of an individual unit is described by a single number and the life of a vintage group is described by a survivor curve, which is a statistical description of the lives of the units of property in the group.

Methods of Allocation

To fully recover capital invested in plant and equipment, the total depreciation charge must equal the depreciation base. When using the allocation of cost concept, the depreciation base is the initial, or original, cost less net salvage. The annual depreciation accrual rate for a unit of property can be (a) constant over life (straight line), (b) high during early years and low in later years (accelerated), or (c) low in early years and high in later years (decelerated). Most methods of allocation fall into one of these three classifications, although it would be possible to develop a method that is a combination of them. The straight line method of allocation is the method of allocation most often used when calculating book depreciation. Accelerated methods of allocation are commonly used for tax purposes. Decelerated methods of allocation are not in common use for book or tax purposes, but they are of historical interest and are used in valuation problems.

Average Life, Equal Life Group, or Probable Life Procedures

The average life and equal life group procedures are two ways of applying a method of allocation to determine the annual accrual. The probable life procedure is similar to the average life procedure, but is not appropriate for depreciation accounting.

A group of property displays a wide range of lives, and the life characteristics of the group must be described statistically. This is in contrast to a unit of property, whose life can be described as a single number. When depreciating a group of property, rather than a unit of property, a major decision must be made whether to base the depreciation accrual rate on the average life of the group (the average life procedure) or whether to divide the group into subgroups of equal life (the equal life group procedure).

In the average life procedure, a constant annual accrual rate based on the average life of all property in the group is applied to the surviving

property. Most retirements occur either before or after, rather than at, the average life, but both short- and long-lived property are depreciated at the same rate. Property having a shorter life than the average will not be fully depreciated by the time of its retirement. Because the accrual rate is based on the average life of the group, the difference between accruals for early retirements and the full cost of the early retirements will be balanced during the life of the property having lives longer than the average. The result is that the group will be fully depreciated by the time of the final retirement.

In the equal life group procedure the property is divided into subgroups that each have a common life. Each subgroup is then depreciated as a unit using an accrual rate based on the common life of the group. Each unit is fully depreciated by the time it is retired. Application of the equal life group procedure is generally considered to better match the consumption of capital with service provided than does application of the average life procedure.

Any of the three methods of allocation (i.e., straight line, accelerated, or decelerated) can be applied to an individual unit or to group property. When the average life procedure is applied, the straight line method of allocation is easily used; application of either an accelerated or a decelerated method becomes more complicated. When the equal life group procedure is used, any of the three methods of allocation can be easily used.

The probable life procedure is a variation of the average life procedure. It is not valid for depreciation accounting or capital recovery because it does not fully depreciate the group. The depreciation charges are allocated over the average life of the property remaining in service (i.e., over the probable life), so that the continually decreasing rate is inadequate to fully recover the depreciable base. Use of this procedure should be restricted to those special situations where it is applicable; for example, it may used in the valuation process.

Methods of Adjustment

Depreciation accrual rates are calculated using estimates of the service life and salvage. Over time, new events that provide additional information occur, and the existing estimates are revised. A revision of the estimates of life and salvage results in the recognition that the accumulated provision for depreciation may now be either higher or lower than necessary, depending upon the magnitude and direction of the revised estimates. This recognition may justify an adjustment to the accumulated provision for depreciation, an adjustment to the annual depreciation rate, or both.

Adjustments to the accumulated provision for depreciation[1] can be made using either a fixed amortization period or the remaining life basis. The term *amortization method of adjustment* is used to describe a general

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equal to the area under the survivor curve and may be written (area under the survivor curve)/AL = original cost, or by rearranging this equation, AL = (the area under the survivor curve)/(original cost). The average life has been shown to equal the area under the survivor curve divided by the original cost (true whether the survivor curve is measured in dollars or units), so the original equality is true. We can conclude that this system will fully recover the initial investment regardless of the shape of the survivor curve.

This equation also shows that if the AL used in the accrual rate is not equal to the actual average life, the sum of the accruals will not equal the original cost. Suppose that the actual life was 8 years, but a life of 6 years was forecast and used in the depreciation rate. The total accruals would equal 8/6 or 133% of the original cost, and the accumulated provision for depreciation would show an overaccrual equal to 133% − 100% or 33% of the original cost at the time of the final retirement. Similarly, a forecast of a life of 10 years would result in total accruals of 8/10 or 80% of the original cost. At the time of the final retirement the accumulated provision for depreciation would show an underaccrual equal to 100% − 80% or 20% of the original cost.

Consider a property group having the survivor curve shown in Figure 5.1. This curve could result from the grouping of two units, one with a cost of $4000 and a 4-year life and the second with a cost of $6000 and an 8-year life. The average life (AL) is the area under the survivor curve divided by the original cost or the AL = [(4000 × 4) + (6000 × 8)]/10000 or 6.4 years. The straight line, average life annual accrual rate is 1/6.4 or 15.625%.

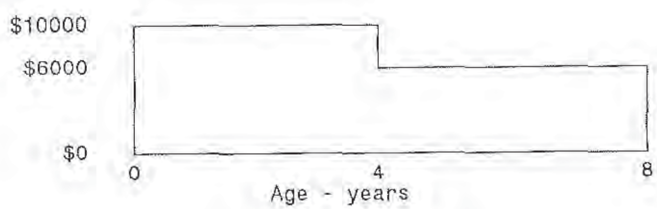

Figure 5.1. A survivor curve with an average life of 6.4 years.

NOTE: To simplify calculations in this section, age intervals will be 0–1, 1–2, 2–3, etc., installations will be assumed to occur at the start of the age interval, and retirements will be assumed to occur at the end of the age interval. The average plant in service during the age interval will then equal the balance at the start of the interval, so that applying the annual accrual rate to the plant in service at the start of the interval is equivalent to applying it to the average balance. In all tables in this section, the balance, accumulated provision for depreciation, and calculated accumulated depreciation are calculated at the beginning of the year. Note that the accumulated provision for depreciation is zero at the beginning of the initial year. Examples using the half-year convention will be shown later.

Table 5.5 (see end of chapter) shows the annual accruals and accumulated provision for depreciation that result from the SL-AL system. Column (b) shows beginning of year balances of $10,000 for the four years 1974 to 1977, and balances of $6000 for the next four years, 1978 to 1981. This follows the survivor curve shown in Figure 5.1. Column (c) shows a $4000 retirement at the end of 1977 and a $6000 retirement at the end of 1981. The annual accrual, column (e), is the product of the rate, column (d), and the plant balance at the start of the year, column (b). As described in the preceding note, retirements are assumed to take place at the end of the year, so that the plant balance at the start of the year is also the average balance during the year. The accumulated provision for depreciation, column (f), is zero at the start of the first year and is then increased by the annual accruals and reduced by the annual retirements. At the time of the final retirement, the accumulated provision for depreciation is zero, showing that the sum of the annual accruals and the annual retirements equals zero and that the property is fully depreciated.

Suppose that at the time of the initial installation of the property, the estimate of the average life was 7.4 years. If the rate 1/7.4 is used throughout the life of the property but the actual life is 6.4 years, then only (6.4/7.4)($10000) or $8649 will be depreciated and $10000 − $8649 or $1351 of invested capital will not be recovered. This is verified by the calculations shown in Table 5.6 (see end of chapter).

An Adjustment Problem—AL Procedure

Now suppose that in January 1977, because of events and activities occurring since 1974, the original forecast of 7.4 years average life is revised to 6.4 years. Table 5.7 (see end of chapter) shows the accumulated provision for depreciation at the start of 1977 is $4054. Unless some corrective action is taken, the annual accruals will not equal the $10,000 original cost, and at the time of the final retirement a total of $1351 will remain unrecovered. The SL-AL system of calculating the annual accruals must be augmented to include a method of adjustment to define a depreciation system that will adapt to the almost certain circumstance that forecasts are revised from time to time.

When there is a revision of the original forecast of service life, it

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NOTES

1. Gas (FERC), electric (FERC), and water companies (NARUC) use the title Accumulated Provision for Depreciation, while telecommunications (FCC) and railroads (ICC) use the title Accumulated Depreciation. Telecommunications used the title Depreciation Reserve until 1986. We will use the term *Accumulated Provision for Depreciation*. It refers to the net balance of either of these accounts.

2. The CADR is also called reserve ratio or the accrued ratio.

3. The balance, usually shown in column (b) of the tables at the end of this chapter, can be defined as the balance at the start of the year or the average balance during the year. The convention used in this chapter results in the start-of-year balance equaling the average balance. When other conventions are used, a small difference between the two annual accruals may result because the remaining life calculation typically applies the rate to the beginning-year balance while the amortization calculation typically applies the rate to the average balance during the year.

4. All of these variables except AL are a function of the age of the property, and a complete notation would include an index for the age. For example, rather than RL, use RL(i) = the remaining life at age i. The following calculations all refer to the same year so the index i is dropped.

5. Remember that both the realized and future salvage ratios are averages. Both the survivor curve and the salvage ratio at each age from zero to the maximum life are required to calculate these average ratios. For convenience, we will use the terms realized or future salvage ratio rather than *average* realized or *average* future salvage ratio.

6 Continuous Property Groups

CHAPTER 5 used elementary survivor curves, a single vintage group, and a simplified format to illustrate the calculation of the annual accrual for depreciation for several systems of depreciation. This chapter will use Iowa survivor curves and formats characteristic of those found in industrial practice to illustrate the application of these systems of depreciation to continuous property groups. The illustrations will include the use of both average salvage and salvage that varies with age.

A continuous property group[1] is created when vintage groups are combined to form a common group. Each year property is removed from service for many reasons and, at the same time, property may be added either to replace property that was retired or to increase service capacity. Over time the operating and physical characteristics of the property change, but the continuous property group continues as long as the service it provides is needed. If new vintages are no longer added, the continuous property group becomes closed, or bounded, and over time becomes smaller and finally vanishes. The continuous property group, which is usually recorded as a plant account or subaccount, is of interest because it represents the most common method of grouping assets for depreciation.

The method of allocation, the procedure for applying the method of allocation, and the method of adjustment (e.g., SL-AL-AM) specify the factors that define a depreciation system for a vintage group. The terms broad group and vintage group describe two ways of viewing the life and salvage characteristics of the vintage groups that have been combined to form a continuous property group for purposes of depreciation. One approach is to view the continuous property group as a collection of vintage

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groups each of which have the same life and salvage characteristics. This is called the *broad group model*. The other approach is to view the continuous property group as a collection of vintage groups each of which can have different life and salvage characteristics. This is called the *vintage group model*. The model being used will be identified by either stating that the broad or vintage group model is being used or by adding the initials BG or VG to those defining the depreciation system (e.g., SL-AL-AM-BG).

THE BROAD GROUP MODEL

The broad group model requires that a single survivor curve and a single salvage schedule be chosen to represent all the vintages in the continuous property group. Though it is likely that individual vintages will have different life and salvage characteristics, the broad group model makes the simplifying assumption that all vintages in the continuous property group have identical life and salvage characteristics. Thus, if the broad group model is used, it must be reasonable to represent the life and salvage characteristics of each vintage with a single survivor curve and salvage schedule.

The Average Life Procedure

The average life (AL) procedure is discussed in this section. First the amortization (i.e., the SL-AL-AM-BG system) and then the remaining life method of adjustment (i.e., the SL-AL-RL-BG system) will be illustrated.[2] Both the life and salvage characteristics must be chosen in a manner consistent with the assumptions of the broad group model to provide reasonable estimates of depreciation. When using the average life procedure, the reasoning used to choose the life and salvage characteristics of the broad group is the same for both the AM and RL methods of adjustment.

Figure 6.1 represents the realized (historical) and future (forecast) life characteristics of a continuous property group. Each row represents a vintage group. The first row represents the oldest surviving vintage and the final row the most recent vintage. Each column represents a calendar year. The first column is the year during which the oldest surviving vintage was installed. The space dividing the realized life and the future life marks the end of the most recent year, and the last column is the year the last unit from the last surviving vintage is retired. Let each cell represent the vintage surviving plant at the end of the calendar year, so that the cells in any row form a survivor curve reflecting the life characteristics of that vintage.

Tables 8.1 and 8.2 in Chapter 8 contain the schedule of additions, retirements, and plant balances for the continuous property group called Account 897—Utility Devices. This property group is used to illustrate the

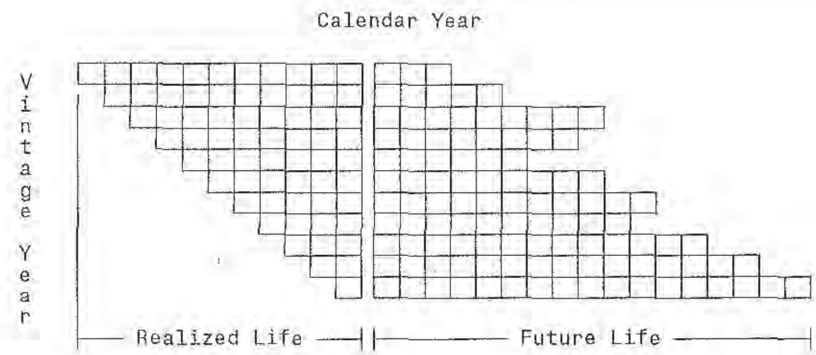

Figure 6.1. A representation of the realized and future life of a continuous property group.

calculation of the annual accrual for depreciation. Tables 8.1 and 8.2 contain the data necessary to construct the triangular-shaped, realized life portion of Figure 6.1. Place yourself in time at the end of the final year for which there is data (i.e., for Account 897 this is December 31, 1990). Look back in time and see the historical record of additions and retirements. Now imagine moving forward in time. As the years pass, visualize the declining vintage balances, with some vintages becoming completely retired while others remain in service. Finally, the last unit of the last vintage is retired. The record of the future balances, contained in the righthand side of the figure, defines the future life. Each row (i.e., each vintage survivor curve) can be divided into either realized life or future life. Older vintages are mostly realized life and newer vintages are mostly future life. The realized life is documented in the aged data while the future life is a forecast. The product of the age at retirement times the amount of the retirement, summed over all retirements and divided by the total retirements is the average life. The job of the depreciation professional is, first, to determine the realized life and, second, to forecast the future life. Then when using the AL procedure and the broad group model, he or she must select the survivor curve that best describes the life characteristics (i.e., a single curve that describes both the realized life and the future life) of the single group of property formed by combining the vintage groups shown in Figure 6.1. The Iowa S1–10 survivor curve will be used to describe the life characteristics of Account 897 for use with the average life procedure and the broad group model.

Avg Life

Figure 6.1 also can be used to help explain the method of estimating the average salvage ratio and the future salvage ratios. For this purpose, let the study date forward), because, under the ELG procedure, property should be fully depreciated at retirement and the accrual rate depends only on the shape of the survivor curve from the age at the study date to maximum life. It is not unusual to estimate a single average net salvage ratio and apply it to each ELG, rather than to use aged salvage with a different salvage ratio for each ELG. The use of an average salvage ratio is often the result of the lack of aged salvage data and the lack of models to estimate future salvage ratios by age.

Remaining life depreciation usually refers to the SL-AL-RL system of depreciation; use of the AL procedure is implied as is use of the same survivor curve for all vintages. Emphasis is placed on forecasting the remaining life or future curve. When calculating the future accruals, the same future salvage ratio is often used for all vintages.

Users of remaining life depreciation often do not explicitly calculate the CAD. As previously discussed, calculation of the CAD is implicit in the use of the remaining life method of adjustment, because the variation between the CAD and the accumulated provision for depreciation is automatically amortized over the remaining life. Explicit calculation of the CAD will allow the depreciation professional to find the portion of the annual accrual associated with amortization of the variation (either positive or negative).

When the ELG procedure is used with the remaining life method of adjustment, a term such as *ELG – remaining life depreciation* may be used to describe the SL-ELG-RL system. A single future survivor curve and future salvage ratio usually are applied to all vintages, although the future curve could be varied. Several pages in Chapter 6 were devoted to a discussion of the allocation of the accumulated provision for depreciation to each vintage when using this depreciation system. It was shown that allocation in proportion to the calculated future accruals resulted in a composite remaining life that is independent of the variation between the CAD and the accumulated provision for depreciation. Then the composite ELG accrual rate is calculated based on that composite remaining life.

Specify each of the four factors of a depreciation system to ensure communication. It is not safe to assume that life and salvage are treated in the same manner. Take care to indicate differences in the manner in which they are treated.

NOTE

1. *Whole life* is also used in a second context in which it is used in contrast to *location life*. When property is reused, the location life is the length of time from installation at a particular location to retirement from that location. The whole life can then be divided into a series of location lives.

8 Actuarial Methods of Developing Life Tables

FOUR basic methods of developing a life table can be used when aged data are available. These include the placement band method,[1] the experience band method, the multiple original group method, and the individual unit method. Each provides special insight to the life characteristics of the property and each has its limitations.

DATA REQUIREMENTS

The term *aged data* is used to describe the information reflecting the initial age distributions, annual additions, and the changes to that property for each year in the history of the account. Original data include the annual additions, retirements, transfers, sales, acquisitions, and other transactions. These data must be checked to ensure they are consistent, accurate, and coded so that they can be used to find the exposures and retirements for each age interval.

The aged data base used in this chapter is an account labeled Account 897–Utility Devices and is shown in Tables 8.1 and 8.2 (see end of chapter). These data contain the initial age distribution and have been simplified by assuming that the only two transactions can occur—the addition of new property and the retirement of installed property. Table 8.1 displays the

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of 6.00/123.00 or 0.0488. Any property that was exposed but not retired during the age interval must have survived or still have been in service at the end of the age interval. Therefore, the survivor ratio, shown in column (e), is 1 minus the retirement ratio. For age interval 8.5–9.5 years the survivor ratio is 1 − 0.0488 or 0.9512.

Column (f) contains the percent surviving at the beginning of the interval. At age 0, 100% of the property is in service and the retirement ratio for the age interval 0.0–0.5 years shows that survivor ratio for the interval is 1 − 0 or 1, leaving 1 × 100.00% or 100.00% surviving at age 0.5 years. During age interval 0.5–1.5 years, the survivor ratio is 1 − 0.0137 or 0.9863, so that the percent surviving at age 1.5 years is 0.9863 × 100.00% or 98.63%. In general, the percent surviving at the beginning of an interval is the percent surviving at the beginning of the previous interval multiplied by the survivor ratio during the interval.

In this example, all the original $146.00 has been retired from service. Often plant is still in service at the start of the oldest age interval resulting in an incomplete or stub survivor curve. The survivor curve is plotted by graphing the percent surviving shown in column (f) as a function of the age, shown in column (a).

The retirement ratios show a tendency to increase with age, although there is randomness as the ratios fluctuate from year to year. The retirement ratio during the age interval 13.5–14.5 years appears significantly larger than its neighbors. Look to the forces of retirement during 1982 for a possible explanation of this large ratio. During the year 1988, all the remaining plant in service is retired and the resulting retirement ratio of 1 brings the percent surviving to zero. Figure 8.2 (see end of chapter) is a graph of this survivor curve.

X

Placement Band Method

Perhaps the most intuitive method of constructing a life table is by the placement band method. First, consider a placement band consisting of a single year, 1968. Experience for this placement band starts in 1968 and ends in 1990, as shown in Figure 8.3 (see end of chapter).

Table 8.3 is the life table developed using the 1968 placement band. Note that in the table heading the placement band is defined 1968–1968, while the experience band runs from 1968, the first year in which property was installed, to 1990, the most recent experience.

When using the placement band method, the resulting survivor curve describes the life characteristics of the group of property installed during that placement year. Suppose there had been no transfers, sales, acquisitions, and other entries, but only the initial installations followed by the regular retirements each year. The survivor curve obtained from the single

placement year would be identical with that obtained by plotting the dollars surviving at the end of each year. Examination of the placement band method when only additions and retirements are considered emphasizes that this method traces the history of the particular group of property installed during a specific placement year or band of years.

Usually placement bands include more than one year, defining a group of property installed during a specific period or era, rather than a single year. Construction of a band of more than one year raises two computational questions. To identify and help answer these questions, consider the placement band formed by combining the 1968, 1969, and 1970 placement years. This band is shown graphically in Figure 8.4 and the resulting life table is shown in Table 8.4 (see end of chapter for figures and tables).

For the typical age interval, retirements and exposures from each of the three placement bands must be combined to obtain a single retirement ratio, and this raises the first computational problem. Two possible methods of combining these numbers come to mind, but first the retirement ratio for each of the three placement bands must be calculated. For the age interval 6.5–7.5 years in the 1968–1970 placement band, retirements come from (a) the 1968 placement band and the 1975 experience year—5 units, (b) the 1969 placement band and the 1976 experience band—2 units, and (c) the 1970 placement band and the 1977 experience band—8 units. These retirements all occur during the same age interval, 6.5–7.5 years. Exposures of 133, 69, and 102 for each of the three years are obtained in a similar manner and retirement ratios of 5/133 or 0.0376, 2/69 or 0.0290, and 8/102 or 0.0784 can then be calculated. The first method to combine these ratios, and the method used in all remaining examples, is to weight each ratio by its exposures/total exposures and calculate a weighted average of (133 × 0.0376 + 69 × 0.0290 + 102 × 0.0784)/(133 + 69 + 102) or 0.0493. This is equivalent to dividing the sum of the retirements by the sum of the exposures, or (5 + 2 + 8)/(133 + 69 + 102) or 0.0493.

The second method is to calculate an unweighted average of the three rates to obtain (0.0375 + 0.0289 + 0.0784)/3 or 0.0483. This weights each placement year equally regardless of the number of units involved, and this method should be considered when it is desirable to give each vintage equal weight. As a practical matter, the two methods will typically yield approximately the same results. The first method is most commonly used because it gives equal weight to each unit.

The second computational problem occurs near the end of the survivor curve. Through the first 19 age intervals, each of the three years contributes exposures and retirements to be included in the calculation of the retirement ratio. However, during the age interval 17.5–18.5 years, the exposures and retirements from the 1970 placement band are from the final experience year, 1990, and this placement cannot contribute information for the

4. Identification of special activities or events that may, in the future, affect retirement of property. This will include only those activities that the analyst encountered during the life analysis process and would not be a comprehensive list.

Life analysis necessarily precedes life forecasting. Analysis of historical data and events gives the forecaster the raw material on which the forecast will be based.

RETIREMENT: A MANAGEMENT DECISION

Early work in the analysis of service life of industrial property was based on models, developed by actuarial scientists, that described age characteristics of human populations. *Forces of mortality* is a descriptive term used by these scientists, and, although "mortality" properly refers to humans, the term has been applied to industrial property. Forces of retirement is perhaps a better phrase to use to describe the various conditions that lead to the retirement of industrial property. Many useful categories of forces of retirement have been made, and the following list is from *Engineering Valuation and Depreciation* (Marston, Winfrey, and Hempstead, 1953).

A. Physical condition
   1. Accident
   2. Catastrophe
   3. Deterioration from time
   4. Wear and tear from use
B. Functional situations
   5. Inadequacy
   6. Obsolescence
      a. Economic
      b. Style and mode
C. Situations unrelated to the property
   7. Termination of the need
   8. Abandonment of the enterprise
   9. Requirement of the public authority

Retirement of industrial property is the result of a management decision. Some of these are easy decisions. When a car, for example, runs into and destroys a telephone pole, management has little choice but to retire and replace the pole. The decision to retire an electric generating system in more difficult and its timing is critical to the financial health of the company.

Decisions to replace industrial property are, to some extent, a function of the laws of nature that determine the physical characteristics and properties of materials and processes used to provide services. Many models used in engineering reflect natural phenomena and are based on the work of mathematicians, physicists, chemists, and other scientists. For example, failure of railroad track caused by wear may be a highly predictable function of the cumulative tons of traffic carried by the track. Models that develop this functional relationship can be very useful in developing maintenance policy, planning operations, and, to some extent, predicting service life.

However, most retirements are more subtle functions of factors such as the economy, changing technology, or government regulations, all of which significantly influence management decisions. Other factors such as maintenance policy or organizational goals are a direct result of management decisions. It is accurate to think of retirement as a management decision and realize that when forecasting service life, the various factors that mold management thought and action must be identified.

FORECASTING: A SYSTEMATIC APPROACH

The process used for developing a forecast of service life characteristics is heavily dependent on the circumstances surrounding the specific group of property under consideration. The appropriate forecasting approach can range from a traditional, well-defined, highly mechanical process to a nontraditional, loosely defined process involving newly developed forecasting techniques. In addition, because forecasting involves significant portions of judgment, experts may disagree upon which process is appropriate.

Most of the forecasting problems encountered in public utilities can be classified into several general categories. One method of developing a systematic approach to forecasting is to define the general categories and then prescribe a forecasting approach to each. When necessary, additional categories can be defined and appropriate forecasting approaches developed. Sometimes new techniques may be needed. The set of categories and prescriptions then defines a systematic approach to forecasting.

Seven Major Categories

This section defines seven general categories of forecasting problems. The characteristics of each category are defined by the results or output from the life analysis process. Each category is broad enough to maintain its own distinct characteristics. Although these seven categories are not

Iowa Engineering Experiment Station.
Winfrey, Robley. (1935). *Statistical analysis of industrial property retirements (Bulletin 125).* Ames, IA, Iowa Engineering Experiment Station.
Winfrey, Robley. (1967). *Statistical analysis of industrial property retirements (Bulletin 125).* Revised edition. Ames, IA, Iowa Engineering Experiment Station.
Winfrey, Robley, and Kurtz, Edwin B. (1931). *Life characteristics of physical property (Bulletin 103).* Ames, IA, Iowa Engineering Experiment Station.
Wolf, Franklin K. (1985). Forecasting forces of mortality. Ames, IA, *Proceedings, Iowa State Regulatory Conference.*

APPENDIX I

Iowa Curve Tables

SOURCE: The table values for the L, S, and R type curves are based on calculations using 10 place logarithms and made under the supervision of Robley Winfrey. The values at 5% intervals were first published in *Depreciation of Group Properties,* Iowa Engineering Experiment Station Bulletin 155, published in 1942 and republished 1969 by the Iowa State University Engineering Research Institute. Winfrey's complete tables, with values at 1% intervals, were first published in *The Estimation of Depreciation* by Fitch, Bissinger, and Wolf in 1975. The values for the O type curves are from Winfrey's *Statistical Analysis of Industrial Property Retirements,* Bulletin 125 as revised in April 1967 by Harold A. Cowles.

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Table A

Iowa Type curve L0

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
0	100.00000	100.0000	0.0000	100.00
1	99.87056	99.1198	0.0088	100.12
2	99.68038	98.3080	0.0169	100.31
3	99.44834	97.5362	0.0246	100.54
4	99.18298	96.7958	0.0320	100.80
5	98.88950	96.0816	0.0392	101.08
6	98.57147	95.3900	0.0461	101.39
7	98.23157	94.7183	0.0528	101.72
8	97.87191	94.0646	0.0594	102.06
9	97.49419	93.4271	0.0657	102.43
10	97.09985	92.8045	0.0720	102.80
11	96.69011	92.1956	0.0780	103.20
12	96.26604	91.5996	0.0840	103.60
13	95.82657	91.0173	0.0898	104.02
14	95.37855	90.4425	0.0956	104.44
15	94.91874	89.8782	0.1012	104.88
16	94.44381	89.3277	0.1067	105.33
17	93.96040	88.7847	0.1122	105.78
18	93.46710	88.2506	0.1175	106.25
19	92.96443	87.7251	0.1227	106.73
20	92.45291	87.2077	0.1279	107.21
21	91.93301	86.6981	0.1330	107.70
22	91.40618	86.1949	0.1381	108.19
23	90.86982	85.7007	0.1430	108.70
24	90.32735	85.2124	0.1479	109.21
25	89.77813	84.7306	0.1527	109.73
26	89.22254	84.2551	0.1574	110.26
27	88.66092	83.7857	0.1621	110.79
28	88.09361	83.3220	0.1668	111.32
29	87.52092	82.8639	0.1714	111.86
30	86.94316	82.4113	0.1759	112.41
31	86.36064	81.9638	0.1804	112.96
32	85.77364	81.5213	0.1848	113.52
33	85.18246	81.0836	0.1892	114.08
34	84.58736	80.6505	0.1935	114.65
35	83.98860	80.2219	0.1978	115.22
36	83.38647	79.7976	0.2020	115.80
37	82.78120	79.3774	0.2062	116.38
38	82.17306	78.9611	0.2104	116.96
39	81.56228	78.5487	0.2145	117.55
40	80.94912	78.1399	0.2186	118.14
41	80.33381	77.7346	0.2227	118.73
42	79.71659	77.3326	0.2267	119.33
43	79.09769	76.9337	0.2307	119.93
44	78.47734	76.5379	0.2346	120.54
45	77.85578	76.1450	0.2386	121.14
46	77.23323	75.7547	0.2425	121.75
47	76.60992	75.3670	0.2463	122.37
48	75.98606	74.9817	0.2502	122.98
49	75.36190	74.5986	0.2540	123.60
50	74.73764	74.2175	0.2578	124.22
51	74.11342	73.8384	0.2616	124.84
52	73.48930	73.4612	0.2654	125.46
53	72.86534	73.0860	0.2691	126.09
54	72.24160	72.7127	0.2729	126.71
55	71.61815	72.3413	0.2766	127.34
56	70.99504	71.9719	0.2803	127.97
57	70.37234	71.6043	0.2840	128.60
58	69.75010	71.2386	0.2876	129.24
59	69.12840	70.8748	0.2913	129.87
60	68.50728	70.5128	0.2949	130.51
61	67.88682	70.1527	0.2985	131.15
62	67.26707	69.7945	0.3021	131.79
63	66.64809	69.4380	0.3056	132.44
64	66.02994	69.0834	0.3092	133.08
65	65.41269	68.7306	0.3127	133.73
66	64.79639	68.3795	0.3162	134.38
67	64.18110	68.0303	0.3197	135.03
68	63.56689	67.6828	0.3232	135.68
69	62.95380	67.3371	0.3266	136.34
70	62.34191	66.9931	0.3301	136.99
71	61.73127	66.6508	0.3335	137.65
72	61.12193	66.3103	0.3369	138.31
73	60.51396	65.9715	0.3403	138.97
74	59.90741	65.6343	0.3437	139.63
75	59.30233	65.2989	0.3470	140.30
76	58.69880	64.9652	0.3503	140.97
77	58.09686	64.6331	0.3537	141.63
78	57.49656	64.3027	0.3570	142.30
79	56.89797	63.9739	0.3603	142.97
80	56.30114	63.6468	0.3635	143.65
81	55.70613	63.3213	0.3668	144.32
82	55.11298	62.9974	0.3700	145.00
83	54.52176	62.6751	0.3732	145.68
84	53.93250	62.3544	0.3765	146.35
85	53.34528	62.0353	0.3796	147.04
86	52.76014	61.7178	0.3828	147.72
87	52.17713	61.4018	0.3860	148.40
88	51.59631	61.0874	0.3891	149.09
89	51.01772	60.7745	0.3923	149.77
90	50.44141	60.4631	0.3954	150.46
91	49.86744	60.1533	0.3985	151.15
92	49.29585	59.8450	0.4016	151.84
93	48.72669	59.5382	0.4046	152.54
94	48.16001	59.2329	0.4077	153.23
95	47.59585	58.9290	0.4107	153.93

Iowa Type curve L0

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
100	44.81457	57.4317	0.4257	157.43
101	44.26652	57.1366	0.4286	158.14
102	43.72131	56.8428	0.4316	158.84
103	43.17898	56.5505	0.4345	159.55
104	42.63958	56.2596	0.4374	160.26
105	42.10313	55.9700	0.4403	160.97
106	41.56968	55.6818	0.4432	161.68
107	41.03927	55.3950	0.4460	162.40
108	40.51194	55.1096	0.4489	163.11
109	39.98773	54.8255	0.4517	163.83
110	39.46666	54.5427	0.4546	164.54
111	38.94879	54.2613	0.4574	165.26
112	38.43413	53.9812	0.4602	165.98
113	37.92274	53.7024	0.4630	166.70
114	37.41463	53.4249	0.4658	167.42
115	36.90985	53.1487	0.4685	168.15
116	36.40843	52.8738	0.4713	168.87
117	35.91039	52.6002	0.4740	169.60
118	35.41577	52.3278	0.4767	170.33
119	34.92459	52.0567	0.4794	171.06
120	34.43690	51.7868	0.4821	171.79
121	33.95270	51.5182	0.4848	172.52
122	33.47204	51.2509	0.4875	173.25
123	32.99494	50.9847	0.4902	173.98
124	32.52142	50.7198	0.4928	174.72
125	32.05150	50.4561	0.4954	175.46
126	31.58522	50.1935	0.4981	176.19
127	31.12260	49.9322	0.5007	176.93
128	30.66365	49.6721	0.5033	177.67
129	30.20840	49.4131	0.5059	178.41
130	29.75687	49.1553	0.5084	179.16
131	29.30908	48.8987	0.5110	179.90
132	28.86504	48.6432	0.5136	180.64
133	28.42478	48.3889	0.5161	181.39
134	27.98832	48.1357	0.5186	182.14
135	27.55566	47.8836	0.5212	182.88
136	27.12683	47.6327	0.5237	183.63
137	26.70184	47.3829	0.5262	184.38
138	26.28070	47.1341	0.5287	185.13
139	25.86343	46.8865	0.5311	185.89
140	25.45004	46.6400	0.5336	186.64
141	25.04053	46.3946	0.5361	187.39
142	24.63493	46.1502	0.5385	188.15
143	24.23324	45.9069	0.5409	188.91
144	23.83547	45.6646	0.5434	189.66
145	23.44162	45.4235	0.5458	190.42
146	23.05171	45.1833	0.5482	191.18
147	22.66573	44.9442	0.5506	191.94
148	22.28371	44.7062	0.5529	192.71
149	21.90563	44.4691	0.5553	193.47
150	21.53151	44.2331	0.5577	194.23
151	21.16135	43.9981	0.5600	195.00
152	20.79514	43.7641	0.5624	195.76
153	20.43290	43.5311	0.5647	196.53
154	20.07462	43.2991	0.5670	197.30
155	19.72030	43.0681	0.5693	198.07
156	19.36994	42.8381	0.5716	198.84
157	19.02354	42.6090	0.5739	199.61
158	18.68110	42.3809	0.5762	200.38
159	18.34261	42.1538	0.5785	201.15
160	18.00807	41.9276	0.5807	201.93
161	17.67748	41.7023	0.5830	202.70
162	17.35082	41.4780	0.5852	203.48
163	17.02810	41.2547	0.5875	204.25
164	16.70931	41.0322	0.5897	205.03
165	16.39443	40.8107	0.5919	205.81
166	16.08347	40.5901	0.5941	206.59
167	15.77640	40.3704	0.5963	207.37
168	15.47323	40.1515	0.5985	208.15
169	15.17394	39.9336	0.6007	208.93
170	14.87851	39.7166	0.6028	209.72
171	14.58695	39.5005	0.6050	210.50
172	14.29923	39.2852	0.6071	211.29
173	14.01534	39.0708	0.6093	212.07
174	13.73527	38.8573	0.6114	212.86
175	13.45901	38.6447	0.6136	213.64
176	13.18653	38.4329	0.6157	214.43
177	12.91783	38.2219	0.6178	215.22
178	12.65288	38.0118	0.6199	216.01
179	12.39168	37.8025	0.6220	216.80
180	12.13419	37.5940	0.6241	217.59
181	11.88041	37.3864	0.6261	218.39
182	11.63032	37.1796	0.6282	219.18
183	11.38389	36.9736	0.6303	219.97
184	11.14111	36.7684	0.6323	220.77
185	10.90195	36.5640	0.6344	221.56
186	10.66640	36.3605	0.6364	222.36
187	10.43444	36.1577	0.6384	223.16
188	10.20603	35.9557	0.6404	223.96
189	9.98117	35.7544	0.6425	224.75
190	9.75982	35.5540	0.6445	225.55
191	9.54197	35.3543	0.6465	226.35
192	9.32758	35.1554	0.6484	227.15
193	9.11664	34.9573	0.6504	227.96
194	8.90912	34.7599	0.6524	228.76
195	8.70500	34.5632	0.6544	229.56

Table A continued

Iowa Type curve L0

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
200	7.73441	33.5911	0.6641	233.59
201	7.55011	33.3989	0.6660	234.40
202	7.36902	33.2073	0.6679	235.21
203	7.19111	33.0165	0.6698	236.02
204	7.01636	32.8264	0.6717	236.83
205	6.84472	32.6370	0.6736	237.64
206	6.67618	32.4483	0.6755	238.45
207	6.51071	32.2603	0.6774	239.26
208	6.34827	32.0729	0.6793	240.07
209	6.18884	31.8863	0.6811	240.89
210	6.03239	31.7003	0.6830	241.70
211	5.87888	31.5150	0.6848	242.52
212	5.72828	31.3304	0.6867	243.33
213	5.58057	31.1464	0.6885	244.15
214	5.43571	30.9632	0.6904	244.96
215	5.29368	30.7805	0.6922	245.78
216	5.15443	30.5985	0.6940	246.60
217	5.01795	30.4172	0.6958	247.42
218	4.88419	30.2365	0.6976	248.24
219	4.75313	30.0564	0.6994	249.06
220	4.62473	29.8770	0.7012	249.88
221	4.49897	29.6982	0.7030	250.70
222	4.37580	29.5201	0.7048	251.52
223	4.25521	29.3425	0.7066	252.34
224	4.13714	29.1656	0.7083	253.17
225	4.02159	28.9893	0.7101	253.99
226	3.90850	28.8136	0.7119	254.81
227	3.79785	28.6385	0.7136	255.64
228	3.68961	28.4640	0.7154	256.46
229	3.58374	28.2901	0.7171	257.29
230	3.48021	28.1168	0.7188	258.12
231	3.37898	27.9441	0.7206	258.94
232	3.28004	27.7720	0.7223	259.77
233	3.18333	27.6005	0.7240	260.60
234	3.08884	27.4295	0.7257	261.43
235	2.99653	27.2591	0.7274	262.26
236	2.90636	27.0893	0.7291	263.09
237	2.81830	26.9201	0.7308	263.92
238	2.73233	26.7514	0.7325	264.75
239	2.64840	26.5833	0.7342	265.58
240	2.56649	26.4158	0.7358	266.42
241	2.48657	26.2487	0.7375	267.25
242	2.40860	26.0823	0.7392	268.08
243	2.33255	25.9163	0.7408	268.92
244	2.25838	25.7511	0.7425	269.75
245	2.18608	25.5862	0.7441	270.59
246	2.11560	25.4219	0.7458	271.42
247	2.04692	25.2581	0.7474	272.26
248	1.98000	25.0949	0.7491	273.09
249	1.91481	24.9322	0.7507	273.93
250	1.85133	24.7700	0.7523	274.77
251	1.78951	24.6084	0.7539	275.61
252	1.72934	24.4472	0.7555	276.45
253	1.67078	24.2866	0.7571	277.29
254	1.61380	24.1264	0.7587	278.13
255	1.55837	23.9668	0.7603	278.97
256	1.50446	23.8077	0.7619	279.81
257	1.45204	23.6491	0.7635	280.65
258	1.40109	23.4909	0.7651	281.49
259	1.35157	23.3333	0.7667	282.33
260	1.30345	23.1763	0.7682	283.18
261	1.25671	23.0196	0.7698	284.02
262	1.21132	22.8635	0.7714	284.86
263	1.16726	22.7076	0.7729	285.71
264	1.12448	22.5525	0.7745	286.55
265	1.08297	22.3978	0.7760	287.40
266	1.04270	22.2435	0.7776	288.24
267	1.00364	22.0897	0.7791	289.09
268	0.96577	21.9363	0.7806	289.94
269	0.92905	21.7835	0.7822	290.78
270	0.89347	21.6311	0.7837	291.63
271	0.85900	21.4790	0.7852	292.48
272	0.82560	21.3277	0.7867	293.33
273	0.79327	21.1766	0.7882	294.18
274	0.76197	21.0259	0.7897	295.03
275	0.73167	20.8760	0.7912	295.88
276	0.70236	20.7263	0.7927	296.73
277	0.67402	20.5767	0.7942	297.58
278	0.64660	20.4281	0.7957	298.43
279	0.62010	20.2797	0.7972	299.28
280	0.59449	20.1318	0.7987	300.13
281	0.56975	19.9843	0.8002	300.98
282	0.54586	19.8370	0.8016	301.84
283	0.52279	19.6903	0.8031	302.69
284	0.50052	19.5442	0.8046	303.54
285	0.47904	19.3981	0.8060	304.40
286	0.45831	19.2529	0.8075	305.25
287	0.43833	19.1077	0.8089	306.11
288	0.41906	18.9633	0.8104	306.96
289	0.40050	18.8190	0.8118	307.82
290	0.38262	18.6750	0.8132	308.68
291	0.36540	18.5316	0.8147	309.53
292	0.34882	18.3886	0.8161	310.39
293	0.33286	18.2464	0.8175	311.25
294	0.31752	18.1037	0.8190	312.10
295	0.30276	17.9619	0.8204	312.96

Iowa Type curve L0

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
300	0.23719	17.2586	0.8274	317.26
301	0.22560	17.1196	0.8288	318.12
302	0.21449	16.9804	0.8302	318.98
303	0.20384	16.8415	0.8316	319.84
304	0.19363	16.7032	0.8330	320.70
305	0.18385	16.5651	0.8343	321.57
306	0.17448	16.4278	0.8357	322.43
307	0.16552	16.2901	0.8371	323.29
308	0.15694	16.1533	0.8385	324.15
309	0.14874	16.0163	0.8398	325.02
310	0.14089	15.8808	0.8412	325.88
311	0.13340	15.7444	0.8426	326.74
312	0.12624	15.6090	0.8439	327.61
313	0.11940	15.4745	0.8453	328.47
314	0.11288	15.3395	0.8466	329.34
315	0.10666	15.2049	0.8480	330.20
316	0.10073	15.0705	0.8493	331.07
317	0.09508	14.9364	0.8506	331.94
318	0.08969	14.8039	0.8520	332.80
319	0.08457	14.6699	0.8533	333.67
320	0.07969	14.5376	0.8546	334.54
321	0.07505	14.4055	0.8559	335.41
322	0.07064	14.2736	0.8573	336.27
323	0.06645	14.1421	0.8586	337.14
324	0.06247	14.0113	0.8599	338.01
325	0.05870	13.8791	0.8612	338.88
326	0.05512	13.7480	0.8625	339.75
327	0.05172	13.6189	0.8638	340.62
328	0.04851	13.4870	0.8651	341.49
329	0.04546	13.3583	0.8664	342.36
330	0.04258	13.2280	0.8677	343.23
331	0.03985	13.1000	0.8690	344.10
332	0.03727	12.9722	0.8703	344.97
333	0.03484	12.8421	0.8716	345.84
334	0.03254	12.7145	0.8729	346.71
335	0.03037	12.5873	0.8741	347.59
336	0.02833	12.4577	0.8754	348.46
337	0.02640	12.3318	0.8767	349.33
338	0.02459	12.2027	0.8780	350.20
339	0.02288	12.0774	0.8792	351.08
340	0.02128	11.9479	0.8805	351.95
341	0.01977	11.8222	0.8818	352.82
342	0.01835	11.6984	0.8830	353.70
343	0.01702	11.5735	0.8843	354.57
344	0.01578	11.4436	0.8856	355.44
345	0.01461	11.3200	0.8868	356.32
346	0.01351	11.2010	0.8880	357.20
347	0.01249	11.0749	0.8893	358.07
348	0.01154	10.9454	0.8905	358.95
349	0.01064	10.8290	0.8917	359.83
350	0.00981	10.7029	0.8930	360.70
351	0.00904	10.5720	0.8943	361.57
352	0.00831	10.4567	0.8954	362.46
353	0.00764	10.3299	0.8967	363.33
354	0.00701	10.2133	0.8979	364.21
355	0.00643	10.0895	0.8991	365.09
356	0.00589	9.9687	0.9003	365.97
357	0.00539	9.8470	0.9015	366.85
358	0.00493	9.7192	0.9028	367.72
359	0.00450	9.6001	0.9040	368.60
360	0.00410	9.4879	0.9051	369.49
361	0.00374	9.3531	0.9065	370.35
362	0.00340	9.2384	0.9076	371.24
363	0.00309	9.1151	0.9088	372.12
364	0.00280	9.0073	0.9099	373.01
365	0.00254	8.8782	0.9112	373.88
366	0.00230	8.7524	0.9125	374.75
367	0.00208	8.6253	0.9137	375.63
368	0.00187	8.5377	0.9146	376.54
369	0.00169	8.3938	0.9161	377.39
370	0.00152	8.2767	0.9172	378.28
371	0.00137	8.1282	0.9187	379.13
372	0.00122	8.0661	0.9193	380.07
373	0.00110	7.8915	0.9211	380.89
374	0.00098	7.7965	0.9220	381.80
375	0.00087	7.7191	0.9228	382.72
376	0.00078	7.5521	0.9245	383.55
377	0.00069	7.4719	0.9253	384.47
378	0.00062	7.2591	0.9274	385.26
379	0.00054	7.2604	0.9274	386.26
380	0.00048	7.1055	0.9289	387.11
381	0.00042	7.0492	0.9295	388.05
382	0.00037	6.9342	0.9307	388.93
383	0.00033	6.7141	0.9329	389.71
384	0.00029	6.5713	0.9343	390.57
385	0.00025	6.5427	0.9346	391.54
386	0.00022	6.3668	0.9363	392.37
387	0.00019	6.2931	0.9371	393.29
388	0.00017	5.9747	0.9403	393.97
389	0.00014	6.1479	0.9385	395.15
390	0.00012	6.0893	0.9391	396.09
391	0.00011	5.5975	0.9440	396.60
392	0.00009	5.7304	0.9427	397.73
393	0.00008	5.3842	0.9462	398.38
394	0.00007	5.0820	0.9492	399.08
395	0.00006	4.8458	0.9515	399.85

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Iowa Type curve L0

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
400	0.00002	5.0390	0.9496	405.04
401	0.00002	4.0396	0.9596	405.04
402	0.00002	3.0402	0.9696	405.04
403	0.00001	4.1190	0.9588	407.12
404	0.00001	3.4820	0.9652	407.48
405	0.00001	2.4831	0.9752	407.48
406	0.00001	1.4843	0.9852	407.48
407	0.00001	0.4855	0.9951	407.49
408	0.00000	0.0000	1.0000	408.00

Iowa Type curve L1

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
0	100.00000	100.0000	0.0000	100.00
1	99.94205	99.0636	0.0094	100.06
2	99.87728	98.1275	0.0187	100.13
3	99.80521	97.1980	0.0280	100.20
4	99.72534	96.2755	0.0372	100.28
5	99.63717	95.3602	0.0464	100.36
6	99.54020	94.4526	0.0555	100.45
7	99.43392	93.5531	0.0645	100.55
8	99.31755	92.6621	0.0734	100.66
9	99.19147	91.7792	0.0822	100.78
10	99.05431	90.9055	0.0909	100.91
11	98.90588	90.0413	0.0996	101.04
12	98.74570	89.1865	0.1081	101.19
13	98.57331	88.3416	0.1166	101.34
14	98.38828	87.5068	0.1249	101.51
15	98.19015	86.6824	0.1332	101.68
16	97.97853	85.8685	0.1413	101.87
17	97.75302	85.0655	0.1493	102.07
18	97.51325	84.2734	0.1573	102.27
19	97.25887	83.4925	0.1651	102.49
20	96.98956	82.7230	0.1728	102.72
21	96.70503	81.9649	0.1804	102.96
22	96.40500	81.2184	0.1878	103.22
23	96.08924	80.4837	0.1952	103.48
24	95.75754	79.7607	0.2024	103.76
25	95.40973	79.0497	0.2095	104.05
26	95.04568	78.3505	0.2165	104.35
27	94.66527	77.6634	0.2234	104.66
28	94.26844	76.9882	0.2301	104.99
29	93.85515	76.3250	0.2367	105.33
30	93.42541	75.6738	0.2433	105.67
31	92.97926	75.0345	0.2497	106.03
32	92.51679	74.4071	0.2559	106.41
33	92.03810	73.7915	0.2621	106.79
34	91.54335	73.1876	0.2681	107.19
35	91.03275	72.5953	0.2740	107.60
36	90.50652	72.0145	0.2799	108.01
37	89.96493	71.4450	0.2856	108.44
38	89.40830	70.8867	0.2911	108.89
39	88.83697	70.3394	0.2966	109.34
40	88.25133	69.8028	0.3020	109.80
41	87.65179	69.2768	0.3072	110.28
42	87.03887	68.7612	0.3124	110.76
43	86.41288	68.2557	0.3174	111.26
44	85.77452	67.7599	0.3224	111.76
45	85.12428	67.2737	0.3273	112.27
46	84.46275	66.7967	0.3320	112.80
47	83.79054	66.3285	0.3367	113.33
48	83.10829	65.8689	0.3413	113.87
49	82.41667	65.4175	0.3458	114.42
50	81.71633	64.9739	0.3503	114.97
51	81.00811	64.5375	0.3546	115.54
52	80.29262	64.1082	0.3589	116.11
53	79.57066	63.6853	0.3631	116.69
54	78.84299	63.2685	0.3673	117.27
55	78.11041	62.8572	0.3714	117.86
56	77.37370	62.4509	0.3755	118.45
57	76.63369	62.0491	0.3795	119.05
58	75.89120	61.6513	0.3835	119.65
59	75.14704	61.2569	0.3874	120.26
60	74.40205	60.8652	0.3913	120.87
61	73.65696	60.4758	0.3952	121.48
62	72.91198	60.0887	0.3991	122.09
63	72.16720	59.7036	0.4030	122.70
64	71.42273	59.3207	0.4068	123.32
65	70.67865	58.9400	0.4106	123.94
66	69.93509	58.5613	0.4144	124.56
67	69.19213	58.1848	0.4182	125.18
68	68.44988	57.8103	0.4219	125.81
69	67.70843	57.4379	0.4256	126.44
70	66.96789	57.0675	0.4293	127.07
71	66.22836	56.6991	0.4330	127.70
72	65.48994	56.3328	0.4367	128.33
73	64.75272	55.9685	0.4403	128.97
74	64.01680	55.6061	0.4439	129.61
75	63.28229	55.2457	0.4475	130.25
76	62.54928	54.8873	0.4511	130.89
77	61.81787	54.5308	0.4547	131.53
78	61.08815	54.1762	0.4582	132.18
79	60.36022	53.8235	0.4618	132.82
80	59.63417	53.4727	0.4653	133.47
81	58.91011	53.1238	0.4688	134.12
82	58.18813	52.7768	0.4722	134.78
83	57.46832	52.4315	0.4757	135.43
84	56.75076	52.0882	0.4791	136.09
85	56.03557	51.7466	0.4825	136.75
86	55.32282	51.4068	0.4859	137.41
87	54.61261	51.0688	0.4893	138.07
88	53.90503	50.7326	0.4927	138.73
89	53.20016	50.3982	0.4960	139.40
90	52.49811	50.0655	0.4993	140.07
91	51.79894	49.7345	0.5027	140.73
92	51.10275	49.4052	0.5059	141.41
93	50.40963	49.0777	0.5092	142.08
94	49.71966	48.7518	0.5125	142.75
95	49.03293	48.4276	0.5157	143.43
96	[illegible]	[illegible]	[illegible]	[illegible]

Iowa Type curve L1

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
100	45.65061	46.8312	0.5317	146.83
101	44.98497	46.5167	0.5348	147.52
102	44.32312	46.2039	0.5380	148.20
103	43.66514	45.8926	0.5411	148.89
104	43.01109	45.5828	0.5442	149.58
105	42.36105	45.2747	0.5473	150.27
106	41.71509	44.9680	0.5503	150.97
107	41.07329	44.6628	0.5534	151.66
108	40.43570	44.3592	0.5564	152.36
109	39.80240	44.0570	0.5594	153.06
110	39.17345	43.7564	0.5624	153.76
111	38.54891	43.4572	0.5654	154.46
112	37.92886	43.1594	0.5684	155.16
113	37.31334	42.8631	0.5714	155.86
114	36.70243	42.5683	0.5743	156.57
115	36.09618	42.2748	0.5773	157.27
116	35.49464	41.9828	0.5802	157.98
117	34.89788	41.6922	0.5831	158.69
118	34.30595	41.4029	0.5860	159.40
119	33.71890	41.1150	0.5888	160.12
120	33.13678	40.8285	0.5917	160.83
121	32.55964	40.5434	0.5946	161.54
122	31.98754	40.2596	0.5974	162.26
123	31.42051	39.9771	0.6002	162.98
124	30.85860	39.6959	0.6030	163.70
125	30.30187	39.4161	0.6058	164.42
126	29.75034	39.1375	0.6086	165.14
127	29.20406	38.8603	0.6114	165.86
128	28.66307	38.5843	0.6142	166.58
129	28.12741	38.3095	0.6169	167.31
130	27.59711	38.0361	0.6196	168.04
131	27.07221	37.7639	0.6224	168.76
132	26.55274	37.4929	0.6251	169.49
133	26.03873	37.2231	0.6278	170.22
134	25.53021	36.9546	0.6305	170.95
135	25.02721	36.6873	0.6331	171.69
136	24.52975	36.4211	0.6358	172.42
137	24.03787	36.1562	0.6384	173.16
138	23.55157	35.8924	0.6411	173.89
139	23.07089	35.6298	0.6437	174.63
140	22.59584	35.3684	0.6463	175.37
141	22.12644	35.1081	0.6489	176.11
142	21.66271	34.8489	0.6515	176.85
143	21.20467	34.5909	0.6541	177.59
144	20.75232	34.3340	0.6567	178.33
145	20.30567	34.0782	0.6592	179.08
146	19.86475	33.8235	0.6618	179.82
147	19.42955	33.5700	0.6643	180.57
148	19.00008	33.3175	0.6668	181.32
149	18.57635	33.0660	0.6693	182.07
150	18.15836	32.8157	0.6718	182.82
151	17.74612	32.5664	0.6743	183.57
152	17.33962	32.3181	0.6768	184.32
153	16.93886	32.0709	0.6793	185.07
154	16.54383	31.8247	0.6818	185.82
155	16.15455	31.5796	0.6842	186.58
156	15.77099	31.3355	0.6866	187.34
157	15.39315	31.0923	0.6891	188.09
158	15.02103	30.8502	0.6915	188.85
159	14.65461	30.6091	0.6939	189.61
160	14.29388	30.3689	0.6963	190.37
161	13.93882	30.1298	0.6987	191.13
162	13.58943	29.8916	0.7011	191.89
163	13.24568	29.6543	0.7035	192.65
164	12.90755	29.4181	0.7058	193.42
165	12.57504	29.1827	0.7082	194.18
166	12.24811	28.9483	0.7105	194.95
167	11.92675	28.7149	0.7129	195.71
168	11.61092	28.4823	0.7152	196.48
169	11.30062	28.2507	0.7175	197.25
170	10.99580	28.0200	0.7198	198.02
171	10.69644	27.7902	0.7221	198.79
172	10.40252	27.5613	0.7244	199.56
173	10.11400	27.3332	0.7267	200.33
174	9.83085	27.1061	0.7289	201.11
175	9.55304	26.8798	0.7312	201.88
176	9.28054	26.6544	0.7335	202.65
177	9.01330	26.4298	0.7357	203.43
178	8.75130	26.2061	0.7379	204.21
179	8.49450	25.9833	0.7402	204.98
180	8.24285	25.7613	0.7424	205.76
181	7.99632	25.5401	0.7446	206.54
182	7.75486	25.3197	0.7468	207.32
183	7.51844	25.1002	0.7490	208.10
184	7.28701	24.8815	0.7512	208.88
185	7.06053	24.6636	0.7534	209.66
186	6.83895	24.4465	0.7555	210.45
187	6.62223	24.2301	0.7577	211.23
188	6.41032	24.0146	0.7599	212.01
189	6.20316	23.7999	0.7620	212.80
190	6.00073	23.5859	0.7641	213.59
191	5.80295	23.3727	0.7663	214.37
192	5.60980	23.1603	0.7684	215.16
193	5.42120	22.9486	0.7705	215.95
194	5.23711	22.7377	0.7726	216.74
195	5.05748	22.5275	0.7747	217.53
196	4.88226	22.3181	0.7768	218.32

Iowa Type curve L1

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
200	4.22433	21.4876	0.7851	221.49
201	4.07030	21.2818	0.7872	222.28
202	3.92035	21.0767	0.7892	223.08
203	3.77442	20.8722	0.7913	223.87
204	3.63244	20.6685	0.7933	224.67
205	3.49436	20.4655	0.7953	225.47
206	3.36011	20.2632	0.7974	226.26
207	3.22965	20.0615	0.7994	227.06
208	3.10291	19.8605	0.8014	227.86
209	2.97983	19.6602	0.8034	228.66
210	2.86035	19.4605	0.8054	229.46
211	2.74440	19.2616	0.8074	230.26
212	2.63194	19.0633	0.8094	231.06
213	2.52290	18.8656	0.8113	231.87
214	2.41722	18.6685	0.8133	232.67
215	2.31483	18.4722	0.8153	233.47
216	2.21568	18.2764	0.8172	234.28
217	2.11971	18.0812	0.8192	235.08
218	2.02685	17.8867	0.8211	235.89
219	1.93704	17.6928	0.8231	236.69
220	1.85022	17.4996	0.8250	237.50
221	1.76634	17.3069	0.8269	238.31
222	1.68532	17.1148	0.8289	239.11
223	1.60711	16.9234	0.8308	239.92
224	1.53165	16.7325	0.8327	240.73
225	1.45888	16.5422	0.8346	241.54
226	1.38873	16.3526	0.8365	242.35
227	1.32115	16.1635	0.8384	243.16
228	1.25607	15.9750	0.8402	243.98
229	1.19344	15.7871	0.8421	244.79
230	1.13320	15.5998	0.8440	245.60
231	1.07529	15.4130	0.8459	246.41
232	1.01965	15.2268	0.8477	247.23
233	0.96622	15.0411	0.8496	248.04
234	0.91495	14.8560	0.8514	248.86
235	0.86577	14.6714	0.8533	249.67
236	0.81864	14.4873	0.8551	250.49
237	0.77348	14.3040	0.8570	251.30
238	0.73026	14.1209	0.8588	252.12
239	0.68891	13.9385	0.8606	252.94
240	0.64937	13.7568	0.8624	253.76
241	0.61160	13.5755	0.8642	254.58
242	0.57554	13.3947	0.8661	255.39
243	0.54114	13.2144	0.8679	256.21
244	0.50835	13.0345	0.8697	257.03
245	0.47711	12.8552	0.8714	257.86
246	0.44737	12.6766	0.8732	258.68
247	0.41909	12.4983	0.8750	259.50
248	0.39221	12.3206	0.8768	260.32
249	0.36669	12.1432	0.8786	261.14
250	0.34247	11.9666	0.8803	261.97
251	0.31952	11.7902	0.8821	262.79
252	0.29778	11.6145	0.8839	263.61
253	0.27721	11.4393	0.8856	264.44
254	0.25777	11.2642	0.8874	265.26
255	0.23940	11.0902	0.8891	266.09
256	0.22208	10.9162	0.8908	266.92
257	0.20575	10.7429	0.8926	267.74
258	0.19038	10.5698	0.8943	268.57
259	0.17592	10.3975	0.8960	269.40
260	0.16234	10.2255	0.8977	270.23
261	0.14960	10.0537	0.8995	271.05
262	0.13765	9.8831	0.9012	271.88
263	0.12647	9.7125	0.9029	272.71
264	0.11602	9.5423	0.9046	273.54
265	0.10627	9.3719	0.9063	274.37
266	0.09717	9.2028	0.9080	275.20
267	0.08870	9.0338	0.9097	276.03
268	0.08082	8.8659	0.9113	276.87
269	0.07351	8.6978	0.9130	277.70
270	0.06673	8.5307	0.9147	278.53
271	0.06046	8.3635	0.9164	279.36
272	0.05467	8.1964	0.9180	280.20
273	0.04932	8.0312	0.9197	281.03
274	0.04441	7.8639	0.9214	281.86
275	0.03989	7.6983	0.9230	282.70
276	0.03574	7.5341	0.9247	283.53
277	0.03195	7.3685	0.9263	284.37
278	0.02848	7.2054	0.9279	285.21
279	0.02532	7.0423	0.9296	286.04
280	0.02245	6.8786	0.9312	286.88
281	0.01985	6.7141	0.9329	287.71
282	0.01749	6.5526	0.9345	288.55
283	0.01536	6.3919	0.9361	289.39
284	0.01345	6.2286	0.9377	290.23
285	0.01173	6.0686	0.9393	291.07
286	0.01019	5.9102	0.9409	291.91
287	0.00882	5.7505	0.9425	292.75
288	0.00761	5.5854	0.9441	293.59
289	0.00653	5.4265	0.9457	294.43
290	0.00558	5.2652	0.9473	295.27
291	0.00474	5.1097	0.9489	296.11
292	0.00401	4.9488	0.9505	296.95
293	0.00337	4.7937	0.9521	297.79
294	0.00282	4.6312	0.9537	298.63
295	0.00234	4.4786	0.9552	299.48
296	0.00193	4.3238	0.9568	300.32

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Table A continued

Iowa Type curve L1

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
300	0.00083	3.6806	0.9632	303.68
301	0.00065	3.5614	0.9644	304.56
302	0.00051	3.4018	0.9660	305.40
303	0.00040	3.1998	0.9680	306.20
304	0.00030	3.0998	0.9690	307.10
305	0.00023	2.8910	0.9711	307.89
306	0.00017	2.7350	0.9726	308.74
307	0.00012	2.6663	0.9733	309.67
308	0.00009	2.3885	0.9761	310.39
309	0.00006	2.3328	0.9767	311.33
310	0.00004	2.2494	0.9775	312.25
311	0.00003	1.8328	0.9817	312.83
312	0.00002	1.4994	0.9850	313.50
313	0.00001	1.4994	0.9850	314.50
314	0.00001	0.5006	0.9950	314.50
315	0.00000	0.0000	1.0000	315.00

Iowa Type curve L2

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
0	100.00000	100.0000	0.0000	100.00
1	99.99981	99.0003	0.0100	100.00
2	99.99889	98.0013	0.0200	100.00
3	99.99658	97.0035	0.0300	100.00
4	99.99221	96.0077	0.0399	100.01
5	99.98516	95.0145	0.0499	100.01
6	99.97481	94.0242	0.0598	100.02
7	99.96060	93.0375	0.0696	100.04
8	99.94197	92.0548	0.0795	100.05
9	99.91840	91.0764	0.0892	100.08
10	99.88940	90.1027	0.0990	100.10
11	99.85449	89.1340	0.1087	100.13
12	99.81324	88.1706	0.1183	100.17
13	99.76522	87.2128	0.1279	100.21
14	99.71004	86.2608	0.1374	100.26
15	99.64733	85.3148	0.1469	100.31
16	99.57673	84.3749	0.1563	100.37
17	99.49792	83.4414	0.1656	100.44
18	99.41060	82.5142	0.1749	100.51
19	99.31448	81.5936	0.1841	100.59
20	99.20931	80.6796	0.1932	100.68
21	99.09483	79.7722	0.2023	100.77
22	98.97083	78.8715	0.2113	100.87
23	98.83711	77.9775	0.2202	100.98
24	98.69348	77.0903	0.2291	101.09
25	98.53978	76.2098	0.2379	101.21
26	98.37585	75.3359	0.2466	101.34
27	98.20157	74.4687	0.2553	101.47
28	98.01682	73.6081	0.2639	101.61
29	97.82151	72.7541	0.2725	101.75
30	97.61555	71.9066	0.2809	101.91
31	97.39888	71.0654	0.2893	102.07
32	97.17137	70.2306	0.2977	102.23
33	96.93262	69.4024	0.3060	102.40
34	96.68183	68.5811	0.3142	102.58
35	96.41791	67.7675	0.3223	102.77
36	96.13962	66.9622	0.3304	102.96
37	95.84571	66.1660	0.3383	103.17
38	95.53422	65.3801	0.3462	103.38
39	95.20560	64.6040	0.3540	103.60
40	94.85656	63.8399	0.3616	103.84
41	94.48666	63.0879	0.3691	104.09
42	94.09481	62.3485	0.3765	104.35
43	93.68001	61.6224	0.3838	104.62
44	93.24139	60.9099	0.3909	104.91
45	92.77820	60.2115	0.3979	105.21
46	92.28984	59.5275	0.4047	105.53
47	91.77585	58.8581	0.4114	105.86
48	91.23588	58.2034	0.4180	106.20
49	90.66975	57.5637	0.4244	106.56
50	90.07738	56.9390	0.4306	106.94
51	89.45886	56.3292	0.4367	107.33
52	88.81436	55.7344	0.4427	107.73
53	88.14419	55.1543	0.4485	108.15
54	87.44876	54.5889	0.4541	108.59
55	86.72360	54.0412	0.4596	109.04
56	85.98432	53.5015	0.4650	109.50
57	85.21661	52.9790	0.4702	109.98
58	84.42626	52.4703	0.4753	110.47
59	83.61413	51.9751	0.4802	110.98
60	82.78113	51.4931	0.4851	111.49
61	81.92824	51.0239	0.4898	112.02
62	81.05647	50.5673	0.4943	112.57
63	80.16690	50.1229	0.4988	113.12
64	79.26062	49.6903	0.5031	113.69
65	78.33876	49.2691	0.5073	114.27
66	77.40247	48.8591	0.5114	114.86
67	76.45290	48.4597	0.5154	115.46
68	75.49123	48.0707	0.5193	116.07
69	74.51863	47.6915	0.5231	116.69
70	73.53626	47.3220	0.5268	117.32
71	72.54528	46.9616	0.5304	117.96
72	71.54685	46.6099	0.5339	118.61
73	70.54209	46.2667	0.5373	119.27
74	69.53212	45.9315	0.5407	119.93
75	68.51802	45.6039	0.5440	120.60
76	67.50085	45.2835	0.5472	121.28
77	66.48165	44.9701	0.5503	121.97
78	65.46140	44.6632	0.5534	122.66
79	64.44107	44.3624	0.5564	123.36
80	63.42158	44.0675	0.5593	124.07
81	62.40383	43.7781	0.5622	124.78
82	61.38866	43.4938	0.5651	125.49
83	60.37687	43.2142	0.5679	126.21
84	59.36924	42.9392	0.5706	126.94
85	58.36648	42.6683	0.5733	127.67
86	57.36928	42.4013	0.5760	128.40
87	56.37827	42.1378	0.5786	129.14
88	55.39407	41.8776	0.5812	129.88
89	54.41722	41.6204	0.5838	130.62
90	53.44823	41.3659	0.5863	131.37
91	52.48759	41.1138	0.5889	132.11
92	51.53574	40.8639	0.5914	132.86
93	50.59305	40.6160	0.5938	133.62
94	49.65990	40.3699	0.5963	134.37
95	48.73660	40.1252	0.5987	135.13

Iowa Type curve L2

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
100	44.27685	38.9174	0.6108	138.92
101	43.41757	38.6777	0.6132	139.68
102	42.56950	38.4383	0.6156	140.44
103	41.73271	38.1990	0.6180	141.20
104	40.90725	37.9597	0.6204	141.96
105	40.09316	37.7203	0.6228	142.72
106	39.29046	37.4807	0.6252	143.48
107	38.49913	37.2408	0.6276	144.24
108	37.71916	37.0006	0.6300	145.00
109	36.95049	36.7599	0.6324	145.76
110	36.19309	36.5187	0.6348	146.52
111	35.44688	36.2769	0.6372	147.28
112	34.71179	36.0346	0.6397	148.03
113	33.98772	35.7916	0.6421	148.79
114	33.27458	35.5480	0.6445	149.55
115	32.57226	35.3037	0.6470	150.30
116	31.88064	35.0587	0.6494	151.06
117	31.19961	34.8130	0.6519	151.81
118	30.52904	34.5667	0.6543	152.57
119	29.86880	34.3198	0.6568	153.32
120	29.21877	34.0721	0.6593	154.07
121	28.57881	33.8239	0.6618	154.82
122	27.94878	33.5751	0.6642	155.58
123	27.32855	33.3258	0.6667	156.33
124	26.71799	33.0759	0.6692	157.08
125	26.11696	32.8256	0.6717	157.83
126	25.52533	32.5748	0.6743	158.57
127	24.94297	32.3237	0.6768	159.32
128	24.36975	32.0722	0.6793	160.07
129	23.80555	31.8205	0.6818	160.82
130	23.25024	31.5686	0.6843	161.57
131	22.70370	31.3165	0.6868	162.32
132	22.16582	31.0643	0.6894	163.06
133	21.63648	30.8120	0.6919	163.81
134	21.11558	30.5598	0.6944	164.56
135	20.60301	30.3077	0.6969	165.31
136	20.09867	30.0556	0.6994	166.06
137	19.60247	29.8038	0.7020	166.80
138	19.11431	29.5522	0.7045	167.55
139	18.63410	29.3008	0.7070	168.30
140	18.16176	29.0499	0.7095	169.05
141	17.69721	28.7993	0.7120	169.80
142	17.24037	28.5492	0.7145	170.55
143	16.79117	28.2996	0.7170	171.30
144	16.34953	28.0505	0.7195	172.05
145	15.91540	27.8020	0.7220	172.80
146	15.48870	27.5542	0.7245	173.55
147	15.06937	27.3070	0.7269	174.31
148	14.65737	27.0605	0.7294	175.06
149	14.25262	26.8148	0.7319	175.81
150	13.85508	26.5698	0.7343	176.57
151	13.46470	26.3256	0.7367	177.33
152	13.08142	26.0823	0.7392	178.08
153	12.70519	25.8399	0.7416	178.84
154	12.33597	25.5983	0.7440	179.60
155	11.97372	25.3576	0.7464	180.36
156	11.61838	25.1179	0.7488	181.12
157	11.26991	24.8791	0.7512	181.88
158	10.92827	24.6412	0.7536	182.64
159	10.59341	24.4043	0.7560	183.40
160	10.26528	24.1684	0.7583	184.17
161	9.94385	23.9335	0.7607	184.93
162	9.62907	23.6995	0.7630	185.70
163	9.32089	23.4666	0.7653	186.47
164	9.01926	23.2347	0.7677	187.23
165	8.72414	23.0037	0.7700	188.00
166	8.43549	22.7738	0.7723	188.77
167	8.15324	22.5449	0.7746	189.54
168	7.87736	22.3169	0.7768	190.32
169	7.60779	22.0900	0.7791	191.09
170	7.34447	21.8640	0.7814	191.86
171	7.08736	21.6391	0.7836	192.64
172	6.83640	21.4151	0.7858	193.42
173	6.59153	21.1920	0.7881	194.19
174	6.35270	20.9699	0.7903	194.97
175	6.11983	20.7489	0.7925	195.75
176	5.89288	20.5287	0.7947	196.53
177	5.67178	20.3095	0.7969	197.31
178	5.45647	20.0911	0.7991	198.09
179	5.24687	19.8738	0.8013	198.87
180	5.04292	19.6573	0.8034	199.66
181	4.84456	19.4417	0.8056	200.44
182	4.65171	19.2270	0.8077	201.23
183	4.46429	19.0132	0.8099	202.01
184	4.28225	18.8002	0.8120	202.80
185	4.10549	18.5881	0.8141	203.59
186	3.93396	18.3768	0.8162	204.38
187	3.76756	18.1663	0.8183	205.17
188	3.60622	17.9567	0.8204	205.96
189	3.44987	17.7478	0.8225	206.75
190	3.29841	17.5398	0.8246	207.54
191	3.15178	17.3326	0.8267	208.33
192	3.00988	17.1261	0.8287	209.13
193	2.87264	16.9205	0.8308	209.92
194	2.73998	16.7155	0.8328	210.72
195	2.61180	16.5113	0.8349	211.51
196	2.48802	16.3079	0.8369	212.31

Iowa Type curve L2

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
200	2.03522	15.5013	0.8450	215.50
201	1.93217	15.3014	0.8470	216.30
202	1.83299	15.1022	0.8490	217.10
203	1.73762	14.9037	0.8510	217.90
204	1.64596	14.7058	0.8529	218.71
205	1.55792	14.5086	0.8549	219.51
206	1.47341	14.3121	0.8569	220.31
207	1.39235	14.1162	0.8588	221.12
208	1.31465	13.9209	0.8608	221.92
209	1.24023	13.7263	0.8627	222.73
210	1.16899	13.5323	0.8647	223.53
211	1.10086	13.3388	0.8666	224.34
212	1.03574	13.1460	0.8685	225.15
213	0.97355	12.9539	0.8705	225.95
214	0.91421	12.7622	0.8724	226.76
215	0.85763	12.5712	0.8743	227.57
216	0.80372	12.3809	0.8762	228.38
217	0.75242	12.1909	0.8781	229.19
218	0.70363	12.0016	0.8800	230.00
219	0.65727	11.8128	0.8819	230.81
220	0.61326	11.6247	0.8838	231.62
221	0.57152	11.4372	0.8856	232.44
222	0.53198	11.2501	0.8875	233.25
223	0.49456	11.0635	0.8894	234.06
224	0.45918	10.8774	0.8912	234.88
225	0.42576	10.6920	0.8931	235.69
226	0.39424	10.5068	0.8949	236.51
227	0.36453	10.3224	0.8968	237.32
228	0.33657	10.1384	0.8986	238.14
229	0.31028	9.9550	0.9004	238.96
230	0.28560	9.7721	0.9023	239.77
231	0.26246	9.5896	0.9041	240.59
232	0.24079	9.4076	0.9059	241.41
233	0.22053	9.2259	0.9077	242.23
234	0.20161	9.0448	0.9096	243.04
235	0.18397	8.8641	0.9114	243.86
236	0.16754	8.6844	0.9132	244.68
237	0.15228	8.5045	0.9150	245.50
238	0.13812	8.3251	0.9167	246.33
239	0.12500	8.1465	0.9185	247.15
240	0.11287	7.9682	0.9203	247.97
241	0.10168	7.7901	0.9221	248.79
242	0.09137	7.6127	0.9239	249.61
243	0.08189	7.4361	0.9256	250.44
244	0.07320	7.2596	0.9274	251.26
245	0.06525	7.0831	0.9292	252.08
246	0.05799	6.9073	0.9309	252.91
247	0.05138	6.7316	0.9327	253.73
248	0.04537	6.5571	0.9344	254.56
249	0.03993	6.3823	0.9362	255.38
250	0.03501	6.2089	0.9379	256.21
251	0.03058	6.0360	0.9396	257.04
252	0.02661	5.8619	0.9414	257.86
253	0.02305	5.6900	0.9431	258.69
254	0.01988	5.5176	0.9448	259.52
255	0.01707	5.3436	0.9466	260.34
256	0.01457	5.1747	0.9483	261.17
257	0.01238	5.0016	0.9500	262.00
258	0.01045	4.8330	0.9517	262.83
259	0.00877	4.6630	0.9534	263.66
260	0.00731	4.4945	0.9551	264.49
261	0.00605	4.3264	0.9567	265.33
262	0.00497	4.1579	0.9584	266.16
263	0.00405	3.9888	0.9601	266.99
264	0.00327	3.8211	0.9618	267.82
265	0.00261	3.6609	0.9634	268.66
266	0.00207	3.4854	0.9651	269.49
267	0.00162	3.3147	0.9669	270.31
268	0.00125	3.1479	0.9685	271.15
269	0.00095	2.9841	0.9702	271.98
270	0.00071	2.8238	0.9718	272.82
271	0.00052	2.6728	0.9733	273.67
272	0.00038	2.4734	0.9753	274.47
273	0.00026	2.3842	0.9762	275.38
274	0.00018	2.2217	0.9778	276.22
275	0.00012	2.0826	0.9792	277.08
276	0.00008	1.8740	0.9813	277.87
277	0.00005	1.6985	0.9830	278.70
278	0.00003	1.4979	0.9850	279.50
279	0.00002	0.9971	0.9900	280.00
280	0.00001	0.4948	0.9951	280.49
281	0.00000	0.0000	1.0000	281.00

Table A continued

Iowa Type curve L3

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
0	100.00000	100.0000	0.0000	100.00
1	100.00000	98.9984	0.0100	100.00
2	100.00000	97.9984	0.0200	100.00
3	100.00000	96.9984	0.0300	100.00
4	100.00000	95.9984	0.0400	100.00
5	100.00000	94.9984	0.0500	100.00
6	100.00000	93.9984	0.0600	100.00
7	100.00000	92.9984	0.0700	100.00
8	100.00000	91.9984	0.0800	100.00
9	99.99994	90.9985	0.0900	100.00
10	99.99965	89.9988	0.1000	100.00
11	99.99892	88.9994	0.1100	100.00
12	99.99747	88.0007	0.1200	100.00
13	99.99501	87.0028	0.1300	100.00
14	99.99120	86.0061	0.1399	100.01
15	99.98571	85.0108	0.1499	100.01
16	99.97817	84.0172	0.1598	100.02
17	99.96820	83.0255	0.1697	100.03
18	99.95545	82.0361	0.1796	100.04
19	99.93952	81.0491	0.1895	100.05
20	99.92003	80.0648	0.1994	100.06
21	99.89660	79.0834	0.2092	100.08
22	99.86885	78.1053	0.2189	100.11
23	99.83641	77.1305	0.2287	100.13
24	99.79891	76.1593	0.2384	100.16
25	99.75597	75.1919	0.2481	100.19
26	99.70725	74.2283	0.2577	100.23
27	99.65238	73.2689	0.2673	100.27
28	99.59104	72.3138	0.2769	100.31
29	99.52289	71.3629	0.2864	100.36
30	99.44761	70.4166	0.2958	100.42
31	99.36486	69.4748	0.3053	100.47
32	99.27435	68.5377	0.3146	100.54
33	99.17576	67.6053	0.3239	100.61
34	99.06876	66.6778	0.3332	100.68
35	98.95305	65.7552	0.3424	100.76
36	98.82826	64.8376	0.3516	100.84
37	98.69405	63.9251	0.3607	100.93
38	98.55001	63.0178	0.3698	101.02
39	98.39571	62.1158	0.3788	101.12
40	98.23067	61.2193	0.3878	101.22
41	98.05433	60.3285	0.3967	101.33
42	97.86608	59.4436	0.4056	101.44
43	97.66522	58.5648	0.4144	101.56
44	97.45097	57.6925	0.4231	101.69
45	97.22245	56.8269	0.4317	101.83
46	96.97867	55.9685	0.4403	101.97
47	96.71856	55.1177	0.4488	102.12
48	96.44093	54.2749	0.4573	102.27
49	96.14450	53.4407	0.4656	102.44
50	95.82788	52.6156	0.4738	102.62
51	95.48963	51.8003	0.4820	102.80
52	95.12819	50.9952	0.4900	103.00
53	94.74196	50.2010	0.4980	103.20
54	94.32930	49.4184	0.5058	103.42
55	93.88854	48.6481	0.5135	103.65
56	93.41800	47.8906	0.5211	103.89
57	92.91602	47.1466	0.5285	104.15
58	92.38099	46.4168	0.5358	104.42
59	91.81137	45.7017	0.5430	104.70
60	91.20570	45.0018	0.5500	105.00
61	90.56264	44.3178	0.5568	105.32
62	89.88101	43.6501	0.5635	105.65
63	89.15978	42.9992	0.5700	106.00
64	88.39810	42.3654	0.5763	106.37
65	87.59534	41.7491	0.5825	106.75
66	86.75108	41.1505	0.5885	107.15
67	85.86515	40.5699	0.5943	107.57
68	84.93760	40.0075	0.5999	108.01
69	83.96877	39.4633	0.6054	108.46
70	82.95922	38.9375	0.6106	108.94
71	81.90980	38.4299	0.6157	109.43
72	80.82158	37.9406	0.6206	109.94
73	79.69591	37.4695	0.6253	110.47
74	78.53435	37.0163	0.6298	111.02
75	77.33872	36.5808	0.6342	111.58
76	76.11102	36.1628	0.6384	112.16
77	74.85345	35.7620	0.6424	112.76
78	73.56839	35.3779	0.6462	113.38
79	72.25836	35.0102	0.6499	114.01
80	70.92603	34.6585	0.6534	114.66
81	69.57415	34.3222	0.6568	115.32
82	68.20555	34.0009	0.6600	116.00
83	66.82314	33.6939	0.6631	116.69
84	65.42982	33.4008	0.6660	117.40
85	64.02853	33.1208	0.6688	118.12
86	62.62216	32.8534	0.6715	118.85
87	61.21358	32.5979	0.6740	119.60
88	59.80558	32.3536	0.6765	120.35
89	58.40088	32.1198	0.6788	121.12
90	57.00209	31.8957	0.6810	121.90
91	55.61170	31.6806	0.6832	122.68
92	54.23208	31.4739	0.6853	123.47
93	52.86543	31.2746	0.6873	124.27
94	51.51383	31.0820	0.6892	125.08
95	50.17916	30.8955	0.6910	125.90
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Iowa Type curve L3

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
100	43.81420	30.0256	0.6997	130.03
101	42.61133	29.8590	0.7014	130.86
102	41.43386	29.6934	0.7031	131.69
103	40.28232	29.5279	0.7047	132.53
104	39.15710	29.3620	0.7064	133.36
105	38.05846	29.1952	0.7080	134.20
106	36.98648	29.0269	0.7097	135.03
107	35.94118	28.8566	0.7114	135.86
108	34.92241	28.6838	0.7132	136.68
109	33.92995	28.5082	0.7149	137.51
110	32.96348	28.3293	0.7167	138.33
111	32.02260	28.1470	0.7185	139.15
112	31.10681	27.9610	0.7204	139.96
113	30.21559	27.7709	0.7223	140.77
114	29.34835	27.5768	0.7242	141.58
115	28.50446	27.3784	0.7262	142.38
116	27.68325	27.1757	0.7282	143.18
117	26.88404	26.9688	0.7303	143.97
118	26.10613	26.7575	0.7324	144.76
119	25.34879	26.5420	0.7346	145.54
120	24.61134	26.3223	0.7368	146.32
121	23.89304	26.0986	0.7390	147.10
122	23.19321	25.8710	0.7413	147.87
123	22.51117	25.6397	0.7436	148.64
124	21.84624	25.4048	0.7460	149.40
125	21.19779	25.1667	0.7483	150.17
126	20.56520	24.9254	0.7507	150.93
127	19.94787	24.6813	0.7532	151.68
128	19.34526	24.4346	0.7557	152.43
129	18.75682	24.1855	0.7581	153.19
130	18.18205	23.9342	0.7607	153.93
131	17.62050	23.6810	0.7632	154.68
132	17.07172	23.4262	0.7657	155.43
133	16.53530	23.1700	0.7683	156.17
134	16.01087	22.9125	0.7709	156.91
135	15.49809	22.6541	0.7735	157.65
136	14.99663	22.3948	0.7761	158.39
137	14.50621	22.1351	0.7786	159.14
138	14.02657	21.8749	0.7813	159.87
139	13.55745	21.6145	0.7839	160.61
140	13.09814	21.3549	0.7865	161.35
141	12.64995	21.0938	0.7891	162.09
142	12.21119	20.8338	0.7917	162.83
143	11.78221	20.5741	0.7943	163.57
144	11.36286	20.3149	0.7969	164.31
145	10.95300	20.0564	0.7994	165.06
146	10.55253	19.7986	0.8020	165.80
147	10.16132	19.5416	0.8046	166.54
148	9.77929	19.2854	0.8071	167.29
149	9.40634	19.0303	0.8097	168.03
150	9.04239	18.7761	0.8122	168.78
151	8.68735	18.5230	0.8148	169.52
152	8.34117	18.2710	0.8173	170.27
153	8.00375	18.0202	0.8198	171.02
154	7.67504	17.7706	0.8223	171.77
155	7.35497	17.5221	0.8248	172.52
156	7.04346	17.2750	0.8273	173.27
157	6.74047	17.0290	0.8297	174.03
158	6.44590	16.7844	0.8322	174.78
159	6.15971	16.5410	0.8346	175.54
160	5.88181	16.2989	0.8370	176.30
161	5.61213	16.0580	0.8394	177.06
162	5.35059	15.8185	0.8418	177.82
163	5.09713	15.5803	0.8442	178.58
164	4.85165	15.3433	0.8466	179.34
165	4.61407	15.1076	0.8489	180.11
166	4.38429	14.8731	0.8513	180.87
167	4.16224	14.6399	0.8536	181.64
168	3.94780	14.4080	0.8559	182.41
169	3.74089	14.1773	0.8582	183.18
170	3.54139	13.9477	0.8605	183.95
171	3.34919	13.7195	0.8628	184.72
172	3.16419	13.4924	0.8651	185.49
173	2.98628	13.2664	0.8673	186.27
174	2.81532	13.0416	0.8696	187.04
175	2.65120	12.8180	0.8718	187.82
176	2.49379	12.5955	0.8740	188.60
177	2.34297	12.3741	0.8763	189.37
178	2.19859	12.1539	0.8785	190.15
179	2.06054	11.9347	0.8807	190.93
180	1.92866	11.7166	0.8828	191.72
181	1.80282	11.4995	0.8850	192.50
182	1.68287	11.2835	0.8872	193.28
183	1.56867	11.0686	0.8893	194.07
184	1.46008	10.8546	0.8915	194.85
185	1.35694	10.6416	0.8936	195.64
186	1.25911	10.4296	0.8957	196.43
187	1.16643	10.2186	0.8978	197.22
188	1.07874	10.0086	0.8999	198.01
189	0.99591	9.7994	0.9020	198.80
190	0.91776	9.5913	0.9041	199.59
191	0.84415	9.3841	0.9062	200.38
192	0.77492	9.1778	0.9082	201.18
193	0.70992	8.9723	0.9103	201.97
194	0.64899	8.7677	0.9123	202.77
195	0.59197	8.5641	0.9144	203.56
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Table A continued

Iowa Type curve L3

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
200	0.36026	7.5582	0.9244	207.56
201	0.32353	7.3595	0.9264	208.36
202	0.28966	7.1615	0.9284	209.16
203	0.25850	6.9645	0.9304	209.96
204	0.22992	6.7681	0.9323	210.77
205	0.20377	6.5725	0.9343	211.57
206	0.17991	6.3778	0.9362	212.38
207	0.15822	6.1836	0.9382	213.18
208	0.13855	5.9905	0.9401	213.99
209	0.12079	5.7978	0.9420	214.80
210	0.10480	5.6061	0.9439	215.61
211	0.09047	5.4149	0.9459	216.41
212	0.07767	5.2249	0.9478	217.22
213	0.06630	5.0351	0.9496	218.04
214	0.05624	4.8464	0.9515	218.85
215	0.04739	4.6580	0.9534	219.66
216	0.03964	4.4710	0.9553	220.47
217	0.03291	4.2830	0.9572	221.28
218	0.02708	4.0975	0.9590	222.10
219	0.02208	3.9121	0.9609	222.91
220	0.01782	3.7278	0.9627	223.73
221	0.01422	3.5460	0.9646	224.54
222	0.01122	3.3592	0.9664	225.36
223	0.00872	3.1789	0.9682	226.18
224	0.00668	2.9970	0.9700	227.00
225	0.00503	2.8161	0.9718	227.82
226	0.00371	2.6401	0.9736	228.64
227	0.00268	2.4627	0.9754	229.46
228	0.00189	2.2830	0.9772	230.28
229	0.00129	2.1123	0.9789	231.11
230	0.00085	1.9470	0.9805	231.95
231	0.00054	1.7776	0.9822	232.78
232	0.00033	1.5907	0.9841	233.59
233	0.00018	1.4996	0.9850	234.50
234	0.00010	1.2994	0.9870	235.30
235	0.00005	1.0990	0.9890	236.10
236	0.00002	0.9977	0.9900	237.00
237	0.00001	0.4959	0.9950	237.50
238	0.00000	0.0000	1.0000	238.00

Iowa Type curve L4

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
0	100.00000	100.0000	0.0000	100.00
1	100.00000	99.0000	0.0100	100.00
2	100.00000	98.0000	0.0200	100.00
3	100.00000	97.0000	0.0300	100.00
4	100.00000	96.0000	0.0400	100.00
5	100.00000	95.0000	0.0500	100.00
6	100.00000	94.0000	0.0600	100.00
7	100.00000	93.0000	0.0700	100.00
8	100.00000	92.0000	0.0800	100.00
9	100.00000	91.0000	0.0900	100.00
10	100.00000	90.0000	0.1000	100.00
11	100.00000	89.0000	0.1100	100.00
12	100.00000	88.0000	0.1200	100.00
13	100.00000	87.0000	0.1300	100.00
14	100.00000	86.0000	0.1400	100.00
15	100.00000	85.0000	0.1500	100.00
16	100.00000	84.0000	0.1600	100.00
17	100.00000	83.0000	0.1700	100.00
18	100.00000	82.0000	0.1800	100.00
19	100.00000	81.0000	0.1900	100.00
20	100.00000	80.0000	0.2000	100.00
21	100.00000	79.0000	0.2100	100.00
22	100.00000	78.0000	0.2200	100.00
23	100.00000	77.0000	0.2300	100.00
24	99.99999	76.0000	0.2400	100.00
25	99.99995	75.0000	0.2500	100.00
26	99.99979	74.0002	0.2600	100.00
27	99.99940	73.0004	0.2700	100.00
28	99.99852	72.0010	0.2800	100.00
29	99.99721	71.0020	0.2900	100.00
30	99.99491	70.0036	0.3000	100.00
31	99.99136	69.0061	0.3099	100.01
32	99.98616	68.0097	0.3199	100.01
33	99.97887	67.0146	0.3299	100.01
34	99.96897	66.0212	0.3398	100.02
35	99.95590	65.0297	0.3497	100.03
36	99.93905	64.0406	0.3596	100.04
37	99.91775	63.0542	0.3695	100.05
38	99.89130	62.0707	0.3793	100.07
39	99.85898	61.0907	0.3891	100.09
40	99.81999	60.1143	0.3989	100.11
41	99.77353	59.1421	0.4086	100.14
42	99.71878	58.1743	0.4183	100.17
43	99.65489	57.2113	0.4279	100.21
44	99.58099	56.2533	0.4375	100.25
45	99.49620	55.3009	0.4470	100.30
46	99.39964	54.3541	0.4565	100.35
47	99.29042	53.4133	0.4659	100.41
48	99.16768	52.4788	0.4752	100.48
49	99.03052	51.5508	0.4845	100.55
50	98.87810	50.6295	0.4937	100.63
51	98.70957	49.7151	0.5028	100.72
52	98.52410	48.8077	0.5119	100.81
53	98.32089	47.9076	0.5209	100.91
54	98.09917	47.0147	0.5299	101.01
55	97.85817	46.1293	0.5387	101.13
56	97.59718	45.2513	0.5475	101.25
57	97.31548	44.3809	0.5562	101.38
58	97.01238	43.5180	0.5648	101.52
59	96.68719	42.6626	0.5734	101.66
60	96.33923	41.8149	0.5819	101.81
61	95.96778	40.9748	0.5903	101.97
62	95.57209	40.1424	0.5986	102.14
63	95.15133	39.3177	0.6068	102.32
64	94.70457	38.5008	0.6150	102.50
65	94.23074	37.6919	0.6231	102.69
66	93.72862	36.8912	0.6311	102.89
67	93.19676	36.0988	0.6390	103.10
68	92.63340	35.3153	0.6468	103.32
69	92.03658	34.5411	0.6546	103.54
70	91.40395	33.7767	0.6622	103.78
71	90.73280	33.0228	0.6698	104.02
72	90.02010	32.2803	0.6772	104.28
73	89.26241	31.5501	0.6845	104.55
74	88.45601	30.8332	0.6917	104.83
75	87.59685	30.1307	0.6987	105.13
76	86.68071	29.4438	0.7056	105.44
77	85.70325	28.7740	0.7123	105.77
78	84.66015	28.1223	0.7188	106.12
79	83.54726	27.4903	0.7251	106.49
80	82.36077	26.8791	0.7312	106.88
81	81.09741	26.2900	0.7371	107.29
82	79.75459	25.7242	0.7428	107.72
83	78.33065	25.1828	0.7482	108.18
84	76.82500	24.6665	0.7533	108.67
85	75.23825	24.1762	0.7582	109.18
86	73.57239	23.7123	0.7629	109.71
87	71.83081	23.2751	0.7672	110.28
88	70.01839	22.8646	0.7714	110.86
89	68.14144	22.4806	0.7752	111.48
90	66.20766	22.1226	0.7788	112.12
91	64.22597	21.7898	0.7821	112.79
92	62.20638	21.4810	0.7852	113.48
93	60.15975	21.1948	0.7881	114.19
94	58.09754	20.9294	0.7907	114.93
95	56.03106	20.6828	0.7932	115.68
96	53.96924	20.4539	0.7955	116.45

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Iowa Type curve L4

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
100	45.92922	19.6873	0.8031	119.69
101	44.00648	19.5257	0.8047	120.53
102	42.13058	19.3728	0.8063	121.37
103	40.30620	19.2270	0.8077	122.23
104	38.53722	19.0867	0.8091	123.09
105	36.82670	18.9500	0.8105	123.95
106	35.17587	18.8153	0.8118	124.82
107	33.58913	18.6811	0.8132	125.68
108	32.06415	18.5458	0.8145	126.55
109	30.60191	18.4080	0.8159	127.41
110	29.20175	18.2667	0.8173	128.27
111	27.86244	18.1207	0.8188	129.12
112	26.58234	17.9693	0.8203	129.97
113	25.35938	17.8117	0.8219	130.81
114	24.19124	17.6477	0.8235	131.65
115	23.07537	17.4769	0.8252	132.48
116	22.00912	17.2993	0.8270	133.30
117	20.98977	17.1152	0.8288	134.12
118	20.01462	16.9247	0.8308	134.92
119	19.08103	16.7283	0.8327	135.73
120	18.18649	16.5265	0.8347	136.53
121	17.32859	16.3200	0.8368	137.32
122	16.50514	16.1093	0.8389	138.11
123	15.71409	15.8950	0.8410	138.90
124	14.95359	15.6780	0.8432	139.68
125	14.22200	15.4587	0.8454	140.46
126	13.51785	15.2379	0.8476	141.24
127	12.83984	15.0162	0.8498	142.02
128	12.18685	14.7940	0.8521	142.79
129	11.55789	14.5718	0.8543	143.57
130	10.95212	14.3502	0.8565	144.35
131	10.36880	14.1293	0.8587	145.13
132	9.80734	13.9096	0.8609	145.91
133	9.26707	13.6914	0.8631	146.69
134	8.74754	13.4748	0.8653	147.47
135	8.24831	13.2601	0.8674	148.26
136	7.76897	13.0474	0.8695	149.05
137	7.30912	12.8368	0.8716	149.84
138	6.86841	12.6284	0.8737	150.63
139	6.44648	12.4223	0.8758	151.42
140	6.04296	12.2184	0.8778	152.22
141	5.65750	12.0168	0.8798	153.02
142	5.28973	11.8175	0.8818	153.82
143	4.93928	11.6205	0.8838	154.62
144	4.60575	11.4258	0.8857	155.43
145	4.28876	11.2333	0.8877	156.23
146	3.98790	11.0431	0.8896	157.04
147	3.70273	10.8551	0.8914	157.86
148	3.43284	10.6692	0.8933	158.67
149	3.17776	10.4855	0.8951	159.49
150	2.93706	10.3038	0.8970	160.30
151	2.71026	10.1242	0.8988	161.12
152	2.49689	9.9466	0.9005	161.95
153	2.29648	9.7710	0.9023	162.77
154	2.10855	9.5973	0.9040	163.60
155	1.93260	9.4256	0.9057	164.43
156	1.76816	9.2557	0.9074	165.26
157	1.61474	9.0875	0.9091	166.09
158	1.47185	8.9212	0.9108	166.92
159	1.33901	8.7567	0.9124	167.76
160	1.21574	8.5939	0.9141	168.59
161	1.10158	8.4327	0.9157	169.43
162	0.99604	8.2732	0.9173	170.27
163	0.89868	8.1153	0.9188	171.12
164	0.80905	7.9590	0.9204	171.96
165	0.72670	7.8043	0.9220	172.80
166	0.65120	7.6511	0.9235	173.65
167	0.58215	7.4993	0.9250	174.50
168	0.51912	7.3492	0.9265	175.35
169	0.46174	7.2003	0.9280	176.20
170	0.40962	7.0529	0.9295	177.05
171	0.36239	6.9069	0.9309	177.91
172	0.31971	6.7622	0.9324	178.76
173	0.28123	6.6190	0.9338	179.62
174	0.24665	6.4769	0.9352	180.48
175	0.21565	6.3361	0.9366	181.34
176	0.18794	6.1966	0.9380	182.20
177	0.16324	6.0585	0.9394	183.06
178	0.14130	5.9216	0.9408	183.92
179	0.12188	5.7854	0.9421	184.79
180	0.10473	5.6510	0.9435	185.65
181	0.08965	5.5174	0.9448	186.52
182	0.07643	5.3853	0.9461	187.39
183	0.06489	5.2540	0.9475	188.25
184	0.05486	5.1232	0.9488	189.12
185	0.04616	4.9946	0.9501	189.99
186	0.03866	4.8665	0.9513	190.87
187	0.03222	4.7393	0.9526	191.74
188	0.02671	4.6138	0.9539	192.61
189	0.02203	4.4877	0.9551	193.49
190	0.01806	4.3643	0.9564	194.36
191	0.01472	4.2412	0.9576	195.24
192	0.01192	4.1200	0.9588	196.12
193	0.00959	3.9995	0.9600	197.00
194	0.00767	3.8755	0.9612	197.88
195	0.00608	3.7582	0.9624	198.76

Iowa Type curve L4

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
200	0.00168	3.1725	0.9683	203.17
201	0.00126	3.0634	0.9694	204.06
202	0.00094	2.9360	0.9706	204.94
203	0.00069	2.8187	0.9718	205.82
204	0.00050	2.6998	0.9730	206.70
205	0.00036	2.5553	0.9744	207.56
206	0.00025	2.4596	0.9754	208.46
207	0.00017	2.3818	0.9762	209.38
208	0.00012	2.1659	0.9783	210.17
209	0.00008	1.9990	0.9800	211.00
210	0.00005	1.8985	0.9810	211.90
211	0.00003	1.8312	0.9817	212.83
212	0.00002	1.4971	0.9850	213.50
213	0.00001	1.4948	0.9851	214.49
214	0.00001	0.4959	0.9950	214.50
215	0.00000	0.0000	1.0000	215.00

Iowa Type curve L5

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
0	100.00000	100.0000	0.0000	100.00
1	100.00000	99.0000	0.0100	100.00
2	100.00000	98.0000	0.0200	100.00
3	100.00000	97.0000	0.0300	100.00
4	100.00000	96.0000	0.0400	100.00
5	100.00000	95.0000	0.0500	100.00
6	100.00000	94.0000	0.0600	100.00
7	100.00000	93.0000	0.0700	100.00
8	100.00000	92.0000	0.0800	100.00
9	100.00000	91.0000	0.0900	100.00
10	100.00000	90.0000	0.1000	100.00
11	100.00000	89.0000	0.1100	100.00
12	100.00000	88.0000	0.1200	100.00
13	100.00000	87.0000	0.1300	100.00
14	100.00000	86.0000	0.1400	100.00
15	100.00000	85.0000	0.1500	100.00
16	100.00000	84.0000	0.1600	100.00
17	100.00000	83.0000	0.1700	100.00
18	100.00000	82.0000	0.1800	100.00
19	100.00000	81.0000	0.1900	100.00
20	100.00000	80.0000	0.2000	100.00
21	100.00000	79.0000	0.2100	100.00
22	100.00000	78.0000	0.2200	100.00
23	100.00000	77.0000	0.2300	100.00
24	100.00000	76.0000	0.2400	100.00
25	100.00000	75.0000	0.2500	100.00
26	100.00000	74.0000	0.2600	100.00
27	100.00000	73.0000	0.2700	100.00
28	100.00000	72.0000	0.2800	100.00
29	100.00000	71.0000	0.2900	100.00
30	100.00000	70.0000	0.3000	100.00
31	100.00000	69.0000	0.3100	100.00
32	100.00000	68.0000	0.3200	100.00
33	100.00000	67.0000	0.3300	100.00
34	100.00000	66.0000	0.3400	100.00
35	100.00000	65.0000	0.3500	100.00
36	100.00000	64.0000	0.3600	100.00
37	100.00000	63.0000	0.3700	100.00
38	100.00000	62.0000	0.3800	100.00
39	100.00000	61.0000	0.3900	100.00
40	100.00000	60.0000	0.4000	100.00
41	100.00000	59.0000	0.4100	100.00
42	100.00000	58.0000	0.4200	100.00
43	100.00000	57.0000	0.4300	100.00
44	99.99999	56.0000	0.4400	100.00
45	99.99994	55.0000	0.4500	100.00
46	99.99975	54.0001	0.4600	100.00
47	99.99925	53.0004	0.4700	100.00
48	99.99814	52.0010	0.4800	100.00
49	99.99600	51.0021	0.4900	100.00
50	99.99221	50.0040	0.5000	100.00
51	99.98600	49.0071	0.5099	100.01
52	99.97639	48.0117	0.5199	100.01
53	99.96220	47.0185	0.5298	100.02
54	99.94206	46.0279	0.5397	100.03
55	99.91441	45.0405	0.5496	100.04
56	99.87749	44.0569	0.5594	100.06
57	99.82938	43.0779	0.5692	100.08
58	99.76802	42.1041	0.5790	100.10
59	99.69121	41.1362	0.5886	100.14
60	99.59667	40.1747	0.5983	100.17
61	99.48202	39.2205	0.6078	100.22
62	99.34487	38.2739	0.6173	100.27
63	99.18277	37.3356	0.6266	100.34
64	98.99333	36.4061	0.6359	100.41
65	98.77418	35.4858	0.6451	100.49
66	98.52303	34.5750	0.6543	100.57
67	98.23757	33.6740	0.6633	100.67
68	97.91601	32.7829	0.6722	100.78
69	97.55607	31.9021	0.6810	100.90
70	97.15593	31.0314	0.6897	101.03
71	96.71371	30.1710	0.6983	101.17
72	96.22749	29.3209	0.7068	101.32
73	95.69515	28.4812	0.7152	101.48
74	95.11423	27.6521	0.7235	101.65
75	94.48180	26.8339	0.7317	101.83
76	93.79419	26.0269	0.7397	102.03
77	93.04688	25.2320	0.7477	102.23
78	92.23432	24.4498	0.7555	102.45
79	91.34990	23.6817	0.7632	102.68
80	90.38589	22.9290	0.7707	102.93
81	89.33357	22.1932	0.7781	103.19
82	88.18348	21.4761	0.7852	103.48
83	86.92572	20.7796	0.7922	103.78
84	85.55036	20.1056	0.7989	104.11
85	84.04805	19.4561	0.8054	104.46
86	82.41051	18.8327	0.8117	104.83
87	80.63117	18.2373	0.8176	105.24
88	78.70573	17.6712	0.8233	105.67
89	76.63271	17.1357	0.8286	106.14
90	74.41381	16.6318	0.8337	106.63
91	72.05424	16.1600	0.8384	107.16
92	69.56276	15.7209	0.8428	107.72
93	66.95171	15.3145	0.8469	108.31
94	64.23671	14.9407	0.8506	108.94
95	61.43633	14.5989	0.8540	109.60

Iowa Type curve L5

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
100	46.93011	13.3373	0.8666	113.34
101	44.09070	13.1641	0.8684	114.16
102	41.32246	13.0124	0.8699	115.01
103	38.64318	12.8800	0.8712	115.88
104	36.06787	12.7639	0.8724	116.76
105	33.60849	12.6614	0.8734	117.66
106	31.27390	12.5692	0.8743	118.57
107	29.06982	12.4843	0.8752	119.48
108	26.99903	12.4035	0.8760	120.40
109	25.06161	12.3237	0.8768	121.32
110	23.25529	12.2421	0.8776	122.24
111	21.57580	12.1561	0.8784	123.16
112	20.01734	12.0636	0.8794	124.06
113	18.57295	11.9629	0.8804	124.96
114	17.23495	11.8528	0.8815	125.85
115	15.99530	11.7326	0.8827	126.73
116	14.84589	11.6023	0.8840	127.60
117	13.77886	11.4620	0.8854	128.46
118	12.78675	11.3126	0.8869	129.31
119	11.86267	11.1549	0.8885	130.15
120	11.00040	10.9900	0.8901	130.99
121	10.19442	10.8194	0.8918	131.82
122	9.43990	10.6442	0.8936	132.64
123	8.73269	10.4657	0.8953	133.47
124	8.06922	10.2851	0.8971	134.29
125	7.44648	10.1035	0.8990	135.10
126	6.86191	9.9216	0.9008	135.92
127	6.31332	9.7403	0.9026	136.74
128	5.79883	9.5601	0.9044	137.56
129	5.31678	9.3815	0.9062	138.38
130	4.86569	9.2049	0.9080	139.20
131	4.44419	9.0305	0.9097	140.03
132	4.05101	8.8585	0.9114	140.86
133	3.68493	8.6889	0.9131	141.69
134	3.34475	8.5217	0.9148	142.52
135	3.02930	8.3570	0.9164	143.36
136	2.73741	8.1948	0.9181	144.19
137	2.46795	8.0350	0.9197	145.03
138	2.21976	7.8775	0.9212	145.88
139	1.99168	7.7223	0.9228	146.72
140	1.78262	7.5693	0.9243	147.57
141	1.59146	7.4184	0.9258	148.42
142	1.41710	7.2697	0.9273	149.27
143	1.25848	7.1229	0.9288	150.12
144	1.11456	6.9781	0.9302	150.98
145	0.98432	6.8353	0.9316	151.84
146	0.86680	6.6942	0.9331	152.69
147	0.75106	6.5549	0.9345	153.55
148	0.66616	6.4174	0.9358	154.42
149	0.58128	6.2815	0.9372	155.28
150	0.50557	6.1473	0.9385	156.15
151	0.43826	6.0146	0.9399	157.01
152	0.37861	5.8835	0.9412	157.88
153	0.32592	5.7538	0.9425	158.75
154	0.27953	5.6257	0.9437	159.63
155	0.23884	5.4989	0.9450	160.50
156	0.20328	5.3734	0.9463	161.37
157	0.17231	5.2493	0.9475	162.25
158	0.14545	5.1263	0.9487	163.13
159	0.12224	5.0047	0.9500	164.00
160	0.10227	4.8844	0.9512	164.88
161	0.08516	4.7653	0.9523	165.77
162	0.07057	4.6471	0.9535	166.65
163	0.05818	4.5302	0.9547	167.53
164	0.04771	4.4147	0.9559	168.41
165	0.03891	4.3000	0.9570	169.30
166	0.03155	4.1865	0.9581	170.19
167	0.02543	4.0737	0.9593	171.07
168	0.02037	3.9615	0.9604	171.96
169	0.01621	3.8498	0.9615	172.85
170	0.01280	3.7422	0.9626	173.74
171	0.01004	3.6334	0.9637	174.63
172	0.00782	3.5230	0.9648	175.52
173	0.00604	3.4139	0.9659	176.41
174	0.00462	3.3095	0.9669	177.31
175	0.00350	3.2085	0.9679	178.21
176	0.00263	3.1045	0.9690	179.10
177	0.00196	2.9948	0.9701	179.99
178	0.00144	2.8958	0.9710	180.90
179	0.00105	2.7856	0.9721	181.79
180	0.00075	2.6999	0.9730	182.70
181	0.00053	2.6130	0.9739	183.61
182	0.00037	2.5268	0.9747	184.53
183	0.00026	2.3843	0.9762	185.38
184	0.00017	2.3819	0.9762	186.38
185	0.00012	2.1660	0.9783	187.17
186	0.00008	1.9991	0.9800	188.00
187	0.00005	1.8987	0.9810	188.90
188	0.00003	1.8316	0.9817	189.83
189	0.00002	1.4977	0.9850	190.50
190	0.00001	1.4959	0.9850	191.50
191	0.00001	0.4971	0.9950	191.50
192	0.00000	0.0000	1.0000	192.00

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Table B

Iowa Type curve S0

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
0	100.00000	100.0000	0.0000	100.00
1	99.97910	99.0208	0.0098	100.02
2	99.92921	98.0700	0.0193	100.07
3	99.85617	97.1414	0.0286	100.14
4	99.76252	96.2321	0.0377	100.23
5	99.64988	95.3403	0.0466	100.34
6	99.51946	94.4646	0.0554	100.46
7	99.37223	93.6038	0.0640	100.60
8	99.20898	92.7570	0.0724	100.76
9	99.03038	91.9234	0.0808	100.92
10	98.83704	91.1022	0.0890	101.10
11	98.62946	90.2929	0.0971	101.29
12	98.40814	89.4948	0.1051	101.49
13	98.17349	88.7076	0.1129	101.71
14	97.92592	87.9306	0.1207	101.93
15	97.66579	87.1634	0.1284	102.16
16	97.39345	86.4058	0.1359	102.41
17	97.10922	85.6572	0.1434	102.66
18	96.81340	84.9174	0.1508	102.92
19	96.50628	84.1861	0.1581	103.19
20	96.18813	83.4629	0.1654	103.46
21	95.85922	82.7475	0.1725	103.75
22	95.51979	82.0398	0.1796	104.04
23	95.17008	81.3394	0.1866	104.34
24	94.81032	80.6462	0.1935	104.65
25	94.44074	79.9598	0.2004	104.96
26	94.06155	79.2801	0.2072	105.28
27	93.67296	78.6069	0.2139	105.61
28	93.27517	77.9400	0.2206	105.94
29	92.86837	77.2792	0.2272	106.28
30	92.45276	76.6244	0.2338	106.62
31	92.02851	75.9753	0.2402	106.98
32	91.59582	75.3319	0.2467	107.33
33	91.15485	74.6939	0.2531	107.69
34	90.70578	74.0612	0.2594	108.06
35	90.24878	73.4337	0.2657	108.43
36	89.78402	72.8112	0.2719	108.81
37	89.31165	72.1937	0.2781	109.19
38	88.83183	71.5809	0.2842	109.58
39	88.34472	70.9729	0.2903	109.97
40	87.85047	70.3693	0.2963	110.37
41	87.34924	69.7703	0.3023	110.77
42	86.84117	69.1755	0.3082	111.18
43	86.32640	68.5851	0.3141	111.59
44	85.80509	67.9987	0.3200	112.00
45	85.27737	67.4164	0.3258	112.42
46	84.74338	66.8381	0.3316	112.84
47	84.20327	66.2636	0.3374	113.26
48	83.65716	65.6929	0.3431	113.69
49	83.10519	65.1259	0.3487	114.13
50	82.54749	64.5625	0.3544	114.56
51	81.98420	64.0027	0.3600	115.00
52	81.41545	63.4463	0.3655	115.45
53	80.84136	62.8933	0.3711	115.89
54	80.26206	62.3436	0.3766	116.34
55	79.67768	61.7972	0.3820	116.80
56	79.08834	61.2540	0.3875	117.25
57	78.49416	60.7138	0.3929	117.71
58	77.89528	60.1768	0.3982	118.18
59	77.29180	59.6427	0.4036	118.64
60	76.68386	59.1116	0.4089	119.11
61	76.07156	58.5834	0.4142	119.58
62	75.45503	58.0580	0.4194	120.06
63	74.83439	57.5353	0.4246	120.54
64	74.20976	57.0154	0.4298	121.02
65	73.58124	56.4981	0.4350	121.50
66	72.94896	55.9835	0.4402	121.98
67	72.31303	55.4714	0.4453	122.47
68	71.67356	54.9619	0.4504	122.96
69	71.03067	54.4548	0.4555	123.45
70	70.38447	53.9502	0.4605	123.95
71	69.73508	53.4479	0.4655	124.45
72	69.08260	52.9480	0.4705	124.95
73	68.42714	52.4504	0.4755	125.45
74	67.76882	51.9551	0.4804	125.96
75	67.10774	51.4619	0.4854	126.46
76	66.44402	50.9710	0.4903	126.97
77	65.77777	50.4822	0.4952	127.48
78	65.10909	49.9955	0.5000	128.00
79	64.43809	49.5109	0.5049	128.51
80	63.76487	49.0284	0.5097	129.03
81	63.08956	48.5478	0.5145	129.55
82	62.41225	48.0693	0.5193	130.07
83	61.73305	47.5926	0.5241	130.59
84	61.05207	47.1179	0.5288	131.12
85	60.36941	46.6451	0.5335	131.65
86	59.68518	46.1741	0.5383	132.17
87	58.99949	45.7049	0.5430	132.70
88	58.31244	45.2375	0.5476	133.24
89	57.62413	44.7719	0.5523	133.77
90	56.93468	44.3080	0.5569	134.31
91	56.24418	43.8458	0.5615	134.85
92	55.55274	43.3853	0.5661	135.39
93	54.86047	42.9265	0.5707	135.93
94	54.16747	42.4693	0.5753	136.47
95	53.47385	42.0137	0.5799	137.01
96	52.77971	41.5596	0.5844	137.56

Iowa Type curve S0

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
100	50.00000	39.7589	0.6024	139.76
101	49.30481	39.3125	0.6069	140.31
102	48.60973	38.8674	0.6113	140.87
103	47.91486	38.4238	0.6158	141.42
104	47.22030	37.9817	0.6202	141.98
105	46.52615	37.5409	0.6246	142.54
106	45.83253	37.1014	0.6290	143.10
107	45.13953	36.6634	0.6334	143.66
108	44.44726	36.2266	0.6377	144.23
109	43.75582	35.7912	0.6421	144.79
110	43.06532	35.3570	0.6464	145.36
111	42.37587	34.9241	0.6508	145.92
112	41.68756	34.4925	0.6551	146.49
113	41.00051	34.0621	0.6594	147.06
114	40.31482	33.6330	0.6637	147.63
115	39.63059	33.2050	0.6679	148.21
116	38.94793	32.7783	0.6722	148.78
117	38.26695	32.3527	0.6765	149.35
118	37.58775	31.9282	0.6807	149.93
119	36.91044	31.5050	0.6850	150.50
120	36.23513	31.0828	0.6892	151.08
121	35.56191	30.6618	0.6934	151.66
122	34.89091	30.2418	0.6976	152.24
123	34.22223	29.8229	0.7018	152.82
124	33.55598	29.4051	0.7059	153.41
125	32.89226	28.9884	0.7101	153.99
126	32.23118	28.5727	0.7143	154.57
127	31.57286	28.1581	0.7184	155.16
128	30.91740	27.7444	0.7226	155.74
129	30.26492	27.3318	0.7267	156.33
130	29.61553	26.9201	0.7308	156.92
131	28.96933	26.5095	0.7349	157.51
132	28.32644	26.0998	0.7390	158.10
133	27.68697	25.6910	0.7431	158.69
134	27.05104	25.2832	0.7472	159.28
135	26.41876	24.8764	0.7512	159.88
136	25.79024	24.4704	0.7553	160.47
137	25.16561	24.0654	0.7593	161.07
138	24.54497	23.6613	0.7634	161.66
139	23.92844	23.2580	0.7674	162.26
140	23.31615	22.8557	0.7714	162.86
141	22.70820	22.4542	0.7755	163.45
142	22.10472	22.0536	0.7795	164.05
143	21.50584	21.6538	0.7835	164.65
144	20.91166	21.2548	0.7875	165.25
145	20.32232	20.8567	0.7914	165.86
146	19.73794	20.4594	0.7954	166.46
147	19.15864	20.0629	0.7994	167.06
148	18.58455	19.6672	0.8033	167.67
149	18.01580	19.2723	0.8073	168.27
150	17.45251	18.8782	0.8112	168.88
151	16.89481	18.4849	0.8152	169.48
152	16.34285	18.0923	0.8191	170.09
153	15.79674	17.7005	0.8230	170.70
154	15.25662	17.3094	0.8269	171.31
155	14.72263	16.9191	0.8308	171.92
156	14.19491	16.5295	0.8347	172.53
157	13.67360	16.1406	0.8386	173.14
158	13.15883	15.7525	0.8425	173.75
159	12.65076	15.3651	0.8463	174.37
160	12.14953	14.9783	0.8502	174.98
161	11.65528	14.5923	0.8541	175.59
162	11.16817	14.2069	0.8579	176.21
163	10.68836	13.8223	0.8618	176.82
164	10.21598	13.4383	0.8656	177.44
165	9.75122	13.0549	0.8695	178.05
166	9.29422	12.6723	0.8733	178.67
167	8.84515	12.2902	0.8771	179.29
168	8.40418	11.9089	0.8809	179.91
169	7.97149	11.5282	0.8847	180.53
170	7.54724	11.1481	0.8885	181.15
171	7.13163	10.7686	0.8923	181.77
172	6.72483	10.3898	0.8961	182.39
173	6.32704	10.0116	0.8999	183.01
174	5.93845	9.6340	0.9037	183.63
175	5.55926	9.2570	0.9074	184.26
176	5.18968	8.8806	0.9112	184.88
177	4.82992	8.5048	0.9150	185.50
178	4.48022	8.1297	0.9187	186.13
179	4.14079	7.7551	0.9224	186.76
180	3.81187	7.3811	0.9262	187.38
181	3.49372	7.0077	0.9299	188.01
182	3.18660	6.6349	0.9337	188.63
183	2.89078	6.2627	0.9374	189.26
184	2.60655	5.8911	0.9411	189.89
185	2.33421	5.5201	0.9448	190.52
186	2.07408	5.1497	0.9485	191.15
187	1.82651	4.7800	0.9522	191.78
188	1.59186	4.4109	0.9559	192.41
189	1.37054	4.0424	0.9596	193.04
190	1.16296	3.6747	0.9633	193.67
191	0.96962	3.3077	0.9669	194.31
192	0.79102	2.9417	0.9706	194.94
193	0.62777	2.5767	0.9742	195.58
194	0.48054	2.2129	0.9779	196.21
195	0.35012	1.8510	0.9815	196.85
196	[illegible]	[illegible]	[illegible]	[illegible]

Table B continued

Iowa Type curve S0

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
200	0.00000	0.0000	1.0000	200.00

Iowa Type curve S1

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
0	100.00000	100.0000	0.0000	100.00
1	99.99975	99.0002	0.0100	100.00
2	99.99824	98.0017	0.0200	100.00
3	99.99449	97.0054	0.0299	100.01
4	99.98765	96.0120	0.0399	100.01
5	99.97697	95.0222	0.0498	100.02
6	99.96169	94.0366	0.0596	100.04
7	99.94116	93.0559	0.0694	100.06
8	99.91476	92.0803	0.0792	100.08
9	99.88188	91.1105	0.0889	100.11
10	99.84198	90.1467	0.0985	100.15
11	99.79453	89.1893	0.1081	100.19
12	99.73904	88.2385	0.1176	100.24
13	99.67505	87.2950	0.1271	100.29
14	99.60210	86.3585	0.1364	100.36
15	99.51978	85.4296	0.1457	100.43
16	99.42771	84.5082	0.1549	100.51
17	99.32552	83.5946	0.1641	100.59
18	99.21285	82.6890	0.1731	100.69
19	99.08937	81.7914	0.1821	100.79
20	98.95479	80.9020	0.1910	100.90
21	98.80882	80.0208	0.1998	101.02
22	98.65118	79.1478	0.2085	101.15
23	98.48164	78.2832	0.2172	101.28
24	98.29995	77.4270	0.2257	101.43
25	98.10590	76.5791	0.2342	101.58
26	97.89930	75.7397	0.2426	101.74
27	97.67997	74.9086	0.2509	101.91
28	97.44773	74.0860	0.2591	102.09
29	97.20244	73.2717	0.2673	102.27
30	96.94397	72.4657	0.2753	102.47
31	96.67219	71.6680	0.2833	102.67
32	96.38701	70.8786	0.2912	102.88
33	96.08832	70.0973	0.2990	103.10
34	95.77605	69.3243	0.3068	103.32
35	95.45014	68.5593	0.3144	103.56
36	95.11053	67.8023	0.3220	103.80
37	94.75719	67.0532	0.3295	104.05
38	94.39009	66.3121	0.3369	104.31
39	94.00922	65.5787	0.3442	104.58
40	93.61458	64.8531	0.3515	104.85
41	93.20617	64.1350	0.3586	105.14
42	92.78401	63.4246	0.3658	105.42
43	92.34815	62.7216	0.3728	105.72
44	91.89862	62.0259	0.3797	106.03
45	91.43547	61.3376	0.3866	106.34
46	90.95878	60.6564	0.3934	106.66
47	90.46861	59.9823	0.4002	106.98
48	89.96505	59.3153	0.4068	107.32
49	89.44819	58.6551	0.4134	107.66
50	88.91814	58.0018	0.4200	108.00
51	88.37500	57.3552	0.4264	108.36
52	87.81891	56.7152	0.4328	108.72
53	87.24999	56.0818	0.4392	109.08
54	86.66838	55.4548	0.4455	109.45
55	86.07422	54.8341	0.4517	109.83
56	85.46768	54.2197	0.4578	110.22
57	84.84891	53.6115	0.4639	110.61
58	84.21809	53.0093	0.4699	111.01
59	83.57540	52.4131	0.4759	111.41
60	82.92102	51.8228	0.4818	111.82
61	82.25514	51.2382	0.4876	112.24
62	81.57797	50.6594	0.4934	112.66
63	80.88971	50.0862	0.4991	113.09
64	80.19057	49.5185	0.5048	113.52
65	79.48078	48.9563	0.5104	113.96
66	78.76056	48.3994	0.5160	114.40
67	78.03015	47.8477	0.5215	114.85
68	77.28977	47.3013	0.5270	115.30
69	76.53969	46.7599	0.5324	115.76
70	75.78013	46.2236	0.5378	116.22
71	75.01136	45.6922	0.5431	116.69
72	74.23364	45.1657	0.5483	117.17
73	73.44723	44.6439	0.5536	117.64
74	72.65240	44.1269	0.5587	118.13
75	71.84942	43.6144	0.5639	118.61
76	71.03858	43.1065	0.5689	119.11
77	70.22014	42.6031	0.5740	119.60
78	69.39441	42.1041	0.5790	120.10
79	68.56167	41.6094	0.5839	120.61
80	67.72221	41.1190	0.5888	121.12
81	66.87633	40.6328	0.5937	121.63
82	66.02433	40.1507	0.5985	122.15
83	65.16651	39.6726	0.6033	122.67
84	64.30319	39.1985	0.6080	123.20
85	63.43467	38.7284	0.6127	123.73
86	62.56125	38.2621	0.6174	124.26
87	61.68327	37.7996	0.6220	124.80
88	60.80103	37.3408	0.6266	125.34
89	59.91485	36.8857	0.6311	125.89
90	59.02506	36.4342	0.6357	126.43
91	58.13198	35.9863	0.6401	126.99
92	57.23592	35.5418	0.6446	127.54
93	56.33723	35.1008	0.6490	128.10
94	55.43622	34.6632	0.6534	128.66
95	54.53322	34.2289	0.6577	129.23
96	53.62857	33.7979	0.6620	129.80
97	[illegible]	[illegible]	[illegible]	[illegible]

Iowa Type curve S1

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
100	50.00000	32.1054	0.6789	132.11
101	49.09203	31.6900	0.6831	132.69
102	48.18439	31.2775	0.6872	133.28
103	47.27741	30.8679	0.6913	133.87
104	46.37144	30.4612	0.6954	134.46
105	45.46678	30.0574	0.6994	135.06
106	44.56378	29.6563	0.7034	135.66
107	43.66277	29.2580	0.7074	136.26
108	42.76408	28.8623	0.7114	136.86
109	41.86803	28.4693	0.7153	137.47
110	40.97494	28.0789	0.7192	138.08
111	40.08515	27.6911	0.7231	138.69
112	39.19897	27.3058	0.7269	139.31
113	38.31673	26.9230	0.7308	139.92
114	37.43875	26.5427	0.7346	140.54
115	36.56534	26.1647	0.7384	141.16
116	35.69681	25.7892	0.7421	141.79
117	34.83349	25.4160	0.7458	142.42
118	33.97567	25.0450	0.7495	143.05
119	33.12367	24.6764	0.7532	143.68
120	32.27779	24.3099	0.7569	144.31
121	31.43833	23.9457	0.7605	144.95
122	30.60559	23.5836	0.7642	145.58
123	29.77986	23.2237	0.7678	146.22
124	28.96142	22.8659	0.7713	146.87
125	28.15058	22.5101	0.7749	147.51
126	27.34760	22.1563	0.7784	148.16
127	26.55277	21.8046	0.7820	148.80
128	25.76636	21.4548	0.7855	149.45
129	24.98864	21.1070	0.7889	150.11
130	24.21987	20.7611	0.7924	150.76
131	23.46032	20.4171	0.7958	151.42
132	22.71023	20.0749	0.7993	152.07
133	21.96985	19.7346	0.8027	152.73
134	21.23944	19.3961	0.8060	153.40
135	20.51922	19.0593	0.8094	154.06
136	19.80943	18.7243	0.8128	154.72
137	19.11030	18.3910	0.8161	155.39
138	18.42203	18.0594	0.8194	156.06
139	17.74486	17.7295	0.8227	156.73
140	17.07899	17.4013	0.8260	157.40
141	16.42460	17.0747	0.8293	158.07
142	15.78191	16.7496	0.8325	158.75
143	15.15109	16.4262	0.8357	159.43
144	14.53232	16.1043	0.8390	160.10
145	13.92578	15.7840	0.8422	160.78
146	13.33163	15.4651	0.8453	161.47
147	12.75001	15.1478	0.8485	162.15
148	12.18109	14.8319	0.8517	162.83
149	11.62500	14.5175	0.8548	163.52
150	11.08187	14.2045	0.8580	164.20
151	10.55181	13.8929	0.8611	164.89
152	10.03495	13.5827	0.8642	165.58
153	9.53139	13.2739	0.8673	166.27
154	9.04122	12.9665	0.8703	166.97
155	8.56453	12.6603	0.8734	167.66
156	8.10138	12.3555	0.8764	168.36
157	7.65185	12.0520	0.8795	169.05
158	7.21599	11.7498	0.8825	169.75
159	6.79383	11.4488	0.8855	170.45
160	6.38542	11.1491	0.8885	171.15
161	5.99078	10.8506	0.8915	171.85
162	5.60991	10.5533	0.8945	172.55
163	5.24281	10.2573	0.8974	173.26
164	4.88947	9.9624	0.9004	173.96
165	4.54986	9.6687	0.9033	174.67
166	4.22395	9.3761	0.9062	175.38
167	3.91168	9.0847	0.9092	176.08
168	3.61300	8.7944	0.9121	176.79
169	3.32781	8.5052	0.9149	177.51
170	3.05603	8.2171	0.9178	178.22
171	2.79756	7.9301	0.9207	178.93
172	2.55227	7.6442	0.9236	179.64
173	2.32003	7.3594	0.9264	180.36
174	2.10070	7.0755	0.9292	181.08
175	1.89410	6.7928	0.9321	181.79
176	1.70005	6.5110	0.9349	182.51
177	1.51836	6.2303	0.9377	183.23
178	1.34882	5.9506	0.9405	183.95
179	1.19118	5.6719	0.9433	184.67
180	1.04521	5.3942	0.9461	185.39
181	0.91063	5.1175	0.9488	186.12
182	0.78716	4.8418	0.9516	186.84
183	0.67448	4.5671	0.9543	187.57
184	0.57229	4.2934	0.9571	188.29
185	0.48022	4.0207	0.9598	189.02
186	0.39790	3.7491	0.9625	189.75
187	0.32495	3.4785	0.9652	190.48
188	0.26096	3.2088	0.9679	191.21
189	0.20547	2.9404	0.9706	191.94
190	0.15802	2.6731	0.9733	192.67
191	0.11812	2.4072	0.9759	193.41
192	0.08524	2.1429	0.9786	194.14
193	0.05884	1.8800	0.9812	194.88
194	0.03831	1.6195	0.9838	195.62
195	0.02303	1.3624	0.9864	196.36
196	0.01234	1.1094	0.9889	197.11
200	0.00000	0.0000	1.0000	200.00

Iowa Type curve S2

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
0	100.00000	100.0000	0.0000	100.00
1	100.00000	99.0000	0.0100	100.00
2	100.00000	98.0000	0.0200	100.00
3	99.99998	97.0000	0.0300	100.00
4	99.99992	96.0001	0.0400	100.00
5	99.99977	95.0002	0.0500	100.00
6	99.99946	94.0005	0.0600	100.00
7	99.99890	93.0010	0.0700	100.00
8	99.99797	92.0019	0.0800	100.00
9	99.99652	91.0032	0.0900	100.00
10	99.99438	90.0052	0.0999	100.01
11	99.99134	89.0079	0.1099	100.01
12	99.98717	88.0116	0.1199	100.01
13	99.98160	87.0165	0.1298	100.02
14	99.97435	86.0227	0.1398	100.02
15	99.96509	85.0306	0.1497	100.03
16	99.95348	84.0405	0.1596	100.04
17	99.93914	83.0525	0.1695	100.05
18	99.92167	82.0669	0.1793	100.07
19	99.90065	81.0840	0.1892	100.08
20	99.87563	80.1042	0.1990	100.10
21	99.84613	79.1278	0.2087	100.13
22	99.81167	78.1549	0.2185	100.15
23	99.77175	77.1860	0.2281	100.19
24	99.72583	76.2213	0.2378	100.22
25	99.67362	75.2609	0.2474	100.26
26	99.61381	74.3058	0.2569	100.31
27	99.54659	73.3557	0.2664	100.36
28	99.47113	72.4109	0.2759	100.41
29	99.38684	71.4719	0.2853	100.47
30	99.29314	70.5389	0.2946	100.54
31	99.18942	69.6121	0.3039	100.61
32	99.07508	68.6919	0.3131	100.69
33	98.94953	67.7784	0.3222	100.78
34	98.81215	66.8720	0.3313	100.87
35	98.66236	65.9727	0.3403	100.97
36	98.49956	65.0809	0.3492	101.08
37	98.32317	64.1968	0.3580	101.20
38	98.13260	63.3205	0.3668	101.32
39	97.92728	62.4522	0.3755	101.45
40	97.70666	61.5921	0.3841	101.59
41	97.47019	60.7403	0.3926	101.74
42	97.21735	59.8970	0.4010	101.90
43	96.94761	59.0622	0.4094	102.06
44	96.66048	58.2362	0.4176	102.24
45	96.35548	57.4190	0.4258	102.42
46	96.03215	56.6106	0.4339	102.61
47	95.69007	55.8112	0.4419	102.81
48	95.32881	55.0208	0.4498	103.02
49	94.94799	54.2395	0.4576	103.24
50	94.54723	53.4673	0.4653	103.47
51	94.12622	52.7042	0.4730	103.70
52	93.68462	51.9502	0.4805	103.95
53	93.22216	51.2055	0.4879	104.21
54	92.73860	50.4699	0.4953	104.47
55	92.23370	49.7434	0.5026	104.74
56	91.70727	49.0261	0.5097	105.03
57	91.15914	48.3179	0.5168	105.32
58	90.58920	47.6187	0.5238	105.62
59	89.99734	46.9286	0.5307	105.93
60	89.38350	46.2474	0.5375	106.25
61	88.74764	45.5752	0.5442	106.58
62	88.08976	44.9118	0.5509	106.91
63	87.40990	44.2573	0.5574	107.26
64	86.70813	43.6114	0.5639	107.61
65	85.98455	42.9742	0.5703	107.97
66	85.23928	42.3456	0.5765	108.35
67	84.47253	41.7254	0.5827	108.73
68	83.68447	41.1136	0.5889	109.11
69	82.87533	40.5101	0.5949	109.51
70	82.04540	39.9149	0.6009	109.91
71	81.19498	39.3277	0.6067	110.33
72	80.32439	38.7485	0.6125	110.75
73	79.43401	38.1773	0.6182	111.18
74	78.52423	37.6138	0.6239	111.61
75	77.59548	37.0580	0.6294	112.06
76	76.64822	36.5098	0.6349	112.51
77	75.68294	35.9691	0.6403	112.97
78	74.70016	35.4357	0.6456	113.44
79	73.70042	34.9096	0.6509	113.91
80	72.68429	34.3907	0.6561	114.39
81	71.65238	33.8788	0.6612	114.88
82	70.60530	33.3738	0.6663	115.37
83	69.54371	32.8756	0.6712	115.88
84	68.46828	32.3841	0.6762	116.38
85	67.37970	31.8992	0.6810	116.90
86	66.27869	31.4208	0.6858	117.42
87	65.16598	30.9488	0.6905	117.95
88	64.04234	30.4830	0.6952	118.48
89	62.90852	30.0234	0.6998	119.02
90	61.76532	29.5699	0.7043	119.57
91	60.61354	29.1222	0.7088	120.12
92	59.45400	28.6805	0.7132	120.68
93	58.28754	28.2444	0.7176	121.24
94	57.11498	27.8140	0.7219	121.81
95	55.93719	27.3891	0.7261	122.39
96	54.75503	26.9697	0.7303	122.97
100	50.00000	25.3441	0.7466	125.34
101	48.80904	24.9503	0.7505	125.95
102	47.61897	24.5614	0.7544	126.56
103	46.43065	24.1772	0.7582	127.18
104	45.24498	23.7977	0.7620	127.80
105	44.06281	23.4227	0.7658	128.42
106	42.88502	23.0523	0.7695	129.05
107	41.71246	22.6862	0.7731	129.69
108	40.54600	22.3245	0.7768	130.32
109	39.38646	21.9670	0.7803	130.97
110	38.23468	21.6137	0.7839	131.61
111	37.09148	21.2644	0.7874	132.26
112	35.95767	20.9192	0.7908	132.92
113	34.83402	20.5778	0.7942	133.58
114	33.72131	20.2403	0.7976	134.24
115	32.62030	19.9066	0.8009	134.91
116	31.53172	19.5766	0.8042	135.58
117	30.45629	19.2502	0.8075	136.25
118	29.39470	18.9274	0.8107	136.93
119	28.34762	18.6080	0.8139	137.61
120	27.31571	18.2921	0.8171	138.29
121	26.29958	17.9795	0.8202	138.98
122	25.29984	17.6702	0.8233	139.67
123	24.31706	17.3642	0.8264	140.36
124	23.35178	17.0613	0.8294	141.06
125	22.40452	16.7615	0.8324	141.76
126	21.47577	16.4648	0.8354	142.46
127	20.56550	16.1714	0.8383	143.17
128	19.67561	15.8802	0.8412	143.88
129	18.80502	15.5922	0.8441	144.59
130	17.95470	15.3070	0.8469	145.31
131	17.12467	15.0246	0.8498	146.02
132	16.31553	14.7450	0.8526	146.74
133	15.52747	14.4679	0.8553	147.47
134	14.76071	14.1935	0.8581	148.19
135	14.01545	13.9217	0.8608	148.92
136	13.29187	13.6523	0.8635	149.65
137	12.59010	13.3854	0.8661	150.39
138	11.91024	13.1209	0.8688	151.12
139	11.25237	12.8588	0.8714	151.86
140	10.61650	12.5990	0.8740	152.60
141	10.00264	12.3416	0.8766	153.34
142	9.41080	12.0863	0.8791	154.09
143	8.84086	11.8332	0.8817	154.83
144	8.29274	11.5823	0.8842	155.58
145	7.76631	11.3335	0.8867	156.33
146	7.26140	11.0868	0.8891	157.09
147	6.77784	10.8421	0.8916	157.84
148	6.31538	10.5994	0.8940	158.60
149	5.87399	10.3583	0.8964	159.36
150	5.45277	10.1199	0.8988	160.12
151	5.05201	9.8830	0.9012	160.88
152	4.67119	9.6479	0.9035	161.65
153	4.30993	9.4147	0.9059	162.41
154	3.96784	9.1833	0.9082	163.18
155	3.64452	8.9536	0.9105	163.95
156	3.33952	8.7257	0.9127	164.73
157	3.05239	8.4994	0.9150	165.50
158	2.78265	8.2749	0.9173	166.27
159	2.52981	8.0519	0.9195	167.05
160	2.29334	7.8306	0.9217	167.83
161	2.07272	7.6109	0.9239	168.61
162	1.86740	7.3927	0.9261	169.39
163	1.67683	7.1761	0.9282	170.18
164	1.50044	6.9609	0.9304	170.96
165	1.33764	6.7473	0.9325	171.75
166	1.18785	6.5351	0.9346	172.54
167	1.05048	6.3243	0.9368	173.32
168	0.92493	6.1148	0.9389	174.11
169	0.81058	5.9069	0.9409	174.91
170	0.70686	5.7003	0.9430	175.70
171	0.61316	5.4950	0.9450	176.50
172	0.52887	5.2911	0.9471	177.29
173	0.45341	5.0885	0.9491	178.09
174	0.38619	4.8871	0.9511	178.89
175	0.32664	4.6870	0.9531	179.69
176	0.27418	4.4881	0.9551	180.49
177	0.22825	4.2906	0.9571	181.29
178	0.18833	4.0941	0.9591	182.09
179	0.15387	3.8990	0.9610	182.90
180	0.12438	3.7049	0.9630	183.70
181	0.09935	3.5123	0.9649	184.51
182	0.07833	3.3206	0.9668	185.32
183	0.06086	3.1303	0.9687	186.13
184	0.04651	2.9418	0.9706	186.94
185	0.03491	2.7532	0.9725	187.75
186	0.02565	2.5667	0.9743	188.57
187	0.01840	2.3810	0.9762	189.38
188	0.01284	2.1955	0.9780	190.20
189	0.00866	2.0138	0.9799	191.01
190	0.00563	1.8286	0.9817	191.83
191	0.00348	1.6494	0.9835	192.65
192	0.00203	1.4704	0.9853	193.47
193	0.00110	1.2909	0.9871	194.29
194	0.00054	1.1111	0.9889	195.11
195	0.00023	0.9347	0.9907	195.93
196	0.00008	0.7497	0.9925	196.75

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Iowa Type curve S3

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
0	100.00000	100.0000	0.0000	100.00
1	100.00000	99.0050	0.0099	100.01
2	100.00000	98.0050	0.0199	100.01
3	100.00000	97.0050	0.0299	100.01
4	100.00000	96.0050	0.0399	100.01
5	100.00000	95.0050	0.0499	100.01
6	100.00000	94.0050	0.0599	100.01
7	100.00000	93.0050	0.0699	100.01
8	100.00000	92.0050	0.0799	100.01
9	99.99999	91.0050	0.0899	100.01
10	99.99998	90.0050	0.0999	100.01
11	99.99996	89.0050	0.1099	100.01
12	99.99991	88.0051	0.1199	100.01
13	99.99984	87.0051	0.1299	100.01
14	99.99973	86.0052	0.1399	100.01
15	99.99955	85.0054	0.1499	100.01
16	99.99927	84.0056	0.1599	100.01
17	99.99886	83.0060	0.1699	100.01
18	99.99826	82.0065	0.1799	100.01
19	99.99742	81.0072	0.1899	100.01
20	99.99626	80.0081	0.1999	100.01
21	99.99468	79.0093	0.2099	100.01
22	99.99257	78.0110	0.2199	100.01
23	99.98980	77.0131	0.2299	100.01
24	99.98620	76.0159	0.2398	100.02
25	99.98158	75.0194	0.2498	100.02
26	99.97573	74.0237	0.2598	100.02
27	99.96840	73.0291	0.2697	100.03
28	99.95930	72.0357	0.2796	100.04
29	99.94813	71.0437	0.2896	100.04
30	99.93451	70.0534	0.2995	100.05
31	99.91805	69.0648	0.3094	100.06
32	99.89831	68.0784	0.3192	100.08
33	99.87481	67.0943	0.3291	100.09
34	99.84701	66.1128	0.3389	100.11
35	99.81435	65.1343	0.3487	100.13
36	99.77620	64.1590	0.3584	100.16
37	99.73190	63.1873	0.3681	100.19
38	99.68073	62.2194	0.3778	100.22
39	99.62192	61.2559	0.3874	100.26
40	99.55468	60.2969	0.3970	100.30
41	99.47816	59.3429	0.4066	100.34
42	99.39145	58.3942	0.4161	100.39
43	99.29363	57.4513	0.4255	100.45
44	99.18371	56.5144	0.4349	100.51
45	99.06070	55.5839	0.4442	100.58
46	98.92355	54.6603	0.4534	100.66
47	98.77119	53.7439	0.4626	100.74
48	98.60251	52.8349	0.4717	100.83
49	98.41639	51.9339	0.4807	100.93
50	98.21171	51.0411	0.4896	101.04
51	97.98729	50.1569	0.4984	101.16
52	97.74199	49.2815	0.5072	101.28
53	97.47464	48.4153	0.5158	101.42
54	97.18408	47.5585	0.5244	101.56
55	96.86915	46.7115	0.5329	101.71
56	96.52871	45.8745	0.5413	101.87
57	96.16165	45.0477	0.5495	102.05
58	95.76687	44.2314	0.5577	102.23
59	95.34330	43.4256	0.5657	102.43
60	94.88992	42.6307	0.5737	102.63
61	94.40574	41.8468	0.5815	102.85
62	93.88982	41.0740	0.5893	103.07
63	93.34129	40.3124	0.5969	103.31
64	92.75933	39.5622	0.6044	103.56
65	92.14318	38.8234	0.6118	103.82
66	91.49216	38.0961	0.6190	104.10
67	90.80566	37.3803	0.6262	104.38
68	90.08316	36.6761	0.6332	104.68
69	89.32423	35.9835	0.6402	104.98
70	88.52851	35.3024	0.6470	105.30
71	87.69574	34.6329	0.6537	105.63
72	86.82577	33.9749	0.6603	105.97
73	85.91855	33.3284	0.6667	106.33
74	84.97412	32.6933	0.6731	106.69
75	83.99263	32.0695	0.6793	107.07
76	82.97434	31.4569	0.6854	107.46
77	81.91963	30.8555	0.6914	107.86
78	80.82896	30.2651	0.6973	108.27
79	79.70293	29.6856	0.7031	108.69
80	78.54224	29.1169	0.7088	109.12
81	77.34768	28.5588	0.7144	109.56
82	76.12018	28.0113	0.7199	110.01
83	74.86076	27.4741	0.7253	110.47
84	73.57053	26.9472	0.7305	110.95
85	72.25073	26.4303	0.7357	111.43
86	70.90268	25.9233	0.7408	111.92
87	69.52779	25.4261	0.7457	112.43
88	68.12758	24.9384	0.7506	112.94
89	66.70364	24.4600	0.7554	113.46
90	65.25765	23.9910	0.7601	113.99
91	63.79136	23.5309	0.7647	114.53
92	62.30659	23.0797	0.7692	115.08
93	60.80523	22.6373	0.7736	115.64
94	59.28922	22.2033	0.7780	116.20
95	57.76056	21.7777	0.7822	116.78
96	56.22128	21.3603	0.7864	117.36

Iowa Type curve S3

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
100	50.00000	19.7687	0.8023	119.77
101	48.43931	19.3896	0.8061	120.39
102	46.88077	19.0175	0.8098	121.02
103	45.32553	18.6525	0.8135	121.65
104	43.77872	18.2943	0.8171	122.29
105	42.73945	17.7270	0.8227	122.73
106	40.71078	17.5854	0.8241	123.59
107	39.19477	17.2463	0.8275	124.25
108	37.69341	16.9133	0.8309	124.91
109	36.20864	16.5863	0.8341	125.59
110	34.74235	16.2653	0.8373	126.27
111	33.29636	15.9499	0.8405	126.95
112	31.87242	15.6402	0.8436	127.64
113	30.47221	15.3358	0.8466	128.34
114	29.09733	15.0369	0.8496	129.04
115	27.74927	14.7431	0.8526	129.74
116	26.42947	14.4543	0.8555	130.45
117	25.13924	14.1705	0.8583	131.17
118	23.87982	13.8915	0.8611	131.89
119	22.65232	13.6171	0.8638	132.62
120	21.45776	13.3474	0.8665	133.35
121	20.29707	13.0820	0.8692	134.08
122	19.17104	12.8211	0.8718	134.82
123	18.08037	12.5643	0.8744	135.56
124	17.02566	12.3117	0.8769	136.31
125	16.00737	12.0631	0.8794	137.06
126	15.02588	11.8184	0.8818	137.82
127	14.08145	11.5775	0.8842	138.58
128	13.17423	11.3403	0.8866	139.34
129	12.30426	11.1068	0.8889	140.11
130	11.47150	10.8768	0.8912	140.88
131	10.67577	10.6502	0.8935	141.65
132	9.91684	10.4270	0.8957	142.43
133	9.19434	10.2071	0.8979	143.21
134	8.50784	9.9903	0.9001	143.99
135	7.85682	9.7767	0.9022	144.78
136	7.24067	9.5661	0.9043	145.57
137	6.65871	9.3585	0.9064	146.36
138	6.11018	9.1537	0.9085	147.15
139	5.59427	8.9518	0.9105	147.95
140	5.11008	8.7526	0.9125	148.75
141	4.65670	8.5561	0.9144	149.56
142	4.23313	8.3622	0.9164	150.36
143	3.83835	8.1708	0.9183	151.17
144	3.47129	7.9819	0.9202	151.98
145	3.13085	7.7955	0.9220	152.80
146	2.81592	7.6114	0.9239	153.61
147	2.52536	7.4297	0.9257	154.43
148	2.25801	7.2501	0.9275	155.25
149	2.01271	7.0728	0.9293	156.07
150	1.78829	6.8977	0.9310	156.90
151	1.58361	6.7245	0.9328	157.72
152	1.39749	6.5535	0.9345	158.55
153	1.22881	6.3845	0.9362	159.38
154	1.07645	6.2174	0.9378	160.22
155	0.93930	6.0522	0.9395	161.05
156	0.81629	5.8889	0.9411	161.89
157	0.70638	5.7274	0.9427	162.73
158	0.60855	5.5678	0.9443	163.57
159	0.52185	5.4097	0.9459	164.41
160	0.44532	5.2535	0.9475	165.25
161	0.37808	5.0988	0.9490	166.10
162	0.31927	4.9460	0.9505	166.95
163	0.26810	4.7945	0.9521	167.79
164	0.22380	4.6446	0.9536	168.64
165	0.18565	4.4963	0.9550	169.50
166	0.15299	4.3494	0.9565	170.35
167	0.12519	4.2042	0.9580	171.20
168	0.10169	4.0602	0.9594	172.06
169	0.08195	3.9178	0.9608	172.92
170	0.06549	3.7768	0.9622	173.78
171	0.05188	3.6365	0.9636	174.64
172	0.04070	3.4980	0.9650	175.50
173	0.03160	3.3614	0.9664	176.36
174	0.02427	3.2256	0.9677	177.23
175	0.01842	3.0912	0.9691	178.09
176	0.01381	2.9562	0.9704	178.96
177	0.01020	2.8255	0.9717	179.83
178	0.00743	2.6924	0.9731	180.69
179	0.00532	2.5620	0.9744	181.56
180	0.00374	2.4331	0.9757	182.43
181	0.00258	2.3023	0.9770	183.30
182	0.00174	2.1723	0.9783	184.17
183	0.00114	2.0525	0.9795	185.05
184	0.00073	1.9245	0.9808	185.92
185	0.00045	1.8109	0.9819	186.81
186	0.00027	1.6848	0.9832	187.68
187	0.00016	1.4994	0.9850	188.50
188	0.00009	1.2768	0.9872	189.28
189	0.00004	1.2480	0.9875	190.25
190	0.00002	0.9965	0.9900	191.00
191	0.00001	0.4936	0.9951	191.49
192	0.00000	0.0000	1.0000	192.00

Iowa Type curve S4

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
0	100.00000	100.0000	0.0000	100.00
1	100.00000	99.0000	0.0100	100.00
2	100.00000	98.0000	0.0200	100.00
3	100.00000	97.0000	0.0300	100.00
4	100.00000	96.0000	0.0400	100.00
5	100.00000	95.0000	0.0500	100.00
6	100.00000	94.0000	0.0600	100.00
7	100.00000	93.0000	0.0700	100.00
8	100.00000	92.0000	0.0800	100.00
9	100.00000	91.0000	0.0900	100.00
10	100.00000	90.0000	0.1000	100.00
11	100.00000	89.0000	0.1100	100.00
12	100.00000	88.0000	0.1200	100.00
13	100.00000	87.0000	0.1300	100.00
14	100.00000	86.0000	0.1400	100.00
15	100.00000	85.0000	0.1500	100.00
16	100.00000	84.0000	0.1600	100.00
17	100.00000	83.0000	0.1700	100.00
18	100.00000	82.0000	0.1800	100.00
19	100.00000	81.0000	0.1900	100.00
20	100.00000	80.0000	0.2000	100.00
21	100.00000	79.0000	0.2100	100.00
22	100.00000	78.0000	0.2200	100.00
23	100.00000	77.0000	0.2300	100.00
24	100.00000	76.0000	0.2400	100.00
25	100.00000	75.0000	0.2500	100.00
26	100.00000	74.0000	0.2600	100.00
27	99.99999	73.0000	0.2700	100.00
28	99.99998	72.0000	0.2800	100.00
29	99.99997	71.0000	0.2900	100.00
30	99.99994	70.0000	0.3000	100.00
31	99.99990	69.0001	0.3100	100.00
32	99.99982	68.0001	0.3200	100.00
33	99.99971	67.0002	0.3300	100.00
34	99.99953	66.0003	0.3400	100.00
35	99.99925	65.0005	0.3500	100.00
36	99.99884	64.0008	0.3600	100.00
37	99.99824	63.0011	0.3700	100.00
38	99.99738	62.0017	0.3800	100.00
39	99.99615	61.0024	0.3900	100.00
40	99.99445	60.0035	0.4000	100.00
41	99.99210	59.0049	0.4100	100.00
42	99.98892	58.0067	0.4199	100.01
43	99.98465	57.0092	0.4299	100.01
44	99.97901	56.0124	0.4399	100.01
45	99.97162	55.0165	0.4498	100.02
46	99.96204	54.0217	0.4598	100.02
47	99.94977	53.0283	0.4697	100.03
48	99.93418	52.0365	0.4796	100.04
49	99.91455	51.0466	0.4895	100.05
50	99.89007	50.0590	0.4994	100.06
51	99.85976	49.0740	0.5093	100.07
52	99.82255	48.0921	0.5191	100.09
53	99.77721	47.1138	0.5289	100.11
54	99.72235	46.1394	0.5386	100.14
55	99.65644	45.1696	0.5483	100.17
56	99.57779	44.2049	0.5580	100.20
57	99.48452	43.2458	0.5675	100.25
58	99.37460	42.2931	0.5771	100.29
59	99.24585	41.3473	0.5865	100.35
60	99.09588	40.4092	0.5959	100.41
61	98.92218	39.4792	0.6052	100.48
62	98.72208	38.5582	0.6144	100.56
63	98.49276	37.6469	0.6235	100.65
64	98.23128	36.7457	0.6325	100.75
65	97.93460	35.8555	0.6414	100.86
66	97.59959	34.9769	0.6502	100.98
67	97.22304	34.1104	0.6589	101.11
68	96.80170	33.2567	0.6674	101.26
69	96.33234	32.4163	0.6758	101.42
70	95.81171	31.5897	0.6841	101.59
71	95.23664	30.7775	0.6922	101.78
72	94.60402	29.9799	0.7002	101.98
73	93.91087	29.1975	0.7080	102.20
74	93.15439	28.4306	0.7157	102.43
75	92.33195	27.6794	0.7232	102.68
76	91.44115	26.9441	0.7306	102.94
77	90.47988	26.2251	0.7377	103.23
78	89.44630	25.5223	0.7448	103.52
79	88.33893	24.8360	0.7516	103.84
80	87.15666	24.1661	0.7583	104.17
81	85.89877	23.5127	0.7649	104.51
82	84.56495	22.8757	0.7712	104.88
83	83.15535	22.2550	0.7775	105.25
84	81.67059	21.6505	0.7835	105.65
85	80.11173	21.0620	0.7894	106.06
86	78.48036	20.4894	0.7951	106.49
87	76.77853	19.9325	0.8007	106.93
88	75.00878	19.3910	0.8061	107.39
89	73.17417	18.8646	0.8114	107.86
90	71.27816	18.3531	0.8165	108.35
91	69.32473	17.8562	0.8214	108.86
92	67.31826	17.3735	0.8263	109.37
93	65.26353	16.9048	0.8310	109.90
94	63.16571	16.4496	0.8355	110.45
95	61.03029	16.0077	0.8399	111.01
96	58.86308	15.5786	0.8442	111.58

Iowa Type curve S4

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
100	50.00000	13.9843	0.8602	113.98
101	47.75789	13.6144	0.8639	114.61
102	45.54234	13.2552	0.8674	115.26
103	43.32988	12.9065	0.8709	115.91
104	41.13692	12.5679	0.8743	116.57
105	38.96971	12.2390	0.8776	117.24
106	36.83429	11.9196	0.8808	117.92
107	34.73647	11.6093	0.8839	118.61
108	32.68174	11.3077	0.8869	119.31
109	30.67527	11.0146	0.8899	120.01
110	28.72184	10.7298	0.8927	120.73
111	26.82583	10.4528	0.8955	121.45
112	24.99122	10.1834	0.8982	122.18
113	23.22147	9.9214	0.9008	122.92
114	21.51964	9.6665	0.9033	123.67
115	19.88827	9.4184	0.9058	124.42
116	18.32942	9.1769	0.9082	125.18
117	16.84465	8.9417	0.9106	125.94
118	15.43505	8.7126	0.9129	126.71
119	14.10123	8.4894	0.9151	127.49
120	12.84334	8.2719	0.9173	128.27
121	11.66107	8.0599	0.9194	129.06
122	10.55370	7.8531	0.9215	129.85
123	9.52012	7.6515	0.9235	130.65
124	8.55885	7.4547	0.9255	131.45
125	7.66805	7.2626	0.9274	132.26
126	6.84561	7.0751	0.9292	133.08
127	6.08913	6.8919	0.9311	133.89
128	5.39599	6.7130	0.9329	134.71
129	4.76336	6.5381	0.9346	135.54
130	4.18829	6.3672	0.9363	136.37
131	3.66766	6.2000	0.9380	137.20
132	3.19830	6.0365	0.9396	138.04
133	2.77697	5.8766	0.9412	138.88
134	2.40041	5.7200	0.9428	139.72
135	2.06540	5.5667	0.9443	140.57
136	1.76872	5.4165	0.9458	141.42
137	1.50724	5.2695	0.9473	142.27
138	1.27792	5.1254	0.9487	143.13
139	1.07782	4.9841	0.9502	143.98
140	0.90412	4.8456	0.9515	144.85
141	0.75416	4.7096	0.9529	145.71
142	0.62540	4.5763	0.9542	146.58
143	0.51548	4.4456	0.9555	147.45
144	0.42222	4.3171	0.9568	148.32
145	0.34356	4.1910	0.9581	149.19
146	0.27765	4.0672	0.9593	150.07
147	0.22279	3.9456	0.9605	150.95
148	0.17745	3.8260	0.9617	151.83
149	0.14024	3.7084	0.9629	152.71
150	0.10994	3.5927	0.9641	153.59
151	0.08545	3.4791	0.9652	154.48
152	0.06582	3.3675	0.9663	155.37
153	0.05023	3.2575	0.9674	156.26
154	0.03796	3.1488	0.9685	157.15
155	0.02838	3.0430	0.9696	158.04
156	0.02099	2.9383	0.9706	158.94
157	0.01535	2.8342	0.9717	159.83
158	0.01108	2.7337	0.9727	160.73
159	0.00790	2.6329	0.9737	161.63
160	0.00555	2.5360	0.9746	162.54
161	0.00385	2.4350	0.9756	163.44
162	0.00262	2.3435	0.9766	164.34
163	0.00176	2.2443	0.9776	165.24
164	0.00116	2.1465	0.9785	166.15
165	0.00075	2.0466	0.9795	167.05
166	0.00047	1.9680	0.9803	167.97
167	0.00029	1.8792	0.9812	168.88
168	0.00018	1.7221	0.9828	169.72
169	0.00010	1.6999	0.9830	170.70
170	0.00006	1.5000	0.9850	171.50
171	0.00003	1.5002	0.9850	172.50
172	0.00002	1.0006	0.9900	173.00
173	0.00001	0.5018	0.9950	173.50
174	0.00000	0.0000	1.0000	174.00

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Table B continued

Iowa Type curve S5

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
0	100.00000	100.0000	0.0000	100.00
1	100.00000	99.0000	0.0100	100.00
2	100.00000	98.0000	0.0200	100.00
3	100.00000	97.0000	0.0300	100.00
4	100.00000	96.0000	0.0400	100.00
5	100.00000	95.0000	0.0500	100.00
6	100.00000	94.0000	0.0600	100.00
7	100.00000	93.0000	0.0700	100.00
8	100.00000	92.0000	0.0800	100.00
9	100.00000	91.0000	0.0900	100.00
10	100.00000	90.0000	0.1000	100.00
11	100.00000	89.0000	0.1100	100.00
12	100.00000	88.0000	0.1200	100.00
13	100.00000	87.0000	0.1300	100.00
14	100.00000	86.0000	0.1400	100.00
15	100.00000	85.0000	0.1500	100.00
16	100.00000	84.0000	0.1600	100.00
17	100.00000	83.0000	0.1700	100.00
18	100.00000	82.0000	0.1800	100.00
19	100.00000	81.0000	0.1900	100.00
20	100.00000	80.0000	0.2000	100.00
21	100.00000	79.0000	0.2100	100.00
22	100.00000	78.0000	0.2200	100.00
23	100.00000	77.0000	0.2300	100.00
24	100.00000	76.0000	0.2400	100.00
25	100.00000	75.0000	0.2500	100.00
26	100.00000	74.0000	0.2600	100.00
27	100.00000	73.0000	0.2700	100.00
28	100.00000	72.0000	0.2800	100.00
29	100.00000	71.0000	0.2900	100.00
30	100.00000	70.0000	0.3000	100.00
31	100.00000	69.0000	0.3100	100.00
32	100.00000	68.0000	0.3200	100.00
33	100.00000	67.0000	0.3300	100.00
34	100.00000	66.0000	0.3400	100.00
35	100.00000	65.0000	0.3500	100.00
36	100.00000	64.0000	0.3600	100.00
37	100.00000	63.0000	0.3700	100.00
38	100.00000	62.0000	0.3800	100.00
39	100.00000	61.0000	0.3900	100.00
40	100.00000	60.0000	0.4000	100.00
41	100.00000	59.0000	0.4100	100.00
42	100.00000	58.0000	0.4200	100.00
43	100.00000	57.0000	0.4300	100.00
44	100.00000	56.0000	0.4400	100.00
45	100.00000	55.0000	0.4500	100.00
46	99.99999	54.0000	0.4600	100.00
47	99.99998	53.0000	0.4700	100.00
48	99.99996	52.0000	0.4800	100.00
49	99.99993	51.0000	0.4900	100.00
50	99.99987	50.0001	0.5000	100.00
51	99.99976	49.0001	0.5100	100.00
52	99.99958	48.0002	0.5200	100.00
53	99.99928	47.0004	0.5300	100.00
54	99.99880	46.0006	0.5400	100.00
55	99.99803	45.0009	0.5500	100.00
56	99.99685	44.0015	0.5600	100.00
57	99.99503	43.0022	0.5700	100.00
58	99.99232	42.0034	0.5800	100.00
59	99.98831	41.0051	0.5899	100.01
60	99.98250	40.0074	0.5999	100.01
61	99.97418	39.0107	0.6099	100.01
62	99.96247	38.0152	0.6198	100.02
63	99.94620	37.0213	0.6298	100.02
64	99.92390	36.0295	0.6397	100.03
65	99.89372	35.0402	0.6496	100.04
66	99.85338	34.0542	0.6595	100.05
67	99.80012	33.0721	0.6693	100.07
68	99.73061	32.0948	0.6791	100.09
69	99.64087	31.1232	0.6888	100.12
70	99.52630	30.1585	0.6984	100.16
71	99.38154	29.2017	0.7080	100.20
72	99.20049	28.2541	0.7175	100.25
73	98.97629	27.3169	0.7268	100.32
74	98.70131	26.3916	0.7361	100.39
75	98.36721	25.4796	0.7452	100.48
76	97.96496	24.5821	0.7542	100.58
77	97.48497	23.7007	0.7630	100.70
78	96.91718	22.8366	0.7716	100.84
79	96.25125	21.9912	0.7801	100.99
80	95.47669	21.1655	0.7883	101.17
81	94.58312	20.3608	0.7964	101.36
82	93.56050	19.5778	0.8042	101.58
83	92.39937	18.8176	0.8118	101.82
84	91.09114	18.0807	0.8192	102.08
85	89.62837	17.3676	0.8263	102.37
86	88.00503	16.6787	0.8332	102.68
87	86.21677	16.0143	0.8399	103.01
88	84.26116	15.3744	0.8463	103.37
89	82.13791	14.7589	0.8524	103.76
90	79.84903	14.1676	0.8583	104.17
91	77.39893	13.6002	0.8640	104.60
92	74.79456	13.0564	0.8694	105.06
93	72.04530	12.5356	0.8746	105.54
94	69.16299	12.0371	0.8796	106.04
95	66.16179	11.5605	0.8844	106.56
96	[illegible]	[illegible]	[illegible]	[illegible]

Iowa Type curve S5

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
100	50.00000	9.4783	0.9052	109.48
101	46.68073	9.1168	0.9088	110.12
102	43.38363	8.7716	0.9123	110.77
103	40.13043	8.4422	0.9156	111.44
104	36.94207	8.1276	0.9187	112.13
105	33.83821	7.8273	0.9217	112.83
106	30.83701	7.5404	0.9246	113.54
107	27.95470	7.2663	0.9273	114.27
108	25.20544	7.0043	0.9300	115.00
109	22.60107	6.7539	0.9325	115.75
110	20.15098	6.5142	0.9349	116.51
111	17.86209	6.2849	0.9372	117.28
112	15.73884	6.0653	0.9393	118.07
113	13.78323	5.8550	0.9415	118.85
114	11.99497	5.6533	0.9435	119.65
115	10.37163	5.4599	0.9454	120.46
116	8.90886	5.2743	0.9473	121.27
117	7.60064	5.0960	0.9490	122.10
118	6.43950	4.9247	0.9508	122.92
119	5.41688	4.7601	0.9524	123.76
120	4.52331	4.6016	0.9540	124.60
121	3.74875	4.4491	0.9555	125.45
122	3.08282	4.3021	0.9570	126.30
123	2.51503	4.1605	0.9584	127.16
124	2.03504	4.0239	0.9598	128.02
125	1.63279	3.8920	0.9611	128.89
126	1.29869	3.7647	0.9624	129.76
127	1.02371	3.6416	0.9636	130.64
128	0.79951	3.5225	0.9648	131.52
129	0.61846	3.4074	0.9659	132.41
130	0.47370	3.2958	0.9670	133.30
131	0.35913	3.1878	0.9681	134.19
132	0.26940	3.0830	0.9692	135.08
133	0.19988	2.9814	0.9702	135.98
134	0.14662	2.8828	0.9712	136.88
135	0.10628	2.7872	0.9721	137.79
136	0.07610	2.6942	0.9731	138.69
137	0.05380	2.6037	0.9740	139.60
138	0.03753	2.5157	0.9748	140.52
139	0.02582	2.4299	0.9757	141.43
140	0.01751	2.3458	0.9765	142.35
141	0.01169	2.2647	0.9774	143.26
142	0.00768	2.1862	0.9781	144.19
143	0.00497	2.1056	0.9789	145.11
144	0.00315	2.0333	0.9797	146.03
145	0.00197	1.9517	0.9805	146.95
146	0.00120	1.8833	0.9812	147.88
147	0.00072	1.8054	0.9819	148.81
148	0.00042	1.7379	0.9826	149.74
149	0.00024	1.6664	0.9833	150.67
150	0.00014	1.4996	0.9850	151.50
151	0.00007	1.4993	0.9850	152.50
152	0.00004	1.2488	0.9875	153.25
153	0.00002	0.9983	0.9900	154.00
154	0.00001	0.4971	0.9950	154.50
155	0.00000	0.0000	1.0000	155.00

Table B continued

Iowa Type curve S6

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
0	100.00000	100.0000	0.0000	100.00
1	100.00000	99.0000	0.0100	100.00
2	100.00000	98.0000	0.0200	100.00
3	100.00000	97.0000	0.0300	100.00
4	100.00000	96.0000	0.0400	100.00
5	100.00000	95.0000	0.0500	100.00
6	100.00000	94.0000	0.0600	100.00
7	100.00000	93.0000	0.0700	100.00
8	100.00000	92.0000	0.0800	100.00
9	100.00000	91.0000	0.0900	100.00
10	100.00000	90.0000	0.1000	100.00
11	100.00000	89.0000	0.1100	100.00
12	100.00000	88.0000	0.1200	100.00
13	100.00000	87.0000	0.1300	100.00
14	100.00000	86.0000	0.1400	100.00
15	100.00000	85.0000	0.1500	100.00
16	100.00000	84.0000	0.1600	100.00
17	100.00000	83.0000	0.1700	100.00
18	100.00000	82.0000	0.1800	100.00
19	100.00000	81.0000	0.1900	100.00
20	100.00000	80.0000	0.2000	100.00
21	100.00000	79.0000	0.2100	100.00
22	100.00000	78.0000	0.2200	100.00
23	100.00000	77.0000	0.2300	100.00
24	100.00000	76.0000	0.2400	100.00
25	100.00000	75.0000	0.2500	100.00
26	100.00000	74.0000	0.2600	100.00
27	100.00000	73.0000	0.2700	100.00
28	100.00000	72.0000	0.2800	100.00
29	100.00000	71.0000	0.2900	100.00
30	100.00000	70.0000	0.3000	100.00
31	100.00000	69.0000	0.3100	100.00
32	100.00000	68.0000	0.3200	100.00
33	100.00000	67.0000	0.3300	100.00
34	100.00000	66.0000	0.3400	100.00
35	100.00000	65.0000	0.3500	100.00
36	100.00000	64.0000	0.3600	100.00
37	100.00000	63.0000	0.3700	100.00
38	100.00000	62.0000	0.3800	100.00
39	100.00000	61.0000	0.3900	100.00
40	100.00000	60.0000	0.4000	100.00
41	100.00000	59.0000	0.4100	100.00
42	100.00000	58.0000	0.4200	100.00
43	100.00000	57.0000	0.4300	100.00
44	100.00000	56.0000	0.4400	100.00
45	100.00000	55.0000	0.4500	100.00
46	100.00000	54.0000	0.4600	100.00
47	100.00000	53.0000	0.4700	100.00
48	100.00000	52.0000	0.4800	100.00
49	100.00000	51.0000	0.4900	100.00
50	100.00000	50.0000	0.5000	100.00
51	100.00000	49.0000	0.5100	100.00
52	100.00000	48.0000	0.5200	100.00
53	100.00000	47.0000	0.5300	100.00
54	100.00000	46.0000	0.5400	100.00
55	100.00000	45.0000	0.5500	100.00
56	100.00000	44.0000	0.5600	100.00
57	100.00000	43.0000	0.5700	100.00
58	100.00000	42.0000	0.5800	100.00
59	100.00000	41.0000	0.5900	100.00
60	100.00000	40.0000	0.6000	100.00
61	100.00000	39.0000	0.6100	100.00
62	100.00000	38.0000	0.6200	100.00
63	99.99999	37.0000	0.6300	100.00
64	99.99998	36.0000	0.6400	100.00
65	99.99996	35.0000	0.6500	100.00
66	99.99991	34.0000	0.6600	100.00
67	99.99981	33.0001	0.6700	100.00
68	99.99960	32.0001	0.6800	100.00
69	99.99917	31.0003	0.6900	100.00
70	99.99835	30.0005	0.7000	100.00
71	99.99682	29.0010	0.7100	100.00
72	99.99405	28.0018	0.7200	100.00
73	99.98917	27.0031	0.7300	100.00
74	99.98079	26.0053	0.7399	100.01
75	99.96681	25.0089	0.7499	100.01
76	99.94405	24.0145	0.7599	100.01
77	99.90793	23.0230	0.7698	100.02
78	99.85194	22.0356	0.7796	100.04
79	99.76716	21.0539	0.7895	100.05
80	99.64167	20.0798	0.7992	100.08
81	99.45998	19.1156	0.8088	100.12
82	99.20251	18.1639	0.8184	100.16
83	98.84525	17.2277	0.8277	100.23
84	98.35961	16.3103	0.8369	100.31
85	97.71260	15.4150	0.8459	100.41
86	96.86741	14.5451	0.8545	100.55
87	95.78452	13.7039	0.8630	100.70
88	94.42320	12.8943	0.8711	100.89
89	92.74358	12.1187	0.8788	101.12
90	90.70910	11.3793	0.8862	101.38
91	88.28921	10.6775	0.8932	101.68
92	85.46213	10.0142	0.8999	102.01
93	82.21749	9.3897	0.9061	102.39
94	78.55859	8.8037	0.9120	102.80
95	74.50386	8.2556	0.9174	103.26
96	[illegible]	[illegible]	[illegible]	[illegible]

Iowa Type curve S6

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
100	50.00000	6.0351	0.9396	106.04
101	44.76385	5.6826	0.9432	106.68
102	39.61599	5.3560	0.9464	107.36
103	34.64028	5.0535	0.9495	108.05
104	29.91234	4.7733	0.9523	108.77
105	25.49614	4.5135	0.9549	109.51
106	21.44141	4.2724	0.9573	110.27
107	17.78251	4.0486	0.9595	111.05
108	14.53787	3.8406	0.9616	111.84
109	11.71079	3.6471	0.9635	112.65
110	9.29090	3.4668	0.9653	113.47
111	7.25642	3.2986	0.9670	114.30
112	5.57680	3.1415	0.9686	115.14
113	4.21548	2.9945	0.9701	115.99
114	3.13259	2.8568	0.9714	116.86
115	2.28740	2.7276	0.9727	117.73
116	1.64039	2.6063	0.9739	118.61
117	1.15475	2.4921	0.9751	119.49
118	0.79749	2.3845	0.9762	120.38
119	0.54002	2.2830	0.9772	121.28
120	0.35833	2.1870	0.9781	122.19
121	0.23284	2.0963	0.9790	123.10
122	0.14806	2.0103	0.9799	124.01
123	0.09207	1.9288	0.9807	124.93
124	0.05595	1.8512	0.9815	125.85
125	0.03319	1.7778	0.9822	126.78
126	0.01921	1.7077	0.9829	127.71
127	0.01084	1.6402	0.9836	128.64
128	0.00595	1.5773	0.9842	129.58
129	0.00318	1.5157	0.9848	130.52
130	0.00165	1.4575	0.9854	131.46
131	0.00083	1.4036	0.9860	132.40
132	0.00040	1.3749	0.9863	133.37
133	0.00019	1.3419	0.9866	134.34
134	0.00009	1.2775	0.9872	135.28
135	0.00004	1.2494	0.9875	136.25
136	0.00002	0.9994	0.9900	137.00
137	0.00001	0.4994	0.9950	137.50
138	0.00000	0.0000	1.0000	138.00

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Table C

Iowa Type curve R1				
Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
0	100.00000	100.0000	0.0000	100.00
1	99.74192	99.2593	0.0074	100.26
2	99.48020	98.5191	0.0148	100.52
3	99.21484	97.7812	0.0222	100.78
4	98.94584	97.0457	0.0295	101.05
5	98.67319	96.3125	0.0369	101.31
6	98.39691	95.5815	0.0442	101.58
7	98.11700	94.8528	0.0515	101.85
8	97.83345	94.1262	0.0587	102.13
9	97.54628	93.4019	0.0660	102.40
10	97.25548	92.6796	0.0732	102.68
11	96.96107	91.9595	0.0804	102.96
12	96.66305	91.2415	0.0876	103.24
13	96.36142	90.5255	0.0947	103.53
14	96.05620	89.8116	0.1019	103.81
15	95.74739	89.0997	0.1090	104.10
16	95.43500	88.3897	0.1161	104.39
17	95.11905	87.6816	0.1232	104.68
18	94.79953	86.9755	0.1302	104.98
19	94.47648	86.2711	0.1373	105.27
20	94.14988	85.5687	0.1443	105.57
21	93.81977	84.8680	0.1513	105.87
22	93.48615	84.1691	0.1583	106.17
23	93.14904	83.4719	0.1653	106.47
24	92.80845	82.7764	0.1722	106.78
25	92.46439	82.0825	0.1792	107.08
26	92.11689	81.3903	0.1861	107.39
27	91.76597	80.6996	0.1930	107.70
28	91.41163	80.0105	0.1999	108.01
29	91.05389	79.3229	0.2068	108.32
30	90.69276	78.6367	0.2136	108.64
31	90.32821	77.9521	0.2205	108.95
32	89.96022	77.2689	0.2273	109.27
33	89.58873	76.5872	0.2341	109.59
34	89.21370	75.9071	0.2409	109.91
35	88.83506	75.2285	0.2477	110.23
36	88.45275	74.5515	0.2545	110.55
37	88.06668	73.8761	0.2612	110.88
38	87.67678	73.2024	0.2680	111.20
39	87.28297	72.5305	0.2747	111.53
40	86.88515	71.8603	0.2814	111.86
41	86.48324	71.1919	0.2881	112.19
42	86.07713	70.5254	0.2947	112.53
43	85.66673	69.8609	0.3014	112.86
44	85.25194	69.1984	0.3080	113.20
45	84.83266	68.5379	0.3146	113.54
46	84.40879	67.8795	0.3212	113.88
47	83.98023	67.2234	0.3278	114.22
48	83.54688	66.5695	0.3343	114.57
49	83.10863	65.9179	0.3408	114.92
50	82.66538	65.2687	0.3473	115.27
51	82.21704	64.6218	0.3538	115.62
52	81.76351	63.9775	0.3602	115.98
53	81.30468	63.3357	0.3666	116.34
54	80.84048	62.6966	0.3730	116.70
55	80.37080	62.0600	0.3794	117.06
56	79.89555	61.4262	0.3857	117.43
57	79.41465	60.7952	0.3920	117.80
58	78.92801	60.1669	0.3983	118.17
59	78.43556	59.5415	0.4046	118.54
60	77.93721	58.9190	0.4108	118.92
61	77.43289	58.2995	0.4170	119.30
62	76.92254	57.6830	0.4232	119.68
63	76.40608	57.0695	0.4293	120.07
64	75.88346	56.4591	0.4354	120.46
65	75.35463	55.8518	0.4415	120.85
66	74.81952	55.2477	0.4475	121.25
67	74.27809	54.6468	0.4535	121.65
68	73.73030	54.0491	0.4595	122.05
69	73.17611	53.4546	0.4655	122.45
70	72.61549	52.8635	0.4714	122.86
71	72.04841	52.2756	0.4772	123.28
72	71.47486	51.6911	0.4831	123.69
73	70.89482	51.1099	0.4889	124.11
74	70.30827	50.5321	0.4947	124.53
75	69.71523	49.9577	0.5004	124.96
76	69.11567	49.3868	0.5061	125.39
77	58.50963	48.8192	0.5118	125.82
78	67.89711	48.2551	0.5174	126.26
79	67.27813	47.6945	0.5231	126.69
80	66.65272	47.1373	0.5286	127.14
81	66.02091	46.5836	0.5342	127.58
82	65.38275	46.0334	0.5397	128.03
83	64.73828	45.4867	0.5451	128.49
84	64.08756	44.9435	0.5506	128.94
85	63.43064	44.4038	0.5560	129.40
86	62.76760	43.8675	0.5613	129.87
87	62.09850	43.3348	0.5667	130.33
88	61.42343	42.8056	0.5719	130.81
89	60.74247	42.2799	0.5772	131.28
90	60.05571	41.7576	0.5824	131.76
91	59.36327	41.2389	0.5876	132.24
92	58.66524	40.7236	0.5928	132.72
93	57.96175	40.2118	0.5979	133.21
94	57.25290	39.7035	0.6030	133.70
95	56.53884	39.1986	0.6080	134.20

Iowa Type curve R1				
Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
100	52.89519	36.7257	0.6327	136.73
101	52.15288	36.2413	0.6376	137.24
102	51.40644	35.7603	0.6424	137.76
103	50.65605	35.2826	0.6472	138.28
104	49.90189	34.8082	0.6519	138.81
105	49.14417	34.3372	0.6566	139.34
106	48.38308	33.8695	0.6613	139.87
107	47.61883	33.4051	0.6659	140.41
108	46.85164	32.9439	0.6706	140.94
109	46.08172	32.4859	0.6751	141.49
110	45.30930	32.0312	0.6797	142.03
111	44.53462	31.5797	0.6842	142.58
112	43.75790	31.1314	0.6887	143.13
113	42.97940	30.6862	0.6931	143.69
114	42.19937	30.2442	0.6976	144.24
115	41.41805	29.8053	0.7019	144.81
116	40.63572	29.3695	0.7063	145.37
117	39.85262	28.9368	0.7106	145.94
118	39.06904	28.5071	0.7149	146.51
119	38.28526	28.0805	0.7192	147.08
120	37.50154	27.6569	0.7234	147.66
121	36.71818	27.2362	0.7276	148.24
122	35.93546	26.8186	0.7318	148.82
123	35.15368	26.4039	0.7360	149.40
124	34.37314	25.9921	0.7401	149.99
125	33.59414	25.5832	0.7442	150.58
126	32.81698	25.1772	0.7482	151.18
127	32.04198	24.7741	0.7523	151.77
128	31.26944	24.3738	0.7563	152.37
129	30.49969	23.9763	0.7602	152.98
130	29.73303	23.5817	0.7642	153.58
131	28.96980	23.1898	0.7681	154.19
132	28.21032	22.8006	0.7720	154.80
133	27.45491	22.4142	0.7759	155.41
134	26.70390	22.0305	0.7797	156.03
135	25.95763	21.6495	0.7835	156.65
136	25.21641	21.2712	0.7873	157.27
137	24.48060	20.8955	0.7910	157.90
138	23.75051	20.5225	0.7948	158.52
139	23.02648	20.1521	0.7985	159.15
140	22.30885	19.7842	0.8022	159.78
141	21.59795	19.4190	0.8058	160.42
142	20.89412	19.0563	0.8094	161.06
143	20.19768	18.6961	0.8130	161.70
144	19.50898	18.3385	0.8166	162.34
145	18.82834	17.9833	0.8202	162.98
146	18.15608	17.6307	0.8237	163.63
147	17.49255	17.2805	0.8272	164.28
148	16.83806	16.9327	0.8307	164.93
149	16.19292	16.5874	0.8341	165.59
150	15.55747	16.2445	0.8376	166.24
151	14.93201	15.9040	0.8410	166.90
152	14.31686	15.5659	0.8443	167.57
153	13.71231	15.2301	0.8477	168.23
154	13.11867	14.8966	0.8510	168.90
155	12.53623	14.5655	0.8543	169.57
156	11.96528	14.2367	0.8576	170.24
157	11.40609	13.9101	0.8609	170.91
158	10.85895	13.5858	0.8641	171.59
159	10.32411	13.2637	0.8674	172.26
160	9.80182	12.9438	0.8706	172.94
161	9.29235	12.6261	0.8737	173.63
162	8.79592	12.3105	0.8769	174.31
163	8.31275	11.9970	0.8800	175.00
164	7.84307	11.6854	0.8831	175.69
165	7.38707	11.3759	0.8862	176.38
166	6.94493	11.0683	0.8893	177.07
167	6.51684	10.7626	0.8924	177.76
168	6.10295	10.4585	0.8954	178.46
169	5.70340	10.1562	0.8984	179.16
170	5.31832	9.8553	0.9014	179.86
171	4.94781	9.5559	0.9044	180.56
172	4.59195	9.2577	0.9074	181.26
173	4.25081	8.9605	0.9104	181.96
174	3.92444	8.6642	0.9134	182.66
175	3.61284	8.3683	0.9163	183.37
176	3.31601	8.0726	0.9193	184.07
177	3.03391	7.7767	0.9222	184.78
178	2.76646	7.4802	0.9252	185.48
179	2.51357	7.1825	0.9282	186.18
180	2.27508	6.8830	0.9312	186.88
181	2.05081	6.5810	0.9342	187.58
182	1.84053	6.2758	0.9372	188.28
183	1.64396	5.9664	0.9403	188.97
184	1.46075	5.6520	0.9435	189.65
185	1.29047	5.3318	0.9467	190.33
186	1.13261	5.0053	0.9499	191.01
187	0.98656	4.6722	0.9533	191.67
188	0.85156	4.3337	0.9567	192.33
189	0.72632	3.9947	0.9601	192.99
190	0.61061	3.6570	0.9634	193.66
191	0.50457	3.3204	0.9668	194.32
192	0.40831	2.9854	0.9701	194.99
193	0.32197	2.6518	0.9735	195.65
194	0.24565	2.3204	0.9768	196.32
195	0.17949	1.9914	0.9801	196.99

Table C continued

Iowa Type curve R1				
Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
200	0.00456	0.5000	0.9950	200.50
201	0.00000	0.0000	1.0000	201.00

Iowa Type curve R2				
Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
0	100.00000	100.0000	0.0000	100.00
1	99.90517	99.0944	0.0091	100.09
2	99.80707	98.1913	0.0181	100.19
3	99.70561	97.2908	0.0271	100.29
4	99.60070	96.3927	0.0361	100.39
5	99.49224	95.4972	0.0450	100.50
6	99.38016	94.6044	0.0540	100.60
7	99.26435	93.7142	0.0629	100.71
8	99.14472	92.8266	0.0717	100.83
9	99.02117	91.9418	0.0806	100.94
10	98.89359	91.0598	0.0894	101.06
11	98.76190	90.1806	0.0982	101.18
12	98.62598	89.3042	0.1070	101.30
13	98.48574	88.4306	0.1157	101.43
14	98.34106	87.5600	0.1244	101.56
15	98.19184	86.6923	0.1331	101.69
16	98.03796	85.8276	0.1417	101.83
17	97.87933	84.9659	0.1503	101.97
18	97.71581	84.1072	0.1589	102.11
19	97.54731	83.2516	0.1675	102.25
20	97.37369	82.3992	0.1760	102.40
21	97.19485	81.5499	0.1845	102.55
22	97.01065	80.7038	0.1930	102.70
23	96.82099	79.8609	0.2014	102.86
24	96.62573	79.0212	0.2098	103.02
25	96.42474	78.1849	0.2182	103.18
26	96.21790	77.3519	0.2265	103.35
27	96.00508	76.5223	0.2348	103.52
28	95.78615	75.6960	0.2430	103.70
29	95.56096	74.8732	0.2513	103.87
30	95.32938	74.0539	0.2595	104.05
31	95.09128	73.2381	0.2676	104.24
32	94.84651	72.4258	0.2757	104.43
33	94.59492	71.6171	0.2838	104.62
34	94.33638	70.8120	0.2919	104.81
35	94.07074	70.0105	0.2999	105.01
36	93.79784	69.2128	0.3079	105.21
37	93.51754	68.4187	0.3158	105.42
38	93.22967	67.6284	0.3237	105.63
39	92.93409	66.8419	0.3316	105.84
40	92.63064	66.0593	0.3394	106.06
41	92.31914	65.2805	0.3472	106.28
42	91.99945	64.5056	0.3549	106.51
43	91.67141	63.7346	0.3627	106.73
44	91.33483	62.9677	0.3703	106.97
45	90.98955	62.2047	0.3780	107.20
46	90.63541	61.4458	0.3855	107.45
47	90.27222	60.6910	0.3931	107.69
48	89.89982	59.9403	0.4006	107.94
49	89.51803	59.1939	0.4081	108.19
50	89.12668	58.4516	0.4155	108.45
51	88.72557	57.7136	0.4229	108.71
52	88.31454	56.9798	0.4302	108.98
53	87.89341	56.2505	0.4375	109.25
54	87.46198	55.5255	0.4447	109.53
55	87.02008	54.8049	0.4520	109.80
56	86.56753	54.0888	0.4591	110.09
57	86.10413	53.3772	0.4662	110.38
58	85.62971	52.6701	0.4733	110.67
59	85.14408	51.9677	0.4803	110.97
60	84.64707	51.2699	0.4873	111.27
61	84.13847	50.5768	0.4942	111.58
62	83.61813	49.8884	0.5011	111.89
63	83.08585	49.2048	0.5080	112.20
64	82.54146	48.5260	0.5147	112.53
65	81.98479	47.8521	0.5215	112.85
66	81.41567	47.1831	0.5282	113.18
67	80.83392	46.5191	0.5348	113.52
68	80.23938	45.8601	0.5414	113.86
69	79.63191	45.2061	0.5479	114.21
70	79.01133	44.5573	0.5544	114.56
71	78.37752	43.9135	0.5609	114.91
72	77.73032	43.2750	0.5672	115.28
73	77.06961	42.6417	0.5736	115.64
74	76.39526	42.0137	0.5799	116.01
75	75.70717	41.3910	0.5861	116.39
76	75.00523	40.7737	0.5923	116.77
77	74.28934	40.1618	0.5984	117.16
78	73.55943	39.5553	0.6044	117.56
79	72.81544	38.9544	0.6105	117.95
80	72.05732	38.3590	0.6164	118.36
81	71.28502	37.7691	0.6223	118.77
82	70.49853	37.1849	0.6282	119.18
83	69.69785	36.6063	0.6339	119.61
84	68.88299	36.0335	0.6397	120.03
85	68.05401	35.4663	0.6453	120.47
86	67.21095	34.9049	0.6510	120.90
87	66.35390	34.3493	0.6565	121.35
88	65.48296	33.7995	0.6620	121.80
89	64.59828	33.2555	0.6674	122.26
90	63.70000	32.7174	0.6728	122.72
91	62.78831	32.1852	0.6781	123.19
92	61.86342	31.6590	0.6834	123.66
93	60.92558	31.1386	0.6886	124.14
94	59.97507	30.6242	0.6938	124.62
95	59.01217	30.1157	0.6988	125.12
96	58.03725	29.6132	0.7039	125.61

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Table C continued

Iowa Type curve R2

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
100	54.02510	27.6629	0.7234	127.66
101	52.99624	27.1902	0.7281	128.19
102	51.95810	26.7235	0.7328	128.72
103	50.91125	26.2627	0.7374	129.26
104	49.85632	25.8078	0.7419	129.81
105	48.79396	25.3588	0.7464	130.36
106	47.72486	24.9157	0.7508	130.92
107	46.64974	24.4784	0.7552	131.48
108	45.56937	24.0469	0.7595	132.05
109	44.48452	23.6211	0.7638	132.62
110	43.39604	23.2011	0.7680	133.20
111	42.30477	22.7867	0.7721	133.79
112	41.21160	22.3778	0.7762	134.38
113	40.11743	21.9745	0.7803	134.97
114	39.02321	21.5767	0.7842	135.58
115	37.92987	21.1842	0.7882	136.18
116	36.83841	20.7971	0.7920	136.80
117	35.74981	20.4151	0.7958	137.42
118	34.66508	20.0383	0.7996	138.04
119	33.58525	19.6665	0.8033	138.67
120	32.51131	19.2996	0.8070	139.30
121	31.44431	18.9375	0.8106	139.94
122	30.38528	18.5801	0.8142	140.58
123	29.33517	18.2274	0.8177	141.23
124	28.29510	17.8790	0.8212	141.88
125	27.26603	17.5349	0.8247	142.53
126	26.24893	17.1950	0.8281	143.19
127	25.24478	16.8590	0.8314	143.86
128	24.25449	16.5270	0.8347	144.53
129	23.27898	16.1986	0.8380	145.20
130	22.31913	15.8737	0.8413	145.87
131	21.37516	15.5526	0.8445	146.55
132	20.44967	15.2339	0.8477	147.23
133	19.54162	14.9185	0.8508	147.92
134	18.65230	14.6060	0.8539	148.61
135	17.78236	14.2961	0.8570	149.30
136	16.93241	13.9886	0.8601	149.99
137	16.10298	13.6833	0.8632	150.68
138	15.29456	13.3802	0.8662	151.38
139	14.50757	13.0789	0.8692	152.08
140	13.74239	12.7793	0.8722	152.78
141	12.99932	12.4812	0.8752	153.48
142	12.27861	12.1844	0.8782	154.18
143	11.58045	11.8889	0.8811	154.89
144	10.90498	11.5943	0.8841	155.59
145	10.25227	11.3006	0.8870	156.30
146	9.62235	11.0077	0.8899	157.01
147	9.01519	10.7154	0.8928	157.72
148	8.43072	10.4235	0.8958	158.42
149	7.86883	10.1322	0.8987	159.13
150	7.32937	9.8411	0.9016	159.84
151	6.81214	9.5504	0.9045	160.55
152	6.31692	9.2599	0.9074	161.26
153	5.84347	8.9696	0.9103	161.97
154	5.39151	8.6796	0.9132	162.68
155	4.96076	8.3899	0.9161	163.39
156	4.55091	8.1004	0.9190	164.10
157	4.16166	7.8113	0.9219	164.81
158	3.79266	7.5226	0.9248	165.52
159	3.44361	7.2345	0.9277	166.23
160	3.11416	6.9469	0.9306	166.95
161	2.80398	6.6601	0.9334	167.66
162	2.51274	6.3740	0.9363	168.37
163	2.24009	6.0890	0.9391	169.09
164	1.98570	5.8050	0.9419	169.81
165	1.74922	5.5222	0.9448	170.52
166	1.53029	5.2407	0.9476	171.24
167	1.32853	4.9607	0.9504	171.96
168	1.14356	4.6822	0.9532	172.68
169	0.97498	4.4053	0.9559	173.41
170	0.82235	4.1301	0.9587	174.13
171	0.68519	3.8568	0.9614	174.86
172	0.56299	3.5854	0.9641	175.59
173	0.45521	3.3159	0.9668	176.32
174	0.36123	3.0485	0.9695	177.05
175	0.28040	2.7832	0.9722	177.78
176	0.21198	2.5201	0.9748	178.52
177	0.15520	2.2592	0.9774	179.26
178	0.10918	2.0007	0.9800	180.00
179	0.07298	1.7451	0.9825	180.75
180	0.04561	1.4923	0.9851	181.49
181	0.02596	1.2435	0.9876	182.24
182	0.01287	0.9996	0.9900	183.00
183	0.00508	0.7657	0.9923	183.77
184	0.00128	0.5547	0.9945	184.55
185	0.00007	0.5003	0.9950	185.50
186	0.00000	0.0000	1.0000	186.00

Iowa Type curve R3

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
0	100.00000	100.0000	0.0000	100.00
1	99.98452	99.0154	0.0098	100.02
2	99.96768	98.0320	0.0197	100.03
3	99.94939	97.0498	0.0295	100.05
4	99.92954	96.0690	0.0393	100.07
5	99.90802	95.0896	0.0491	100.09
6	99.88474	94.1117	0.0589	100.11
7	99.85958	93.1352	0.0686	100.14
8	99.83241	92.1605	0.0784	100.16
9	99.80311	91.1874	0.0881	100.19
10	99.77155	90.2161	0.0978	100.22
11	99.73759	89.2456	0.1075	100.25
12	99.70109	88.2791	0.1172	100.28
13	99.66191	87.3136	0.1269	100.31
14	99.61990	86.3502	0.1365	100.35
15	99.57488	85.3890	0.1461	100.39
16	99.52671	84.4301	0.1557	100.43
17	99.47520	83.4736	0.1653	100.47
18	99.42019	82.5195	0.1748	100.52
19	99.36149	81.5679	0.1843	100.57
20	99.29891	80.6190	0.1938	100.62
21	99.23227	79.6728	0.2033	100.67
22	99.16136	78.7294	0.2127	100.73
23	99.08598	77.7890	0.2221	100.79
24	99.00592	76.8515	0.2315	100.85
25	98.92096	75.9170	0.2408	100.92
26	98.83088	74.9858	0.2501	100.99
27	98.73545	74.0578	0.2594	101.06
28	98.63445	73.1331	0.2687	101.13
29	98.52763	72.2118	0.2779	101.21
30	98.41475	71.2941	0.2871	101.29
31	98.29557	70.3799	0.2962	101.38
32	98.16982	69.4694	0.3053	101.47
33	98.03725	68.5627	0.3144	101.56
34	97.89760	67.6598	0.3234	101.66
35	97.75059	66.7608	0.3324	101.76
36	97.59596	65.8658	0.3413	101.87
37	97.43343	64.9748	0.3503	101.97
38	97.26271	64.0880	0.3591	102.09
39	97.08351	63.2054	0.3679	102.21
40	96.89555	62.3270	0.3767	102.33
41	96.69853	61.4530	0.3855	102.45
42	96.49214	60.5833	0.3942	102.58
43	96.27608	59.7182	0.4028	102.72
44	96.05003	58.8575	0.4114	102.86
45	95.81369	58.0015	0.4200	103.00
46	95.56673	57.1501	0.4285	103.15
47	95.30882	56.3034	0.4370	103.30
48	95.03964	55.4614	0.4454	103.46
49	94.75883	54.6243	0.4538	103.62
50	94.46607	53.7920	0.4621	103.79
51	94.16099	52.9647	0.4704	103.96
52	93.84325	52.1423	0.4786	104.14
53	93.51246	51.3250	0.4867	104.33
54	93.16827	50.5128	0.4949	104.51
55	92.81029	49.7057	0.5029	104.71
56	92.43813	48.9038	0.5110	104.90
57	92.05139	48.1072	0.5189	105.11
58	91.64965	47.3158	0.5268	105.32
59	91.23249	46.5299	0.5347	105.53
60	90.79948	45.7494	0.5425	105.75
61	90.35018	44.9744	0.5503	105.97
62	89.88412	44.2050	0.5579	106.21
63	89.40083	43.4413	0.5656	106.44
64	88.89983	42.6833	0.5732	106.68
65	88.38061	41.9311	0.5807	106.93
66	87.84267	41.1848	0.5882	107.18
67	87.28547	40.4446	0.5956	107.44
68	86.70847	39.7104	0.6029	107.71
69	86.11111	38.9824	0.6102	107.98
70	85.49283	38.2607	0.6174	108.26
71	84.85304	37.5454	0.6245	108.55
72	84.19116	36.8366	0.6316	108.84
73	83.50659	36.1345	0.6387	109.13
74	82.79872	35.4392	0.6456	109.44
75	82.06693	34.7507	0.6525	109.75
76	81.31063	34.0693	0.6593	110.07
77	80.52919	33.3950	0.6660	110.40
78	79.72202	32.7281	0.6727	110.73
79	78.88853	32.0686	0.6793	111.07
80	78.02814	31.4167	0.6858	111.42
81	77.14030	30.7725	0.6923	111.77
82	76.22450	30.1362	0.6986	112.14
83	75.28024	29.5080	0.7049	112.51
84	74.30707	28.8879	0.7111	112.89
85	73.30461	28.2761	0.7172	113.28
86	72.27252	27.6727	0.7233	113.67
87	71.21053	27.0780	0.7292	114.08
88	70.11845	26.4919	0.7351	114.49
89	68.99618	25.9147	0.7409	114.91
90	67.84370	25.3464	0.7465	115.35
91	66.66112	24.7872	0.7521	115.79
92	65.44864	24.2371	0.7576	116.24
93	64.20660	23.6963	0.7630	116.70
94	62.93546	23.1648	0.7684	117.16
95	61.63582	22.6427	0.7736	117.64

Table C continued

Iowa Type curve R3

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
100	54.74208	20.1748	0.7983	120.17
101	53.29286	19.7098	0.8029	120.71
102	51.82380	19.2543	0.8075	121.25
103	50.33681	18.8084	0.8119	121.81
104	48.83395	18.3718	0.8163	122.37
105	47.31746	17.9446	0.8206	122.94
106	45.78970	17.5266	0.8247	123.53
107	44.25318	17.1178	0.8288	124.12
108	42.71054	16.7180	0.8328	124.72
109	41.16453	16.3271	0.8367	125.33
110	39.61801	15.9449	0.8406	125.94
111	38.07390	15.5713	0.8443	126.57
112	36.53519	15.2060	0.8479	127.21
113	35.00491	14.8489	0.8515	127.85
114	33.48610	14.4997	0.8550	128.50
115	31.98179	14.1582	0.8584	129.16
116	30.49498	13.8242	0.8618	129.82
117	29.02861	13.4972	0.8650	130.50
118	27.58555	13.1771	0.8682	131.18
119	26.16856	12.8636	0.8714	131.86
120	24.78027	12.5562	0.8744	132.56
121	23.42315	12.2548	0.8775	133.25
122	22.09951	11.9588	0.8804	133.96
123	20.81147	11.6680	0.8833	134.67
124	19.56094	11.3820	0.8862	135.38
125	18.34962	11.1003	0.8890	136.10
126	17.17897	10.8227	0.8918	136.82
127	16.05021	10.5486	0.8945	137.55
128	14.96433	10.2778	0.8972	138.28
129	13.92208	10.0098	0.8999	139.01
130	12.92397	9.7443	0.9026	139.74
131	11.97027	9.4808	0.9052	140.48
132	11.06104	9.2190	0.9078	141.22
133	10.19612	8.9586	0.9104	141.96
134	9.37516	8.6993	0.9130	142.70
135	8.59763	8.4408	0.9156	143.44
136	7.86284	8.1829	0.9182	144.18
137	7.16997	7.9253	0.9207	144.93
138	6.51807	7.6680	0.9233	145.67
139	5.90611	7.4107	0.9259	146.41
140	5.33296	7.1534	0.9285	147.15
141	4.79745	6.8961	0.9310	147.90
142	4.29836	6.6388	0.9336	148.64
143	3.83446	6.3814	0.9362	149.38
144	3.40449	6.1242	0.9388	150.12
145	3.00720	5.8673	0.9413	150.87
146	2.64136	5.6107	0.9439	151.61
147	2.30572	5.3546	0.9465	152.35
148	1.99908	5.0993	0.9490	153.10
149	1.72022	4.8448	0.9516	153.84
150	1.46794	4.5915	0.9541	154.59
151	1.24106	4.3395	0.9566	155.34
152	1.03836	4.0890	0.9591	156.09
153	0.85865	3.8402	0.9616	156.84
154	0.70069	3.5932	0.9641	157.59
155	0.56322	3.3481	0.9665	158.35
156	0.44495	3.1052	0.9689	159.11
157	0.34453	2.8645	0.9714	159.86
158	0.26059	2.6262	0.9737	160.63
159	0.19170	2.3902	0.9761	161.39
160	0.13638	2.1570	0.9784	162.16
161	0.09313	1.9265	0.9807	162.93
162	0.06042	1.6988	0.9830	163.70
163	0.03668	1.4746	0.9853	164.47
164	0.02038	1.2542	0.9875	165.25
165	0.00998	1.0401	0.9896	166.04
166	0.00404	0.8341	0.9917	166.83
167	0.00118	0.6440	0.9936	167.64
168	0.00017	0.4996	0.9950	168.50
169	0.00000	0.0000	1.0000	169.00

Iowa Type curve R4

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
0	100.00000	100.0000	0.0000	100.00
1	99.99917	99.0013	0.0100	100.00
2	99.99821	98.0023	0.0200	100.00
3	99.99710	97.0033	0.0300	100.00
4	99.99581	96.0046	0.0400	100.00
5	99.99432	95.0060	0.0499	100.01
6	99.99261	94.0076	0.0699	100.01
7	99.99064	93.0095	0.0699	100.01
8	99.98839	92.0116	0.0799	100.01
9	99.98581	91.0139	0.0899	100.01
10	99.98287	90.0166	0.0998	100.02
11	99.97952	89.0196	0.1098	100.02
12	99.97570	88.0230	0.1198	100.02
13	99.97137	87.0267	0.1297	100.03
14	99.96646	86.0310	0.1397	100.03
15	99.96090	85.0358	0.1496	100.04
16	99.95463	84.0411	0.1596	100.04
17	99.94755	83.0470	0.1695	100.05
18	99.93959	82.0536	0.1795	100.05
19	99.93064	81.0609	0.1894	100.06
20	99.92061	80.0689	0.1993	100.07
21	99.90936	79.0779	0.2092	100.08
22	99.89679	78.0878	0.2191	100.09
23	99.88275	77.0987	0.2290	100.10
24	99.86710	76.1107	0.2389	100.11
25	99.84968	75.1239	0.2488	100.12
26	99.83032	74.1384	0.2586	100.14
27	99.80883	73.1542	0.2685	100.15
28	99.78501	72.1716	0.2783	100.17
29	99.75865	71.1905	0.2881	100.19
30	99.72953	70.2111	0.2979	100.21
31	99.69738	69.2336	0.3077	100.23
32	99.66196	68.2580	0.3174	100.26
33	99.62298	67.2846	0.3272	100.28
34	99.58013	66.3133	0.3369	100.31
35	99.53311	65.3444	0.3466	100.34
36	99.48157	64.3780	0.3562	100.38
37	99.42514	63.4142	0.3659	100.41
38	99.36345	62.4533	0.3755	100.45
39	99.29610	61.4953	0.3850	100.50
40	99.22264	60.5405	0.3946	100.54
41	99.14264	59.5889	0.4041	100.59
42	99.05561	58.6408	0.4136	100.64
43	98.96106	57.6964	0.4230	100.70
44	98.85847	56.7557	0.4324	100.76
45	98.74727	55.8191	0.4418	100.82
46	98.62690	54.8866	0.4511	100.89
47	98.49675	53.9585	0.4604	100.96
48	98.35620	53.0349	0.4697	101.03
49	98.20459	52.1160	0.4788	101.12
50	98.04125	51.2020	0.4880	101.20
51	97.86547	50.2930	0.4971	101.29
52	97.67652	49.3894	0.5061	101.39
53	97.47365	48.4911	0.5151	101.49
54	97.25609	47.5985	0.5240	101.60
55	97.02304	46.7116	0.5329	101.71
56	96.77368	45.8307	0.5417	101.83
57	96.50717	44.9559	0.5504	101.96
58	96.22265	44.0873	0.5591	102.09
59	95.91925	43.2252	0.5677	102.23
60	95.59608	42.3696	0.5763	102.37
61	95.25224	41.5207	0.5848	102.52
62	94.88682	40.6787	0.5932	102.68
63	94.49889	39.8437	0.6016	102.84
64	94.08753	39.0157	0.6098	103.02
65	93.65182	38.1949	0.6181	103.19
66	93.19084	37.3813	0.6262	103.38
67	92.70366	36.5752	0.6342	103.58
68	92.18937	35.7764	0.6422	103.78
69	91.64709	34.9851	0.6501	103.99
70	91.07592	34.2014	0.6580	104.20
71	90.47502	33.4252	0.6657	104.43
72	89.84356	32.6566	0.6734	104.66
73	89.18073	31.8957	0.6810	104.90
74	88.48579	31.1422	0.6886	105.14
75	87.75801	30.3963	0.6960	105.40
76	86.99674	29.6579	0.7034	105.66
77	86.20134	28.9270	0.7107	105.93
78	85.37129	28.2034	0.7180	106.20
79	84.50608	27.4870	0.7251	106.49
80	83.60532	26.7778	0.7322	106.78
81	82.66866	26.0755	0.7392	107.08
82	81.69587	25.3801	0.7462	107.38
83	80.68667	24.6912	0.7531	107.69
84	79.64019	24.0091	0.7599	108.01
85	78.55460	23.3340	0.7667	108.33
86	77.42712	22.6665	0.7733	108.67
87	76.25416	22.0075	0.7799	109.01
88	75.03161	21.3579	0.7864	109.36
89	73.75505	20.7189	0.7928	109.72
90	72.42015	20.0916	0.7991	110.09
91	71.02281	19.4771	0.8052	110.48
92	69.55955	18.8763	0.8112	110.88
93	68.02768	18.2901	0.8171	111.29
94	66.42545	17.7192	0.8228	111.72
95	64.75218	17.1642	0.8284	112.16

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Table C continued

Iowa Type curve R4

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
100	55.38004	14.6344	0.8537	114.63
101	53.33268	14.1770	0.8582	115.18
102	51.24196	13.7350	0.8626	115.74
103	49.11568	13.3080	0.8669	116.31
104	46.96222	12.8953	0.8710	116.90
105	44.79042	12.4963	0.8750	117.50
106	42.60935	12.1104	0.8789	118.11
107	40.42821	11.7368	0.8826	118.74
108	38.25614	11.3747	0.8863	119.37
109	36.10212	11.0236	0.8898	120.02
110	33.97478	10.6825	0.8932	120.68
111	31.88233	10.3508	0.8965	121.35
112	29.83244	10.0277	0.8997	122.03
113	27.83215	9.7124	0.9029	122.71
114	25.88782	9.4043	0.9060	123.40
115	24.00508	9.1027	0.9090	124.10
116	22.18878	8.8069	0.9119	124.81
117	20.44302	8.5163	0.9148	125.52
118	18.77112	8.2303	0.9177	126.23
119	17.17567	7.9483	0.9205	126.95
120	15.65853	7.6700	0.9233	127.67
121	14.22090	7.3948	0.9261	128.39
122	12.86339	7.1225	0.9288	129.12
123	11.58601	6.8526	0.9315	129.85
124	10.38833	6.5850	0.9341	130.59
125	9.26945	6.3195	0.9368	131.32
126	8.22813	6.0560	0.9394	132.06
127	7.26282	5.7945	0.9421	132.79
128	6.37174	5.5349	0.9447	133.53
129	5.55286	5.2774	0.9472	134.28
130	4.80402	5.0221	0.9498	135.02
131	4.12291	4.7691	0.9523	135.77
132	3.50709	4.5187	0.9548	136.52
133	2.95402	4.2712	0.9573	137.27
134	2.46101	4.0266	0.9597	138.03
135	2.02527	3.7854	0.9621	138.79
136	1.64387	3.5476	0.9645	139.55
137	1.31370	3.3136	0.9669	140.31
138	1.03149	3.0834	0.9692	141.08
139	0.79375	2.8572	0.9714	141.86
140	0.59682	2.6349	0.9737	142.63
141	0.43685	2.4167	0.9758	143.42
142	0.30983	2.2025	0.9780	144.20
143	0.21164	1.9924	0.9801	144.99
144	0.13815	1.7863	0.9821	145.79
145	0.08525	1.5845	0.9842	146.58
146	0.04899	1.3871	0.9861	147.39
147	0.02564	1.1950	0.9880	148.20
148	0.01181	1.0089	0.9899	149.01
149	0.00451	0.8326	0.9917	149.83
150	0.00128	0.6719	0.9933	150.67
151	0.00021	0.5480	0.9945	151.55
152	0.00001	0.5087	0.9949	152.51
153	0.00000	0.0000	1.0000	153.00

Iowa Type curve R5

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
0	100.00000	100.0000	0.0000	100.00
1	100.00000	98.9993	0.0100	100.00
2	100.00000	97.9993	0.0200	100.00
3	100.00000	96.9993	0.0300	100.00
4	100.00000	95.9993	0.0400	100.00
5	100.00000	94.9993	0.0500	100.00
6	100.00000	93.9993	0.0600	100.00
7	100.00000	92.9993	0.0700	100.00
8	100.00000	91.9993	0.0800	100.00
9	100.00000	90.9993	0.0900	100.00
10	100.00000	89.9993	0.1000	100.00
11	100.00000	88.9993	0.1100	100.00
12	100.00000	87.9993	0.1200	100.00
13	100.00000	86.9993	0.1300	100.00
14	100.00000	85.9993	0.1400	100.00
15	100.00000	84.9993	0.1500	100.00
16	100.00000	83.9993	0.1600	100.00
17	100.00000	82.9993	0.1700	100.00
18	100.00000	81.9993	0.1800	100.00
19	100.00000	80.9993	0.1900	100.00
20	100.00000	79.9993	0.2000	100.00
21	100.00000	78.9993	0.2100	100.00
22	100.00000	77.9993	0.2200	100.00
23	100.00000	76.9993	0.2300	100.00
24	100.00000	75.9993	0.2400	100.00
25	100.00000	74.9993	0.2500	100.00
26	100.00000	73.9993	0.2600	100.00
27	100.00000	72.9993	0.2700	100.00
28	100.00000	71.9993	0.2800	100.00
29	100.00000	70.9993	0.2900	100.00
30	100.00000	69.9993	0.3000	100.00
31	100.00000	68.9993	0.3100	100.00
32	100.00000	67.9993	0.3200	100.00
33	100.00000	66.9993	0.3300	100.00
34	100.00000	65.9993	0.3400	100.00
35	99.99998	64.9993	0.3500	100.00
36	99.99995	63.9993	0.3600	100.00
37	99.99989	62.9993	0.3700	100.00
38	99.99977	61.9994	0.3800	100.00
39	99.99954	60.9996	0.3900	100.00
40	99.99916	59.9998	0.4000	100.00
41	99.99854	59.0002	0.4100	100.00
42	99.99758	58.0007	0.4200	100.00
43	99.99612	57.0016	0.4300	100.00
44	99.99401	56.0027	0.4400	100.00
45	99.99100	55.0044	0.4500	100.00
46	99.98684	54.0067	0.4599	100.01
47	99.98120	53.0097	0.4699	100.01
48	99.97368	52.0137	0.4799	100.01
49	99.96384	51.0187	0.4898	100.02
50	99.95114	50.0251	0.4997	100.03
51	99.93499	49.0331	0.5097	100.03
52	99.91470	48.0430	0.5196	100.04
53	99.88951	47.0550	0.5295	100.05
54	99.85855	46.0694	0.5393	100.07
55	99.82089	45.0866	0.5491	100.09
56	99.77649	44.1069	0.5589	100.11
57	99.72121	43.1306	0.5687	100.13
58	99.65684	42.1582	0.5784	100.16
59	99.58107	41.1899	0.5881	100.19
60	99.49249	40.2261	0.5977	100.23
61	99.38962	39.2672	0.6073	100.27
62	99.27089	38.3136	0.6169	100.31
63	99.13466	37.3655	0.6263	100.37
64	98.97921	36.4234	0.6358	100.42
65	98.80275	35.4876	0.6451	100.49
66	98.60337	34.5584	0.6544	100.56
67	98.37911	33.6360	0.6636	100.64
68	98.12785	32.7208	0.6728	100.72
69	97.84733	31.8132	0.6819	100.81
70	97.53512	30.9134	0.6909	100.91
71	97.18856	30.0219	0.6998	101.02
72	96.80476	29.1389	0.7086	101.14
73	96.38056	28.2650	0.7174	101.26
74	95.91248	27.4005	0.7260	101.40
75	95.39673	26.5459	0.7345	101.55
76	94.82918	25.7018	0.7430	101.70
77	94.20535	24.8687	0.7513	101.87
78	93.52043	24.0472	0.7595	102.05
79	92.76930	23.2378	0.7676	102.24
80	91.94656	22.4413	0.7756	102.44
81	91.04658	21.6582	0.7834	102.66
82	90.06358	20.8891	0.7911	102.89
83	88.99168	20.1347	0.7987	103.13
84	87.82505	19.3955	0.8060	103.40
85	86.55794	18.6721	0.8133	103.67
86	85.18490	17.9650	0.8203	103.97
87	83.70084	17.2747	0.8273	104.27
88	82.10122	16.6015	0.8340	104.60
89	80.38218	15.9458	0.8405	104.95
90	78.54068	15.3080	0.8469	105.31
91	76.57472	14.6882	0.8531	105.69
92	74.48341	14.0865	0.8591	106.09
93	72.26719	13.5032	0.8650	106.50
94	69.92791	12.9382	0.8706	106.94
95	67.46899	12.3915	0.8761	107.39

Table C continued

Iowa Type curve R5

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
100	53.61839	9.9293	0.9007	109.93
101	50.60957	9.4898	0.9051	110.49
102	47.55343	9.0676	0.9093	111.07
103	44.46694	8.6623	0.9134	111.66
104	41.36834	8.2737	0.9173	112.27
105	38.27696	7.9015	0.9210	112.90
106	35.21285	7.5455	0.9245	113.55
107	32.19655	7.2056	0.9279	114.21
108	29.24865	6.8814	0.9312	114.88
109	26.38945	6.5728	0.9343	115.57
110	23.63847	6.2796	0.9372	116.28
111	21.01406	6.0014	0.9400	117.00
112	18.53286	5.7379	0.9426	117.74
113	16.20943	5.4887	0.9451	118.49
114	14.05572	5.2531	0.9475	119.25
115	12.08071	5.0302	0.9497	120.03
116	10.29003	4.8185	0.9518	120.82
117	8.68570	4.6162	0.9538	121.62
118	7.26591	4.4205	0.9558	122.42
119	6.02505	4.2280	0.9577	123.23
120	4.95374	4.0342	0.9597	124.03
121	4.03925	3.8343	0.9617	124.83
122	3.26602	3.6237	0.9638	125.62
123	2.61654	3.3991	0.9660	126.40
124	2.07253	3.1600	0.9684	127.16
125	1.61653	2.9104	0.9709	127.91
126	1.23390	2.6579	0.9734	128.66
127	0.91519	2.4093	0.9759	129.41
128	0.65554	2.1656	0.9783	130.17
129	0.44994	1.9267	0.9807	130.93
130	0.29276	1.6926	0.9831	131.69
131	0.17779	1.4638	0.9854	132.46
132	0.09844	1.2408	0.9876	133.24
133	0.04783	1.0246	0.9898	134.02
134	0.01903	0.8184	0.9918	134.82
135	0.00536	0.6306	0.9937	135.63
136	0.00070	0.5001	0.9950	136.50
137	0.00000	0.0000	1.0000	137.00

Table D

Iowa Type curve O1

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
0	100.00000	100.0000	0.0000	100.00
1	99.50000	99.5000	0.0050	100.50
2	99.00000	99.0000	0.0100	101.00
3	98.50000	98.5000	0.0150	101.50
4	98.00000	98.0000	0.0200	102.00
5	97.50000	97.5000	0.0250	102.50
6	97.00000	97.0000	0.0300	103.00
7	96.50000	96.5000	0.0350	103.50
8	96.00000	96.0000	0.0400	104.00
9	95.50000	95.5000	0.0450	104.50
10	95.00000	95.0000	0.0500	105.00
11	94.50000	94.5000	0.0550	105.50
12	94.00000	94.0000	0.0600	106.00
13	93.50000	93.5000	0.0650	106.50
14	93.00000	93.0000	0.0700	107.00
15	92.50000	92.5000	0.0750	107.50
16	92.00000	92.0000	0.0800	108.00
17	91.50000	91.5000	0.0850	108.50
18	91.00000	91.0000	0.0900	109.00
19	90.50000	90.5000	0.0950	109.50
20	90.00000	90.0000	0.1000	110.00
21	89.50000	89.5000	0.1050	110.50
22	89.00000	89.0000	0.1100	111.00
23	88.50000	88.5000	0.1150	111.50
24	88.00000	88.0000	0.1200	112.00
25	87.50000	87.5000	0.1250	112.50
26	87.00000	87.0000	0.1300	113.00
27	86.50000	86.5000	0.1350	113.50
28	86.00000	86.0000	0.1400	114.00
29	85.50000	85.5000	0.1450	114.50
30	85.00000	85.0000	0.1500	115.00
31	84.50000	84.5000	0.1550	115.50
32	84.00000	84.0000	0.1600	116.00
33	83.50000	83.5000	0.1650	116.50
34	83.00000	83.0000	0.1700	117.00
35	82.50000	82.5000	0.1750	117.50
36	82.00000	82.0000	0.1800	118.00
37	81.50000	81.5000	0.1850	118.50
38	81.00000	81.0000	0.1900	119.00
39	80.50000	80.5000	0.1950	119.50
40	80.00000	80.0000	0.2000	120.00
41	79.50000	79.5000	0.2050	120.50
42	79.00000	79.0000	0.2100	121.00
43	78.50000	78.5000	0.2150	121.50
44	78.00000	78.0000	0.2200	122.00
45	77.50000	77.5000	0.2250	122.50
46	77.00000	77.0000	0.2300	123.00
47	76.50000	76.5000	0.2350	123.50
48	76.00000	76.0000	0.2400	124.00
49	75.50000	75.5000	0.2450	124.50
50	75.00000	75.0000	0.2500	125.00
51	74.50000	74.5000	0.2550	125.50
52	74.00000	74.0000	0.2600	126.00
53	73.50000	73.5000	0.2650	126.50
54	73.00000	73.0000	0.2700	127.00
55	72.50000	72.5000	0.2750	127.50
56	72.00000	72.0000	0.2800	128.00
57	71.50000	71.5000	0.2850	128.50
58	71.00000	71.0000	0.2900	129.00
59	70.50000	70.5000	0.2950	129.50
60	70.00000	70.0000	0.3000	130.00
61	69.50000	69.5000	0.3050	130.50
62	69.00000	69.0000	0.3100	131.00
63	68.50000	68.5000	0.3150	131.50
64	68.00000	68.0000	0.3200	132.00
65	67.50000	67.5000	0.3250	132.50
66	67.00000	67.0000	0.3300	133.00
67	66.50000	66.5000	0.3350	133.50
68	66.00000	66.0000	0.3400	134.00
69	65.50000	65.5000	0.3450	134.50
70	65.00000	65.0000	0.3500	135.00
71	64.50000	64.5000	0.3550	135.50
72	64.00000	64.0000	0.3600	136.00
73	63.50000	63.5000	0.3650	136.50
74	63.00000	63.0000	0.3700	137.00
75	62.50000	62.5000	0.3750	137.50
76	62.00000	62.0000	0.3800	138.00
77	61.50000	61.5000	0.3850	138.50
78	61.00000	61.0000	0.3900	139.00
79	60.50000	60.5000	0.3950	139.50
80	60.00000	60.0000	0.4000	140.00
81	59.50000	59.5000	0.4050	140.50
82	59.00000	59.0000	0.4100	141.00
83	58.50000	58.5000	0.4150	141.50
84	58.00000	58.0000	0.4200	142.00
85	57.50000	57.5000	0.4250	142.50
86	57.00000	57.0000	0.4300	143.00
87	56.50000	56.5000	0.4350	143.50
88	56.00000	56.0000	0.4400	144.00
89	55.50000	55.5000	0.4450	144.50
90	55.00000	55.0000	0.4500	145.00
91	54.50000	54.5000	0.4550	145.50
92	54.00000	54.0000	0.4600	146.00
93	53.50000	53.5000	0.4650	146.50
94	53.00000	53.0000	0.4700	147.00
95	52.50000	52.5000	0.4750	147.50

Iowa Type curve O1

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
100	50.00000	50.0000	0.5000	150.00
101	49.50000	49.5000	0.5050	150.50
102	49.00000	49.0000	0.5100	151.00
103	48.50000	48.5000	0.5150	151.50
104	48.00000	48.0000	0.5200	152.00
105	47.50000	47.5000	0.5250	152.50
106	47.00000	47.0000	0.5300	153.00
107	46.50000	46.5000	0.5350	153.50
108	46.00000	46.0000	0.5400	154.00
109	45.50000	45.5000	0.5450	154.50
110	45.00000	45.0000	0.5500	155.00
111	44.50000	44.5000	0.5550	155.50
112	44.00000	44.0000	0.5600	156.00
113	43.50000	43.5000	0.5650	156.50
114	43.00000	43.0000	0.5700	157.00
115	42.50000	42.5000	0.5750	157.50
116	42.00000	42.0000	0.5800	158.00
117	41.50000	41.5000	0.5850	158.50
118	41.00000	41.0000	0.5900	159.00
119	40.50000	40.5000	0.5950	159.50
120	40.00000	40.0000	0.6000	160.00
121	39.50000	39.5000	0.6050	160.50
122	39.00000	39.0000	0.6100	161.00
123	38.50000	38.5000	0.6150	161.50
124	38.00000	38.0000	0.6200	162.00
125	37.50000	37.5000	0.6250	162.50
126	37.00000	37.0000	0.6300	163.00
127	36.50000	36.5000	0.6350	163.50
128	36.00000	36.0000	0.6400	164.00
129	35.50000	35.5000	0.6450	164.50
130	35.00000	35.0000	0.6500	165.00
131	34.50000	34.5000	0.6550	165.50
132	34.00000	34.0000	0.6600	166.00
133	33.50000	33.5000	0.6650	166.50
134	33.00000	33.0000	0.6700	167.00
135	32.50000	32.5000	0.6750	167.50
136	32.00000	32.0000	0.6800	168.00
137	31.50000	31.5000	0.6850	168.50
138	31.00000	31.0000	0.6900	169.00
139	30.50000	30.5000	0.6950	169.50
140	30.00000	30.0000	0.7000	170.00
141	29.50000	29.5000	0.7050	170.50
142	29.00000	29.0000	0.7100	171.00
143	28.50000	28.5000	0.7150	171.50
144	28.00000	28.0000	0.7200	172.00
145	27.50000	27.5000	0.7250	172.50
146	27.00000	27.0000	0.7300	173.00
147	26.50000	26.5000	0.7350	173.50
148	26.00000	26.0000	0.7400	174.00
149	25.50000	25.5000	0.7450	174.50
150	25.00000	25.0000	0.7500	175.00
151	24.50000	24.5000	0.7550	175.50
152	24.00000	24.0000	0.7600	176.00
153	23.50000	23.5000	0.7650	176.50
154	23.00000	23.0000	0.7700	177.00
155	22.50000	22.5000	0.7750	177.50
156	22.00000	22.0000	0.7800	178.00
157	21.50000	21.5000	0.7850	178.50
158	21.00000	21.0000	0.7900	179.00
159	20.50000	20.5000	0.7950	179.50
160	20.00000	20.0000	0.8000	180.00
161	19.50000	19.5000	0.8050	180.50
162	19.00000	19.0000	0.8100	181.00
163	18.50000	18.5000	0.8150	181.50
164	18.00000	18.0000	0.8200	182.00
165	17.50000	17.5000	0.8250	182.50
166	17.00000	17.0000	0.8300	183.00
167	16.50000	16.5000	0.8350	183.50
168	16.00000	16.0000	0.8400	184.00
169	15.50000	15.5000	0.8450	184.50
170	15.00000	15.0000	0.8500	185.00
171	14.50000	14.5000	0.8550	185.50
172	14.00000	14.0000	0.8600	186.00
173	13.50000	13.5000	0.8650	186.50
174	13.00000	13.0000	0.8700	187.00
175	12.50000	12.5000	0.8750	187.50
176	12.00000	12.0000	0.8800	188.00
177	11.50000	11.5000	0.8850	188.50
178	11.00000	11.0000	0.8900	189.00
179	10.50000	10.5000	0.8950	189.50
180	10.00000	10.0000	0.9000	190.00
181	9.50000	9.5000	0.9050	190.50
182	9.00000	9.0000	0.9100	191.00
183	8.50000	8.5000	0.9150	191.50
184	8.00000	8.0000	0.9200	192.00
185	7.50000	7.5000	0.9250	192.50
186	7.00000	7.0000	0.9300	193.00
187	6.50000	6.5000	0.9350	193.50
188	6.00000	6.0000	0.9400	194.00
189	5.50000	5.5000	0.9450	194.50
190	5.00000	5.0000	0.9500	195.00
191	4.50000	4.5000	0.9550	195.50
192	4.00000	4.0000	0.9600	196.00
193	3.50000	3.5000	0.9650	196.50
194	3.00000	3.0000	0.9700	197.00
195	2.50000	2.5000	0.9750	197.50

Table D continued

Iowa Type curve O1

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
200	0.00000	0.0000	1.0000	200.00

Iowa Type curve O2

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
0	100.00000	100.0000	0.0000	100.00
1	99.43868	99.1126	0.0089	100.11
2	98.87721	98.6726	0.0133	100.67
3	98.31559	98.2334	0.0177	101.23
4	97.75384	97.7950	0.0220	101.80
5	97.19199	97.3575	0.0264	102.36
6	96.63003	96.9208	0.0308	102.92
7	96.06800	96.4849	0.0352	103.48
8	95.50589	96.0498	0.0395	104.05
9	94.94372	95.6155	0.0438	104.62
10	94.38150	95.1821	0.0482	105.18
11	93.81923	94.7496	0.0525	105.75
12	93.25692	94.3179	0.0568	106.32
13	92.69459	93.8870	0.0611	106.89
14	92.13222	93.4570	0.0654	107.46
15	91.56984	93.0279	0.0697	108.03
16	91.00744	92.5997	0.0740	108.60
17	90.44502	92.1725	0.0783	109.17
18	89.88259	91.7461	0.0825	109.75
19	89.32016	91.3206	0.0868	110.32
20	88.75771	90.8962	0.0910	110.90
21	88.19526	90.4727	0.0953	111.47
22	87.63280	90.0501	0.0995	112.05
23	87.07034	89.6286	0.1037	112.63
24	86.50788	89.2081	0.1079	113.21
25	85.94542	88.7886	0.1121	113.79
26	85.38295	88.3703	0.1163	114.37
27	84.82048	87.9530	0.1205	114.95
28	84.25802	87.5367	0.1246	115.54
29	83.69555	87.1217	0.1288	116.12
30	83.13308	86.7077	0.1329	116.71
31	82.57061	86.2950	0.1371	117.29
32	82.00815	85.8834	0.1412	117.88
33	81.44568	85.4731	0.1453	118.47
34	80.88321	85.0640	0.1494	119.06
35	80.32074	84.6562	0.1534	119.66
36	79.75827	84.2497	0.1575	120.25
37	79.19580	83.8445	0.1616	120.84
38	78.63334	83.4406	0.1656	121.44
39	78.07087	83.0382	0.1696	122.04
40	77.50840	82.6372	0.1736	122.64
41	76.94593	82.2376	0.1776	123.24
42	76.38346	81.8395	0.1816	123.84
43	75.82099	81.4429	0.1856	124.44
44	75.25852	81.0478	0.1895	125.05
45	74.69606	80.6544	0.1935	125.65
46	74.13359	80.2625	0.1974	126.26
47	73.57112	79.8723	0.2013	126.87
48	73.00865	79.4838	0.2052	127.48
49	72.44618	79.0970	0.2090	128.10
50	71.88371	78.7120	0.2129	128.71
51	71.32125	78.3288	0.2167	129.33
52	70.75878	77.9475	0.2205	129.95
53	70.19631	77.5681	0.2243	130.57
54	69.63385	77.1906	0.2281	131.19
55	69.07138	76.8151	0.2318	131.82
56	68.50892	76.4417	0.2356	132.44
57	67.94645	76.0703	0.2393	133.07
58	67.38399	75.7011	0.2430	133.70
59	66.82154	75.3341	0.2467	134.33
60	66.25909	74.9693	0.2503	134.97
61	65.69664	74.6069	0.2539	135.61
62	65.13420	74.2468	0.2575	136.25
63	64.57177	73.8892	0.2611	136.89
64	64.00936	73.5340	0.2647	137.53
65	63.44696	73.1814	0.2682	138.18
66	62.88457	72.8314	0.2717	138.83
67	62.32221	72.4840	0.2752	139.48
68	61.75987	72.1395	0.2786	140.14
69	61.19757	71.7977	0.2820	140.80
70	60.63530	71.4589	0.2854	141.46
71	60.07308	71.1230	0.2888	142.12
72	59.51091	70.7901	0.2921	142.79
73	58.94880	70.4604	0.2954	143.46
74	58.38676	70.1338	0.2987	144.13
75	57.82481	69.8105	0.3019	144.81
76	57.26295	69.4906	0.3051	145.49
77	56.70121	69.1741	0.3083	146.17
78	56.13959	68.8611	0.3114	146.86
79	55.57812	68.5517	0.3145	147.55
80	55.01681	68.2460	0.3175	148.25
81	54.45568	67.9441	0.3206	148.94
82	53.89477	67.6460	0.3235	149.65
83	53.33410	67.3519	0.3265	150.35
84	52.77370	67.0618	0.3294	151.06
85	52.21359	66.7758	0.3322	151.78
86	51.65382	66.4940	0.3351	152.49
87	51.09442	66.2165	0.3378	153.22
88	50.53544	65.9434	0.3406	153.94
89	49.97692	65.6748	0.3433	154.67
90	49.41890	65.4107	0.3459	155.41
91	48.86144	65.1513	0.3485	156.15
92	48.30460	64.8966	0.3510	156.90
93	47.74843	64.6467	0.3535	157.65
94	47.19301	64.4016	0.3560	158.40
95	46.63840	64.1615	0.3584	159.16

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
Iowa Type curve 02				
100	43.88026	63.0376	0.3696	163.04
101	43.33224	62.8285	0.3717	163.83
102	42.78565	62.6247	0.3738	164.62
103	42.24059	62.4264	0.3757	165.43
104	41.69720	62.2334	0.3777	166.23
105	41.15557	62.0458	0.3795	167.05
106	40.61584	61.8637	0.3814	167.86
107	40.07814	61.6870	0.3831	168.69
108	39.54259	61.5156	0.3848	169.52
109	39.00935	61.3497	0.3865	170.35
110	38.47853	61.1891	0.3881	171.19
111	37.95031	61.0339	0.3897	172.03
112	37.42481	60.8838	0.3912	172.88
113	36.90220	60.7390	0.3926	173.74
114	36.38262	60.5993	0.3940	174.60
115	35.86624	60.4645	0.3954	175.46
116	35.35322	60.3347	0.3967	176.33
117	34.84370	60.2097	0.3979	177.21
118	34.33786	60.0893	0.3991	178.09
119	33.83584	59.9734	0.4003	178.97
120	33.33782	59.8618	0.4014	179.86
121	32.84393	59.7545	0.4025	180.75
122	32.35435	59.6511	0.4035	181.65
123	31.86922	59.5515	0.4045	182.55
124	31.38869	59.4556	0.4054	183.46
125	30.91290	59.3630	0.4064	184.36
126	30.44199	59.2735	0.4073	185.27
127	29.97610	59.1870	0.4081	186.19
128	29.51535	59.1031	0.4090	187.10
129	29.05986	59.0217	0.4098	188.02
130	28.60975	58.9424	0.4106	188.94
131	28.16512	58.8650	0.4114	189.86
132	27.72606	58.7892	0.4121	190.79
133	27.29268	58.7148	0.4129	191.71
134	26.86504	58.6415	0.4136	192.64
135	26.44322	58.5689	0.4143	193.57
136	26.02729	58.4969	0.4150	194.50
137	25.61729	58.4252	0.4157	195.43
138	25.21326	58.3534	0.4165	196.35
139	24.81526	58.2813	0.4172	197.28
140	24.42329	58.2086	0.4179	198.21
141	24.03737	58.1351	0.4186	199.14
142	23.65752	58.0605	0.4194	200.06
143	23.28374	57.9845	0.4202	200.98
144	22.91600	57.9070	0.4209	201.91
145	22.55431	57.8276	0.4217	202.83
146	22.19862	57.7462	0.4225	203.75
147	21.84892	57.6624	0.4234	204.66
148	21.50515	57.5762	0.4242	205.58
149	21.16729	57.4872	0.4251	206.49
150	20.83526	57.3954	0.4260	207.40
151	20.50902	57.3004	0.4270	208.30
152	20.18851	57.2021	0.4280	209.20
153	19.87365	57.1005	0.4290	210.10
154	19.56438	56.9952	0.4300	211.00
155	19.26062	56.8862	0.4311	211.89
156	18.96230	56.7733	0.4323	212.77
157	18.66932	56.6564	0.4334	213.66
158	18.38161	56.5354	0.4346	214.54
159	18.09908	56.4101	0.4359	215.41
160	17.82163	56.2805	0.4372	216.28
161	17.54919	56.1465	0.4385	217.15
162	17.28166	56.0079	0.4399	218.01
163	17.01894	55.8648	0.4414	218.86
164	16.76094	55.7170	0.4428	219.72
165	16.50757	55.5645	0.4444	220.56
166	16.25873	55.4073	0.4459	221.41
167	16.01432	55.2453	0.4476	222.25
168	15.77426	55.0784	0.4492	223.08
169	15.53845	54.9067	0.4509	223.91
170	15.30680	54.7300	0.4527	224.73
171	15.07921	54.5485	0.4545	225.55
172	14.85559	54.3621	0.4564	226.36
173	14.63585	54.1708	0.4583	227.17
174	14.41990	53.9746	0.4603	227.97
175	14.20766	53.7734	0.4623	228.77
176	13.99903	53.5673	0.4643	229.57
177	13.79393	53.3564	0.4664	230.36
178	13.59228	53.1405	0.4686	231.14
179	13.39398	52.9199	0.4708	231.92
180	13.19897	52.6944	0.4731	232.69
181	13.00715	52.4641	0.4754	233.46
182	12.81846	52.2290	0.4777	234.23
183	12.63280	51.9893	0.4801	234.99
184	12.45012	51.7448	0.4826	235.74
185	12.27033	51.4956	0.4850	236.50
186	12.09336	51.2419	0.4876	237.24
187	11.91915	50.9835	0.4902	237.98
188	11.74762	50.7206	0.4928	238.72
189	11.57870	50.4533	0.4955	239.45
190	11.41233	50.1815	0.4982	240.18
191	11.24845	49.9053	0.5009	240.91
192	11.08699	49.6248	0.5038	241.62
193	10.92789	49.3401	0.5066	242.34
194	10.77109	49.0510	0.5095	243.05
195	10.61654	48.7578	0.5124	243.76

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
Iowa Type curve 02				
200	9.87552	47.2309	0.5277	247.23
201	9.73330	46.9137	0.5309	247.91
202	9.59295	46.5928	0.5341	248.59
203	9.45444	46.2680	0.5373	249.27
204	9.31771	45.9397	0.5406	249.94
205	9.18273	45.6076	0.5439	250.61
206	9.04944	45.2720	0.5473	251.27
207	8.91780	44.9329	0.5507	251.93
208	8.78778	44.5903	0.5541	252.59
209	8.65934	44.2443	0.5576	253.24
210	8.53243	43.8949	0.5611	253.89
211	8.40702	43.5422	0.5646	254.54
212	8.28307	43.1863	0.5681	255.19
213	8.16055	42.8272	0.5717	255.83
214	8.03942	42.4650	0.5754	256.46
215	7.91964	42.0997	0.5790	257.10
216	7.80120	41.7312	0.5827	257.73
217	7.68404	41.3599	0.5864	258.36
218	7.56815	40.9856	0.5901	258.99
219	7.45348	40.6084	0.5939	259.61
220	7.34002	40.2284	0.5977	260.23
221	7.22774	39.8456	0.6015	260.85
222	7.11660	39.4601	0.6054	261.46
223	7.00657	39.0719	0.6093	262.07
224	6.89764	38.6810	0.6132	262.68
225	6.78978	38.2875	0.6171	263.29
226	6.68296	37.8915	0.6211	263.89
227	6.57716	37.4930	0.6251	264.49
228	6.47235	37.0921	0.6291	265.09
229	6.36851	36.6887	0.6331	265.69
230	6.26562	36.2830	0.6372	266.28
231	6.16365	35.8750	0.6413	266.87
232	6.06260	35.4646	0.6454	267.46
233	5.96242	35.0521	0.6495	268.05
234	5.86312	34.6372	0.6536	268.64
235	5.76466	34.2203	0.6578	269.22
236	5.66702	33.8013	0.6620	269.80
237	5.57019	33.3802	0.6662	270.38
238	5.47415	32.9570	0.6704	270.96
239	5.37889	32.5319	0.6747	271.53
240	5.28438	32.1047	0.6790	272.10
241	5.19061	31.6757	0.6832	272.68
242	5.09756	31.2448	0.6876	273.24
243	5.00521	30.8120	0.6919	273.81
244	4.91356	30.3774	0.6962	274.38
245	4.82259	29.9410	0.7006	274.94
246	4.73227	29.5029	0.7050	275.50
247	4.64260	29.0631	0.7094	276.06
248	4.55357	28.6216	0.7138	276.62
249	4.46516	28.1784	0.7182	277.18
250	4.37735	27.7336	0.7227	277.73
251	4.29014	27.2872	0.7271	278.29
252	4.20350	26.8393	0.7316	278.84
253	4.11744	26.3898	0.7361	279.39
254	4.03194	25.9389	0.7406	279.94
255	3.94697	25.4865	0.7451	280.49
256	3.86255	25.0326	0.7497	281.03
257	3.77866	24.5773	0.7542	281.58
258	3.69525	24.1207	0.7588	282.12
259	3.61236	23.6627	0.7634	282.66
260	3.52997	23.2034	0.7680	283.20
261	3.44805	22.7428	0.7726	283.74
262	3.36661	22.2808	0.7772	284.28
263	3.28563	21.8176	0.7818	284.82
264	3.20510	21.3533	0.7865	285.35
265	3.12501	20.8877	0.7911	285.89
266	3.04536	20.4209	0.7958	286.42
267	2.96614	19.9530	0.8005	286.95
268	2.88733	19.4840	0.8052	287.48
269	2.80894	19.0138	0.8099	288.01
270	2.73094	18.5425	0.8146	288.54
271	2.65334	18.0702	0.8193	289.07
272	2.57613	17.5968	0.8240	289.60
273	2.49929	17.1224	0.8288	290.12
274	2.42283	16.6470	0.8335	290.65
275	2.34673	16.1706	0.8383	291.17
276	2.27098	15.6933	0.8431	291.69
277	2.19559	15.2150	0.8478	292.22
278	2.12054	14.7358	0.8526	292.74
279	2.04583	14.2557	0.8574	293.26
280	1.97145	13.7747	0.8623	293.77
281	1.89739	13.2928	0.8671	294.29
282	1.82365	12.8101	0.8719	294.81
283	1.75023	12.3265	0.8767	295.33
284	1.67711	11.8421	0.8816	295.84
285	1.60429	11.3570	0.8864	296.36
286	1.53177	10.8710	0.8913	296.87
287	1.45953	10.3843	0.8962	297.38
288	1.38759	9.8967	0.9010	297.90
289	1.31592	9.4085	0.9059	298.41
290	1.24452	8.9196	0.9108	298.92
291	1.17340	8.4299	0.9157	299.43
292	1.10254	7.9396	0.9206	299.94
293	1.03194	7.4486	0.9255	300.45
294	0.96160	6.9568	0.9304	300.96
295	0.89151	6.4645	0.9354	301.46

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
Iowa Type curve 02				
300	0.54464	3.9937	0.9601	303.99
301	0.47596	3.4978	0.9650	304.50
302	0.40751	3.0013	0.9700	305.00
303	0.33926	2.5045	0.9750	305.50
304	0.27124	2.0072	0.9799	306.01
305	0.20342	1.5097	0.9849	306.51
306	0.13581	1.0124	0.9899	307.01
307	0.06840	0.5174	0.9948	307.52
308	0.00119	0.5000	0.9950	308.50
309	0.00000	0.0000	1.0000	309.00

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
Iowa Type curve 03				
0	100.00000	100.0000	0.0000	100.00
1	99.17103	98.4323	0.0157	99.43
2	98.34314	98.2567	0.0174	100.26
3	97.51638	98.0855	0.0191	101.09
4	96.69082	97.9187	0.0208	101.92
5	95.86652	97.7564	0.0224	102.76
6	95.04354	97.5985	0.0240	103.60
7	94.22196	97.4452	0.0255	104.45
8	93.40185	97.2964	0.0270	105.30
9	92.58328	97.1522	0.0285	106.15
10	91.76634	97.0127	0.0299	107.01
11	90.95110	96.8778	0.0312	107.88
12	90.13764	96.7475	0.0325	108.75
13	89.32605	96.6220	0.0338	109.62
14	88.51642	96.5012	0.0350	110.50
15	87.70882	96.3852	0.0361	111.39
16	86.90337	96.2739	0.0373	112.27
17	86.10014	96.1673	0.0383	113.17
18	85.29924	96.0656	0.0393	114.07
19	84.50076	95.9686	0.0403	114.97
20	83.70479	95.8765	0.0412	115.88
21	82.91146	95.7891	0.0421	116.79
22	82.12084	95.7065	0.0429	117.71
23	81.33306	95.6286	0.0437	118.63
24	80.54821	95.5555	0.0444	119.56
25	79.76641	95.4872	0.0451	120.49
26	78.98776	95.4236	0.0458	121.42
27	78.21237	95.3646	0.0464	122.36
28	77.44035	95.3103	0.0469	123.31
29	76.67182	95.2607	0.0474	124.26
30	75.90688	95.2156	0.0478	125.22
31	75.14565	95.1751	0.0482	126.18
32	74.38824	95.1391	0.0486	127.14
33	73.63476	95.1075	0.0489	128.11
34	72.88532	95.0803	0.0492	129.08
35	72.14004	95.0574	0.0494	130.06
36	71.39902	95.0388	0.0496	131.04
37	70.66238	95.0243	0.0498	132.02
38	69.93023	95.0139	0.0499	133.01
39	69.20266	95.0076	0.0499	134.01
40	68.47980	95.0052	0.0499	135.01
41	67.76174	95.0067	0.0499	136.01
42	67.04858	95.0119	0.0499	137.01
43	66.34043	95.0208	0.0498	138.02
44	65.63739	95.0332	0.0497	139.03
45	64.93954	95.0490	0.0495	140.05
46	64.24699	95.0682	0.0493	141.07
47	63.55981	95.0907	0.0491	142.09
48	62.87810	95.1162	0.0488	143.12
49	62.20194	95.1447	0.0486	144.14
50	61.53142	95.1761	0.0482	145.18
51	60.86660	95.2102	0.0479	146.21
52	60.20756	95.2469	0.0475	147.25
53	59.55436	95.2861	0.0471	148.29
54	58.90708	95.3276	0.0467	149.33
55	58.26577	95.3713	0.0463	150.37
56	57.63049	95.4171	0.0458	151.42
57	57.00130	95.4649	0.0454	152.46
58	56.37823	95.5144	0.0449	153.51
59	55.76133	95.5655	0.0443	154.57
60	55.15065	95.6182	0.0438	155.62
61	54.54622	95.6722	0.0433	156.67
62	53.94807	95.7274	0.0427	157.73
63	53.35622	95.7837	0.0422	158.78
64	52.77070	95.8410	0.0416	159.84
65	52.19152	95.8990	0.0410	160.90
66	51.61870	95.9576	0.0404	161.96
67	51.05225	96.0168	0.0398	163.02
68	50.49217	96.0763	0.0392	164.08
69	49.93847	96.1360	0.0386	165.14
70	49.39113	96.1958	0.0380	166.20
71	48.85016	96.2556	0.0374	167.26
72	48.31555	96.3151	0.0368	168.32
73	47.78727	96.3743	0.0363	169.37
74	47.26531	96.4331	0.0357	170.43
75	46.74965	96.4912	0.0351	171.49
76	46.24027	96.5487	0.0345	172.55
77	45.73714	96.6053	0.0339	173.61
78	45.24022	96.6609	0.0334	174.66
79	44.74949	96.7154	0.0328	175.72
80	44.26490	96.7687	0.0323	176.77
81	43.78642	96.8207	0.0318	177.82
82	43.31400	96.8712	0.0313	178.87
83	42.84761	96.9202	0.0308	179.92
84	42.38719	96.9676	0.0303	180.97
85	41.93269	97.0132	0.0299	182.01
86	41.48407	97.0569	0.0294	183.06
87	41.04127	97.0987	0.0290	184.10
88	40.60424	97.1384	0.0286	185.14
89	40.17293	97.1759	0.0282	186.18
90	39.74727	97.2112	0.0279	187.21
91	39.32720	97.2442	0.0276	188.24
92	38.91267	97.2748	0.0273	189.27
93	38.50362	97.3029	0.0270	190.30
94	38.09998	97.3285	0.0267	191.33
95	37.70170	97.3514	0.0265	192.35

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Iowa Type curve O3

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
100	35.78829	97.4240	0.0258	197.42
101	35.42076	97.4297	0.0257	198.43
102	35.05812	97.4324	0.0257	199.43
103	34.70031	97.4319	0.0257	200.43
104	34.34726	97.4282	0.0257	201.43
105	33.99891	97.4213	0.0258	202.42
106	33.65519	97.4112	0.0259	203.41
107	33.31603	97.3977	0.0260	204.40
108	32.98137	97.3810	0.0262	205.38
109	32.65114	97.3608	0.0264	206.36
110	32.32527	97.3373	0.0266	207.34
111	32.00370	97.3103	0.0269	208.31
112	31.68637	97.2798	0.0272	209.28
113	31.37321	97.2458	0.0275	210.25
114	31.06415	97.2084	0.0279	211.21
115	30.75914	97.1673	0.0283	212.17
116	30.45811	97.1227	0.0288	213.12
117	30.16099	97.0746	0.0293	214.07
118	29.86773	97.0228	0.0298	215.02
119	29.57826	96.9674	0.0303	215.97
120	29.29252	96.9084	0.0309	216.91
121	29.01046	96.8458	0.0315	217.85
122	28.73202	96.7795	0.0322	218.78
123	28.45712	96.7095	0.0329	219.71
124	28.18573	96.6359	0.0336	220.64
125	27.91777	96.5586	0.0344	221.56
126	27.65320	96.4777	0.0352	222.48
127	27.39195	96.3931	0.0361	223.39
128	27.13398	96.3047	0.0370	224.30
129	26.87923	96.2127	0.0379	225.21
130	26.62764	96.1171	0.0388	226.12
131	26.37916	96.0178	0.0398	227.02
132	26.13374	95.9148	0.0409	227.91
133	25.89133	95.8081	0.0419	228.81
134	25.65188	95.6977	0.0430	229.70
135	25.41533	95.5838	0.0442	230.58
136	25.18165	95.4661	0.0453	231.47
137	24.95077	95.3449	0.0466	232.34
138	24.72266	95.2200	0.0478	233.22
139	24.49727	95.0915	0.0491	234.09
140	24.27454	94.9594	0.0504	234.96
141	24.05444	94.8237	0.0518	235.82
142	23.83692	94.6845	0.0532	236.68
143	23.62194	94.5416	0.0546	237.54
144	23.40946	94.3952	0.0560	238.40
145	23.19942	94.2453	0.0575	239.25
146	22.99180	94.0918	0.0591	240.09
147	22.78654	93.9349	0.0607	240.93
148	22.58362	93.7745	0.0623	241.77
149	22.38298	93.6106	0.0639	242.61
150	22.18460	93.4432	0.0656	243.44
151	21.98843	93.2724	0.0673	244.27
152	21.79443	93.0982	0.0690	245.10
153	21.60257	92.9206	0.0708	245.92
154	21.41282	92.7396	0.0726	246.74
155	21.22513	92.5552	0.0744	247.56
156	21.03947	92.3675	0.0763	248.37
157	20.85582	92.1765	0.0782	249.18
158	20.67412	91.9822	0.0802	249.98
159	20.49436	91.7846	0.0822	250.78
160	20.31650	91.5838	0.0842	251.58
161	20.14050	91.3797	0.0862	252.38
162	19.96634	91.1724	0.0883	253.17
163	19.79398	90.9620	0.0904	253.96
164	19.62340	90.7483	0.0925	254.75
165	19.45456	90.5316	0.0947	255.53
166	19.28744	90.3117	0.0969	256.31
167	19.12201	90.0887	0.0991	257.09
168	18.95824	89.8626	0.1014	257.86
169	18.79610	89.6334	0.1037	258.63
170	18.63556	89.4013	0.1060	259.40
171	18.47660	89.1661	0.1083	260.17
172	18.31920	88.9280	0.1107	260.93
173	18.16332	88.6869	0.1131	261.69
174	18.00895	88.4428	0.1156	262.44
175	17.85605	88.1958	0.1180	263.20
176	17.70460	87.9460	0.1205	263.95
177	17.55459	87.6933	0.1231	264.69
178	17.40598	87.4377	0.1256	265.44
179	17.25875	87.1794	0.1282	266.18
180	17.11289	86.9182	0.1308	266.92
181	16.96836	86.6542	0.1335	267.65
182	16.82515	86.3876	0.1361	268.39
183	16.68324	86.1181	0.1388	269.12
184	16.54260	85.8460	0.1415	269.85
185	16.40322	85.5712	0.1443	270.57
186	16.26508	85.2937	0.1471	271.29
187	16.12814	85.0137	0.1499	272.01
188	15.99241	84.7310	0.1527	272.73
189	15.85785	84.4457	0.1555	273.45
190	15.72445	84.1579	0.1584	274.16
191	15.59219	83.8675	0.1613	274.87
192	15.46105	83.5746	0.1643	275.57
193	15.33101	83.2793	0.1672	276.28
194	15.20206	82.9814	0.1702	276.98
195	15.07418	82.6812	0.1732	277.68

Iowa Type curve O3

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
200	14.45027	81.1438	0.1886	281.14
201	14.32847	80.8293	0.1917	281.83
202	14.20763	80.5125	0.1949	282.51
203	14.08773	80.1935	0.1981	283.19
204	13.96876	79.8722	0.2013	283.87
205	13.85071	79.5487	0.2045	284.55
206	13.73356	79.2230	0.2078	285.22
207	13.61730	78.8951	0.2110	285.90
208	13.50192	78.5651	0.2143	286.57
209	13.38739	78.2329	0.2177	287.23
210	13.27371	77.8987	0.2210	287.90
211	13.16087	77.5623	0.2244	288.56
212	13.04886	77.2238	0.2278	289.22
213	12.93765	76.8833	0.2312	289.88
214	12.82725	76.5407	0.2346	290.54
215	12.71762	76.1962	0.2380	291.20
216	12.60878	75.8496	0.2415	291.85
217	12.50070	75.5010	0.2450	292.50
218	12.39337	75.1506	0.2485	293.15
219	12.28679	74.7981	0.2520	293.80
220	12.18093	74.4438	0.2556	294.44
221	12.07579	74.0876	0.2591	295.09
222	11.97137	73.7295	0.2627	295.73
223	11.86764	73.3696	0.2663	296.37
224	11.76460	73.0078	0.2699	297.01
225	11.66224	72.6442	0.2736	297.64
226	11.56055	72.2788	0.2772	298.28
227	11.45952	71.9116	0.2809	298.91
228	11.35914	71.5427	0.2846	299.54
229	11.25940	71.1720	0.2883	300.17
230	11.16030	70.7995	0.2920	300.80
231	11.06181	70.4255	0.2957	301.43
232	10.96394	70.0496	0.2995	302.05
233	10.86668	69.6721	0.3033	302.67
234	10.77001	69.2930	0.3071	303.29
235	10.67393	68.9122	0.3109	303.91
236	10.57843	68.5299	0.3147	304.53
237	10.48350	68.1459	0.3185	305.15
238	10.38914	67.7603	0.3224	305.76
239	10.29533	67.3731	0.3263	306.37
240	10.20207	66.9845	0.3302	306.98
241	10.10934	66.5943	0.3341	307.59
242	10.01716	66.2025	0.3380	308.20
243	9.92549	65.8093	0.3419	308.81
244	9.83435	65.4146	0.3459	309.41
245	9.74372	65.0184	0.3498	310.02
246	9.65359	64.6207	0.3538	310.62
247	9.56396	64.2217	0.3578	311.22
248	9.47483	63.8211	0.3618	311.82
249	9.38617	63.4192	0.3658	312.42
250	9.29800	63.0159	0.3698	313.02
251	9.21029	62.6112	0.3739	313.61
252	9.12305	62.2051	0.3779	314.21
253	9.03627	61.7977	0.3820	314.80
254	8.94994	61.3890	0.3861	315.39
255	8.86406	60.9789	0.3902	315.98
256	8.77862	60.5675	0.3943	316.57
257	8.69361	60.1549	0.3985	317.15
258	8.60903	59.7410	0.4026	317.74
259	8.52488	59.3258	0.4067	318.33
260	8.44114	58.9094	0.4109	318.91
261	8.35782	58.4915	0.4151	319.49
262	8.27490	58.0728	0.4193	320.07
263	8.19238	57.6527	0.4235	320.65
264	8.11026	57.2314	0.4277	321.23
265	8.02854	56.8088	0.4319	321.81
266	7.94719	56.3852	0.4361	322.39
267	7.86623	55.9604	0.4404	322.96
268	7.78564	55.5345	0.4447	323.53
269	7.70543	55.1074	0.4489	324.11
270	7.62558	54.6792	0.4532	324.68
271	7.54610	54.2498	0.4575	325.25
272	7.46697	53.8194	0.4618	325.82
273	7.38819	53.3880	0.4661	326.39
274	7.30976	52.9554	0.4704	326.96
275	7.23168	52.5218	0.4748	327.52
276	7.15394	52.0871	0.4791	328.09
277	7.07653	51.6514	0.4835	328.65
278	6.99945	51.2147	0.4879	329.21
279	6.92270	50.7769	0.4922	329.78
280	6.84627	50.3382	0.4966	330.34
281	6.77016	49.8985	0.5010	330.90
282	6.69436	49.4578	0.5054	331.46
283	6.61888	49.0161	0.5098	332.02
284	6.54370	48.5735	0.5143	332.57
285	6.46883	48.1299	0.5187	333.13
286	6.39425	47.6855	0.5231	333.69
287	6.31998	47.2400	0.5276	334.24
288	6.24599	46.7937	0.5321	334.79
289	6.17229	46.3464	0.5365	335.35
290	6.09888	45.8983	0.5410	335.90
291	6.02575	45.4492	0.5455	336.45
292	5.95290	44.9993	0.5500	337.00
293	5.88032	44.5486	0.5545	337.55
294	5.80801	44.0970	0.5590	338.10
295	5.73697	43.6445	0.5636	338.64

Iowa Type curve O3

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
300	5.37970	41.3698	0.5863	341.37
301	5.30921	40.9124	0.5909	341.91
302	5.23896	40.4543	0.5955	342.45
303	5.16895	39.9955	0.6000	343.00
304	5.09919	39.5358	0.6046	343.54
305	5.02967	39.0753	0.6092	344.08
306	4.96038	38.6142	0.6139	344.61
307	4.89132	38.1523	0.6185	345.15
308	4.82250	37.6896	0.6231	345.69
309	4.75390	37.2263	0.6277	346.23
310	4.68552	36.7623	0.6324	346.76
311	4.61737	36.2975	0.6370	347.30
312	4.54943	35.8321	0.6417	347.83
313	4.48172	35.3659	0.6463	348.37
314	4.41422	34.8990	0.6510	348.90
315	4.34693	34.4315	0.6557	349.43
316	4.27984	33.9634	0.6604	349.96
317	4.21297	33.4946	0.6651	350.49
318	4.14630	33.0251	0.6697	351.03
319	4.07984	32.5549	0.6745	351.55
320	4.01357	32.0842	0.6792	352.08
321	3.94750	31.6128	0.6839	352.61
322	3.88163	31.1408	0.6886	353.14
323	3.81595	30.6682	0.6933	353.67
324	3.75046	30.1950	0.6981	354.19
325	3.68516	29.7212	0.7028	354.72
326	3.62005	29.2468	0.7075	355.25
327	3.55513	28.7717	0.7123	355.77
328	3.49038	28.2962	0.7170	356.30
329	3.42582	27.8200	0.7218	356.82
330	3.36144	27.3432	0.7265	357.34
331	3.29723	26.8660	0.7313	357.87
332	3.23320	26.3881	0.7361	358.39
333	3.16934	25.9098	0.7409	358.91
334	3.10565	25.4309	0.7457	359.43
335	3.04213	24.9514	0.7505	359.95
336	2.97878	24.4714	0.7553	360.47
337	2.91560	23.9909	0.7601	360.99
338	2.85258	23.5098	0.7649	361.51
339	2.78972	23.0283	0.7697	362.03
340	2.72702	22.5463	0.7745	362.55
341	2.66448	22.0638	0.7794	363.06
342	2.60209	21.5808	0.7842	363.58
343	2.53986	21.0973	0.7890	364.10
344	2.47779	20.6133	0.7939	364.61
345	2.41587	20.1288	0.7987	365.13
346	2.35409	19.6439	0.8036	365.64
347	2.29247	19.1585	0.8084	366.16
348	2.23099	18.6727	0.8133	366.67
349	2.16966	18.1864	0.8181	367.19
350	2.10848	17.6995	0.8230	367.70
351	2.04743	17.2124	0.8279	368.21
352	1.98653	16.7247	0.8328	368.72
353	1.92576	16.2367	0.8376	369.24
354	1.86514	15.7482	0.8425	369.75
355	1.80465	15.2593	0.8474	370.26
356	1.74429	14.7700	0.8523	370.77
357	1.68407	14.2803	0.8572	371.28
358	1.62399	13.7901	0.8621	371.79
359	1.56403	13.2996	0.8670	372.30
360	1.50420	12.8087	0.8719	372.81
361	1.44450	12.3174	0.8768	373.32
362	1.38493	11.8257	0.8817	373.83
363	1.32549	11.3336	0.8867	374.33
364	1.26616	10.8413	0.8916	374.84
365	1.20697	10.3484	0.8965	375.35
366	1.14789	9.8553	0.9014	375.86
367	1.08893	9.3618	0.9064	376.36
368	1.03010	8.8680	0.9113	376.87
369	0.97138	8.3738	0.9163	377.37
370	0.91278	7.8793	0.9212	377.88
371	0.85429	7.3845	0.9262	378.38
372	0.79592	6.8894	0.9311	378.89
373	0.73767	6.3939	0.9361	379.39
374	0.67952	5.8983	0.9410	379.90
375	0.62149	5.4024	0.9460	380.40
376	0.56357	4.9062	0.9509	380.91
377	0.50575	4.4100	0.9559	381.41
378	0.44805	3.9135	0.9609	381.91
379	0.39045	3.4170	0.9658	382.42
380	0.33295	2.9208	0.9708	382.92
381	0.27557	2.4249	0.9758	383.42
382	0.21828	1.9301	0.9807	383.93
383	0.16110	1.4377	0.9856	384.44
384	0.10402	0.9522	0.9905	384.95
385	0.04704	0.5000	0.9950	385.50
386	0.00000	0.0000	1.0000	386.00

Iowa Type curve O4

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
0	100.00000	100.0000	0.0000	100.00
1	98.87130	100.4034	-0.0040	101.40
2	97.74568	100.5538	-0.0055	102.55
3	96.62338	100.7160	-0.0072	103.72
4	95.50465	100.8899	-0.0089	104.89
5	94.38974	101.0757	-0.0108	106.08
6	93.27892	101.2734	-0.0127	107.27
7	92.17248	101.4831	-0.0148	108.48
8	91.07069	101.7048	-0.0170	109.70
9	89.97385	101.9385	-0.0194	110.94
10	88.88226	102.1843	-0.0218	112.18
11	87.79625	102.4421	-0.0244	113.44
12	86.71613	102.7119	-0.0271	114.71
13	85.64223	102.9936	-0.0299	115.99
14	84.57489	103.2871	-0.0329	117.29
15	83.51443	103.5922	-0.0359	118.59
16	82.46120	103.9090	-0.0391	119.91
17	81.41555	104.2371	-0.0424	121.24
18	80.37782	104.5764	-0.0458	122.58
19	79.34834	104.9267	-0.0493	123.93
20	78.32747	105.2878	-0.0529	125.29
21	77.31554	105.6593	-0.0566	126.66
22	76.31289	106.0409	-0.0604	128.04
23	75.31983	106.4324	-0.0643	129.43
24	74.33669	106.8334	-0.0683	130.83
25	73.36379	107.2435	-0.0724	132.24
26	72.40141	107.6624	-0.0766	133.66
27	71.44985	108.0896	-0.0809	135.09
28	70.50937	108.5247	-0.0852	136.52
29	69.58025	108.9671	-0.0897	137.97
30	68.66272	109.4166	-0.0942	139.42
31	67.75702	109.8724	-0.0987	140.87
32	66.86334	110.3343	-0.1033	142.33
33	65.98189	110.8016	-0.1080	143.80
34	65.11284	111.2737	-0.1127	145.27
35	64.25634	111.7503	-0.1175	146.75
36	63.41253	112.2307	-0.1223	148.23
37	62.58152	112.7143	-0.1271	149.71
38	61.76340	113.2007	-0.1320	151.20
39	60.95826	113.6893	-0.1369	152.69
40	60.16614	114.1795	-0.1418	154.18
41	59.38708	114.6707	-0.1467	155.67
42	58.62110	115.1626	-0.1516	157.16
43	57.86820	115.6544	-0.1565	158.65
44	57.12835	116.1457	-0.1615	160.15
45	56.40153	116.6360	-0.1664	161.64
46	55.68767	117.1247	-0.1712	163.12
47	54.98671	117.6115	-0.1761	164.61
48	54.29856	118.0957	-0.1810	166.10
49	53.62313	118.5769	-0.1858	167.58
50	52.96030	119.0547	-0.1905	169.05
51	52.30995	119.5286	-0.1953	170.53
52	51.67195	119.9983	-0.2000	172.00
53	51.04616	120.4633	-0.2046	173.46
54	50.43241	120.9232	-0.2092	174.92
55	49.83055	121.3777	-0.2138	176.38
56	49.24040	121.8264	-0.2183	177.83
57	48.66179	122.2690	-0.2227	179.27
58	48.09455	122.7052	-0.2271	180.71
59	47.53847	123.1347	-0.2313	182.13
60	46.99337	123.5572	-0.2356	183.56
61	46.45905	123.9725	-0.2397	184.97
62	45.93532	124.3802	-0.2438	186.38
63	45.42197	124.7803	-0.2478	187.78
64	44.91881	125.1724	-0.2517	189.17
65	44.42562	125.5565	-0.2556	190.56
66	43.94221	125.9322	-0.2593	191.93
67	43.46837	126.2995	-0.2630	193.30
68	43.00390	126.6582	-0.2666	194.66
69	42.54859	127.0083	-0.2701	196.01
70	42.10225	127.3494	-0.2735	197.35
71	41.66457	127.6816	-0.2768	198.68
72	41.23566	128.0048	-0.2800	200.00
73	40.81502	128.3189	-0.2832	201.32
74	40.40256	128.6238	-0.2862	202.62
75	39.99808	128.9194	-0.2892	203.92
76	39.60140	129.2058	-0.2921	205.21
77	39.21233	129.4828	-0.2948	206.48
78	38.83069	129.7505	-0.2975	207.75
79	38.45630	130.0088	-0.3001	209.01
80	38.08898	130.2577	-0.3026	210.26
81	37.72857	130.4973	-0.3050	211.50
82	37.37488	130.7275	-0.3073	212.73
83	37.02776	130.9483	-0.3095	213.95
84	36.68704	131.1598	-0.3116	215.16
85	36.35257	131.3620	-0.3136	216.36
86	36.02418	131.5549	-0.3155	217.55
87	35.70172	131.7386	-0.3174	218.74
88	35.38506	131.9130	-0.3191	219.91
89	35.07403	132.0784	-0.3208	221.08
90	34.76850	132.2346	-0.3223	222.23
91	34.46834	132.3818	-0.3238	223.38
92	34.17339	132.5201	-0.3252	224.52
93	33.88355	132.6494	-0.3265	225.65
94	33.59867	132.7699	-0.3277	226.77
95	33.31862	132.8816	-0.3288	227.88

Table D continued

Iowa Type curve 04

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
100	31.98687	133.3114	-0.3331	233.31
101	31.73341	133.3721	-0.3337	234.37
102	31.48401	133.4247	-0.3342	235.42
103	31.23856	133.4691	-0.3347	236.47
104	30.99696	133.5055	-0.3351	237.51
105	30.75912	133.5340	-0.3353	238.53
106	30.52493	133.5546	-0.3355	239.55
107	30.29432	133.5675	-0.3357	240.57
108	30.06719	133.5727	-0.3357	241.57
109	29.84346	133.5703	-0.3357	242.57
110	29.62304	133.5604	-0.3356	243.56
111	29.40585	133.5432	-0.3354	244.54
112	29.19181	133.5187	-0.3352	245.52
113	28.98086	133.4869	-0.3349	246.49
114	28.77290	133.4481	-0.3345	247.45
115	28.56787	133.4023	-0.3340	248.40
116	28.36569	133.3496	-0.3335	249.35
117	28.16631	133.2900	-0.3329	250.29
118	27.96965	133.2236	-0.3322	251.22
119	27.77564	133.1507	-0.3315	252.15
120	27.58423	133.0712	-0.3307	253.07
121	27.39534	132.9853	-0.3299	253.99
122	27.20893	132.8929	-0.3289	254.89
123	27.02493	132.7943	-0.3279	255.79
124	26.84329	132.6895	-0.3269	256.69
125	26.66395	132.5786	-0.3258	257.58
126	26.48685	132.4617	-0.3246	258.46
127	26.31195	132.3389	-0.3234	259.34
128	26.13919	132.2103	-0.3221	260.21
129	25.96853	132.0758	-0.3208	261.08
130	25.79992	131.9357	-0.3194	261.94
131	25.63330	131.7901	-0.3179	262.79
132	25.46864	131.6389	-0.3164	263.64
133	25.30588	131.4823	-0.3148	264.48
134	25.14499	131.3204	-0.3132	265.32
135	24.98593	131.1532	-0.3115	266.15
136	24.82864	130.9809	-0.3098	266.98
137	24.67310	130.8035	-0.3080	267.80
138	24.51927	130.6210	-0.3062	268.62
139	24.36710	130.4336	-0.3043	269.43
140	24.21655	130.2413	-0.3024	270.24
141	24.06761	130.0442	-0.3004	271.04
142	23.92022	129.8424	-0.2984	271.84
143	23.77435	129.6360	-0.2964	272.64
144	23.62997	129.4251	-0.2943	273.43
145	23.48705	129.2096	-0.2921	274.21
146	23.34556	128.9897	-0.2899	274.99
147	23.20547	128.7653	-0.2877	275.77
148	23.06674	128.5368	-0.2854	276.54
149	22.92935	128.3039	-0.2830	277.30
150	22.79328	128.0670	-0.2807	278.07
151	22.65846	127.8259	-0.2783	278.83
152	22.52491	127.5808	-0.2758	279.58
153	22.39259	127.3318	-0.2733	280.33
154	22.26147	127.0788	-0.2708	281.08
155	22.13152	126.8221	-0.2682	281.82
156	22.00273	126.5615	-0.2656	282.56
157	21.87507	126.2971	-0.2630	283.30
158	21.74851	126.0292	-0.2603	284.03
159	21.62303	125.7576	-0.2576	284.76
160	21.49862	125.4825	-0.2548	285.48
161	21.37524	125.2039	-0.2520	286.20
162	21.25288	124.9219	-0.2492	286.92
163	21.13152	124.6364	-0.2464	287.64
164	21.01113	124.3477	-0.2435	288.35
165	20.89170	124.0557	-0.2406	289.06
166	20.77321	123.7605	-0.2376	289.76
167	20.65564	123.4620	-0.2346	290.46
168	20.53897	123.1605	-0.2316	291.16
169	20.42318	122.8560	-0.2286	291.86
170	20.30826	122.5483	-0.2255	292.55
171	20.19419	122.2377	-0.2224	293.24
172	20.08096	121.9242	-0.2192	293.92
173	19.96853	121.6078	-0.2161	294.61
174	19.85691	121.2886	-0.2129	295.29
175	19.74608	120.9666	-0.2097	295.97
176	19.63601	120.6419	-0.2064	296.64
177	19.52671	120.3143	-0.2031	297.31
178	19.41814	119.9842	-0.1998	297.98
179	19.31030	119.6515	-0.1965	298.65
180	19.20317	119.3162	-0.1932	299.32
181	19.09675	118.9784	-0.1898	299.98
182	18.99101	118.6380	-0.1864	300.64
183	18.88594	118.2953	-0.1830	301.30
184	18.78154	117.9501	-0.1795	301.95
185	18.67779	117.6025	-0.1760	302.60
186	18.57467	117.2526	-0.1725	303.25
187	18.47218	116.9004	-0.1690	303.90
188	18.37030	116.5459	-0.1655	304.55
189	18.26903	116.1892	-0.1619	305.19
190	18.16834	115.8303	-0.1583	305.83
191	18.06824	115.4693	-0.1547	306.47
192	17.96871	115.1061	-0.1511	307.11
193	17.86974	114.7408	-0.1474	307.74
194	17.77132	114.3735	-0.1437	308.37
195	17.67344	114.0042	-0.1400	309.00
196	17.57600	[illegible]	-0.1363	309.62

Iowa Type curve 04

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
200	17.19183	112.1281	-0.1213	312.13
201	17.09700	111.7473	-0.1175	312.75
202	17.00264	111.3647	-0.1136	313.36
203	16.90876	110.9802	-0.1098	313.98
204	16.81534	110.5940	-0.1059	314.59
205	16.72237	110.2061	-0.1021	315.21
206	16.62984	109.8165	-0.0982	315.82
207	16.53776	109.4252	-0.0943	316.43
208	16.44610	109.0323	-0.0903	317.03
209	16.35487	108.6377	-0.0864	317.64
210	16.26405	108.2415	-0.0824	318.24
211	16.17365	107.8437	-0.0784	318.84
212	16.08364	107.4445	-0.0744	319.44
213	15.99403	107.0436	-0.0704	320.04
214	15.90481	106.6413	-0.0664	320.64
215	15.81597	106.2375	-0.0624	321.24
216	15.72751	105.8322	-0.0583	321.83
217	15.63942	105.4255	-0.0543	322.43
218	15.55169	105.0174	-0.0502	323.02
219	15.46432	104.6079	-0.0461	323.61
220	15.37730	104.1971	-0.0420	324.20
221	15.29062	103.7849	-0.0378	324.78
222	15.20429	103.3714	-0.0337	325.37
223	15.11829	102.9566	-0.0296	325.96
224	15.03262	102.5404	-0.0254	326.54
225	14.94728	102.1230	-0.0212	327.12
226	14.86225	101.7044	-0.0170	327.70
227	14.77754	101.2846	-0.0128	328.28
228	14.69314	100.8635	-0.0086	328.86
229	14.60904	100.4413	-0.0044	329.44
230	14.52525	100.0178	-0.0002	330.02
231	14.44174	99.5933	0.0041	330.59
232	14.35853	99.1675	0.0083	331.17
233	14.27560	98.7407	0.0126	331.74
234	14.19296	98.3127	0.0169	332.31
235	14.11059	97.8837	0.0212	332.88
236	14.02849	97.4536	0.0255	333.45
237	13.94666	97.0225	0.0298	334.02
238	13.86510	96.5903	0.0341	334.59
239	13.78380	96.1570	0.0384	335.16
240	13.70275	95.7228	0.0428	335.72
241	13.62195	95.2876	0.0471	336.29
242	13.54141	94.8514	0.0515	336.85
243	13.46111	94.4142	0.0559	337.41
244	13.38105	93.9761	0.0602	337.98
245	13.30122	93.5372	0.0646	338.54
246	13.22163	93.0972	0.0690	339.10
247	13.14227	92.6564	0.0734	339.66
248	13.06314	92.2146	0.0779	340.21
249	12.98423	91.7720	0.0823	340.77
250	12.90555	91.3284	0.0867	341.33
251	12.82708	90.8841	0.0912	341.88
252	12.74882	90.4389	0.0956	342.44
253	12.67077	89.9929	0.1001	342.99
254	12.59294	89.5460	0.1045	343.55
255	12.51530	89.0984	0.1090	344.10
256	12.43787	88.6500	0.1135	344.65
257	12.36064	88.2008	0.1180	345.20
258	12.28360	87.7508	0.1225	345.75
259	12.20676	87.3000	0.1270	346.30
260	12.13010	86.8486	0.1315	346.85
261	12.05364	86.3963	0.1360	347.40
262	11.97735	85.9434	0.1406	347.94
263	11.90126	85.4897	0.1451	348.49
264	11.82534	85.0354	0.1496	349.04
265	11.74960	84.5803	0.1542	349.58
266	11.67403	84.1246	0.1588	350.12
267	11.59864	83.6681	0.1633	350.67
268	11.52341	83.2111	0.1679	351.21
269	11.44836	82.7533	0.1725	351.75
270	11.37347	82.2949	0.1771	352.29
271	11.29874	81.8359	0.1816	352.84
272	11.22417	81.3763	0.1862	353.38
273	11.14976	80.9160	0.1908	353.92
274	11.07551	80.4551	0.1954	354.46
275	11.00141	79.9937	0.2001	354.99
276	10.92746	79.5316	0.2047	355.53
277	10.85366	79.0690	0.2093	356.07
278	10.78001	78.6058	0.2139	356.61
279	10.70651	78.1420	0.2186	357.14
280	10.63315	77.6777	0.2232	357.68
281	10.55993	77.2128	0.2279	358.21
282	10.48686	76.7473	0.2325	358.75
283	10.41392	76.2813	0.2372	359.28
284	10.34111	75.8149	0.2419	359.81
285	10.26845	75.3478	0.2465	360.35
286	10.19591	74.8804	0.2512	360.88
287	10.12351	74.4123	0.2559	361.41
288	10.05123	73.9438	0.2606	361.94
289	9.97908	73.4748	0.2653	362.47
290	9.90706	73.0053	0.2699	363.01
291	9.83517	72.5353	0.2746	363.54
292	9.76339	72.0649	0.2794	364.06
293	9.69174	71.5940	0.2841	364.59
294	9.62021	71.1226	0.2888	365.12
295	9.54879	70.6508	0.2935	365.65
296	9.47749	70.1785	0.2982	366.18

Table D continued

Iowa Type curve 04

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
300	9.19343	68.2852	0.3171	368.29
301	9.12270	67.8107	0.3219	368.81
302	9.05206	67.3360	0.3266	369.34
303	8.98154	66.8608	0.3314	369.86
304	8.91112	66.3852	0.3361	370.39
305	8.84081	65.9092	0.3409	370.91
306	8.77060	65.4328	0.3457	371.43
307	8.70048	64.9561	0.3504	371.96
308	8.63047	64.4790	0.3552	372.48
309	8.56056	64.0015	0.3600	373.00
310	8.49075	63.5236	0.3648	373.52
311	8.42103	63.0454	0.3695	374.05
312	8.35141	62.5668	0.3743	374.57
313	8.28188	62.0878	0.3791	375.09
314	8.21244	61.6086	0.3839	375.61
315	8.14309	61.1290	0.3887	376.13
316	8.07384	60.6490	0.3935	376.65
317	8.00468	60.1687	0.3983	377.17
318	7.93560	59.6881	0.4031	377.69
319	7.86661	59.2072	0.4079	378.21
320	7.79771	58.7260	0.4127	378.73
321	7.72889	58.2444	0.4176	379.24
322	7.66016	57.7625	0.4224	379.76
323	7.59151	57.2803	0.4272	380.28
324	7.52294	56.7979	0.4320	380.80
325	7.45445	56.3151	0.4368	381.32
326	7.38604	55.8321	0.4417	381.83
327	7.31772	55.3487	0.4465	382.35
328	7.24947	54.8651	0.4513	382.87
329	7.18130	54.3811	0.4562	383.38
330	7.11320	53.8970	0.4610	383.90
331	7.04518	53.4125	0.4659	384.41
332	6.97724	52.9277	0.4707	384.93
333	6.90937	52.4427	0.4756	385.44
334	6.84157	51.9575	0.4804	385.96
335	6.77384	51.4720	0.4853	386.47
336	6.70619	50.9862	0.4901	386.99
337	6.63860	50.5002	0.4950	387.50
338	6.57109	50.0139	0.4999	388.01
339	6.50364	49.5274	0.5047	388.53
340	6.43627	49.0406	0.5096	389.04
341	6.36896	48.5536	0.5145	389.55
342	6.30171	48.0664	0.5193	390.07
343	6.23453	47.5790	0.5242	390.58
344	6.16742	47.0912	0.5291	391.09
345	6.10037	46.6033	0.5340	391.60
346	6.03339	46.1152	0.5388	392.12
347	5.96647	45.6268	0.5437	392.63
348	5.89961	45.1382	0.5486	393.14
349	5.83281	44.6494	0.5535	393.65
350	5.76607	44.1604	0.5584	394.16
351	5.69939	43.6712	0.5633	394.67
352	5.63278	43.1818	0.5682	395.18
353	5.56622	42.6921	0.5731	395.69
354	5.49971	42.2024	0.5780	396.20
355	5.43327	41.7123	0.5829	396.71
356	5.36688	41.2221	0.5878	397.22
357	5.30055	40.7317	0.5927	397.73
358	5.23428	40.2411	0.5976	398.24
359	5.16806	39.7503	0.6025	398.75
360	5.10189	39.2594	0.6074	399.26
361	5.03578	38.7682	0.6123	399.77
362	4.96972	38.2769	0.6172	400.28
363	4.90371	37.7854	0.6221	400.79
364	4.83776	37.2937	0.6271	401.29
365	4.77186	36.8018	0.6320	401.80
366	4.70601	36.3098	0.6369	402.31
367	4.64020	35.8177	0.6418	402.82
368	4.57445	35.3253	0.6467	403.33
369	4.50875	34.8328	0.6517	403.83
370	4.44310	34.3400	0.6566	404.34
371	4.37749	33.8472	0.6615	404.85
372	4.31193	33.3543	0.6665	405.35
373	4.24642	32.8611	0.6714	405.86
374	4.18096	32.3678	0.6763	406.37
375	4.11554	31.8743	0.6813	406.87
376	4.05017	31.3807	0.6862	407.38
377	3.98485	30.8869	0.6911	407.89
378	3.91956	30.3931	0.6961	408.39
379	3.85433	29.8990	0.7010	408.90
380	3.78913	29.4049	0.7060	409.40
381	3.72398	28.9106	0.7109	409.91
382	3.65888	28.4160	0.7158	410.42
383	3.59381	27.9215	0.7208	410.92
384	3.52879	27.4268	0.7257	411.43
385	3.46381	26.9319	0.7307	411.93
386	3.39887	26.4369	0.7356	412.44
387	3.33397	25.9418	0.7406	412.94
388	3.26911	25.4466	0.7455	413.45
389	3.20430	24.9511	0.7505	413.95
390	3.13952	24.4557	0.7554	414.46
391	3.07478	23.9601	0.7604	414.96
392	3.01008	23.4643	0.7654	415.46
393	2.94542	22.9684	0.7703	415.97
394	2.88079	22.4725	0.7753	416.47
395	2.81621	21.9764	0.7802	416.98
[illegible]	[illegible]	[illegible]	[illegible]	417.48

Iowa Type curve 04

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
400	2.49382	19.4943	0.8051	419.49
401	2.42945	18.9976	0.8100	420.00
402	2.36512	18.5007	0.8150	420.50
403	2.30082	18.0038	0.8200	421.00
404	2.23656	17.5067	0.8249	421.51
405	2.17233	17.0096	0.8299	422.01
406	2.10813	16.5123	0.8349	422.51
407	2.04397	16.0150	0.8399	423.01
408	1.97983	15.5176	0.8448	423.52
409	1.91574	15.0200	0.8498	424.02
410	1.85167	14.5224	0.8548	424.52
411	1.78764	14.0247	0.8598	425.02
412	1.72364	13.5268	0.8647	425.53
413	1.65966	13.0290	0.8697	426.03
414	1.59573	12.5310	0.8747	426.53
415	1.53182	12.0329	0.8797	427.03
416	1.46794	11.5348	0.8847	427.53
417	1.40409	11.0366	0.8896	428.04
418	1.34027	10.5383	0.8946	428.54
419	1.27649	10.0399	0.8996	429.04
420	1.21273	9.5415	0.9046	429.54
421	1.14900	9.0430	0.9096	430.04
422	1.08530	8.5444	0.9146	430.54
423	1.02162	8.0458	0.9195	431.05
424	0.95798	7.5471	0.9245	431.55
425	0.89436	7.0484	0.9295	432.05
426	0.83078	6.5495	0.9345	432.55
427	0.76722	6.0507	0.9395	433.05
428	0.70369	5.5518	0.9445	433.55
429	0.64018	5.0530	0.9495	434.05
430	0.57670	4.5542	0.9545	434.55
431	0.51325	4.0553	0.9594	435.06
432	0.44982	3.5567	0.9644	435.56
433	0.38642	3.0582	0.9694	436.06
434	0.32305	2.5600	0.9744	436.56
435	0.25970	2.0625	0.9794	437.06
436	0.19637	1.5665	0.9843	437.57
437	0.13307	1.0738	0.9893	438.07
438	0.06980	0.5938	0.9941	438.59
439	0.00655	0.5000	0.9950	439.50
440	0.00000	0.0000	1.0000	440.00

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Table D continued

Iowa Type curve SQ

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
0	100.00000	100.0000	0.0000	100.00
1	100.00000	99.5000	0.0050	100.50
2	100.00000	98.5000	0.0150	100.50
3	100.00000	97.5000	0.0250	100.50
4	100.00000	96.5000	0.0350	100.50
5	100.00000	95.5000	0.0450	100.50
6	100.00000	94.5000	0.0550	100.50
7	100.00000	93.5000	0.0650	100.50
8	100.00000	92.5000	0.0750	100.50
9	100.00000	91.5000	0.0850	100.50
10	100.00000	90.5000	0.0950	100.50
11	100.00000	89.5000	0.1050	100.50
12	100.00000	88.5000	0.1150	100.50
13	100.00000	87.5000	0.1250	100.50
14	100.00000	86.5000	0.1350	100.50
15	100.00000	85.5000	0.1450	100.50
16	100.00000	84.5000	0.1550	100.50
17	100.00000	83.5000	0.1650	100.50
18	100.00000	82.5000	0.1750	100.50
19	100.00000	81.5000	0.1850	100.50
20	100.00000	80.5000	0.1950	100.50
21	100.00000	79.5000	0.2050	100.50
22	100.00000	78.5000	0.2150	100.50
23	100.00000	77.5000	0.2250	100.50
24	100.00000	76.5000	0.2350	100.50
25	100.00000	75.5000	0.2450	100.50
26	100.00000	74.5000	0.2550	100.50
27	100.00000	73.5000	0.2650	100.50
28	100.00000	72.5000	0.2750	100.50
29	100.00000	71.5000	0.2850	100.50
30	100.00000	70.5000	0.2950	100.50
31	100.00000	69.5000	0.3050	100.50
32	100.00000	68.5000	0.3150	100.50
33	100.00000	67.5000	0.3250	100.50
34	100.00000	66.5000	0.3350	100.50
35	100.00000	65.5000	0.3450	100.50
36	100.00000	64.5000	0.3550	100.50
37	100.00000	63.5000	0.3650	100.50
38	100.00000	62.5000	0.3750	100.50
39	100.00000	61.5000	0.3850	100.50
40	100.00000	60.5000	0.3950	100.50
41	100.00000	59.5000	0.4050	100.50
42	100.00000	58.5000	0.4150	100.50
43	100.00000	57.5000	0.4250	100.50
44	100.00000	56.5000	0.4350	100.50
45	100.00000	55.5000	0.4450	100.50
46	100.00000	54.5000	0.4550	100.50
47	100.00000	53.5000	0.4650	100.50
48	100.00000	52.5000	0.4750	100.50
49	100.00000	51.5000	0.4850	100.50
50	100.00000	50.5000	0.4950	100.50
51	100.00000	49.5000	0.5050	100.50
52	100.00000	48.5000	0.5150	100.50
53	100.00000	47.5000	0.5250	100.50
54	100.00000	46.5000	0.5350	100.50
55	100.00000	45.5000	0.5450	100.50
56	100.00000	44.5000	0.5550	100.50
57	100.00000	43.5000	0.5650	100.50
58	100.00000	42.5000	0.5750	100.50
59	100.00000	41.5000	0.5850	100.50
60	100.00000	40.5000	0.5950	100.50
61	100.00000	39.5000	0.6050	100.50
62	100.00000	38.5000	0.6150	100.50
63	100.00000	37.5000	0.6250	100.50
64	100.00000	36.5000	0.6350	100.50
65	100.00000	35.5000	0.6450	100.50
66	100.00000	34.5000	0.6550	100.50
67	100.00000	33.5000	0.6650	100.50
68	100.00000	32.5000	0.6750	100.50
69	100.00000	31.5000	0.6850	100.50
70	100.00000	30.5000	0.6950	100.50
71	100.00000	29.5000	0.7050	100.50
72	100.00000	28.5000	0.7150	100.50
73	100.00000	27.5000	0.7250	100.50
74	100.00000	26.5000	0.7350	100.50
75	100.00000	25.5000	0.7450	100.50
76	100.00000	24.5000	0.7550	100.50
77	100.00000	23.5000	0.7650	100.50
78	100.00000	22.5000	0.7750	100.50
79	100.00000	21.5000	0.7850	100.50
80	100.00000	20.5000	0.7950	100.50
81	100.00000	19.5000	0.8050	100.50
82	100.00000	18.5000	0.8150	100.50
83	100.00000	17.5000	0.8250	100.50
84	100.00000	16.5000	0.8350	100.50
85	100.00000	15.5000	0.8450	100.50
86	100.00000	14.5000	0.8550	100.50
87	100.00000	13.5000	0.8650	100.50
88	100.00000	12.5000	0.8750	100.50
89	100.00000	11.5000	0.8850	100.50
90	100.00000	10.5000	0.8950	100.50
91	100.00000	9.5000	0.9050	100.50
92	100.00000	8.5000	0.9150	100.50
93	100.00000	7.5000	0.9250	100.50
94	100.00000	6.5000	0.9350	100.50
95	100.00000	5.5000	0.9450	100.50

Iowa Type curve SQ

Age %AL	Percent Surviving	Remaining Life	CAD Ratio	Prob. Life
100	100.00000	0.5000	0.9950	100.50
101	0.00000	0.0000	1.0000	101.00

APPENDIX II

Moments of Iowa Curves

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ENGINEERING VALUATION AND DEPRECIATION

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(3) as a means of checking the adequacy of the depreciation reserve balance or in obtaining an adjustment factor in appraisal procedure.

The process of smoothing and extending stub survivor curves by use of type curves is explained in Sec. 7.15; the procedure for estimating the probable life of a single unit by use of a type survivor curve is given in Secs. 7.21 and 7.22; and the use of type survivor curves and expectancy-life ratios is presented in Secs. 11.6 and 11.7.

METHODS OF CALCULATING SURVIVOR CURVES

Three systematic actuarial methods for calculating property survivor curves are discussed, the *annual rate*, or *retirement rate*, method, the *original group* method, and the *individual unit* method. Of these, the retirement rate method is much the best, because it is based on the collection and compilation of the data of all property in service during a period of recent years, both property retired and that still in service. The original group method uses only the data of one vintage installation. The individual unit method is a last-resort method because it uses only the data of property which has been retired.

7.9. Retirement Rate Method. The retirement data collected for use in the retirement rate method should be those for a recent normal period of, say 3 to 30 years, which will give retirement rates fairly representative of present and probable future policies and service conditions. The ideal period is one so short that it reflects only present policies and standards; yet it is long enough for sufficient retirements to have been made at each age to give reliable average retirement rates over a period that averages the ups and downs of the enterprise.

For any type of industrial property, the steps in the retirement rate method of compiling a survivor curve are as follows:

1. Determine the numbers of units, or their total costs, and the ages of the property retired each year of the experience band of years chosen for study. Table 7.3 illustrates the compilation of this information for the retirements from 1940 through 1950 for centrifugal gas pumps.

2. Determine the numbers of units, or their total costs, and ages of the property in service at the beginning of each year of the experience band. Table 7.4 illustrates the compilation of this information on exposures for the same property for which the retirements are given in Table 7.3. The experience band is 1940 to 1950, although the placement band is 1919 to 1950.

3. By using the retirements for each experience year from each vintage group as obtained in step 1, determine the total retirements during each age interval. Column (3) of Table 7.5 illustrates the result of this step. The total retirements at each age are obtained by adding the retirements from Table 7.3 on the diagonal stair-step line. Retirements on all such diagonals were made at the same average integral age. The line illustrated is for the retirements at an average age of 11 years or for the age interval of 10½–11½. The two retirements at this age are 689 from the 1939 vintage in 1950 and 10,609 from the 1935 vintage in 1946. The total retirements for the age interval are 11,298. Only one retirement, 618, was made during the age interval 0–0½ years.

4. From the property in service each year as obtained in step 2, determine the total number of units exposed to retirement at the beginning of each age interval. Column (2) of Table 7.5 illustrates the result of this step. The total exposures at each age are obtained by adding the property in service from Table 7.4 on the diagonal stair-step line in the same manner as for the retirements. For the age interval of 10½–11½ this sum is the total of 3,199, 962, 0, 996, . . . , 58,690, and 70,812, or 244,448. The exposure at the beginning of the age interval 0–0½ is the sum of the installations for each year of the experience band. For the example, this sum is obtained from column (2) of Table 7.4 by adding the installations from 1950 back to and including 1940, which are 90,676, 102,434, 180,111, . . . , 20,606, and 15,215. The total is 546,214.

5. Using the retirements at each age, as determined in step 3, and the amounts of property of each age in service determined as in step 4, calculate the retirement rate of the property at each age. These rates are illustrated in column (4) of Table 7.5 and result from the division of column (3) by column (2).

6. Calculate the percentage surviving at the beginning of each age interval by multiplying the retirement rate for each age interval by the percentage surviving at the beginning of that age interval and subtracting this product from the percentage surviving at the beginning of the same interval. Thus, using the retirement rate for the age interval of 10½–11½, the percent surviving for age 11½ is calculated as follows: $93.37 - (93.37)(0.046218) = 89.05$. These surviving percentages are illustrated in column (5) of Table 7.5.

7. Plot the survivor curve from the survivor percentages found in step (6), as far as they extend. These percentages are illustrated in column (5) of Table 7.5 and plotted in Fig. 7.4. In Fig. 7.5 this curve is compared with survivor curves calculated by the original group and individual unit methods.

8. Determine the average service life from the area under the survivor curve. When the original survivor curve is not reasonably smooth or when it is a stub curve, the curve should be first smoothed and extended as explained in Secs. 7.12 to 7.15.

7.10. Original Group Method. The original group method of calculating survivor curves is applicable to vintage groups. When applied

Survey: Market Risk Premium and Risk-Free Rate used for 80 countries in 2023

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ABSTRACT

This paper contains the statistics of a survey about the Risk-Free Rate (**RF**) and the Market Risk Premium (**MRP**) used in 2023 for **80 countries**. We got answers for 102 countries, but we only report the results for 80 countries with more than 6 answers.

The paper also contains the links to previous years surveys, from 2008 to 2022.

JEL Classification: G12, G31, M21

Keywords: equity premium; required equity premium; expected equity premium; risk-free rate

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1. Market Risk Premium (MRP), Risk Free Rate (RF) and Km [RF + MRP] used in 2022 in 95 countries

We sent a short email (see exhibit 1) on March, 2023 to more than 15,000 email addresses of finance and economics professors, analysts and managers of companies obtained from previous correspondence, papers and webs of companies and universities. We asked about the Risk-Free Rate (**RF**) and the Market Risk Premium (**MRP**) **used *"to calculate the required return to equity in different countries"*.**

By March 31, 2023, we had received 1,717 emails. 194 persons answered that they do not use MRP (see table 1), most of them use Km (required return to equity) but do not use MRP nor RF. The remaining emails had specific Risk-Free Rates and MRPs used in 2023 for one or more countries.[1] We would like to sincerely thank everyone who took the time to answer us.

Table 1. MRP and RF used in 2022: 1,624 emails

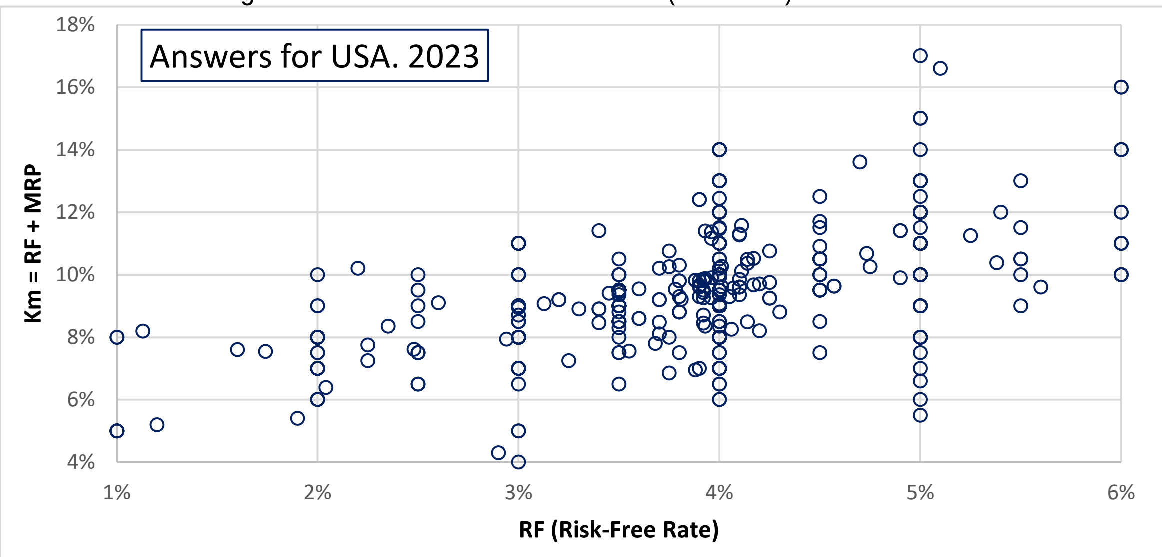
	Total
Answers reported (MRP figures)	3,812
Answers for countries with less than 6 answera	61
Outliers	74
"I can't provide you those figures: now are confidential"	47
Only MRP or RF (not both)	36
"We do not use MRP"	194

Table 2 contains the statistics of the **MRP** used in 2023 **for 80 countries**. We got answers for 102 countries, but we only report the results for 80 countries with more than 6 answers.

Table 3 contains the statistics of the Risk-Free Rate (**RF**) used in 2023 in the 80 countries[2] and **Table 4** contains the average of **Km** (required return to equity: Km = Risk-Free Rate + MRP).

Figure 1 is a graphic representation of the answers (Km and RF) we got for USA.

Figure 1. Answers for USA. RF and Km (RF + MRP) used in 2023



[1] We considered 74 of them as outliers because they provided a very small MRP (below 2%)

[2] Fernandez, P. (2020), "'Normalized' Risk-Free Rate: Fiction or Science Fiction?" Available at: https://ssrn.com/abstract=3708863

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Table 2. Market Risk Premium (MRP) used for 80 countries in 2023

MRP	Number of Answers	Average	Median	MAX	min
USA	1378	5,7%	5,5%	15,0%	2,0%
Spain 2023	428	6,6%	6,3%	15,0%	3,0%
Andorra	8	8,9%	8,8%	10,2%	7,8%
Argentina	15	28,1%	26,7%	39,8%	7,5%
Australia	39	6,2%	6,0%	15,0%	3,3%
Austria	67	6,8%	6,6%	9,0%	5,0%
Belgium	63	6,4%	7,0%	8,2%	4,0%
Bolivia	10	14,3%	14,8%	17,0%	9,0%
Bosnia	9	16,6%	16,5%	18,9%	14,6%
Brazil	43	9,3%	9,7%	20,0%	4,0%
Bulgaria	10	8,1%	8,3%	9,6%	6,5%
Canada	41	6,0%	6,0%	8,0%	4,0%
Chile	25	6,9%	7,0%	8,1%	5,5%
China	25	8,6%	8,7%	12,0%	4,0%
Colombia	15	9,0%	9,2%	20,0%	3,0%
Costa Rica	9	14,2%	14,7%	17,0%	9,0%
Croatia	13	8,7%	9,0%	10,1%	7,0%
Czech Republic	24	6,6%	6,7%	9,0%	5,3%
Denmark	27	6,2%	5,9%	8,7%	4,8%
Dominican Rep.	8	11,7%	11,6%	13,4%	10,3%
Ecuador	19	20,9%	23,2%	32,2%	3,0%
Egypt	9	14,4%	14,7%	17,0%	10,8%
Estonia	19	6,9%	6,8%	8,9%	6,1%
Ethiopia	8	20,7%	20,5%	23,6%	18,3%
Finland	31	6,2%	6,6%	7,8%	3,5%
France	88	6,0%	6,3%	8,3%	0,3%
Germany	264	5,7%	5,9%	9,0%	0,0%
Greece	38	10,9%	12,2%	14,9%	5,5%
Hong Kong	17	6,8%	7,0%	7,6%	5,5%
Hungary	14	8,4%	9,0%	11,1%	3,2%
Iceland	6	7,1%	7,1%	8,4%	6,1%
India	19	8,5%	9,7%	14,0%	1,5%
Indonesia	12	8,0%	9,1%	11,4%	3,2%
Ireland	41	6,7%	7,2%	8,6%	3,5%
Israel	19	6,9%	7,1%	8,5%	5,1%
Italy	79	7,1%	6,8%	11,5%	1,9%
Japan	38	6,1%	6,0%	9,9%	2,0%
Kenya	11	14,7%	15,4%	18,3%	9,0%
Korea, (South)	17	6,4%	6,5%	8,0%	5,0%
Kuwait	15	6,9%	6,8%	9,9%	5,1%
Latvia	11	7,7%	7,7%	9,8%	5,8%
Lithuania	35	7,1%	7,1%	9,1%	6,3%
Luxembourg	36	5,9%	5,9%	7,1%	4,5%
Malaysia	10	7,6%	8,0%	9,2%	5,5%
Mexico	35	7,7%	8,2%	12,1%	2,0%
Mongolia	9	17,2%	16,5%	22,5%	14,6%
Morocco	18	9,9%	9,8%	12,3%	8,1%
Mozambique	8	20,7%	20,5%	23,6%	18,3%
Netherlands	56	5,6%	5,8%	6,9%	2,5%
New Zealand	10	6,3%	5,9%	8,5%	5,1%
Nigeria	9	16,8%	17,2%	18,6%	14,0%
Norway	14	5,8%	5,9%	8,0%	4,5%
Pakistan	9	19,5%	18,9%	24,0%	16,6%
Panama	9	8,5%	8,8%	10,2%	5,5%

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Peru	16	8,4%	8,7%	12,0%	6,0%
Phillipines	9	8,6%	8,8%	10,2%	6,5%
Poland	28	7,2%	7,4%	10,0%	5,0%
Portugal	42	8,2%	7,9%	15,0%	3,8%
Qatar	8	6,7%	6,7%	7,7%	5,9%
Romania	17	9,4%	9,7%	11,5%	6,0%
Russia	14	18,2%	18,9%	35,0%	6,6%
Saudi Arabia	18	6,9%	6,8%	7,9%	6,1%
Serbia	10	10,9%	11,1%	12,8%	7,5%
Singapore	17	5,6%	5,7%	6,5%	4,0%
Slovakia	13	7,5%	7,4%	9,0%	5,5%
Slovenia	10	7,6%	8,0%	9,1%	5,5%
South Africa	17	8,7%	7,9%	13,4%	5,0%
Sweden	52	5,7%	5,4%	8,5%	5,0%
Switzerland	65	5,6%	5,9%	8,0%	2,8%
Taiwan	24	6,7%	7,0%	10,0%	3,0%
Tanzania	8	14,9%	14,8%	17,0%	13,1%
Thailand	12	8,1%	8,7%	10,1%	5,5%
Turkey	10	18,3%	17,2%	35,0%	9,0%
Uganda	6	14,9%	14,8%	18,0%	12,1%
Ukraine	9	22,7%	23,2%	25,1%	19,0%
United Arab Emirates	10	6,4%	6,5%	8,5%	4,5%
United Kingdom	79	6,0%	6,0%	9,0%	2,0%
Uruguay	11	9,3%	9,2%	11,2%	7,7%
Venezuela	8	29,5%	29,3%	33,7%	26,0%
Vietnam	9	10,8%	10,7%	12,2%	9,5%

Table 3. Risk Free Rate (RF) used for 80 countries in 2023

RF	Number of Answers	Average	Median	MAX	min
USA	1378	3,8%	4,0%	6,0%	1,0%
Spain 2023	428	3,5%	3,5%	8,5%	1,2%
Andorra	8	2,9%	3,0%	4,0%	1,1%
Argentina	15	29,6%	27,9%	42,3%	8,0%
Australia	39	3,8%	3,9%	6,0%	3,0%
Austria	67	2,7%	3,0%	4,1%	1,3%
Belgium	63	3,8%	3,4%	12,0%	0,3%
Bolivia	10	5,7%	5,8%	10,0%	1,3%
Bosnia	9	4,5%	5,0%	6,3%	1,1%
Brazil	43	12,2%	13,5%	16,0%	6,3%
Bulgaria	10	3,3%	3,7%	6,5%	-0,1%
Canada	41	3,5%	3,3%	6,0%	2,0%
Chile	25	4,9%	4,9%	11,0%	-1,0%
China	25	4,2%	3,9%	7,1%	1,9%
Colombia	15	11,6%	12,6%	14,9%	3,5%
Costa Rica	9	4,2%	3,5%	11,0%	-0,9%
Croatia	13	3,7%	4,0%	4,8%	2,5%
Czech Republic	24	4,3%	4,5%	5,9%	2,7%
Denmark	27	2,9%	3,0%	3,4%	2,0%
Dominican Rep.	8	7,4%	7,5%	9,5%	5,1%
Ecuador	19	13,6%	14,0%	15,6%	10,8%
Egypt	9	14,9%	14,8%	19,9%	10,7%
Estonia	19	1,5%	1,5%	4,0%	-0,4%
Ethiopia	8	11,4%	11,7%	20,5%	3,7%
Finland	31	3,2%	3,3%	3,5%	3,0%
France	88	3,0%	3,0%	4,0%	0,5%

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Germany	264	2,5%	2,6%	7,0%	1,0%
Greece	38	4,1%	4,4%	5,3%	2,5%
Hong Kong	17	3,8%	3,9%	4,5%	2,5%
Hungary	14	8,3%	8,9%	10,3%	5,7%
Iceland	6	6,2%	6,1%	7,4%	5,0%
India	19	7,1%	7,3%	8,2%	5,0%
Indonesia	12	6,9%	7,0%	7,3%	6,3%
Ireland	41	2,9%	3,2%	4,2%	0,6%
Israel	19	3,9%	4,0%	5,0%	1,5%
Italy	79	4,0%	4,0%	5,3%	2,0%
Japan	38	1,1%	0,5%	4,0%	-0,6%
Kenya	11	14,1%	14,3%	14,9%	12,8%
Korea, (South)	17	2,9%	3,5%	4,5%	0,3%
Kuwait	15	1,9%	2,0%	3,5%	-0,4%
Latvia	11	1,2%	0,9%	3,5%	-0,6%
Lithuania	35	1,7%	1,5%	4,7%	-1,5%
Luxembourg	36	3,0%	3,0%	3,0%	3,0%
Malaysia	10	4,1%	4,1%	5,3%	3,5%
Mexico	35	8,3%	9,0%	12,0%	4,0%
Mongolia	9	9,6%	9,5%	13,0%	6,2%
Morocco	18	3,3%	3,4%	7,0%	0,0%
Mozambique	8	7,0%	7,1%	9,0%	4,4%
Netherlands	56	3,0%	3,0%	5,0%	2,3%
New Zealand	10	4,7%	4,7%	5,0%	4,5%
Nigeria	9	13,7%	14,2%	16,1%	10,0%
Norway	14	3,4%	3,3%	5,0%	2,0%
Pakistan	9	16,3%	15,7%	21,0%	13,3%
Panama	9	6,9%	6,9%	8,8%	4,4%
Peru	16	6,5%	7,0%	8,0%	3,5%
Phillipines	9	5,2%	6,3%	8,2%	0,6%
Poland	28	6,1%	6,4%	9,0%	2,0%
Portugal	42	3,4%	3,6%	5,0%	1,3%
Qatar	8	2,9%	3,0%	5,0%	0,2%
Romania	17	7,2%	7,8%	9,4%	3,3%
Russia	14	9,4%	10,5%	13,6%	3,5%
Saudi Arabia	18	5,1%	5,1%	7,0%	2,8%
Serbia	10	7,2%	8,0%	10,1%	3,1%
Singapore	17	2,6%	3,0%	4,4%	0,1%
Slovakia	13	3,4%	3,7%	4,1%	2,5%
Slovenia	10	3,6%	3,8%	4,4%	2,5%
South Africa	17	9,4%	10,1%	12,6%	4,0%
Sweden	52	1,9%	2,1%	3,5%	-0,5%
Switzerland	65	1,7%	1,5%	3,0%	0,8%
Taiwan	24	1,4%	1,2%	2,0%	0,9%
Tanzania	8	8,1%	8,2%	13,0%	4,9%
Thailand	12	3,0%	2,6%	6,0%	1,3%
Turkey	10	14,4%	11,5%	45,0%	-2,6%
Uganda	6	11,3%	11,4%	13,2%	8,7%
Ukraine	9	30,6%	29,0%	41,6%	12,7%
United Arab Emirates	10	3,7%	4,5%	5,6%	0,0%
United Kingdom	79	3,9%	3,8%	8,3%	2,0%
Uruguay	11	8,3%	8,0%	13,0%	3,0%
Venezuela	8	34,8%	32,2%	70,4%	10,4%
Vietnam	9	4,1%	4,5%	5,9%	1,7%

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Table 4. Km [Required return to equity (market): RF + MRP)] used for 80 countries in 2023

USA	9,5%	Greece	15,0%	Peru	14,9%
Spain 2023	10,1%	Hong Kong	10,6%	Phillipines	13,9%
Andorra	11,8%	Hungary	16,7%	Poland	13,4%
Argentina	57,7%	Iceland	13,4%	Portugal	11,6%
Australia	10,0%	India	15,5%	Qatar	9,6%
Austria	9,5%	Indonesia	14,9%	Romania	16,6%
Belgium	10,2%	Ireland	9,6%	Russia	27,6%
Bolivia	20,1%	Israel	10,8%	Saudi Arabia	12,0%
Bosnia	21,1%	Italy	11,1%	Serbia	18,1%
Brazil	21,5%	Japan	7,1%	Singapore	8,2%
Bulgaria	11,5%	Kenya	28,7%	Slovakia	10,9%
Canada	9,5%	Korea, (South)	9,3%	Slovenia	11,2%
Chile	11,8%	Kuwait	8,8%	South Africa	18,1%
China	12,8%	Latvia	8,9%	Sweden	7,5%
Colombia	20,6%	Lithuania	8,9%	Switzerland	7,4%
Costa Rica	18,4%	Luxembourg	8,9%	Taiwan	8,1%
Croatia	12,4%	Malaysia	11,7%	Tanzania	23,0%
Czech Republic	10,9%	Mexico	16,0%	Thailand	11,1%
Denmark	9,0%	Mongolia	26,8%	Turkey	32,7%
Dominican Rep.	19,2%	Morocco	13,2%	Uganda	26,2%
Ecuador	34,5%	Mozambique	27,7%	Ukraine	53,3%
Egypt	29,3%	Netherlands	8,7%	United Arab Emirates	10,1%
Estonia	8,4%	New Zealand	10,9%	United Kingdom	9,8%
Ethiopia	32,2%	Nigeria	30,5%	Uruguay	17,7%
Finland	9,4%	Norway	9,2%	Venezuela	64,3%
France	9,0%	Pakistan	35,8%	Vietnam	14,8%
Germany	8,2%	Panama	15,4%		

2. Changes from 2015 to 2018, 2019, 2020, 2021, 2022 and 2023

Tables 5 and 6 compare the results of the 2023 survey with the results of the surveys published in 2015, 2018, 2019, 2020, 2021 and 2022.

Table 5. Km [Required return to equity (market): RF + MRP)]
Averages of the surveys of 2023, 2022, 2021, 2020, 2019, 2018 and 2015

	average Km (RF + MRP)						
	2023	2022	2021	2020	2019	2018	2015
USA	9,5	8,3	7,3	7,5	8,3	8,2	7,9
Spain	10,1	8,8	7,4	7,6	8,1	8,8	8,1
Argentina	57,7	58,3	41,6	29,6	25,0	23,2	35,5
Australia	10,0	9,7	9,0	10,3	9,3	9,7	9,1
Austria	9,5	7,6	6,5	7,1	7,4	8,2	8,5
Belgium	10,2	7,2	6,5	7,1	7,4	7,8	6,8
Brazil	21,5	20,1	14,2	12,7	15,4	15,7	16,5
Canada	9,5	8,5	7,5	7,5	8,3	8,7	8,2
Chile	11,8	13,1	10,2	10,2	10,5	10,2	10,4
China	12,8	12,6	9,0	9,8	11,5	10,1	12,6
Colombia	20,6	16,5	13,8	14,5	13,9	15,4	12,1
Czech Rep.	10,9	10,1	7,8	8,2	8,7	8,5	7,4
Denmark	9,0	7,2	6,5	7,0	7,2	7,6	6,8
Finland	9,4	7,0	6,5	7,5	7,3	7,6	6,9
France	9,0	7,6	6,6	7,0	7,2	7,5	7,1
Germany	8,2	6,9	6,4	6,6	6,8	6,7	6,6
Greece	15,0	8,2	7,8	19,1	19,7	20,6	29,3
Hungary	16,7	11,6	10,4	10,5	11,9	11,5	9,4

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India	15,5	12,5	12,9	11,8	14,8	14,7	15,8
Indonesia	14,9	13,2	12,9	13,9	16,2	15,6	16,4
Ireland	9,6	7,3	6,6	7,9	7,4	8,1	6,8
Israel	10,8	8,7	6,8	7,8	8,4	7,7	6,1
Italy	11,1	7,7	7,0	7,5	7,9	8,4	6,9
Japan	7,1	6,4	5,7	7,1	7,2	6,0	6,5
Korea (South)	9,3	9,7	8,3	8,1	9,1	8,8	8,5
Mexico	16,0	14,8	12,2	13,7	15,4	15,3	12,3
Netherlands	8,7	7,5	6,7	7,5	7,3	7,5	7,7
New Zealand	10,9	9,5	8,0	8,6	8,9	8,9	9,5
Norway	9,2	7,5	7,2	7,0	7,4	8,1	6,9
Peru	14,9	13,3	11,1	10,7	13,1	12,6	11,2
Poland	13,4	9,7	8,2	9,0	9,7	9,4	7,9
Portugal	11,6	7,8	8,2	8,7	10,1	10,4	7,3
Russia	27,6	20,0	13,8	13,7	16,8	16,5	17,1
South Africa	18,1	16,4	15,1	14,6	16,4	14,5	15,9
Sweden	7,5	7,4	8,4	7,1	7,4	8,9	6,5
Switzerland	7,4	7,2	5,3	7,0	7,3	8,0	6,5
Thailand	11,1	10,1	9,5	10,2	11,3	12,4	16,0
Turkey	32,7	33,6	27,2	21,2	20,8	18,0	17,1
UK	9,8	8,5	6,9	6,9	8,3	7,5	7,3
Uruguay	17,7	12,7	11,3	15,2	12,8	13,6	10,7
Venezuela	64,3	58,8	60,2	34,5	36,3	28,6	23,1

Table 6. Market Risk Premium (MRP) and Risk Free Rate (R_F) (%)
Averages of the surveys of 2023, 2022, 2021, 2020, 2019, 2018 and 2015

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	Av. 2023		Av. 2022		Av. 2021		Av. 2020		Av. 2019		Av. 2018		Av. 2015	
	RF	MRP	RF	MRP	RF	MRP	RF	MRP	RF	MRP	RF	MRP	RF	MRP
USA	3,8	5,7	2,7	5,6	1,8	5,5	1,9	5,6	2,7	5,6	2,8	5,4	2,4	5,5
Spain	3,5	6,6	2,1	6,7	1,0	6,4	1,3	6,3	1,7	6,4	2,1	6,7	2,2	5,9
Argentina	29,6	28,1	28,4	29,9	24,2	17,4	12,3	17,3	10,1	14,9	9,3	13,9	12,6	22,9
Australia	3,8	6,2	3,4	6,3	2,6	6,4	2,4	7,9	2,8	6,5	3,1	6,6	3,1	6,0
Austria	2,7	6,8	1,8	5,8	0,6	5,9	0,9	6,2	1,3	6,1	2,0	6,2	2,8	5,7
Belgium	3,8	6,4	1,4	5,8	0,6	5,9	0,9	6,2	1,2	6,2	1,6	6,2	1,3	5,5
Brazil	12,2	9,3	10,3	9,8	6,5	7,7	4,8	7,9	7,2	8,2	7,3	8,4	9,0	7,5
Canada	3,5	6,0	2,8	5,7	1,9	5,6	1,8	5,7	2,5	5,8	2,9	5,8	2,3	5,9
Chile	4,9	6,9	5,7	7,4	3,9	6,3	3,6	6,6	4,2	6,3	4,1	6,1	3,9	6,5
China	4,2	8,6	3,9	8,7	2,8	6,2	3,1	6,7	4,0	7,5	3,8	6,3	4,5	8,1
Colombia	11,6	9,0	9,8	6,7	6,9	6,9	6,3	8,2	6,2	7,7	6,7	8,7	3,8	8,3
Czech Rep.	4,3	6,6	4,1	6,0	2,0	5,8	1,8	6,4	2,4	6,3	2,6	5,9	1,8	5,6
Denmark	2,9	6,2	1,4	5,8	0,7	5,8	0,9	6,1	1,2	6,0	1,6	6,0	1,3	5,5
Finland	3,2	6,2	1,4	5,6	0,6	5,9	1,0	6,5	1,1	6,2	1,7	5,9	1,2	5,7
France	3,0	6,0	1,3	6,3	0,8	5,8	0,8	6,2	1,2	6,0	1,6	5,9	1,5	5,6
Germany	2,5	5,7	1,2	5,7	0,6	5,8	0,8	5,8	1,1	5,7	1,4	5,3	1,3	5,3
Greece	4,1	10,9	1,6	6,6	0,9	6,9	6,4	12,7	4,3	15,4	4,8	15,8	15,0	14,3
Hungary	8,3	8,4	4,9	6,7	3,3	7,1	3,1	7,4	4,0	7,9	3,6	7,9	0,6	8,8
India	7,1	8,5	5,6	6,9	5,6	7,3	4,8	7,0	6,5	8,3	6,8	7,9	7,4	8,4
Indonesia	6,9	8,0	5,5	7,7	5,9	7,0	6,3	7,6	7,2	9,0	6,8	8,8	7,5	8,9
Ireland	2,9	6,7	1,5	5,8	0,7	5,9	1,3	6,6	1,4	6,0	1,6	6,5	1,3	5,5
Israel	3,9	6,9	2,7	6,0	1,1	5,7	1,5	6,3	2,0	6,4	1,9	5,8	0,9	5,2
Italy	4,0	7,1	1,7	6,0	1,0	6,0	1,3	6,2	1,6	6,3	2,3	6,1	1,5	5,4
Japan	1,1	6,1	0,5	5,9	0,5	5,2	0,9	6,2	1,1	6,1	0,3	5,7	0,7	5,8
Korea (South)	2,9	6,4	3,7	6,0	2,4	5,9	2,0	6,1	2,5	6,6	2,4	6,4	2,3	6,2
Mexico	8,3	7,7	7,4	7,4	5,8	6,4	5,4	8,3	7,1	8,3	6,8	8,5	4,3	8,0
Netherlands	3,0	5,6	1,3	6,2	0,9	5,8	1,6	5,9	1,3	6,0	1,7	5,8	1,8	5,9
New Zealand	4,7	6,3	3,8	5,7	2,0	6,0	2,4	6,2	3,0	5,9	3,1	5,8	2,9	6,6
Norway	3,4	5,8	1,7	5,8	1,8	5,4	1,2	5,8	1,4	6,0	2,4	5,7	1,4	5,5
Peru	6,5	8,4	6,4	6,9	4,3	6,8	3,7	7,0	5,6	7,5	5,3	7,3	4,0	7,2
Poland	6,1	7,2	4,0	5,7	2,7	5,5	2,4	6,6	3,1	6,6	3,4	6,0	2,7	5,2
Portugal	3,4	8,2	1,6	6,2	1,4	6,8	1,6	7,1	2,6	7,5	3,2	7,2	1,6	5,7
Russia	9,4	18,2	5,8	14,2	5,7	8,1	5,9	7,8	8,3	8,5	7,8	8,7	7,4	9,7
South Africa	9,4	8,7	9,1	7,3	8,1	7,0	6,7	7,9	8,0	8,4	7,6	6,9	8,2	7,7
Sweden	1,9	5,7	1,4	6,0	0,9	7,5	1,0	6,1	1,3	6,1	1,8	7,1	1,1	5,4
Switzerland	1,7	5,6	1,4	5,8	0,1	5,2	0,9	6,1	1,1	6,2	1,1	6,9	1,1	5,4
Thailand	3,0	8,1	3,1	7,0	2,2	7,3	4,5	5,7	3,1	8,2	3,5	8,9	8,7	7,3
Turkey	14,4	18,3	22,6	11,0	17,7	9,5	10,9	10,3	11,2	9,6	10,3	7,7	7,8	9,3
UK	3,9	6,0	2,4	6,1	1,3	5,6	1,1	5,8	2,1	6,2	2,0	5,5	2,1	5,2
Uruguay	8,3	9,3	5,4	7,3	4,2	7,1	6,1	9,1	4,4	8,4	5,3	8,3	3,6	7,1
Venezuela	34,8	29,5	32,7	26,1	40,4	19,8	11,4	23,1	12,6	23,7	11,7	16,9	3,5	19,6

3. Previous surveys

2008	http://ssrn.com/abstract=1344209
2010	http://ssrn.com/abstract=1606563; http://ssrn.com/abstract=1609563
2011	http://ssrn.com/abstract=1822182; http://ssrn.com/abstract=1805852
2012	http://ssrn.com/abstract=2084213
2013	http://ssrn.com/abstract=914160
2014	http://ssrn.com/abstract=1609563
2015	https://ssrn.com/abstract=2598104
2016	https://ssrn.com/abstract=2776636
2017	https://ssrn.com/abstract=2954142
2018	https://ssrn.com/abstract=3155709
2019	https://ssrn.com/abstract=3358901
2020	https://ssrn.com/abstract=3560869
2021	https://ssrn.com/abstract=3861152
2022	https://ssrn.com/abstract=3803990

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2023	https://ssrn.com/abstract=4407839

Welch (2000) performed two surveys with finance professors in 1997 and 1998, asking them what they thought the Expected MRP would be over the next 30 years. He obtained 226 replies, ranging from 1% to 15%, with an average arithmetic EEP of 7% above T-Bonds.[3] Welch (2001) presented the results of a survey of 510 finance and economics professors performed in August 2001 and the consensus for the 30-year arithmetic EEP was 5.5%, much lower than just 3 years earlier. In an update published in 2008 Welch reports that the MRP "used in class" in December 2007 by about 400 finance professors was on average 5.89%, and 90% of the professors used equity premiums between 4% and 8.5%.

Johnson et al (2007) report the results of a survey of 116 finance professors in North America done in March 2007: 90% of the professors believed the Expected MRP during the next 30 years to range from 3% to 7%.

Graham and Harvey (2007) indicate that U.S. CFOs reduced their average EEP from 4.65% in September 2000 to 2.93% by September 2006 (st. dev. of the 465 responses = 2.47%). In the 2008 survey, they report an average EEP of 3.80%, ranging from 3.1% to 11.5% at the tenth percentile at each end of the spectrum. They show that average EEP changes through time. Goldman Sachs (O'Neill, Wilson and Masih 2002) conducted a survey of its global clients in July 2002 and the average long-run EEP was 3.9%, with most responses between 3.5% and 4.5%.

Ilmanen (2003) argues that surveys tend to be optimistic: *"survey-based expected returns may tell us more about hoped-for returns than about required returns".* Damodaran (2008) points out that *"the risk premiums in academic surveys indicate how far removed most academics are from the real world of valuation and corporate finance and how much of their own thinking is framed by the historical risk premiums... The risk premiums that are presented in classroom settings are not only much higher than the risk premiums in practice but also contradict other academic research".*

Table 4 of Fernandez et al (2011a) shows the evolution of the Market Risk Premium used for the USA in 2011, 2010, 2009 and 2008 according to previous surveys (Fernandez et al, 2009, 2010a and 2010b).

The magazine *Pensions and Investments* (12/1/1998) carried out a survey among professionals working for institutional investors: the average EEP was 3%. Shiller[4] publishes and updates an index of investor sentiment since the crash of 1987. While neither survey provides a direct measure of the equity risk premium, they yield a broad measure of where investors or professors expect stock prices to go in the near future. The 2004 survey of the Securities Industry Association (SIA) found that the median EEP of 1500 U.S. investors was about 8.3%. Merrill Lynch surveys more than 300 institutional investors globally in July 2008: the average EEP was 3.5%.

A main difference of this survey with previous ones is that this survey asks about the **Required** MRP, while most surveys are interested in the **Expected** MRP.

4. Expected and Required Equity Premium: different concepts

Fernandez and F. Acín (2015) claim and show that Expected Return and Required Return are two very different concepts. Fernandez (2007, 2009b) claims that the term "equity premium" is used to designate four different concepts:

1. **Historical** equity premium (HEP): historical differential return of the stock market over treasuries.
2. **Expected** equity premium (EEP): expected differential return of the stock market over treasuries.

[3] At that time, the most recent Ibbotson Associates Yearbook reported an arithmetic HEP versus T-bills of 8.9% (1926–1997).
[4] See http://icf.som.yale.edu/Confidence.Index

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3. **Required** equity premium (REP): incremental return of a diversified portfolio (the market) over the risk-free rate required by an investor. It is used for calculating the required return to equity.
4. **Implied** equity premium (IEP): the required equity premium that arises from assuming that the market price is correct.

The four concepts (HEP, REP, EEP and IEP) designate different realities. The **HEP** is easy to calculate and is equal for all investors, provided they use the same time frame, the same market index, the same risk-free instrument and the same average (arithmetic or geometric). But the **EEP,** the **REP** and the **IEP** may be different for different investors and are not observable.

The **HEP** is the historical average differential return of the market portfolio over the risk-free debt. The most widely cited sources are Ibbotson Associates and Dimson *et al.* (2007).

Numerous papers and books assert or imply that there is a "market" EEP. However, it is obvious that investors and professors do not share "homogeneous expectations" and have different assessments of the **EEP**. As Brealey et al. (2005, page 154) affirm, *"Do not trust anyone who claims to know what returns investors expect".*

The **REP** is the answer to the following question: What incremental return do I require for investing in a diversified portfolio of shares over the risk-free rate? It is a crucial parameter because the REP is the key to determining the company's required return to equity and the WACC. Different companies may use, and in fact do use, different **REPs**.

The **IEP** is the implicit REP used in the valuation of a stock (or market index) that matches the current market price. The most widely used model to calculate the IEP is the dividend discount model: the current price per share (P_0) is the present value of expected dividends discounted at the required rate of return (Ke). If d_1 is the dividend per share expected to be received in year 1, and g the expected long term growth rate in dividends per share,

$$P_0 = d_1 / (Ke - g), \text{ which implies: } IEP = d_1/P_0 + g - R_F \quad (1)$$

The estimates of the IEP depend on the particular assumption made for the expected growth (g). Even if market prices are correct for all investors, there is not an IEP common for all investors: there are many pairs (IEP, g) that accomplish equation (1). Even if equation (1) holds for every investor, there are many *required* returns (as many as expected growths, g) in the market. Many papers in the financial literature report different estimates of the IEP with great dispersion, as for example, Claus and Thomas (2001, IEP = 3%), Harris and Marston (2001, IEP = 7.14%) and Ritter and Warr (2002, IEP = 12% in 1980 and -2% in 1999). There is no a common **IEP** for all investors.

For a particular investor, the **EEP** is not necessary equal to the REP (unless he considers that the market price is equal to the value of the shares). Obviously, an investor will hold a diversified portfolio of shares if his EEP is higher (or equal) than his REP and will not hold it otherwise.

We can find out the REP and the EEP of an investor by asking him, although for many investors the REP is not an explicit parameter but, rather, it is implicit in the price they are prepared to pay for the shares. However, it is not possible to determine the REP for the market as a whole, because it does not exist: even if we knew the REPs of all the investors in the market, it would be meaningless to talk of a REP for the market as a whole. There is a distribution of REPs and we can only say that some percentage of investors have REPs contained in a range. The average of that distribution cannot be interpreted as the REP of the market nor as the REP of a representative investor.

Much confusion arises from not distinguishing among the four concepts that the phrase *equity premium* designates: Historical equity premium, Expected equity premium, Required equity premium and Implied equity premium. 129 of the books reviewed by Fernandez (2009b) identify Expected and Required equity premium and 82 books identify Expected and Historical equity premium.

Finance textbooks should clarify the MRP by incorporating distinguishing definitions of the four different concepts and conveying a clearer message about their sensible magnitudes.

5. Conclusion

Most previous surveys have been interested in the Expected MRP, but this survey asks about the Required MRP.

Electronic copy available at: https://ssrn.com/abstract=4407839

This paper contains the statistics of a survey about the Risk-Free Rate (**RF**) and the Market Risk Premium (**MRP**) used in 2023 for **80 countries**. We got answers for 102 countries, but we only report the results for countries with more than 6 answers.

This survey links with the *Equity Premium Puzzle*: Fernandez et al (2009), argue that the equity premium puzzle may be explained by the fact that many market participants (equity investors, investment banks, analysts, companies…) do not use standard theory (such as a standard representative consumer asset pricing model…) for determining their Required Equity Premium, but rather, they use historical data and advice from textbooks and finance professors. Many investors still use historical data and textbook prescriptions to estimate the required and the expected equity premium.

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EXHIBIT 1. Mail sent in March 2023

Survey Market Risk Premium and Risk-Free Rate 2023

We are doing a **survey** about the **Market Risk Premium** (MRP or Equity Premium) and **Risk-Free Rate** that companies, analysts, regulators and professors use to calculate the **required return on equity** in different countries.

I would be grateful if you would kindly answer the following 2 questions. No companies, individuals or universities will be identified, and only aggregate data will be made public. I will send you the results in a month.

Best regards and thanks,

Pablo Fernandez. Professor of Finance. IESE Business School. Spain.

2 questions:

1. The Market Risk Premium that I am using in 2023
for USA is: ______%
for__________ is: ______ %
for__________ is: ______ %

2. The Risk-Free rate that I am using in 2023
for USA is: ______ %
for __________ is: ______ %
for __________ is: ______ %

EXHIBIT 2. Some comments and webs recommended by respondents.

Equity premium: http://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/ctryprem.html
http://www.market-risk-premia.com/market-risk-premia.html
http://www.marktrisikoprämie.de/marktrisikopraemien.html

US risk free rate: http://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=yieldYear&year=2015

risk free rate: http://www.basiszinskurve.de/basiszinssatz-gemaess-idw.html
http://www.econ.yale.edu/~shiller/
http://www.cfosurvey.org/pastresults.htm
http://alephblog.com/

I'm not much use for you because I don't add a market risk premium to a risk free rate to get a basic equity rate of return. Many years ago, I took your lessons to heart and stopped using any sort of build-up method, principally because it is backwards looking. Instead, I rely on the Pepperdine survey, along with my understanding of how investors think and my best judgement of the risks of a particular asset. I have not found any better way to do this.

Islamic Development Bank works under development mandate and therefore does not follow market based premium on pricing, and uses its internal costs as benchmark. In short, all of our member countries are given financing at the same pricing.

Our commercial bank can invest overnight funds in our excess balance account with the U.S. Federal Reserve Bank at 2.5%. Our overall cost of funds is 0.2%, yielding a spread of 2.3%. Our leverage ratio (equity/assets) is 9.63%. Hence, our pre-tax risk-free rate is 23.88% of equity. Our target is to earn a net interest margin (interest income less interest expense as a percentage of earning assets) of 4.00%, which yields a targeted asset yield of 4.2%, or 43.61% of equity.

Market risk premium = actual equity return - risk free rate

I want to explain the unusually high risk premium I am using in the US market (7%). In my opinion, the way that costs whether they be raw materials, labor, interest etc. process through the economy differently than a simple "add on" cost. I believe that as any cost increase requires a greater capital base to hold inventory or to produce goods and services, that the pass through is not just the actual cost but the cost plus an increment for a return on the greater capital base. Accordingly, the "cost" of money with interest rates so low is more likely than not to be higher in the future. Labor also with unemployment so low is more likely than not to be higher in the future. Therefore although I do not see traditional commodity inflation and labor costs have been unusually stable for this unemployment level, I believe the probability is higher of an increase than a decrease. Thus I have a higher than would be expected market risk premium to address the direction I think the pressures will move on the discount rate. Conversely, If wrong on the upward pressures on capital

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returns; it would likely be due to slowing global growth and/or trade disruption of longer duration. In that event I again want a higher discount rate to reflect that greater risk potential. Interesting times we live in.

I do not use a MRP or a RF rate for three reasons:
1) I am retired.
2) I do not accept their validity.
3) The "new normal" makes no economic or financial sense.

I am an academic in a public university – I don't know of any University discount rate.

"The subject who is truly loyal to the Chief Magistrate will neither advise nor submit to arbitrary measures." Junius

Prima de riesgo que utilizo en España: diferencia de rentabilidad que ofrece el bono español respecto al alemán. Tipos de interés sin riesgo: los extraídos día a día del boletín de deuda pública española en operaciones de compra-venta al contado.

I don't value companies on this basis. I prefer to use price to earnings ratio.

In the Netherlands there is a discussion with the fiscal authorities. A lot of valuation experts use the MRP from your Survey. The Fiscal authorities accept that but want consequently also the use of the Rf from your survey. There is a lot of discussion when we use a normalized adjusted Rf.

Por tipo de interés sin riesgo se entiende en el corto plazo, pe 3 meses, al tipo de interés interbancario al plazo correspondiente para el área de referencia. En caso del euro, sería el EURIBOR y en caso de EEUU el Libor USD. Hablando de riesgo soberano USA y Alemania son considerados Benchmarks, por lo que su prima de riesgo es 0 y por tanto se les considera que son libres de riesgo. (Excepto entre ellos cuando se habla de riesgo entre EUR y USD) Por ello, cuando hablamos de prima de riesgo de un país, pe. España, hablamos del diferencial de tipos que hay el bono español con el de Alemania, tomando el mismo plazo. Normalmente se utiliza el plazo estándar del 10 años.

Sigo las recomendaciones de Credit Swiss Global Investment Return Yearbook, en este caso, 2018, con un 3,5% de PRM. No me gustan las recomendaciones de Damodaran, cuando incluye un riesgo país a España mayor que el de, creo, Perú o Ecuador, El tipo de interés sin riesgo que utilizo es, para España, el de el bono alemán a 10 años, según leo es de 0,17%, aunque Credit Swiss, creo recordar utiliza otro....el de EEUU es de 2,73%.

The risk free rate is determined on the historical present value-equivalent base interest rates on the basis of a series of payments increasing with the selected growth rate over a period of 1,000 years. For the calculations, the spot rate from year 30 to year 1,000 is updated constantly based upon the valuation date.

Germany

Risk free rate	0.9%	20 y Bund	Investing.com/rates-bonds/germany-20-year-bond-yield (1-1-2018)
Adjustment	1.8%	Credit Suisse	Credit Suisse Global Investment Source book and Yield book 2016 – Range of estimated long term real rate government bonds 1900-2015 - globally diversified
Risk free rate Adjusted	**2.7%**		

I don't use the market risk premium. I use a hurdle rate of return and won't invest in investments that don't achieve that hurdle. I aspire to a 25% rate of return on my investments but will generally settle for 15%.

I use the relevant rate from each country/currency "risk-free" yield curve to discount the respective expected future cash flow: V0 = CF1/(1 + Rf1 + risk prem)^1 + CF2/(1 + Rf2 + risk prem)^2 + ... + CFt/(1 + Rft + risk prem)^t

The Rf that I am using in 2019 for USA is: 10 year historical average, US Treasuries 20-year notes.

I use the US Equity premium of Damodaran to avoid explanations or justifications to clients.

We only use ROS (Return on Sales).

Rf: 3%, of which 2% is a premium for the risk of manipulation of the interest rate market operated by the ECB with the Quantitative Easing.

Al tener limitación nacional al hacer inversiones, debemos emplear un tipo de interés sin riesgo alto. Al operar en mercados muy consolidados, con pocos operadores y con fuertes barreras de entrada, la prima de riesgo de mercado es muy alta.

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En anteriores encuestas intenté ofreceros un tipo orientativo pero estos últimos años, después de la “experimentación” de tipos, de diferentes QE con tipos negativos… sólo tengo una certeza, que ya hemos comentado en muchas ocasiones: es muy difícil, o de dudosa utilidad, establecer un tipo de interés sin riesgo. Porque ¿Es normal que la Deuda Griega pague menos que la Deuda de USA? ¿Emisiones de Deuda del gobierno argentino a periodos larguísimos? ¿Deuda alemana o suiza en tipos negativos?…
Respecto a establecer una tasa que sirva como referencia, mantendría dos premisas: 1) El horizonte de inversión (una Tasa de referencia con el mismo plazo); 2) La seguridad en las estimaciones de los flujos de caja futuros del proyecto o inversión: en caso de menor confianza o duda en las estimaciones, mayor tasa de Descuento
Como norma, siempre tenemos en cuenta que la Renta variable ha sido en periodos muy largos el activo más rentable y, por tanto, a muy largo plazo es el Activo de “Menor riesgo”

Fascinating results. It is always interesting how investors and fund managers interpret the risk free rate of countries who have a negative prevailing long-term bond rate.

I am sure you that you are analysing the data and asking more questions that data can answer. It´s time to improve theory! I hope you will advance on it.

In my DCF valuation I use a global perspective of the marginal investor hence a global MRP.

I match rf with currency/inflation of cash flows being discounted and do not rely too much on current interest rates due to imperfections in the market. The MRP is made consistent with the level of interest rate I use in my model (E(Rm)-Rf) end end up with 6%

For equities we use a 10% as a cost of opportunity independently of the level of interest.

Rf: average last 5-year 10 year Treasury

I would like to help you with these two questions, but the problem is that in no any literature sources or analytical reports I met the calculation of Market Risk Premium and Risk Free rate for Uzbekistan.

The risk free rate that I use depends upon the timing of the future cash flows. I refer to the interest rate swap market and the US treasury market for starters. These days, one has to bear in mind currency volatility as that has a bigger effect on PV than market cost-of-capital.

We use the same Market Risk Premium for any country: 5,75% (source: Damodaran). Only Rf changes.

I am happy that you are asking the second question, because it accounts for what I consider to be a historical anomaly in the reply to the first question. I've concluded that the ERP was recently 3-4 percent. But I think US monetary policy (the various "QE" programs) have in the past couple of years distorted the traditional relationship between expected total market returns and the risk free rate. QE has been driving the US Treasury rate down, while the expected total market return has held steady, leading to a larger than usual market risk premium. This higher market risk premium is not a sign of higher market equity risk, but of the perverse impact of aggressive monetary policy.

For the US in 2015: MRP: 14% (as US equities are even more highly priced than last year).

Interest rates are artificially well below historic levels. Thus, bonds and equities values are artificially inflated.

I do not use "canned" rates applicable for a whole year. The rates I use are time-specific and case-specific, depending on conditions prevailing as of the valuation date.

I must confess I am still surprised with the rates suggested that are at the upper bound of respondent answers.

One hint: It might make sense to ask more precisely about the premium before/after personal income tax. For Germany the premium would differ and I am not sure how people would interpret the question.

The Risk-Free Rate we use is based on rates published by the Federal Reserve. We use the 20 year rate, currently 2.73%. The Equity Risk Premium we use is based on Duff & Phelps Annual Valuation Handbook.

For foreign countries, I generally look at it in dollar terms and assume that purchasing power parity held; hence, I’d use US rates. If I had to do it in a foreign currency, I would use the local 10-year treasury for the risk-free rate. I would use the US equity risk premium, adjust for inflation to real terms, and then adjust for foreign inflation to put it in local nominal terms.

USA. MRP   6.4% - essentially bloomberg/ibbotson number. RF   10 year U.S. treasury yield.

Exijo un mínimo de un 15% de retorno neto de impuestos a cualquier acción, independientemente de su nacionalidad.

No existe un activo libre de riesgo en absoluto. Y menos en estos distorsionados entornos debido a la intervención de los bancos centrales. En mi modesta opinión, creo que nunca sido tan riesgosa la renta fija como lo es ahora.

No creo especialmente en el modelo de CAPM y prefiero usar una cifra basada en el sentido común.

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Market Risk Premium for any market is not salubrious for peace or mind.
https://comcom.govt.nz/__data/assets/pdf_file/0029/282674/5B20225D-NZCC-12-Cost-of-capital-determination-EDBs-and-WIAL-3-May-2022.pdf.
https://indialogue.io/clients/reports/public/5d9da61986db2894649a7ef2/5d9da63386db2894649a7ef5

The CAPM is wrongly derived from very beginning (basically, CAPM is the first order condition for optimal portfolio decision (which must have a unique solution of mean-variance efficient portfolio) with its unique solution of market portfolio. CAPM is, of course, a tautology even the market portfolio is mean-variance efficient, not an asset pricing no matter market portfolio is mean-variance efficient or no. In sum, CAPM is theoretical useless.

En Uruguay la práctica más aceptada es descontar flujos convertidos a USD dada la debilidad de la moneda local y dolarizacion de la economía.

Your research over the years has been enlightening. It would be interesting to see the "meta" research on your data, that is, an analysis of the cross-section / time series to determine if there is any information embedded in the disperse responses that you receive, e.g. for forecasting or determining whether the consensus is correct over time.

I am guessing you already know my answers:
1. I do not use CAPM, the build-up-method or similar strategies to figure out required rates of return, and I pay no attention to the so-called “Market Risk Premium”. Instead I rely mostly on the Pepperdine Cost of Capital Survey in my work.
2. I acknowledge current and changing U.S. Treasury bond rates because it’s probably true they have some effect on investors’ Required Rates of Return. But I don’t use any specific number at any given time so I don’t have an answer to your second question either.

We use a WACC of 8.0% for our pan-European industrial coverage, including UK, CH. We are not explicitly modeling Rf, beta or premium.

I just wanted to thank you for your annual surveys. I work in the intersection between academic theory and economic policy, and your annual surveys provide me with an excellent tool for explaining the market environment for debt-financed government spending. I am especially pleased with the opportunity that your survey provides, to point to the risk-free rates in relation to where par yields are on treasury debt, trends in inflation-adjusted securities and government bond rating.

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References

Brealey, R.A., S.C. Myers and F. Allen (2005), *Principles of Corporate Finance*, 8th edition, McGraw-Hill/Irwin.

Claus, J.J. and J.K. Thomas (2001), "Equity Premia as Low as Three Percent? Evidence from Analysts' Earnings Forecasts for Domestic and International Stock Markets," *Journal of Finance*. 55, (5), pp. 1629-66.

Damodaran, A. (2008), "Equity Risk Premiums (ERP): Determinants, Estimation and Implications", Working Paper.

Dimson, E., P. Marsh and M. Staunton (2007), "The Worldwide Equity Premium: A Smaller Puzzle," in *Handbook of investments: Equity risk premium*, R. Mehra, Elsevier.

Fernandez, P. (2007), "Equity Premium: Historical, Expected, Required and Implied", http://ssrn.com/abstract=933070

Fernandez, P. (2009a), "Market Risk Premium Used in 2008 by Professors: A Survey with 1,400 Answers", http://ssrn.com/abstract=1344209

Fernandez, P. (2009b), "The Equity Premium in 150 Textbooks", http://ssrn.com/abstract=1473225

Fernandez, P., J. Aguirremalloa and H. Liechtenstein (2009), "The Equity Premium Puzzle: High Required Premium, Undervaluation and Self Fulfilling Prophecy". IESE Business School WP. http://ssrn.com/abstract=1274816

Fernandez, P. and J. del Campo (2010a), "Market Risk Premium used in 2010 by Analysts and Companies: a survey with 2,400 answers*",* downloadable in http://ssrn.com/abstract=1609563

Fernandez, P. and J. del Campo (2010b), "Market Risk Premium Used in 2010 by Professors: A Survey with 1,500 Answers", downloadable in http://ssrn.com/abstract=1606563

Fernandez, P., J. Aguirreamalloa and L. Corres (2011a), "US Market Risk Premium Used in 2011 by Professors, Analysts and Companies: A Survey with 5.731 Answers", downloadable in http://ssrn.com/abstract=1805852

Fernandez, P., J. Aguirreamalloa and L. Corres (2011b), "The Equity Premium in Spain: Survey 2011 (in Spanish)", downloadable in http://ssrn.com/abstract=1822422

Fernandez, P., J. Aguirreamalloa and L. Corres (2011c), "Market Risk Premium Used in 56 Countries in 2011: A Survey with 6,014 Answers", downloadable in http://ssrn.com/abstract=1822182

Fernandez, P., J. Aguirreamalloa and P. Linares (2014), "Market Risk Premium and Risk Free Rate Used for 51 Countries in 2013: A Survey with 6,237 Answers", downloadable in http://ssrn.com/abstract=914160

Fernandez, P., J. Aguirreamalloa and L. Corres (2012), "Market Risk Premium Used in 82 Countries in 2012: A Survey with 7,192 Answers", downloadable in http://ssrn.com/abstract=2084213

Fernandez, P. and I. F. Acín (2015), "Expected and Required Returns: Very Different Concepts", downloadable in http://ssrn.com/abstract=2591319

Fernandez, P., P. Linares and I. F. Acín (2014), "Market Risk Premium Used in 88 Countries in 2014: A Survey with 8,228 Answers", downloadable in http://ssrn.com/abstract=2450452

Fernandez, P., A. Ortiz and I. F. Acín (2015), "Discount Rate (Risk-Free Rate and Market Risk Premium) Used for 41 Countries in 2015: A Survey", Available at: https://ssrn.com/abstract=2598104

Fernandez, P., A. Ortiz and I. F. Acín (2016), "Market Risk Premium Used in 71 Countries in 2016: A Survey with 6,932 Answers", Available at: https://ssrn.com/abstract=2776636

Fernandez, P., V. Pershin and I.F. Acín (2017), "Discount Rate (Risk-Free Rate and Market Risk Premium) Used for 41 Countries in 2017: A Survey", Available at: https://ssrn.com/abstract=2954142

Fernandez, P. (2020), "'Normalized' Risk-Free Rate: Fiction or Science Fiction?" Available at: https://ssrn.com/abstract=3708863

Graham, J.R. and C.R. Harvey (2007), "The Equity Risk Premium in January 2007: Evidence from the Global CFO Outlook Survey," *Icfai Journal of Financial Risk Management,* Vol. IV, No. 2, pp. 46-61.

Harris, R.S. and F.C. Marston (2001), "The Market Risk Premium: Expectational Estimates Using Analysts' Forecasts," *Journal of Applied Finance*, Vol. 11.

Ilmanen, A. (2003), "Expected returns on stocks and bonds", *Journal of Portfolio Management* 29, pp. 7-27.

Johnson, D. T., T. Kochanek, T and J. Alexander (2007), "The Equity Premium Puzzle: A New Look", *Journal of the Academy of Finance,* Vol. 5, No. 1, pp. 61-71.

O'Neill, J., D. Wilson and R. Masih (2002), "The Equity Risk Premium from an Economics Perspective", Goldman Sachs, Global Economics Paper No. 84.

Ritter, J.R. and R. Warr (2002), "The Decline of Inflation and the Bull Market of 1982 to 1999," *Journal of Financial and Quantitative Analysis*, Vol. 37, No. 1, pp. 29-61.

Welch, I. (2000), "Views of Financial Economists on the Equity Premium and on Professional Controversies", *Journal of Business*, Vol. 73, No. 4, pp. 501-537.

Welch, I. (2001), "The Equity Premium Consensus Forecast Revisited", Cowles Foundation Discussion Paper No. 1325.

Welch, I. (2007), "A Different Way to Estimate the Equity Premium (for CAPM and One-Factor Model Use Only)," SSRN n. 1077876.

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KROLL

Kroll Recommended U.S. Equity Risk Premium (ERP) and Corresponding Risk-free Rates (R_f); January 2008–Present

For additional information, please visit kroll.com/cost-of-capital-resource-center

Date	Risk-free Rate (R_f)	R_f (%)	Kroll Recommended U.S. ERP (%)	What Changed
Current Guidance:				
October 18, 2022 – UNTIL FURTHER NOTICE*	Normalized 20-year U.S. Treasury yield*	3.50*	6.00	*ERP*
June 16, 2022 – October 17, 2022	Normalized 20-year U.S. Treasury yield	3.50	5.50	*Rf*
April 7, 2022 – June 15, 2022	Normalized 20-year U.S. Treasury yield	3.00	5.50	*Rf*
December 7, 2020 – April 6, 2022	Normalized 20-year U.S. Treasury yield	2.50	5.50	*ERP*
June 30, 2020 – December 6, 2020	Normalized 20-year U.S. Treasury yield	2.50	6.00	*Rf*
March 25, 2020 – June 29, 2020	Normalized 20-year U.S. Treasury yield	3.00	6.00	*ERP*
December 19, 2019 – March 24, 2020	Normalized 20-year U.S. Treasury yield	3.00	5.00	*ERP*
September 30, 2019 – December 18, 2019	Normalized 20-year U.S. Treasury yield	3.00	5.50	R_f
December 31, 2018 – September 29, 2019	Normalized 20-year U.S. Treasury yield	3.50	5.50	*ERP*
September 5, 2017 – December 30, 2018	Normalized 20-year U.S. Treasury yield	3.50	5.00	*ERP*
November 15, 2016 – September 4, 2017	Normalized 20-year U.S. Treasury yield	3.50	5.50	R_f
January 31, 2016 – November 14, 2016	Normalized 20-year U.S. Treasury yield	4.00	5.50	*ERP*
December 31, 2015	Normalized 20-year U.S. Treasury yield	4.00	5.00	
December 31, 2014	Normalized 20-year U.S. Treasury yield	4.00	5.00	
December 31, 2013	Normalized 20-year U.S. Treasury yield	4.00	5.00	
February 28, 2013 – January 30, 2016	Normalized 20-year U.S. Treasury yield	4.00	5.00	*ERP*
December 31, 2012	Normalized 20-year U.S. Treasury yield	4.00	5.50	
January 15, 2012 – February 27, 2013	Normalized 20-year U.S. Treasury yield	4.00	5.50	*ERP*
December 31, 2011	Normalized 20-year U.S. Treasury yield	4.00	6.00	
September 30, 2011 – January 14, 2012	Normalized 20-year U.S. Treasury yield	4.00	6.00	*ERP*
July 1 2011 – September 29, 2011	Normalized 20-year U.S. Treasury yield	4.00	5.50	R_f
June 1, 2011 – June 30, 2011	Spot 20-year U.S. Treasury yield	Spot	5.50	R_f
May 1, 2011 – May 31, 2011	Normalized 20-year U.S. Treasury yield	4.00	5.50	R_f
December 31, 2010	Spot 20-year U.S. Treasury yield	Spot	5.50	
December 1, 2010 – April 30, 2011	Spot 20-year U.S. Treasury yield	Spot	5.50	R_f
June 1, 2010 – November 30, 2010	Normalized 20-year U.S. Treasury yield	4.00	5.50	R_f
December 31, 2009	Spot 20-year U.S. Treasury yield	Spot	5.50	
December 1, 2009 – May 31, 2010	Spot 20-year U.S. Treasury yield	Spot	5.50	*ERP*
June 1, 2009 – November 30, 2009	Spot 20-year U.S. Treasury yield	Spot	6.00	R_f
December 31, 2008	Normalized 20-year U.S. Treasury yield	4.50	6.00	
November 1, 2008 – May 31, 2009	Normalized 20-year U.S. Treasury yield	4.50	6.00	R_f
October 27, 2008 – October 31, 2008	Spot 20-year U.S. Treasury yield	Spot	6.00	*ERP*
January 1, 2008 – October 26, 2008	Spot 20-year U.S. Treasury yield	Spot	5.00	*Initialized*

*** We recommend using the spot 20-year U.S. Treasury yield as the proxy for the risk-free rate, if the prevailing yield as of the valuation date is higher than our recommended U.S. normalized risk-free rate of 3.5%. This guidance is effective when developing USD-denominated discount rates as of June 16, 2022 and thereafter.**

"Normalized" in this context means that in months where the risk-free rate is deemed to be abnormally low, a proxy for a longer-term sustainable risk-free rate is used.

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Public Utility

Depreciation Practices

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strictly to original cost terms. In all cases, some measure of depreciation occurring between estimates can be determined. The customary method is for a competent appraiser to study the effect of factors such as obsolescence, inadequacy, and public requirements, as well as to conduct a physical inspection of the property, or a scientific sample of it, to determine its loss in value since it was first constructed. Regardless of the method employed, in order to achieve consistency, the successive estimates must be made in the same way.

It would, however, be a staggering undertaking to attempt such estimates on an annual basis for complex and extensive utility plant. Therefore, the practice of conducting annual estimates has found little application in the utility industry. It is particularly cumbersome and inadequate because utilities need to record depreciation on a monthly basis for earnings and expense reports. A further complication, of course, is that major technological improvements tend to make questionable any year-to-year measure of depreciation that is determined by this process.

Cost Allocation Concept

This concept recognizes the original cost of the asset as a prepaid expense. As such, it must be allocated to specific accounting periods and realized on income statements during the time the asset is providing service. The unallocated amount, often called net plant or net book (gross plant less accumulated depreciation), is recorded on the asset side of the balance sheet. The cost allocation concept satisfies the accounting principle of matching expense and revenues.

On the income statement, the inflow of resources is revenue. The outflow is expense. Using up the productive capacity of assets in an accounting period is recorded in accounting records as depreciation expense.

As used above, "cost" is based on the cost valuation principle of accounting, with cost being a surrogate for value. The amount of money used to purchase the asset is the basis for the entry in accounting records. This amount is regarded as being definite and immediately determinable. The accounting objectives of verifiability and neutrality are also satisfied.

Equally important to the proper estimation of current net income is the recovery of the investment over its useful life. Depreciation accounting cannot, automatically and of itself, result in the recovery of investment in property. However, if revenues are adequate to cover depreciation expense in addition to other current expense, the investment will be recovered. On the other hand, if revenues are not sufficient to cover the depreciation expense, the investment will not be fully recovered. Recognition of depreciation merely records the fact that costs are being incurred.

Definitions

Before proceeding into an investigation of some of the associated procedures and problems, let us examine some important definitions of depreciation.

According to the Supreme Court of the United States:

Broadly speaking, depreciation is the loss, not restored by current maintenance, which is due to all the factors causing the ultimate retirement of the property. These factors embrace wear and tear, decay, inadequacy and obsolescence. Annual depreciation is the loss which takes place in a year.[1]

The Interstate Commerce Commission defines depreciation as:

Depreciation is the loss in service value not restored by current maintenance and incurred in connection with the consumption or prospective retirement of property in the course of service from causes against which the carrier is not protected by insurance, which are known to be in current operation, and whose effect can be forecast with a reasonable approach to accuracy.[2]

The National Association of Railroad and Utilities Commissioners in 1958 sanctioned the following definition:

'Depreciation,' as applied to depreciable utility plant, means the loss in service value not restored by current maintenance, incurred in connection with the consumption or prospective retirement of utility plant in the course of service from causes which are known to be in current operation and against which the utility is not protected by insurance. Among the causes to be given consideration are wear and tear, decay, action of the elements, inadequacy, obsolescence, changes in the art, changes in demand, and requirements of public authorities.[3]

The Federal Communications Commission uses a definition in Part 32 of its rules that is almost identical to NARUC's, except that it applies to "telephone plant" instead of "utility plant," and it requires that the causes of depreciation "can be forecast with a reasonable approach to accuracy."

The definitions used by the Federal Energy Regulatory Commission for electric (Part 101 of the Code of Federal Regulations) and gas (Part 201 of the Code of Federal Regulations) companies are essentially the same as that used by NARUC. The only difference is that the definition for gas companies recognizes the exhaustion of natural resources as a cause of depreciation for natural gas companies.

Sec. 167 of the Internal Revenue Code states:

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[1] *Lindheimer v. Illinois Bell Telephone Company,* 292 U.S. 151, 167 (1934).

[2] 177 ICC 351, 422 (1931), 14700 Depreciation Charges of Telephone Companies, 15100 Depreciation Charges of Steam Railroad Companies.

[3] *Uniform System of Accounts for Class A and Class B Electric Utilities,* 1958, rev., 1962.

> There shall be allowed as a depreciation deduction a reasonable allowance for the exhaustion, wear and tear (including a reasonable allowance for obsolescence)—(1) of property used in the trade or business, or (2) the property held for the production of income.

Some of the definitions refer to depreciation as a loss in service value. "Service value" is used in a special sense, meaning the cost of plant less net salvage (net salvage is gross salvage less the cost of removal). The Uniform System of Accounts for electric utilities recommended by NARUC defines "service value" as follows:

> The difference between the original cost and the net salvage value of the utility plant.

"Loss in service value," therefore, must be understood and construed in light of its specially defined meaning.

The American Institute of Certified Public Accountants in Accounting Research and Terminology Bulletin #1 defines depreciation accounting as follows:

> Depreciation accounting is a system of accounting which aims to distribute cost or other basic value of tangible capital assets, less salvage (if any), over the estimated useful life of the unit (which may be a group of assets) in a systematic and rational manner. It is a process of allocation, not of valuation. Depreciation for the year is the portion of the total charge under such a system that is allocated to the year. Although the allocation may properly take into account occurrences during the year, it is not intended to be a measurement of the effect of all such occurrences.

This definition of depreciation accounting brings the "allocation of cost" concept into much clearer focus. It de-emphasizes the concept of depreciation expense as a "loss in service value" or an "allowance" and emphasizes the concept of depreciation expense as the cost of an asset which is allocable to a particular accounting period. This definition also clearly illustrates that the goal is recognizing cost, not providing funds for replacement of the asset.

Factors Which Affect the Retirement of Property

The sole reason for concern about depreciation is that all plant devoted to the pursuit of a business enterprise will ultimately reach the end of its useful life. Several factors cause property to be retired. They include:

1. Physical Factors
   a. Wear and tear
   b. Decay or deterioration
   c. Action of the elements and accidents

2. Functional Factors
    a. Inadequacy
    b. Obsolescence
    c. Changes in the art and technology
    d. Changes in demand
    e. Requirements of public authorities
    f. Management discretion

3. Contingent Factors
    a. Casualties or disasters
    b. Extraordinary obsolescence

Physical factors are the most readily observed causes of retirement. However, functional factors sometimes are the more frequent causes.

Inadequacy is a lack of capacity to supply what is required or demanded. For example, a telephone company's central office switch may not have sufficient capacity to process the traffic generated, or it may be unable to provide certain information services desired by customers. Thus, it may be more prudent to replace the entire switch in lieu of making additions.

Obsolescence may bring about retirements by rendering plant uneconomical, inefficient, or otherwise unfit for service because of improvements in technology or because of changes in function. Equipment manufacturers may contribute to obsolescence by discontinuing production of replacement parts or de-emphasizing maintenance, software, or other kinds of support for older equipment.

Technological advances have increased the frequency in which obsolescence causes the retirement of utility plant. Computers, the electronic chip, remote controlled operation and supervision of power distribution stations and natural gas regulating equipment, remote meter reading, fiber optic cable, as well as interest in nonutility power production and demand-side management are technological developments that have impacted utility operations.

Changes in demand reflect changing customer preferences requiring the replacement of plant which no longer permits the utility to fulfill its obligation to provide service. An example is the replacement of electric kilowatt hour meters with meters that also record usage by time of day.

Public authorities may require utility plant to be relocated because of its interference with public uses, such as highway relocations. They also may require utility plant to be replaced or refurbished because its design fails to meet current service, environmental, or safety standards. An example is the imminent expiration of operating licenses for hydraulic production plants. This has often resulted in an extensive review of the safety, environmental, recreational, as well as power generation aspects of these projects. Substantial requirements for additional maintenance and capital expenditures may be required to satisfy the concerns of regulatory agencies and their constituencies.

Although not included in the previous definitions, management discretion clearly is also a factor in the retirement of plant. This can occur when management decides to:

consumption with time (the age-life methods). The various age-life methods are presented below in accordance with the manner in which they spread depreciation expense over the life of property.

The Straight-Line Method

The straight-line method ratably charges a like amount to each accounting period over the service life of a plant item or plant group. Thus, it directly meets the depreciation objective, which perhaps accounts for its wide acceptance in utility practice. The basic formula is:

$$\text{Annual Depreciation Accrual} = \frac{\text{Depreciable Cost}}{\text{Service Life}} \qquad (1)$$

where Depreciable Cost is original or gross plant cost less estimated net salvage. In actual practice a depreciation rate is applied to the book cost of plant.

The straight-line method is sometimes spoken of as the method of equal annual depreciation charges. For item or unit accounting, this is true if the service life and net salvage are correctly estimated from the beginning of placement in service. However, because of changes in depreciation rates, which reflect changing conditions of service and causes of retirement during the service life, the equal annual charges are not usually made even for unit depreciation. With group properties, equal annual charges seldom occur because, although the rate may be constant, the rate is applied to a changing plant balance by virtue of retirements and additions. Thus, the straight-line method is best described as the method of constant rate applied to the book cost of plant in service between depreciation review periods.

The following formula is used to determine the depreciation rate to be applied to the original or gross plant cost:

$$d = \frac{100 - c}{L} \qquad (2)$$

where d is the depreciation rate in percent
where c is the estimated average net salvage in percent
where L is the estimated average service life

The formula requires two basic estimates—service life and anticipated net salvage. With group properties, care must be exercised to be sure the life and net salvage estimates reflect averages for the entire group to which the rate will be applied. This is because the estimates are often based on consideration of the more prominent items within the account. The selection of depreciation categories discussed in Chapter III and the methods of weighting discussed in Chapter IX are factors to consider. With estimates related to an account or group of accounts, the straight-line

In utility rate making, the sinking fund (compound interest) method can be applied with either a depreciated or undepreciated rate base. The depreciation expense used with the depreciated rate base is the total accrual of the annuity plus interest. This is sometimes termed the modified sinking fund method. The depreciation expense to be used with the undepreciated rate base is the annuity only. The two results will give the same total cost of service if the interest rate and the rate of return are the same. If an interest rate less than the rate of return is used, only the modified sinking fund method avoids an overallowance for return.

Equalizing return and depreciation under the sinking fund method ignores the many other utility costs which are seldom equal from year to year. Compared to the straight-line method, the sinking fund method produces lower early accruals and higher accruals in the later years. This difference increases with an increase in interest rate. Conversely, sinking fund advocates say that the straight-line method is a sinking fund solution with an interest rate of zero. The heavy accruals due to greater interest toward the end of a property's life can produce wide differences between the accumulated accruals and the cost being recovered if retirements occur only a year or two from the estimated time. In other words, the sinking fund method requires closer accuracy in service life and net salvage estimates.

The sinking fund and related interest methods were widely adopted at the time retirement and replacement accounting were being discontinued. At that time, they caused substantial increases in depreciation expenses for many companies. The sinking fund method is rarely used today due to the advance of tax depreciation, first on a straight-line basis and now with more "liberalized" methods; problems of annuity mathematics; and difficulties of proper accruals near the end of a property's life.

Summary

The straight-line method is almost universally used in the utility rate making process. The particular procedure used will vary depending upon the regulatory jurisdiction involved.

The accelerated methods identified above are not generally used for regulatory purposes. The Internal Revenue Service has permitted their use, and modifications of them, in computing tax depreciation, along with other specialized depreciation procedures for taxes. Interest methods, such as the sinking fund method, are no longer in general use.

Category Grouping Procedures

The group plan of depreciation accounting is particularly adaptable to utility property but raises many questions concerning the makeup of the group or category selected for analysis. Rather than one single group containing all utility plant, each group should contain homogeneous units of plant that are generally alike in character, used in the same manner throughout the plant, and operated under the same general conditions. However, even within the framework of this definition, it must be realized that there will be differences in the lives of the individual units.

Consider the case of poles. Some poles will be retired because of storms or other casualties, some because of public convenience or decay, some because of the substitution of underground for aerial facilities, and many more for a combination of the several causes of retirement. There will be a wide dispersion of retirements by age. What then is the proper grouping for a study of poles? Should it be all of the poles owned by the company analyzed en masse? This has not always proven satisfactory because there was a time when it was evident that the life characteristics of untreated poles differed materially from those of treated poles. Accordingly, during the time when untreated poles were substantial in number, it was appropriate to study poles in two separate categories: untreated and treated.

Regardless of which depreciation method is used, several alternatives are available for grouping individual plant units within a depreciation category. The most commonly used grouping procedures are as follows:

1. The Single Unit. Under this procedure each unit of property is depreciated separately. Because the procedure requires separate record-keeping for each unit, it is not practical for most types of property. Thus, it is not widely used by utilities.

2. The Broad Group. Under this procedure all units of plant within a particular depreciation category, usually a plant account or subaccount, are considered to be one group. The Broad Group is widely used and produces reasonably stable depreciation rates from year to year because of its averaging effects. It is a procedure that requires at least accounting records of annual additions and balances. Retirements by vintage are desirable.

3. The Vintage Group. Under this procedure each vintage or placement year within the depreciation category is considered to be a separate group. This combines, into one group, all of the poles placed in a single calendar year, or vintage. Even within each vintage group there will be dispersions of retirements by age, due to the many causes of retirements mentioned above. This requires that each vintage group be analyzed separately to determine its average life; all vintages are composited to produce the average service life for the plant class. Then the depreciation rate may be based on this estimated average service life of the units making up the group.

4. The Equal Life Group (ELG). Under this procedure the plant units are grouped according to their service lives, with the units from each vintage expected to experience the same service life being included in the same life group. This procedure permits accruing the full cost of the shorter-lived units to the depreciation reserve while they are in service. Thus the longer-lived units bear only their own costs. This is accomplished by dividing each vintage group (plant placed in a single year) into smaller groups, each of which is limited to units that are expected to have the same life. This distribution is based on life tables developed from the recorded experience, with respect to the mortality of utility plant. While it is not possible to identify the individual units of plant that will have a given life, it is possible to estimate statistically the number of units or dollars of plant in each equal life group, provided

> mortality data were accumulated. The prediction of future retirement patterns is also necessary in application of the vintage group procedure. However, ELG is much more sensitive to these predictions. ELG may be expected to produce greater fluctuations in depreciation expense from year to year than the broad group procedure.

The Broad Group procedure does not require that an assumption be made concerning the shape of the appropriate survivor curve (see Chapter VI) in the grouping process. However, Vintage Group, as generally applied, and ELG require such a determination. ELG depends upon the survivor curve forecast to determine the subgroups. With the FCC's agreement, the ELG procedure has been widely adopted by telephone companies subject to FCC jurisdiction. Some of the state commissions, however, have disallowed its use for intrastate rate making on both practical and technical grounds. The Vintage Group and Equal Life Group procedures are discussed in more detail in Chapter XII.

Application Techniques

There are two techniques commonly used to determine the depreciation rate to be applied to a utility's plant depreciation categories: Whole Life and Remaining Life.

Whole Life

The Whole Life technique bases the depreciation rate on the estimated average service life of the plant category. Whole life depreciation results in the allocation of a gross plant base over the total life of the investment. However, to the extent that the estimated average service life assigned turns out to be incorrect, (and precision in these estimates cannot reasonably be expected), the Whole Life technique will result in a depreciation reserve imbalance. For example, such over-accrual or under-accrual may remain in the reserve indefinitely unless offset by later overages or underages in the opposite direction. However, when a depreciation reserve excess or deficiency is reasonably certain, the Whole Life technique may be modified to include an adjustment to the accrual rate designed to eliminate the reserve imbalance in the future. For example, a special amortization of the difference may be allowed.

Remaining Life

The Remaining Life technique seeks to recover the undepreciated original cost less future net salvage over its remaining life. With this technique, the gross plant less book depreciation reserve is used as the depreciable cost and the remaining life or future life expectancy is used in the denominator. The formula is:

$$D = \frac{B - U - C'}{E} \tag{11}$$

where D is the depreciation expense or annual accrual
where B is the book cost of the Gross Plant
where U is the book depreciation reserve at start of the year
where C′ is the Estimated Future Net Salvage in dollars
where E is the Estimated Average Remaining Life

The following formula is used to arrive at the depreciation rate in percent:

$$\text{depreciation rate } d = \frac{D}{B} \times 100 \tag{12}$$

This rate may also be derived by dealing entirely in percentages as follows:

$$\text{depreciation rate } d = \frac{100 - u - c'}{E} \tag{13}$$

$$\text{where, in percent reserve, } u = \frac{U}{B} \times 100 \qquad (14)$$

$$\text{where, in percent future net salvage, } c' = \frac{C'}{B} \qquad (15)$$

A review of the depreciation reserve is appropriate at the commencement of use of the remaining life technique to ensure consistency with prior accounting and regulatory policies. The desirability of using the remaining life technique is that any necessary adjustments of depreciation reserves, because of changes to the estimates of life on net salvage, are accrued automatically over the remaining life of the property. Once commenced, adjustments to the depreciation reserve, outside of those inherent in the remaining life rate would require regulatory approval.

The Depreciation Model

The foregoing sections of this chapter discussed several depreciation Methods (e.g., Unit of Production, Straight-Line, Declining Balance), Procedures (e.g., Broad Group, Vintage Group, Equal Life Group) and Techniques (Whole Life and Remaining Life). A complete "depreciation model" is composed of a Method, a Procedure and a Technique, e.g., Straight-Line, Vintage Group, and the Remaining Life techniques. Subsequent chapters will also utilize this terminology.

CHAPTER VI

MORTALITY CONCEPTS

Introduction

From the previous discussions of depreciation, it is evident that an estimate of the life of property is essential to most of the common methods of computing depreciation accruals. Estimates may range from somewhat arbitrary assumptions of average life by management to informed judgment based upon highly technical mathematical models derived from actuarial science.

Through observation and classification of peoples' ages at death, actuaries have developed mortality tables. These tables reveal the death rate and life expectancy for people at different ages as a basis for determining life insurance premiums and reserves.

Mortality tables reflect the various risks affecting groups of people. While many people die purely from chance, the great majority of deaths are related to age. This age relationship is shown by the increasing death rate as age increases. Although the life of an individual cannot be predicted with surety, the number of people of a given age who will die in any year can be predicted fairly accurately.

Analogously, physical property is subject to forces of retirement. These forces include those related to the property's physical condition (e.g., wear and tear, accident), functional obsolescence or inadequacy, or termination of the need or enterprise. Industrial counterparts to insurance actuaries assemble and classify the ages at retirement of different types of industrial property in order to study the property's life characteristics.

For life analysis purposes, the ages at retirement are usually expressed in the form of retirement or survivor curves. The graph of the number of retirements at each age is termed the *retirement frequency curve*. The sum of the points on the retirement frequency curve from a specified age to maximum life represents the survivors from the original placements at the specified age. The graph of these survivors at each age is known as the *survivor curve*.

If a group is fully retired, the survivor curve will extend to the maximum life; if the group is not fully retired, the survivor curve is incomplete and is termed a stub survivor curve. Typically, a generalized survivor curve is used. Here, the survivors are expressed as percentages of the total number of units or dollars installed and the points on the curve are referred to as percents surviving.

The survivor curve may be used to obtain an indication of the average of the lives of all the units, or dollars, in the group, i.e., the *average life* of the property. The average life is found by dividing the area under the survivor curve from age zero to maximum life by 100%.

Since the survivor curve must reach maximum life in order for the average life calculation to be made, a stub survivor curve may be extended to maximum life using curve fitting techniques (see Chapter VIII). The vintage average lives may be composited to generate an average life for a group of vintages (e.g., an account) (see Chapter IX).

In lieu of extending the survivor curve, the area under the future portion of the curve, termed the *unrealized life*, may be estimated directly and added to the area under the stub curve,

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referred to as the *realized life*. The future area may be estimated by multiplying the percent surviving at any age by the vintage's forecasted average remaining life. As explained herein, unrealized life is not synonymous with remaining life nor is realized life synonymous with age.

Average remaining life represents the future years of service expected for the surviving property. The average remaining life for a vintage of any age is found by dividing the area under the estimated future portion of the survivor curve by the percent surviving at that age. Vintage average remaining lives may be composited to generate a remaining life for a group of vintages (e.g., an account).

The *probable life* of a vintage at a given age is the total years of service expected from the survivors. It is found by summing the vintage's age and remaining life.

Ratios may be calculated from the property records to describe the life characteristics of property. A *retirement ratio* for an age interval is the ratio of retirements during the interval to the property exposed to retirement at the beginning of the interval.

Retirement ratios calculated from the property records may be used to develop the observed life table, as discussed in Chapter VIII. In lieu of calculating the observed life table directly from the retirement ratios, *survival ratios* calculated from the retirement ratios may be used to calculate the percents surviving. A survival ratio is the complement of a retirement ratio.

Physical property retirements generally follow definable patterns that can be standardized. The *Iowa curves* are standard curves that were empirically developed to describe the life characteristics of most industrial and utility property. They are used throughout the utility industry, as well as in other applications[1] where life characteristics are sought. Their use in extending stub survivor curves and forecasting life characteristics is discussed in Chapter VIII.

The curves were placed into *L, R,* or *S* families depending upon whether the highest point (mode) of the retirement frequency curve was *left* of, *right* of, or *symmetrical* to the curve's average life. The curves in each family were then ordered according to the magnitude of the mode from low (e.g., *L0*) to high (e.g., *L5*).

The Iowa curve set was expanded to 31 curve types. This was accomplished by combining the original curves to form *half* curves (e.g., *S0.5*) and adding the *O* curves, so-called because their mode is at the origin. For any one of the 31 curve types, curves with different average lives may be generated by varying the area under curves of a given type. The development and validation of the curves are discussed in Appendix A, part 3.

Standard curves other than the Iowa curves may be used to describe history and predict the future. One such set of curves is the *New York h curves*. These curves are not empirical but were developed by truncating the normal frequency curve. The *h* curves are used by the New York Department of Public Service and most New York utilities, as well as some other utilities and several consultants. The development and application of the *h* curves are discussed further in Appendix A, part 5.

Another mortality formula, the Gompertz-Makeham formula, was not developed from empirical testing of industrial property but was formulated to describe human mortality. The

[1] An example is their use to describe the life of bank accounts.

development of the formula and its application to utility data are discussed in Appendix A, parts 1 and 2.

Retirement and Survivor Curves

Fundamental to the appropriate use of the survivor curve methodology is an understanding of the development and underlying properties of survivor curves and other curves associated with them. The retirement frequency and survivor curves are defined and developed in this section.

Retirement Frequency Curve

For a group of property, retirements do not typically occur at a single age but are distributed from age zero to the group's maximum age (i.e., maximum life). The graph of the number of retirements at each age is termed the retirement frequency curve.

The age at which the greatest number of retirements occurs is termed the modal age, and the associated point on the retirement frequency curve is referred to as the mode of the curve. Generally, the modal age is positioned near the average of all the retirement ages (i.e., average life) (see Figure 6-1).

A retirement frequency curve may be expressed in units or dollars. Alternatively, the curve may be generalized by expressing the retirements at each age as percentages of the total number of units or dollars (see Figure 6-1). The area under such a generalized curve from age zero to maximum life is 100%. The ages may also be generalized by expressing them as percentages of average life (see Iowa curve discussion in Appendix A, part 3).

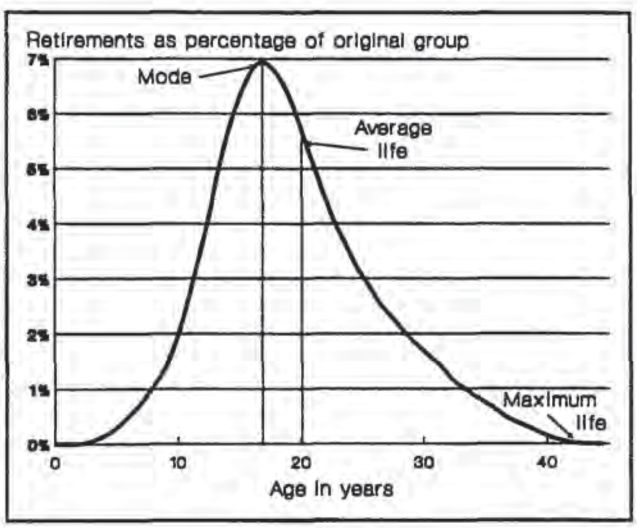

Figure 6-1. Retirement Frequency Curve.

Survivor Curve

The sum of the points on the retirement frequency curve from a specified age to maximum life represents the plant remaining in service (i.e., the survivors from the original placements) at the specified age. The graph of the survivors at each age beginning with age zero is known as the survivor curve. If a group is fully retired, the survivor curve will extend to maximum life; otherwise, it is referred to as a *stub* survivor curve.

The survivors may be expressed in units or dollars. Typically, a generalized survivor curve is used; here the survivors are expressed as percentages of the total number of units or dollars installed and the points on the curve are referred to as percents surviving (see Figure 6-2). The ages may also be generalized by expressing them as percentages of average life (see Iowa curve discussion in Appendix A, part 3).

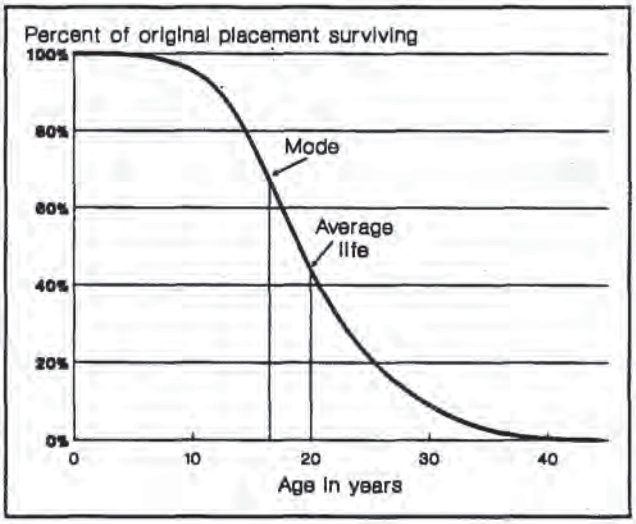

Figure 6-2. Survivor Curve.

The greatest decrease in percent surviving (i.e., the steepest slope of the curve) occurs at the age that is the modal age of the retirement frequency curve. Generally, this point of inflection of the survivor curve is positioned near the group's average life.

If the survivor curve is known, the retirement frequency curve may then be calculated. The number retired (or percent retired) during an age interval (e.g., 2.5 years to 3.5 years) is the difference between the number surviving (or percent surviving) at the beginning and the end of the age interval.

Types of Lives

Various types of average lives may be calculated to describe the life characteristics of property. The following terms are used to refer to the types of lives discussed in this section: *average, realized, unrealized, remaining, probable.*

Average Life

A commonly used statistic in life analysis and life estimation is the average life[2] of the property. This is the average of the lives of all the units, or dollars, in the group from age zero to maximum life. The average life **(AL)** is calculated by weighting each age **(i)** at which property was retired by the number retired **(R)** at that age and dividing the sum of these products by the total installed, as shown below:

$$AL = \frac{\sum_{i=0}^{\text{max life}} (i * R_i)}{\text{total installed}} \qquad \textbf{(1)}$$

Where sufficient mortality data are available, an indication of average life may be determined from a survivor curve constructed for the property group. To calculate average life, the area under a survivor curve **(SC)** from age zero to maximum life is divided by the total installed (or 100% for a generalized curve):

$$AL = \frac{\text{area under SC from age 0 to max life}}{100\%} \qquad \textbf{(2)}$$

The average life calculated above is a direct weighted average. To illustrate this averaging, consider a set of horizontal trapezoids constructed so as to cover the area under the survivor curve from age zero to maximum life. The trapezoids are formed by breaking the y

[2] When an account is considered as a single group, the terms average life and average service life are interchangeable.

anomalies, or adjustments present in the data; how they may affect the result; and how the result of the analysis is going to be used.

Retirements Subject to Reimbursement

Retirements may be subject to reimbursement from various sources. For example, wood poles in either the telephone or electric industries may be retired subject to reimbursement from an insurance company (e.g., a pole damaged by an automobile) or the government (e.g., a line of poles that must be retired due to street or highway work). Depending on the accounting treatment for reimbursements related to retired property, the analyst may need to remove such plant from the database. If the reimbursement is recorded as salvage, no adjustment of retirement data would be necessary, assuming that such salvage is also considered in establishing future depreciation rates. Consistent treatment is the rule.

Banding

Banding is the compositing of a number of years of data in order to merge them into a single data set for further analysis. Often, several bands are analyzed. By making determinations of the life and retirement dispersion indicated in successive bands, the analyst can get a clear indication of whether there is a trend in either the life of the plant or in the dispersion of the retirements.

In general, there are three reasons to use bands:

1. *Increase the sample size.* In statistical analyses, the larger the sample size in relation to the universe (the body of all data), the greater the reliability of the result (i.e., the greater the probability that the results will be applicable to the universe as a whole).

2. *Smooth the observed data.* Generally, the data obtained from a single activity or vintage year will not produce an observed life table that can be easily fit.

3. *Identify trends.* By looking at successive bands, the analyst may identify broad trends in the data that may be useful in projecting the future life characteristics of the property.

The following sections discuss placement bands and experience bands, as well as different types of bands—rolling, shrinking, and fixed.

Placement Bands

Placement bands show, for a group of vintages, the composite retirement history from the property's placement in service to the present. Placement bands allow the analyst to isolate the effects of changes in technology and materials that occur in successive generations of plant. For example, consider a telephone company that installed air-core buried cable before a given year and jelly-filled cable thereafter. In order to identify the differences in service life and retirement dispersion between the two types of cable, one might want to look at a placement band consisting of all vintages prior to the changeover and a second band of all vintages after the changeover.

An advantage of placement bands is that they generally yield smooth curves when based on fairly narrow bands. Unfortunately, placement bands yield fairly complete curves only for the oldest vintages. The newest vintages, presumably of greater interest in forecasting, yield the shortest stub curves.

Experience Bands

Experience bands show the composite retirement history for all vintages during a select set of activity years. These bands allow the analyst to isolate the effects of the operating environment over time.

Experience bands yield the most complete curves for the recent bands because they have the greatest number of vintages (ages) included. However, they may require significant smoothing because the data for each age is independent of the data for other ages. This independence can result in an erratic retirement dispersion.

Experience bands require that during the experience band, in order to construct an observed life table, at least one vintage in the band must be at age zero.

Types of Bands

There are several ways to select placement and experience bands. Rolling bands and shrinking bands may be useful in identifying trends in the data. These bands, along with fixed bands, are discussed below.

Rolling. To set up rolling bands, the analyst selects beginning and ending years for the initial band. The second band has beginning and ending points x years (usually one year) later than those of the first band; the third band has beginning and ending points each x years (usually one year) later than those of the second band; and so on. The result is a series of "rolling" bands of identical width as shown in the sample three-year rolling bands below:

Band 1:	1990	1991	1992		
Band 2:		1991	1992	1993	
Band 3:			1992	1993	1994

Simulated Plant-Record Model (SPR)
A trial-and-error model used to estimate the average service life of a depreciable group. The SPR model simulates retirements and the resultant plant balances for combinations of standardized survivor curves and average service lives and compares the results to the historical data until a good match is found.

Sinking Fund Method
Under this method the depreciation accrual is comprised of two parts: an annuity and interest on the accumulated depreciation. As compared with the straight-line method, the sinking fund method produces lower early accruals and higher accruals in the latter part of the service life.

Statistical Aging
See **Computed Mortality**.

Straight-Line Method
A depreciation method by which the service value of plant is charged to depreciation expense (or a clearing account) and credited to the accumulated depreciation account through equal annual charges over its service life. See **Depreciation Rate**.

Survivor Curve
A plot representing the percent surviving at each age.

Survival Ratio
The ratio of the number of units (or dollars) surviving in a group at the end of a period to the number of units (or dollars) in the group at the beginning of that period. The ratio is equal to one minus the retirement ratio. See **Proportion Surviving**.

T-cut
A truncation of the observed life table values which is generally used in a mathematical fitting of a curve to the observed values.

Theoretical Depreciation Reserve
The calculated balance that would be in the accumulated depreciation account at a point in time using current depreciation parameters, such as average service and net salvage. Also known as "reserve requirement" or "calculated accumulated depreciation (CAD)." See **Accumulated Depreciation Account**.

Turnover Methods
Methods of estimating service life based on the time it takes the plant to "turn over," that is, the time it takes for the actual retirements to exhaust a previous plant balance. See **Computed Mortality**.

Statistical Analyses of Industrial Property Retirements

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group methods of constructing survivor curves, which methods frequently result in stub curves.

The 18 type curves can be used for this purpose. The probable average life and type of distribution are selected without computation other than the calculation and plotting of the stub curve for which the probable average life is wanted. The method involves simply plotting the survivor curve (stub or completed curve) to the same scale that the 18 type curves are plotted using the ordinates in percent of the total number of units and the abscissas in *years*. For this method the type curves need to be drawn for definite average lives, say, for each 5-year interval from 5 to 50, making about 10 patterns of the same type curve on a sheet, as illustrated by Fig. 29.

If these type survivor curves and the stub survivor curve for which the probable average life is wanted are each drawn on transparent graphs, the individual stub curve can be superimposed on each of the 18 type sheets in turn until a satisfactory agreement is found. The stub curve is classified by the type curve which it fits best, and the probable average life estimated according to the position the individual curve occupies when superimposed upon the type sheet.

Figure 29 shows curves 13–2 and 56–1 plotted on the L_3 type sheet. By the location of curve 56–1 approximately parallel to the 15-year average-life type curve, it is readily seen that the stub curve is an L_3 type of about 13 years probable average life. As shown, stub curve 13–2 does not fit the L_3 type. In the upper right corner are curves for the R_1 type with curves 13–2 and 56–1 drawn in. Here it is seen that curve 13–2 parallels the R_1 10-year average-life curve at about 11.5 years probable average life and that curve 56–1 does not fit. In comparing a stub curve with these type sheets it is best first to smooth it by eye, so that its location and shape are more definite than can be judged from the plotted points only.

It is not feasible to work in percent of average life because the individual stub curve whose probable average life is sought cannot be so expressed. But since a given type curve has the same relative distribution for any average life it is possible to plot it for any number of average lives and then to compare the individual stub curve with this series of curves to determine the probable average life and type of the stub curve. The standards for each type curve as illustrated in Fig. 29 can be drawn to any suitable scale. That found satisfactory here was 10 inches to 100 percent surviving for the vertical scale and ¼ inch to 1 year for the horizontal scale. The graph paper used was a standard sheet, 11x16½ inches, ruled 20 divisions to the inch both ways.

The 18 type curves are expected to represent quite well all survivor curves commonly encountered in utility and industrial practices. For a given large organization, or for a group of smaller organizations of similar purposes, it may prove feasible to develop a set of standard curves embodying the company's own experience.